



**The XXVIII International Conference on
Supersymmetry and Unification of Fundamental
Interactions (SUSY 2021)**



暨南大學
JINAN UNIVERSITY

Solving Flavor Anomalies in the 2HDM with Flavor Symmetries

Fanrong Xu
Jinan University

Beijing, August 26, 2021

Junmou Chen, Qiaoyi Wen, FX, Mengchao Zhang
arXiv: 2104.03699

Outline

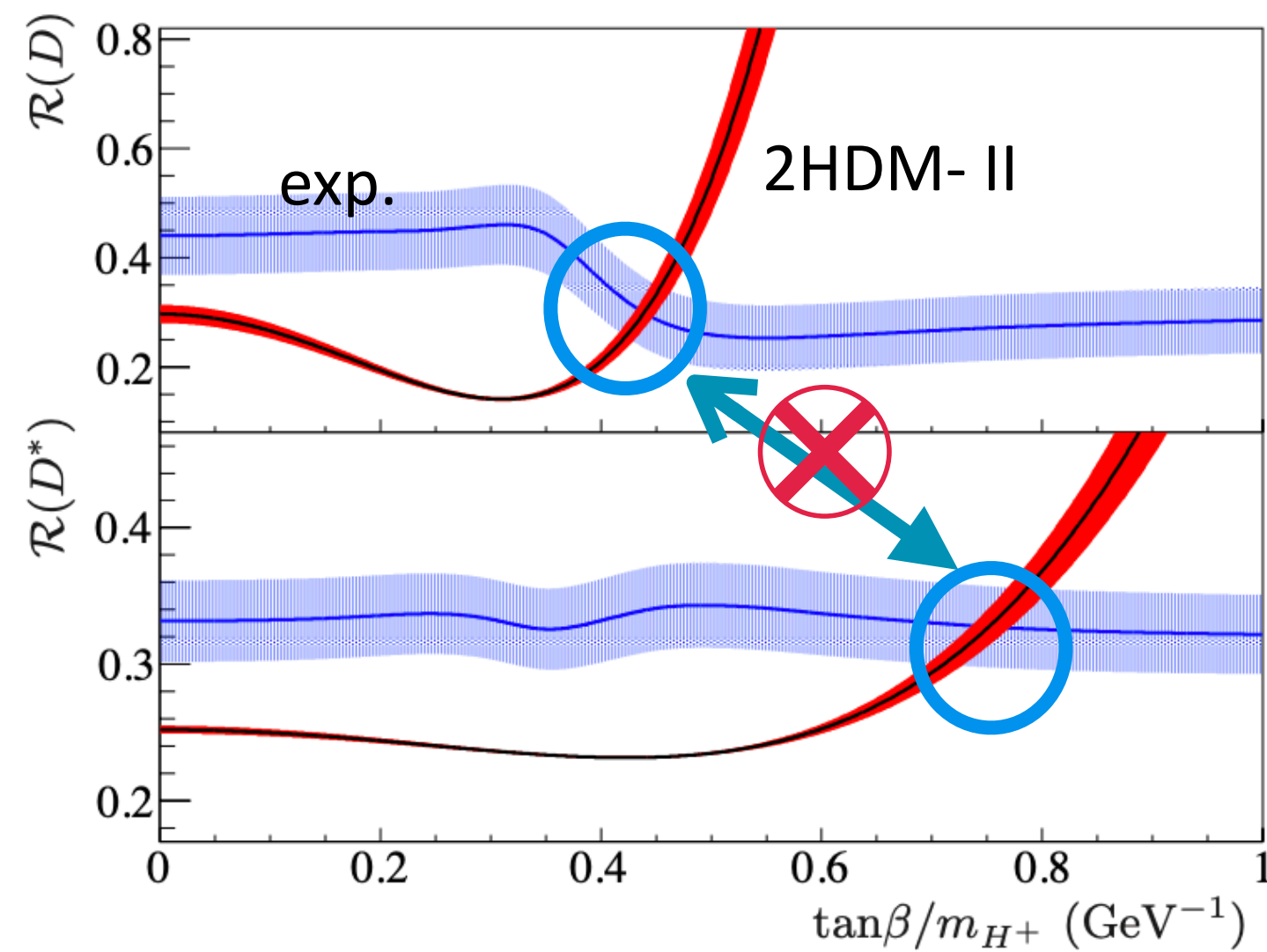
- Introduction: flavor anomalies
- FG₂HDM
- Model solutions to flavor anomalies
- Summary and Outlook

$R_{D^{(*)}}$ anomalies

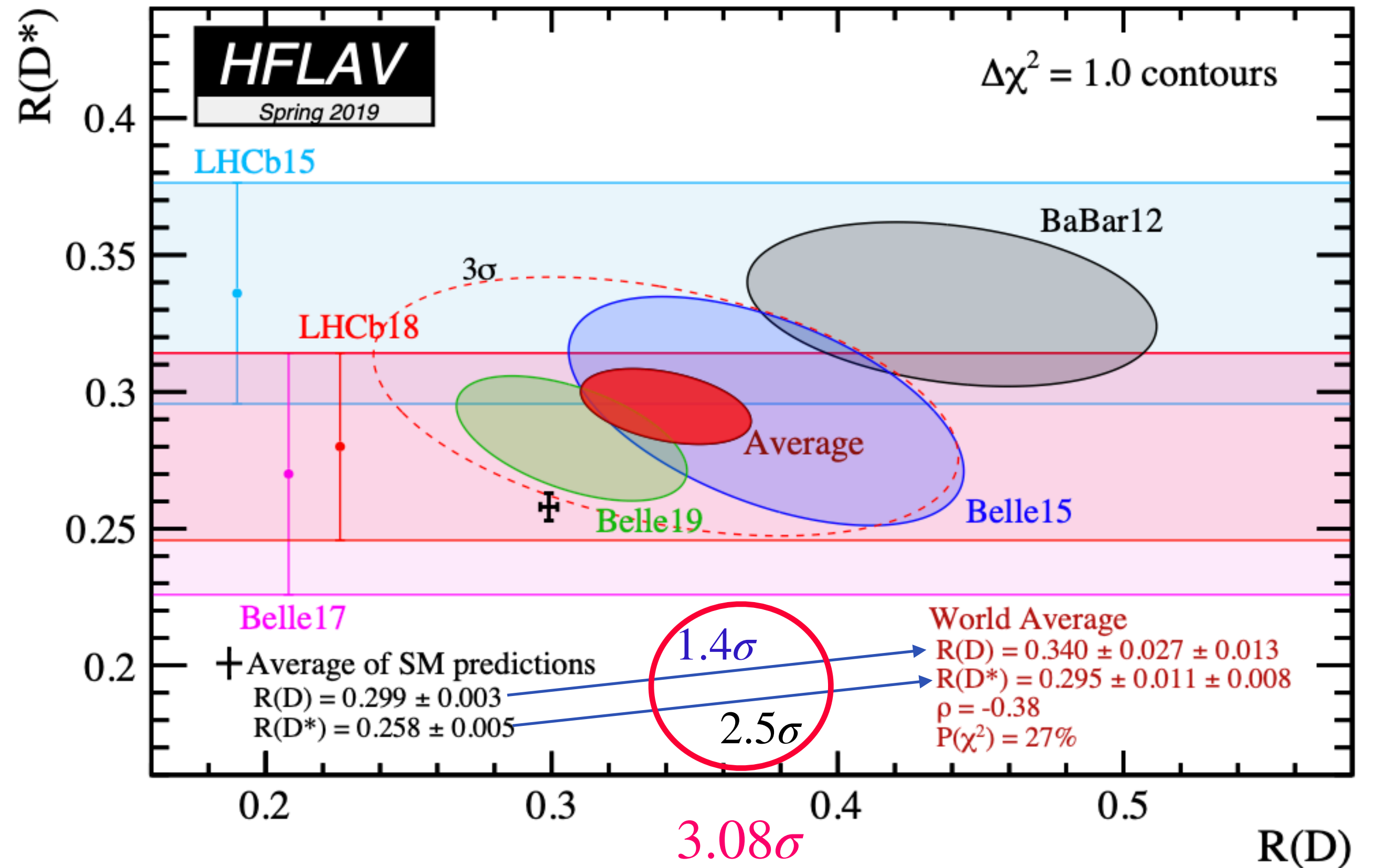
$$R_{D^{(*)}} = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \bar{\nu})}{\mathcal{B}(B \rightarrow D^{(*)} \ell \bar{\nu})} \Big|_{\ell=e,\mu}$$

EPJC (2021) 81: 226

$$\begin{aligned} \mathcal{R}(D) &= 0.440 \pm 0.058 \pm 0.042 && 2.0\sigma \\ \mathcal{R}(D^*) &= 0.332 \pm 0.024 \pm 0.018 && 2.7\sigma \end{aligned} \xrightarrow{\text{SM}} \text{SM}$$



