

A Supersymmetric Flavor Clockwork

W. Altmannshofer, S. A. Gadam arXiv:2106.09869

S. Aditya Gadam August 25, 2021

Santa Cruz Institute for Particle Physics

- 1. Clockwork
- 2. SUSY
- 3. Validity
- 4. Conclusion

Introduction to Clockwork

$$\mathcal{L}_{Yukawa} \supset Y_{ij}^d \overline{q}_L^i H d_R^j + Y_{ij}^u \overline{q}_L^i \tilde{H} u_R^j.$$
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⁽¹⁾

These terms are responsible for the quark masses we see in the SM, after the Electro-Weak symmetry breaking occurs.

• Yukawa Couplings: Increasing per generation - *Hierarchic*

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- Why? \in SM Flavor Puzzle

Fermion Mass Hierarchy

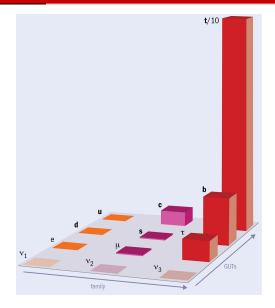


Figure 1: https://cerncourier.com/a/who-ordered-all-of-that/

The Clockwork mechanism [Giudice et. al 1610.07962] can generate small (or large) scales from $\mathcal{O}(1)$ numbers. As a quick introduction, let us consider the simplest SM Clockwork, for the RH up quark $u_R \sim (3, 1, \frac{2}{3})$:

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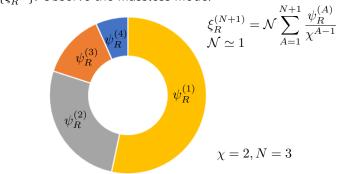
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$$\mathcal{L}_{breaking} = -m \sum_{A=1}^{N} \left(\overline{\psi}_L^{(A)} \psi_R^{(A)} - \chi \overline{\psi}_L^{(A)} \psi_R^{(A+1)} + \text{h.c.} \right) \,. \tag{2}$$

The exponential suppression is made apparent by going to the mass eigenbasis $\{\xi_R^{(A)}\}$. Observe the massless mode:

Clockwork: The Execution

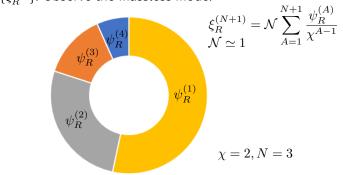
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$$\mathcal{L} \supset -\tilde{Y}^u \,\overline{q}_L \tilde{H} \,\psi_R^{(N+1)} \supset -\frac{1}{\chi^N} \tilde{Y}^u \,\overline{q}_L \tilde{H} \,u_R \tag{3}$$

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$$Y_x^{ik} \simeq \tilde{Y}_x^{ik} \chi^{-N_q^i - N_x^k}, \qquad x \in \{u, d\}$$

$$N_q = \begin{pmatrix} 3\\2\\0 \end{pmatrix}, \qquad N_u = \begin{pmatrix} 5\\2\\0 \end{pmatrix}, \qquad N_d = \begin{pmatrix} 4\\3\\3 \end{pmatrix}$$
(5)

SUSY



https://www.symmetrymagarine.org/article/the-status-of-supersymmetry

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$$W \supset \boldsymbol{\Phi}_{Q_i} \boldsymbol{m}_{Q_i} \boldsymbol{\Phi}_{Q_i^c} + \boldsymbol{\Phi}_{U_i} \boldsymbol{m}_{U_i} \boldsymbol{\Phi}_{U_i^c} + \boldsymbol{\Phi}_{D_i} \boldsymbol{m}_{D_i} \boldsymbol{\Phi}_{D_i^c}$$
(6)
+ $\tilde{Y}_u^{ij} H_u \boldsymbol{\Phi}_{Q_i} \boldsymbol{\Delta}_u^{ij} \boldsymbol{\Phi}_{U_j^c} + \tilde{Y}_d^{ij} H_d \boldsymbol{\Phi}_{Q_i} \boldsymbol{\Delta}_d^{ij} \boldsymbol{\Phi}_{D_j^c} ,$

where Φ_{ψ_i} , the chiral super-field corresponding to the SM ψ_i , is a vector in Clockwork space, while Δ_x^{ij} is a projection operator, selecting the $(N_q^i + 1, N_x^j + 1)$ gear fields.