BaBar 2012

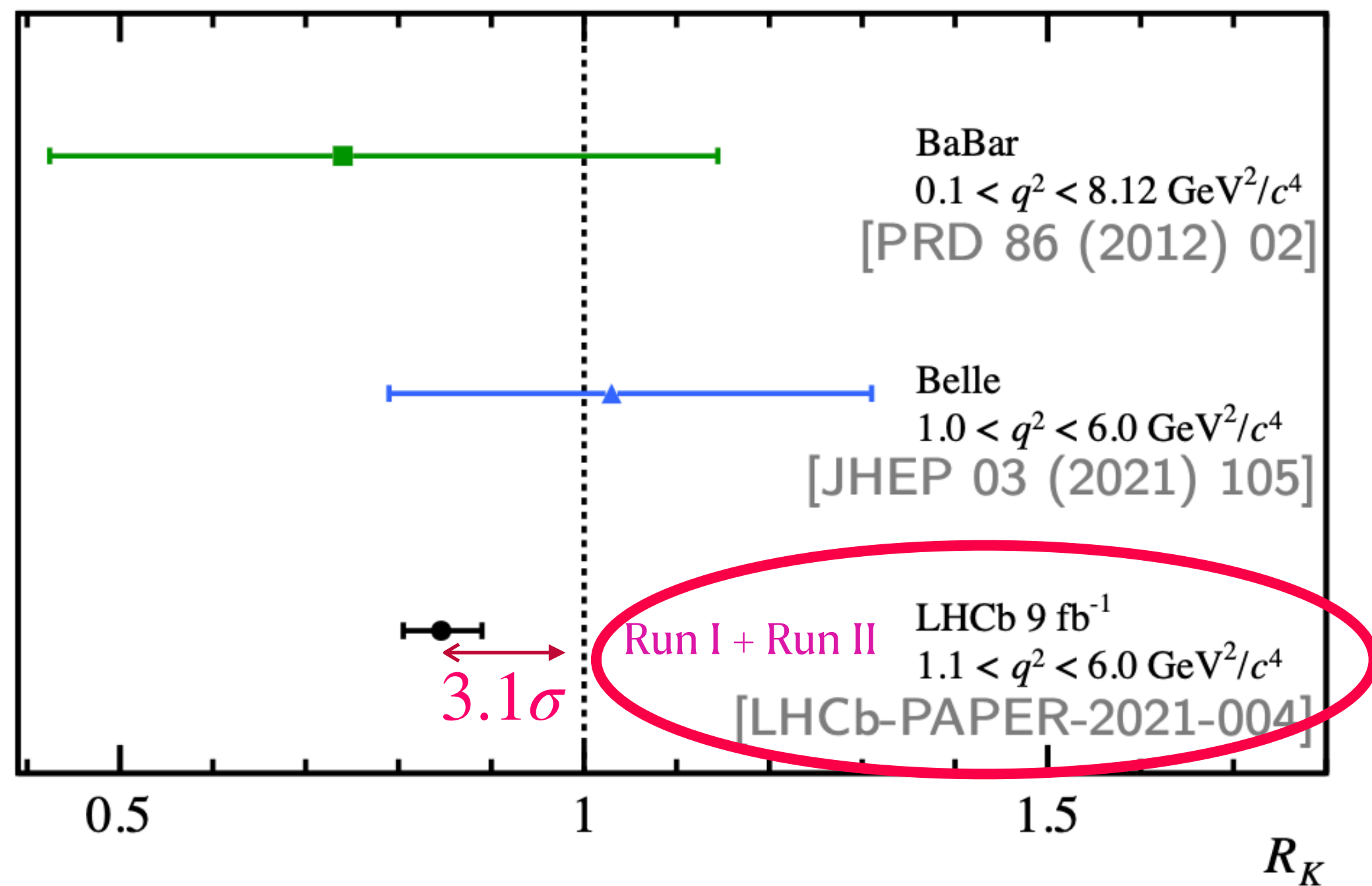


HFLAV, 2019

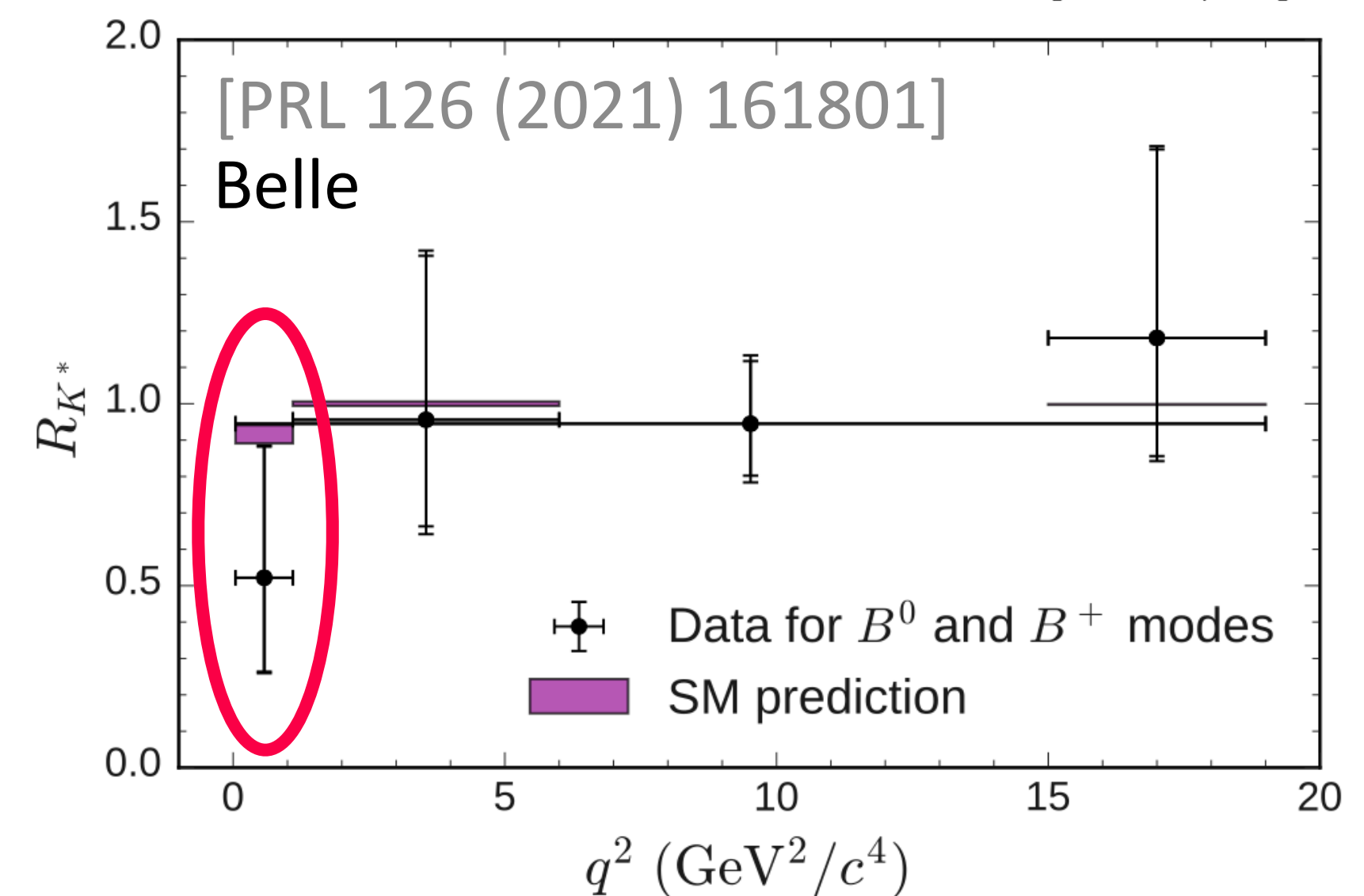
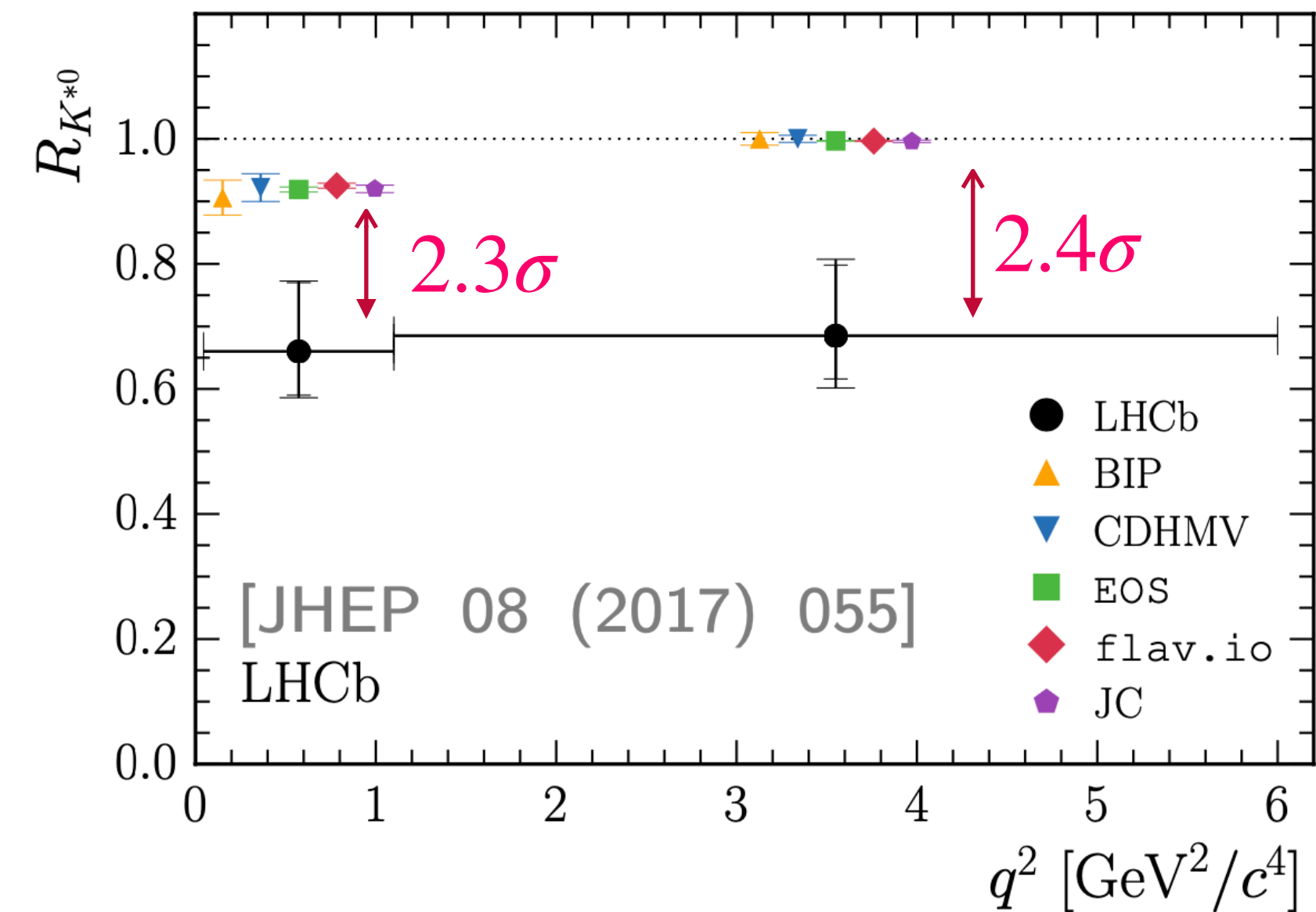
$R_{K^{(*)}}$ anomalies

$$R_{K^{(*)}} = \frac{\mathcal{B}(B \rightarrow K^{(*)} \mu^+ \mu^-)}{\mathcal{B}(B \rightarrow K^{(*)} e^+ e^-)}$$

$$R_K = 0.846^{+0.042}_{-0.039} \text{ (stat.) } ^{+0.013}_{-0.012} \text{ (syst.)}$$



R. D. Moise, Moriond 2021



What we learnt about

If existed, the lepton non-universality (LFU)

☑ exists in B meson decays

• $b \rightarrow c\ell\bar{\nu}$: τ vs (μ, e)

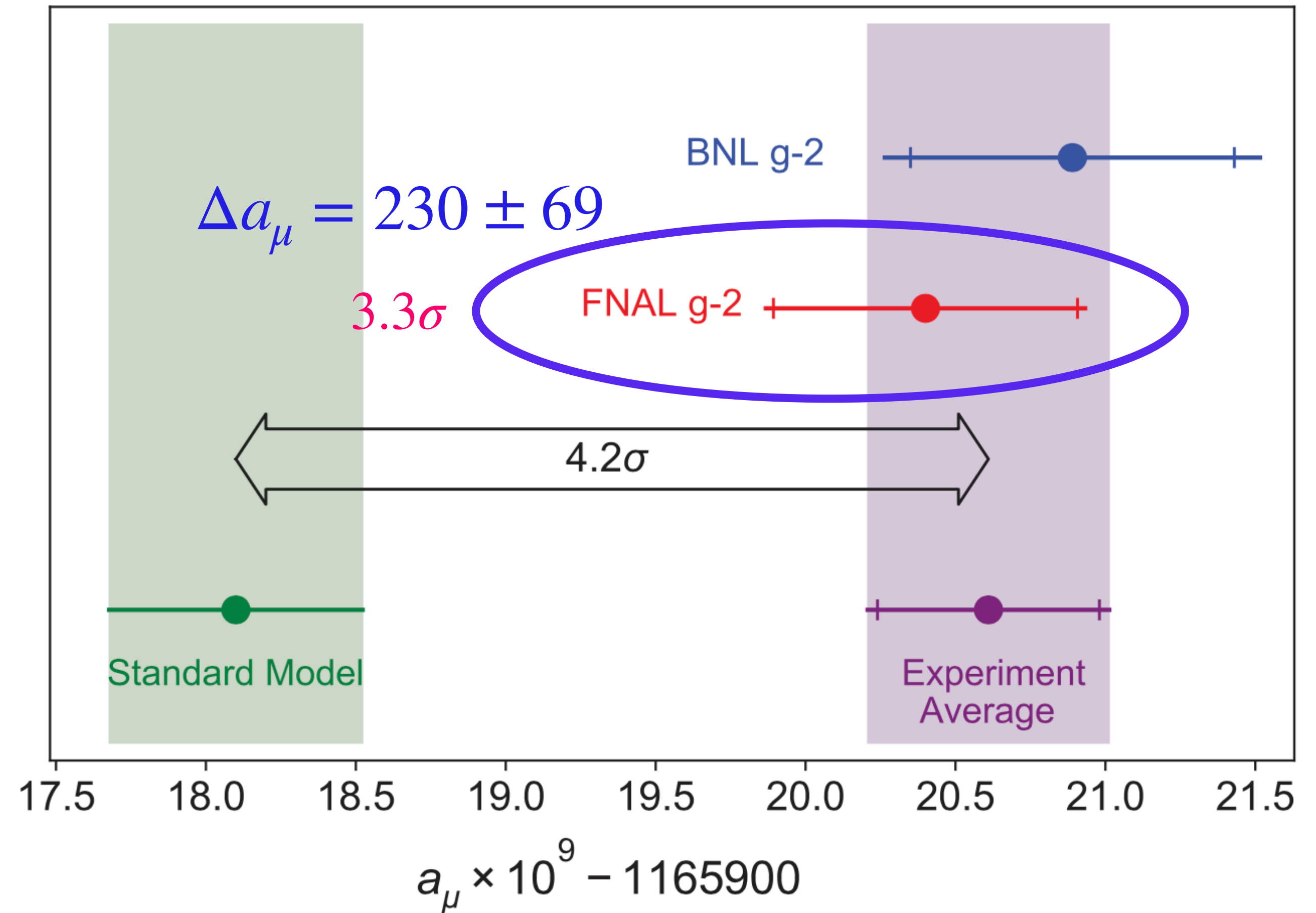
• $b \rightarrow s\ell\bar{\ell}$: μ vs e

► how about in pure lepton sector ?

Muon g-2

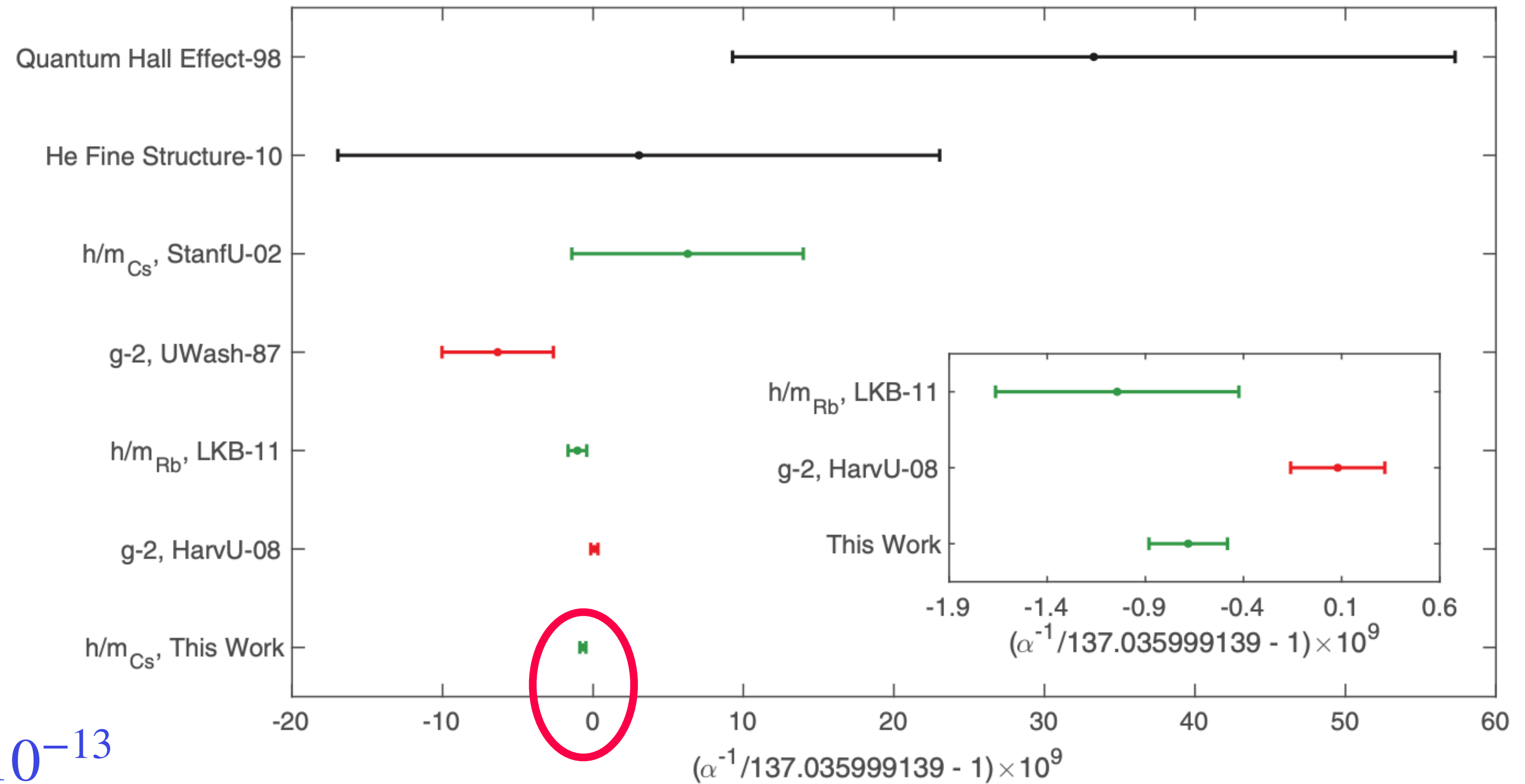
Contribution	$a_\mu \times 10^{11}$
QED (order $\mathcal{O}(\alpha^5)$)	116 584 718.93 \pm 0.10
Electroweak	153.6 \pm 1.0
QCD	
HVP (LO)	6 931 \pm 40
HVP (NLO)	-98.3 \pm 0.7
HVP (NNLO)	12.4 \pm 0.1
HLbL	94 \pm 19
Total (theory)	116 591 810 \pm 43

T. Aoyama *et al.*,
Phys. Rept. 887(2020) 1-166



muon g-2 Collaboration, PRL 2021

Electron g-2



$$\Delta a_e = \underline{(-8.8 \pm 3.6)} \times 10^{-13}$$

2.4 σ

Parker *et al.*,
Science 360, 191-195 (2018)

What we learnt about

If existed, the lepton non-universality (LFU)

☑ exists in B meson decays

☑ how about in pure lepton sector ?