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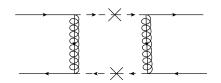
$$\begin{split} \mathcal{L}_{soft} \supset \tilde{\mathbf{Q}}_{i}^{\dagger} \boldsymbol{\mu}_{Q_{i}}^{2} \tilde{\mathbf{Q}}_{i} + \tilde{\mathbf{Q}}_{i}^{c\dagger} \boldsymbol{\mu}_{Q_{i}^{c}}^{2} \tilde{\mathbf{Q}}_{i}^{c} + (M_{Q}^{2})_{ij} \tilde{\mathbf{Q}}_{i}^{\dagger} \boldsymbol{\Delta}_{Q}^{ij} \tilde{\mathbf{Q}}_{j} \\ &+ \tilde{\mathbf{U}}_{i}^{\dagger} \boldsymbol{\mu}_{U_{i}}^{2} \tilde{\mathbf{U}}_{i} + \tilde{\mathbf{U}}_{i}^{c\dagger} \boldsymbol{\mu}_{U_{i}^{c}}^{2} \tilde{\mathbf{U}}_{i}^{c} + (M_{U}^{2})_{ij} \tilde{\mathbf{U}}_{i}^{c\dagger} \boldsymbol{\Delta}_{U}^{ij} \tilde{\mathbf{U}}_{j}^{c} \\ &+ \tilde{\mathbf{D}}_{i}^{\dagger} \boldsymbol{\mu}_{D_{i}}^{2} \tilde{\mathbf{D}}_{i} + \tilde{\mathbf{D}}_{i}^{c\dagger} \boldsymbol{\mu}_{D_{i}^{c}}^{2} \tilde{\mathbf{D}}_{i}^{c} + (M_{D}^{2})_{ij} \tilde{\mathbf{D}}_{i}^{c\dagger} \boldsymbol{\Delta}_{D}^{ij} \tilde{\mathbf{D}}_{j}^{c} \end{split}$$

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$$m_{\tilde{q}}^{2} \simeq m_{S}^{2} \begin{pmatrix} 1 & \chi^{-1} & \chi^{-3} \\ \chi^{-1} & 1 & \chi^{-2} \\ \chi^{-3} & \chi^{-2} & 1 \end{pmatrix},$$
(7)
$$m_{\tilde{u}}^{2} \simeq m_{S}^{2} \begin{pmatrix} 1 & \chi^{-1} & \chi^{-3} \\ \chi^{-1} & 1 & \chi^{-2} \\ \chi^{-3} & \chi^{-2} & 1 \end{pmatrix}, m_{\tilde{d}}^{2} \simeq m_{S}^{2} \begin{pmatrix} 1 & \chi^{-1} & \chi^{-1} \\ \chi^{-1} & 1 & 1 \\ \chi^{-1} & 1 & 1 \end{pmatrix}$$
(8)

Validity



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$$\begin{split} \langle K^0 | \,\overline{d}_L^{\alpha} \gamma_\mu s_L^{\alpha} \overline{d}_L^{\beta} \gamma^\mu s_L^{\beta} \, | \,\overline{K}^0 \rangle &= \frac{1}{3} B_1(\mu) m_K f_K^2, \\ \langle K^0 | \,\overline{d}_R^{\alpha} s_L^{\alpha} \overline{d}_L^{\beta} s_R^{\beta} \, | \,\overline{K}^0 \rangle &= \frac{1}{4} B_4(\mu) m_K f_K^2 \frac{m_K^2}{(m_d(\mu) + m_s(\mu))^2}, \\ \langle K^0 | \,\overline{d}_R^{\alpha} s_L^{\beta} \overline{d}_L^{\beta} s_R^{\alpha} \, | \,\overline{K}^0 \rangle &= \frac{1}{12} B_5(\mu) m_K f_K^2 \frac{m_K^2}{(m_d(\mu) + m_s(\mu))^2} \\ C_i^{SUSY} \propto \alpha_s^2 / \Lambda_{SUSY}^2 \end{split}$$

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- The real part is Δm_{K}^{SUSY} , and the imaginary, ϵ_{K}^{SUSY}

Valid Parameter Space

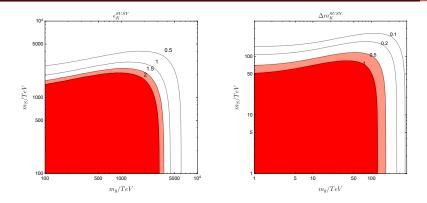


Figure 2: The ϵ_K plot indicates some contours labelling the ratio of the Clockwork SUSY contribution to ϵ_K to the theoretical uncertainty of the SM ϵ_K prediction in [Brod et. al, 1911.06822]. The Δm_K plot tracks the ratio of the Clockwork SUSY contribution to Δm_K to the Standard Model experimental value found in the PDG review.

Conclusion

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- Further flavor mixing and fine-tuning can be incorporated and tweaked by having non-universal masses and gear ratios

Questions?