☑ anomaly exists both in muon and electron

☑ non-universality appears

▶ What kind of New Physics we expect ?

2HDMs: Yukawa

$$-\mathcal{L}_Y = \overline{Q}_L^0 (Y_1^d \Phi_1 + Y_2^d \Phi_2) d_R^0 + \overline{Q}_L^0 (Y_1^u \tilde{\Phi}_1 + Y_2^u \tilde{\Phi}_2) u_R^0 + \overline{L}_L^0 (Y_1^\ell \Phi_1 + Y_2^\ell \Phi_2) e_R^0 + \overline{L}_L^0 (Y_1^\nu \tilde{\Phi}_1 + Y_2^\nu \tilde{\Phi}_2) \nu_R^0 + h.c.$$

$$-\mathcal{L}_m = \bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \bar{\ell}_L M_\ell \ell_R + \bar{\nu}_L M_\nu \nu_R + h.c.$$

$$M_f = U_{fL}^\dagger \tilde{M}_f U_{fR},$$

$$\tilde{M}_f = \frac{1}{\sqrt{2}} (v_1 Y_1^f + v_2 Y_2^f)$$

Model	u_R^i	d_R^i	e_R^i	$Y_1^u = 0$
Type I	Φ_2	Φ_2	Φ_2	(Y_2^d, Y_2^ℓ)
Type II	Φ_2	Φ_1	Φ_1	(Y_1^d, Y_1^ℓ)
Lepton-specific (type X)	Φ_2	Φ_2	Φ_1	(Y_2^d, Y_1^ℓ)
Flipped (type Y)	Φ_2	Φ_1	Φ_2	(Y_1^d, Y_2^ℓ)

2HDM-III

$$-\mathcal{L}_Y = \overline{Q_L^0}(Y_1^d\Phi_1 + Y_2^d\Phi_2)d_R^0 + \overline{Q_L^0}(Y_1^u\tilde{\Phi}_1 + Y_2^u\tilde{\Phi}_2)u_R^0 \\ + \overline{L_L^0}(Y_1^\ell\Phi_1 + Y_2^\ell\Phi_2)e_R^0 + \overline{L_L^0}(Y_1^\nu\tilde{\Phi}_1 + Y_2^\nu\tilde{\Phi}_2)\nu_R^0 + h.c.$$

- ▶ general 2HDM-III: too many Yukawa parameters
- ▶ the price: dangerous FCNHs brought in
- ▶ one attempt: Cheng-Sher ansatz
- ▶ designed symmetry:
extended but limited parameters & protected FCNHs

FG2HDM

Flavor Gauged Two-Higgs Doublet Model (BGL-like model)

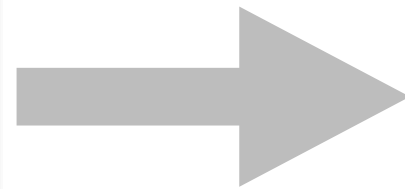
Flavor-dependent U(1) gauge symmetry

$$\phi \rightarrow \phi' = e^{i\theta X_\phi} \phi$$

Anomaly cancellation condition

2 model parameters

$$\begin{aligned}
 X_{Q_L} &= \frac{1}{2} \begin{pmatrix} Q_{u_R} + Q_{d_R} & & \\ & Q_{u_R} + Q_{d_R} & \\ & & Q_{t_R} + Q_{d_R} \end{pmatrix}, \\
 X_{u_R} &= \begin{pmatrix} Q_{u_R} & & \\ & Q_{u_R} & \\ & & Q_{t_R} \end{pmatrix}, \quad X_{d_R} = \begin{pmatrix} Q_{d_R} & & \\ & Q_{d_R} & \\ & & Q_{d_R} \end{pmatrix}, \\
 X_\Phi &= \frac{1}{2} \begin{pmatrix} Q_{u_R} - Q_{d_R} & & \\ & Q_{t_R} - Q_{d_R} & \\ & & \end{pmatrix}, \\
 X_{L_L} &= \begin{pmatrix} Q_{e_L} & & \\ & Q_{\mu_L} & \\ & & Q_{\tau_L} \end{pmatrix}, \\
 X_{l_R} &= \begin{pmatrix} Q_{e_R} & & \\ & Q_{\mu_R} & \\ & & Q_{\tau_R} \end{pmatrix}, \quad X_{\nu_R} = 0.
 \end{aligned}$$



$$\begin{aligned}
 Q_{u_R} &= -Q_{d_R} - \frac{1}{3}Q_{\mu_R}, & Q_{t_R} &= -4Q_{d_R} + \frac{2}{3}Q_{\mu_R} \\
 Q_{\tau_L} &= Q_{d_R} + \frac{1}{6}Q_{\mu_R}, & Q_{\mu_L} &= -Q_{d_R} + \frac{5}{6}Q_{\mu_R}, & Q_{e_L} &= \frac{9}{2}Q_{d_R} - Q_{\mu_R}, \\
 Q_{\tau_R} &= 2Q_{d_R} + \frac{1}{3}Q_{\mu_R}, & Q_{e_R} &= 7Q_{d_R} - \frac{4}{3}Q_{\mu_R}
 \end{aligned}$$

$$Y_1^u = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_2^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}, \quad Y_1^d = \begin{pmatrix} * & * & * \\ * & * & * \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_2^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & * & * \end{pmatrix}$$

$$Y_1^\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad Y_2^\ell = \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_1^\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & * & * \end{pmatrix}, \quad Y_2^\nu = 0,$$

FG2HDM: scalar

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2$$

$$+ \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_5}{2} [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2],$$

$$X_\Phi = \frac{1}{2} \begin{pmatrix} Q_{uR} - Q_{dR} & \\ & Q_{tR} - Q_{dR} \end{pmatrix}$$

$$V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2$$

$$+ \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1),$$

$$\mathcal{L}_\eta = 0$$

$$\mathcal{L}_{\phi^\pm} = -\frac{1}{2} \lambda_4 v_1 v_2 (\phi_1^-, \phi_2^-) \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

$$\mathcal{L}_\rho = -\frac{1}{2} (\rho_1, \rho_2) \begin{pmatrix} \lambda_1 v_1^2 & \lambda_{34} v_1 v_2 \\ \lambda_{34} v_1 v_2 & \lambda_2 v_2^2 \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

$$\begin{pmatrix} h \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix},$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}.$$

$$\Phi_i = \left(\phi_i^+, \frac{1}{\sqrt{2}}(\rho_i + i\eta_i + v_i) \right)^T$$

- 3 Goldstone bosons: eaten by W^\pm, Z, Z' ;
- 2 extra physical scalar left: H^0 and H^\pm

FG2HDM: Yukawa

$$\begin{aligned}
 -\mathcal{L} = & \frac{\sqrt{2}}{v} H^+ \left[\bar{u} \left(V_{\text{CKM}} N_d \mathbb{P}_R - N_u^\dagger V_{\text{CKM}} \mathbb{P}_L \right) d + \bar{\nu} \left(V_{\text{PMNS}} N_\ell \mathbb{P}_R - N_\nu^\dagger V_{\text{PMNS}} \mathbb{P}_L \right) \ell \right] + h.c. \\
 & + \frac{1}{v} \left[\cos(\beta - \alpha) H^0 - \sin(\beta - \alpha) h \right] \left[\bar{u} N_u u + \bar{d} N_d d + \bar{\ell} N_\ell \ell + \bar{\nu} N_\nu \nu \right] \\
 & + \frac{1}{v} \left[\sin(\beta - \alpha) H^0 + \cos(\beta - \alpha) h \right] \left[\bar{u} M_u u + \bar{d} M_d d + \bar{\ell} M_\ell \ell + \bar{\nu} M_\nu \nu \right]
 \end{aligned}$$

$$N_u = -\frac{v_2}{v_1} \text{diag}(m_u, m_c, 0) + \frac{v_1}{v_2} \text{diag}(0, 0, m_t),$$

$$(N_d)_{ij} = -\frac{v_2}{v_1} (M_d)_{ij} + \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) V_{i3}^\dagger V_{3j} (M_d)_{jj},$$

$$N_\nu = -\frac{v_2}{v_1} M_\nu,$$

$$N_\ell = -\frac{v_2}{v_1} \text{diag}(0, m_\mu, m_\tau) + \frac{v_1}{v_2} \text{diag}(m_e, 0, 0)$$

FCNH occurs only in down-type quark with CKM suppression.

FG2HDM: gauge bosons

$$\mathcal{L}_m^G = \begin{pmatrix} B & W^3 & \hat{Z}' \end{pmatrix} \tilde{M} \begin{pmatrix} B \\ W^3 \\ \hat{Z}' \end{pmatrix} = \begin{pmatrix} A & Z & Z' \end{pmatrix} M_d \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix}$$

$$\tilde{M} = \frac{1}{2} m_Z^2 \begin{pmatrix} \sin^2 \theta_W & -\sin \theta_W \cos \theta_W & a \sin \theta_W \\ -\sin \theta_W \cos \theta_W & \cos^2 \theta_W & -a \cos \theta_W \\ a \sin \theta_W & -a \cos \theta_W & b \end{pmatrix}$$

$$M_d = \frac{1}{2} m_Z^2 \begin{pmatrix} 0 & & \\ & \mu_Z & \\ & & \mu_{Z'} \end{pmatrix},$$

$$\begin{pmatrix} A \\ Z \\ Z' \end{pmatrix} = U \begin{pmatrix} B \\ W^3 \\ \hat{Z}' \end{pmatrix}, \quad U = \begin{pmatrix} \cos \theta_W & \sin \theta_W & 0 \\ -\sin \theta_W \sin \theta_1 & \cos \theta_W \sin \theta_1 & \cos \theta_1 \\ -\sin \theta_W \sin \theta_2 & \cos \theta_W \sin \theta_2 & \cos \theta_2 \end{pmatrix}.$$

$$\begin{aligned} \mathcal{L}_{FG} = & e Q_f A_\mu \bar{f} \gamma^\mu f + \frac{g_2 \sin \theta_1}{\cos \theta_W} Z_\mu \bar{f} [I_f^3 \gamma^\mu \mathbb{P}_L - Q_f \sin^2 \theta_W \gamma^\mu] f + g' \cos \theta_1 Z_\mu \bar{f} [\mathcal{Q}_{fL} \gamma^\mu \mathbb{P}_L + \mathcal{Q}_{fR} \gamma^\mu \mathbb{P}_R] f \\ & + \frac{g_2 \sin \theta_2}{\cos \theta_W} Z'_\mu \bar{f} [I_f^3 \gamma^\mu \mathbb{P}_L - Q_f \sin^2 \theta_W \gamma^\mu] f + g' \cos \theta_2 Z'_\mu \bar{f} [\mathcal{Q}_{fL} \gamma^\mu \mathbb{P}_L + \mathcal{Q}_{fR} \gamma^\mu \mathbb{P}_R] f \end{aligned}$$

FG2HDM: gauge bosons

In an extreme case: $Q_1 = -Q_2 \tan^2 \beta$.

$$\mu_Z \rightarrow 1, \quad \mu_{Z'} \rightarrow b$$

$$\sin \theta_1 \rightarrow 1, \quad \cos \theta_1 \rightarrow 0, \quad \sin \theta_2 \rightarrow 0, \quad \cos \theta_2 \rightarrow 1$$

$$U \rightarrow \begin{pmatrix} \cos \theta_W & \sin \theta_W & 0 \\ -\sin \theta_W & \cos \theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{L}_{Z'} = J^\mu Z'_\mu,$$

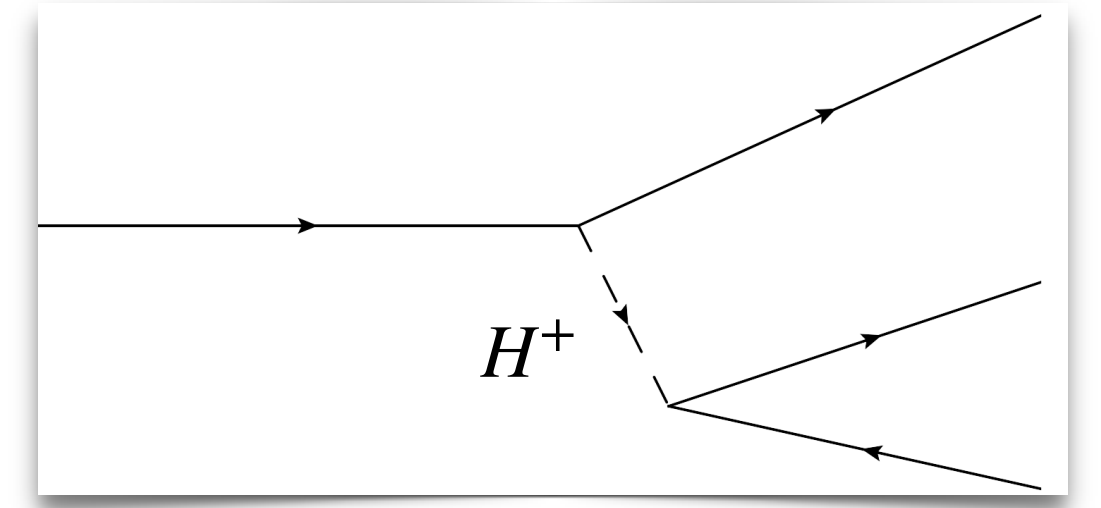
$$J^\mu = g' \bar{f} [Q_{fL} \gamma^\mu \mathbb{P}_L + Q_{fR} \gamma^\mu \mathbb{P}_R] f$$

$$Q_{dL} = \frac{1}{2}(Q_{uR} + Q_{dR}) \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{1}{2}(Q_{tR} - Q_{uR}) \begin{pmatrix} |c_1|^2 & c_1^* c_2 & c_1^* c_3 \\ c_2^* c_1 & |c_2|^2 & c_2^* c_3 \\ c_3^* c_1 & c_3^* c_2 & |c_3|^2 \end{pmatrix}$$

FCNC only occurs in down-type quark sector

Solution I

$R_{D^{(*)}}$

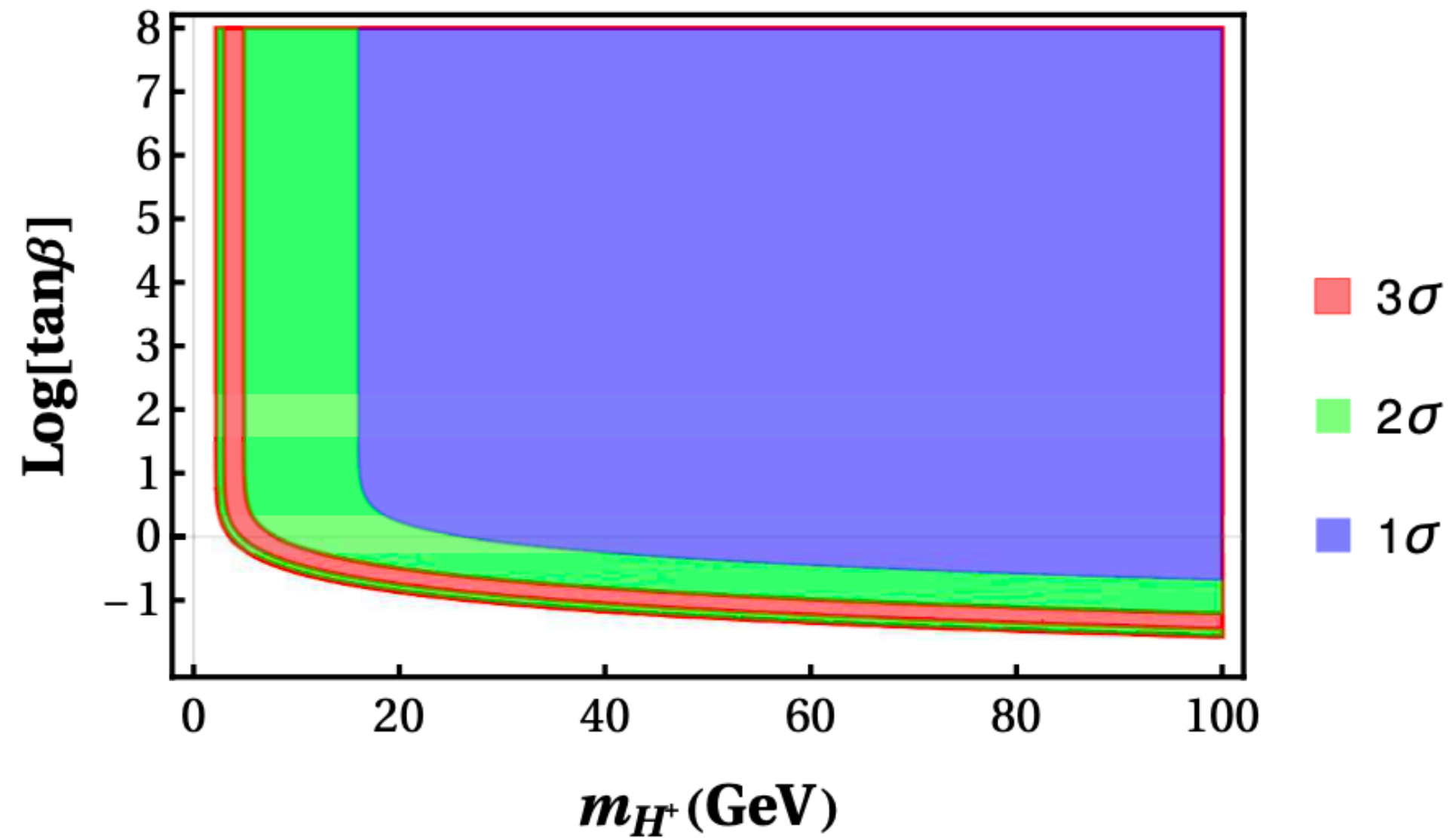
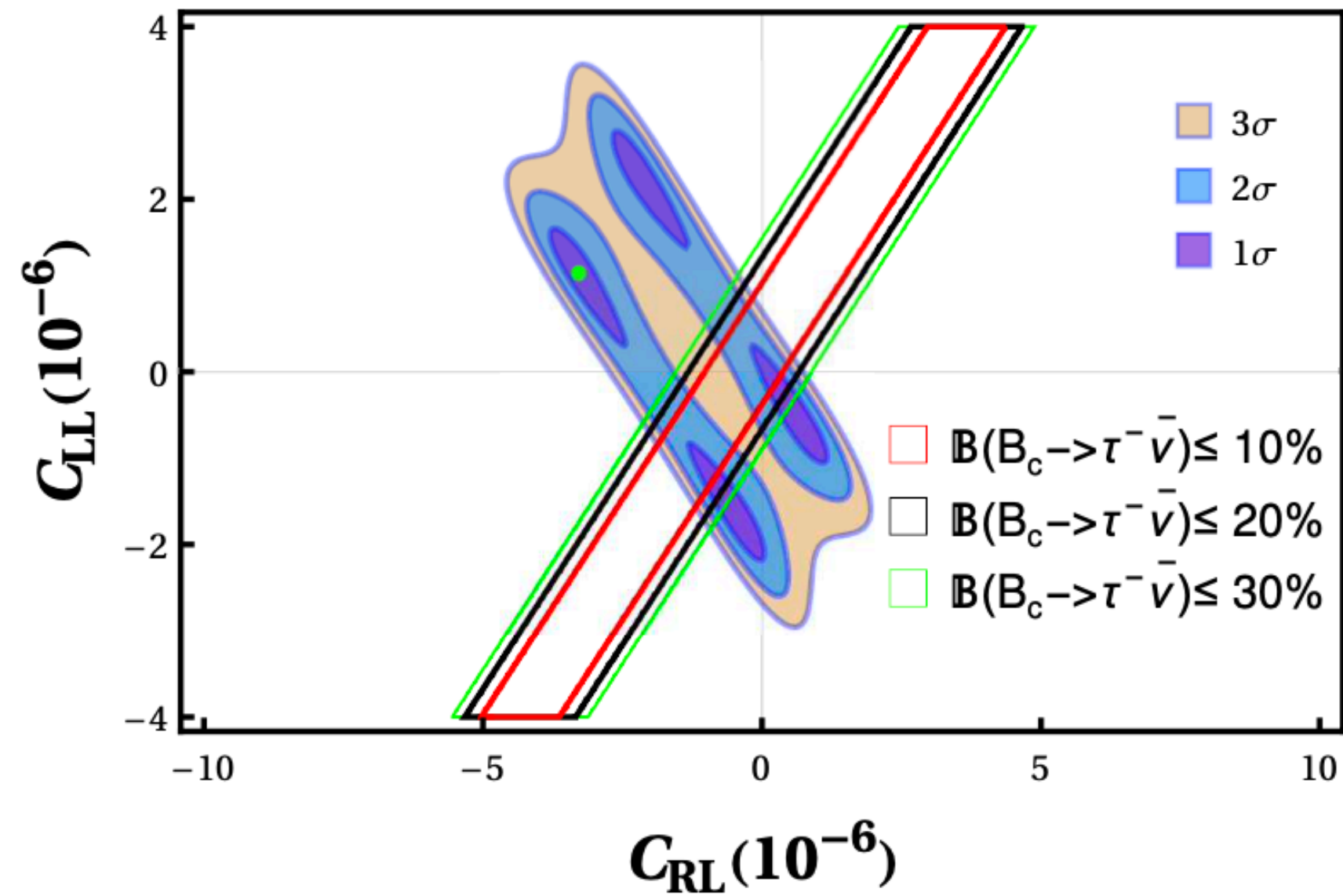


$$R_D = R_D^{\text{SM}} \left[1 + 1.5 \text{Re} \left(\frac{C_{RL}^{\tau 3} + C_{LL}^{\tau 3}}{C_{\text{SM}}^{cb}} \right) + 1.0 \left| \frac{C_{RL}^{\tau 3} + C_{LL}^{\tau 3}}{C_{\text{SM}}^{cb}} \right|^2 \right],$$

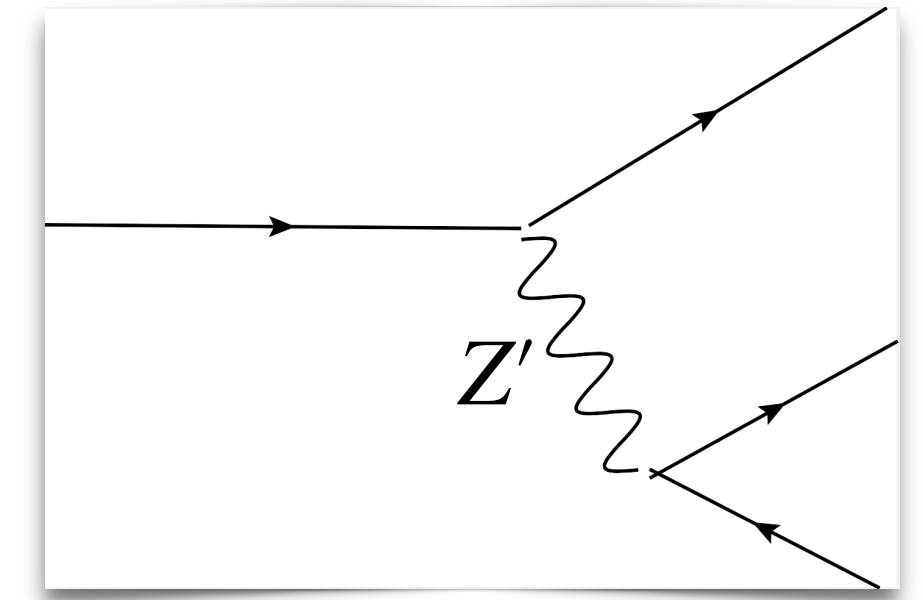
$$R_{D^*} = R_{D^*}^{\text{SM}} \left[1 + 0.12 \text{Re} \left(\frac{C_{RL}^{\tau 3} - C_{LL}^{\tau 3}}{C_{\text{SM}}^{cb}} \right) + 0.05 \left| \frac{C_{RL}^{\tau 3} - C_{LL}^{\tau 3}}{C_{\text{SM}}^{cb}} \right|^2 \right],$$

$$C_{RL}^{\tau 3} \approx -2\sqrt{2}G_F V_{cb} \frac{m_b m_\tau}{m_H^2} \left(\frac{2}{\tan^2 \beta} + 1 \right)$$

$$C_{LL}^{\tau 3} = -2\sqrt{2}G_F V_{cb} \frac{m_c m_\tau}{m_H^2}.$$



Global fit of ΔC_i^ℓ



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[86] J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl and D. M. Straub, *B-decay discrepancies after Moriond 2019*, *Eur. Phys. J. C* **80** (2020) 252 [1903.10434].

[87] M. Algueró, B. Capdevila, A. Crivellin, S. Descotes-Genon, P. Masjuan, J. Matias et al., *Emerging patterns of New Physics with and without Lepton Flavour Universal contributions*, *Eur. Phys. J. C* **79** (2019) 714 [1903.09578].

[88] A. K. Alok, A. Dighe, S. Gangal and D. Kumar, *Continuing search for new physics in $b \rightarrow s\mu\mu$ decays: two operators at a time*, *JHEP* **06** (2019) 089 [1903.09617].

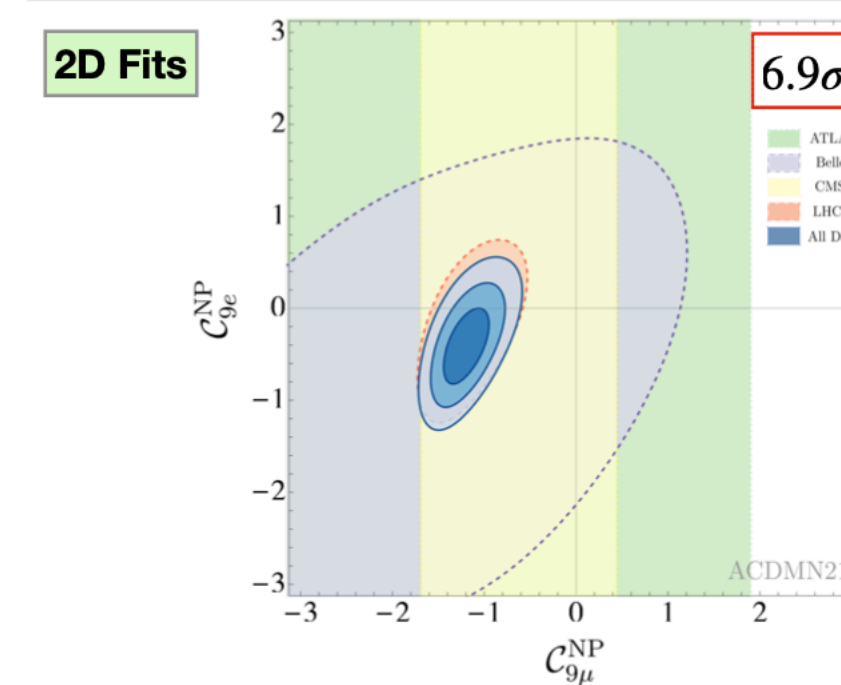
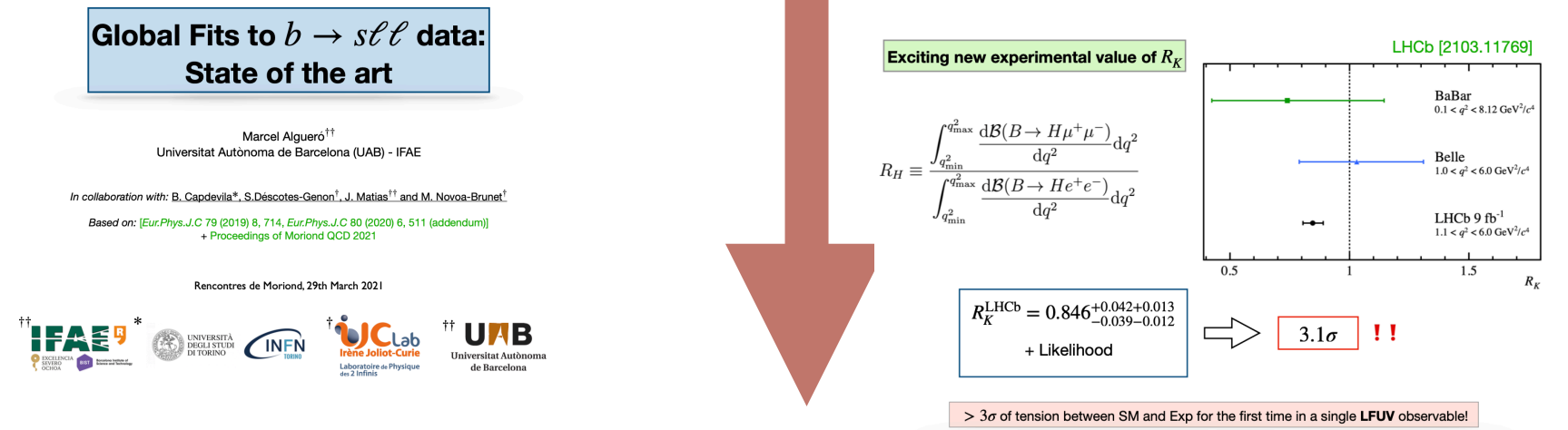
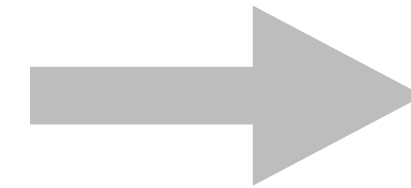
[89] K. Kowalska, D. Kumar and E. M. Sessolo, *Implications for new physics in $b \rightarrow s\mu\mu$ transitions after recent measurements by Belle and LHCb*, *Eur. Phys. J. C* **79** (2019) 840 [1903.10932].

$$\mathcal{O}_9 = (\bar{s}\gamma_\mu \mathbb{P}_L b)(\bar{\ell}\gamma^\mu \ell),$$

$$\mathcal{O}_{10} = (\bar{s}\gamma_\mu \mathbb{P}_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

- i) large and negative δC_9^μ (best fit ~ -1)
- ii) relative small and positive ΔC_{10}^μ (best fit ~ 0.5)

- i) positive and relative large ΔC_9^e (best fit ~ 0.8)
- ii) negative and relative large ΔC_{10}^e (best fit ~ -0.78)



change of the sign of central value

$$\Delta C_9^\mu = -1.21 \pm 0.20, \quad \Delta C_9^e = -0.40 \pm 0.40,$$

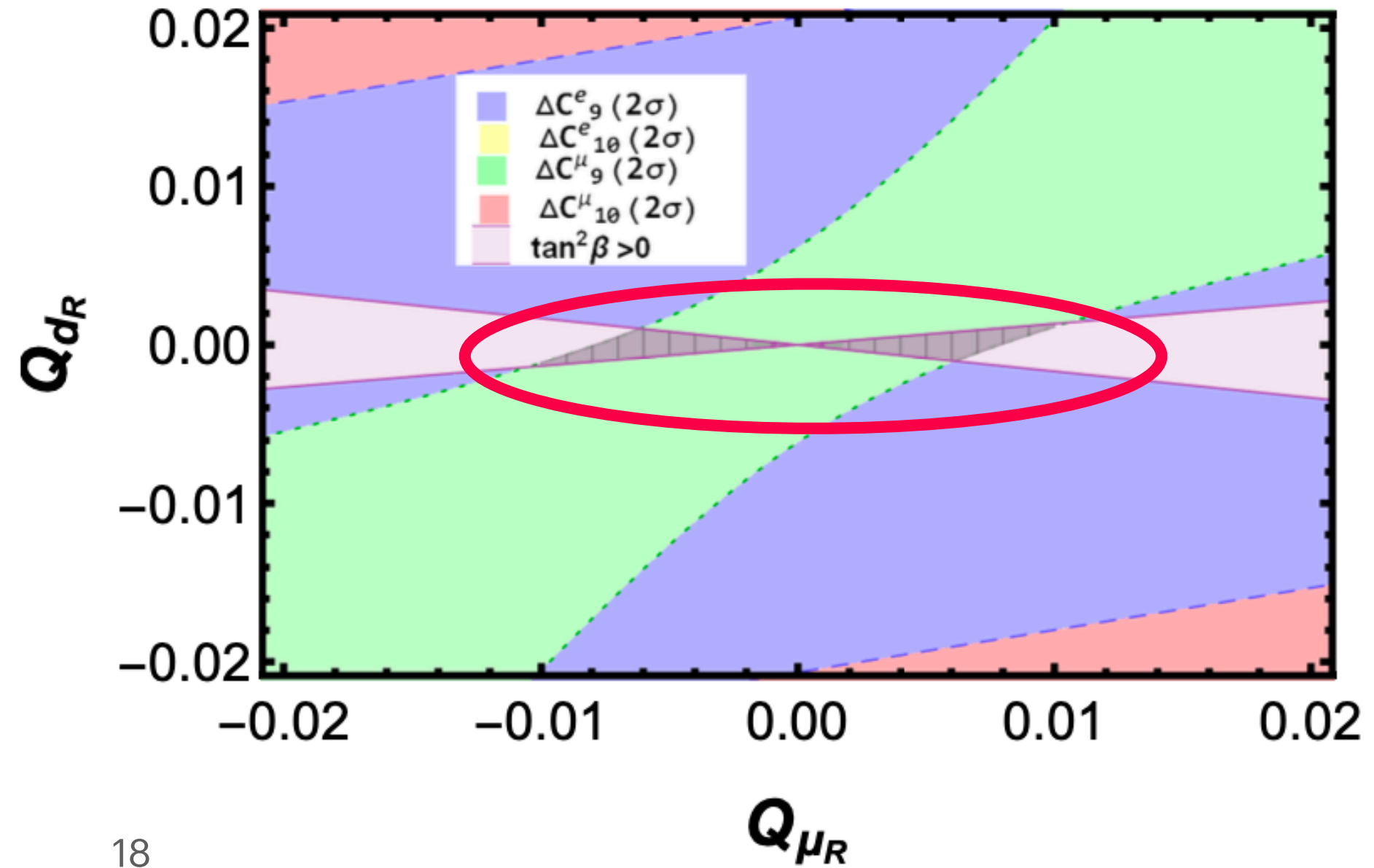
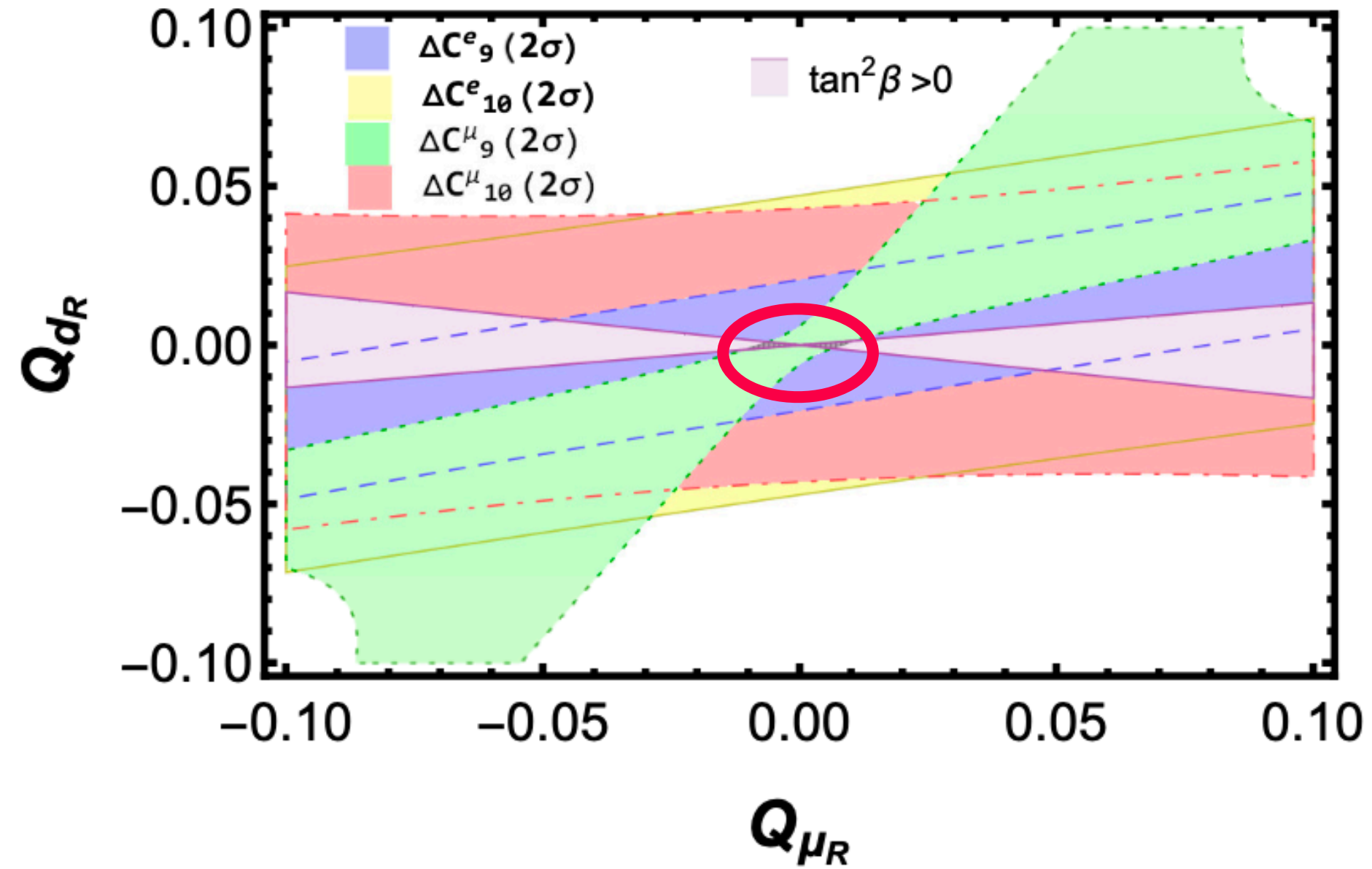
$$\Delta C_{10}^\mu = 0.15 \pm 0.20, \quad \Delta C_{10}^e = -0.78 \pm 0.40.$$

Solution II

$R_{K^{(*)}}$

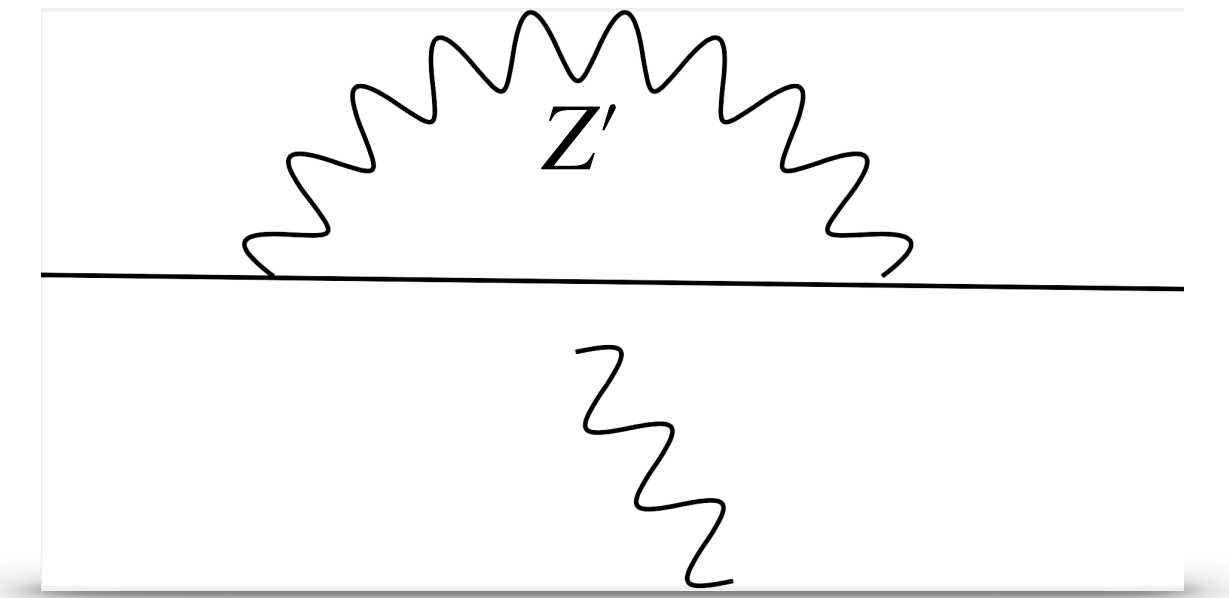
$$\begin{aligned} \Delta C_9^\mu &= -1.21 \pm 0.20, & \Delta C_9^e &= -0.40 \pm 0.40, \\ \Delta C_{10}^\mu &= 0.15 \pm 0.20, & \Delta C_{10}^e &= -0.78 \pm 0.40. \end{aligned}$$

$$\begin{aligned} \Delta C_9^\ell &= \frac{1}{N} \frac{g'^2}{m_{Z'}^2} \frac{1}{4} V_{ts}^* V_{tb} (Q_{tR} - Q_{uR})(Q_{lR} + Q_{lL}), \\ \Delta C_{10}^\ell &= \frac{1}{N} \frac{g'^2}{m_{Z'}^2} \frac{1}{4} V_{ts}^* V_{tb} (Q_{tR} - Q_{uR})(Q_{lR} - Q_{lL}), \end{aligned}$$

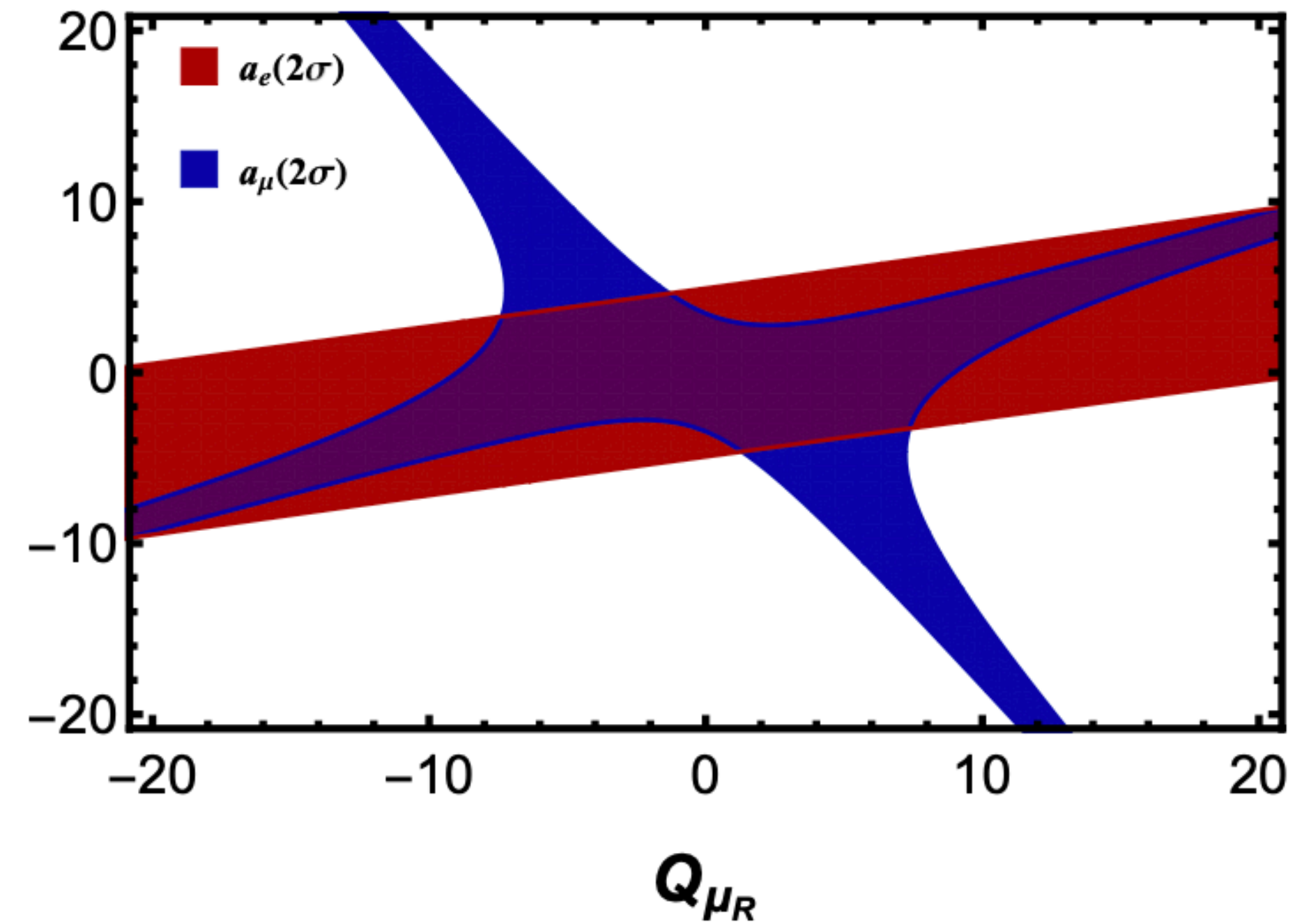
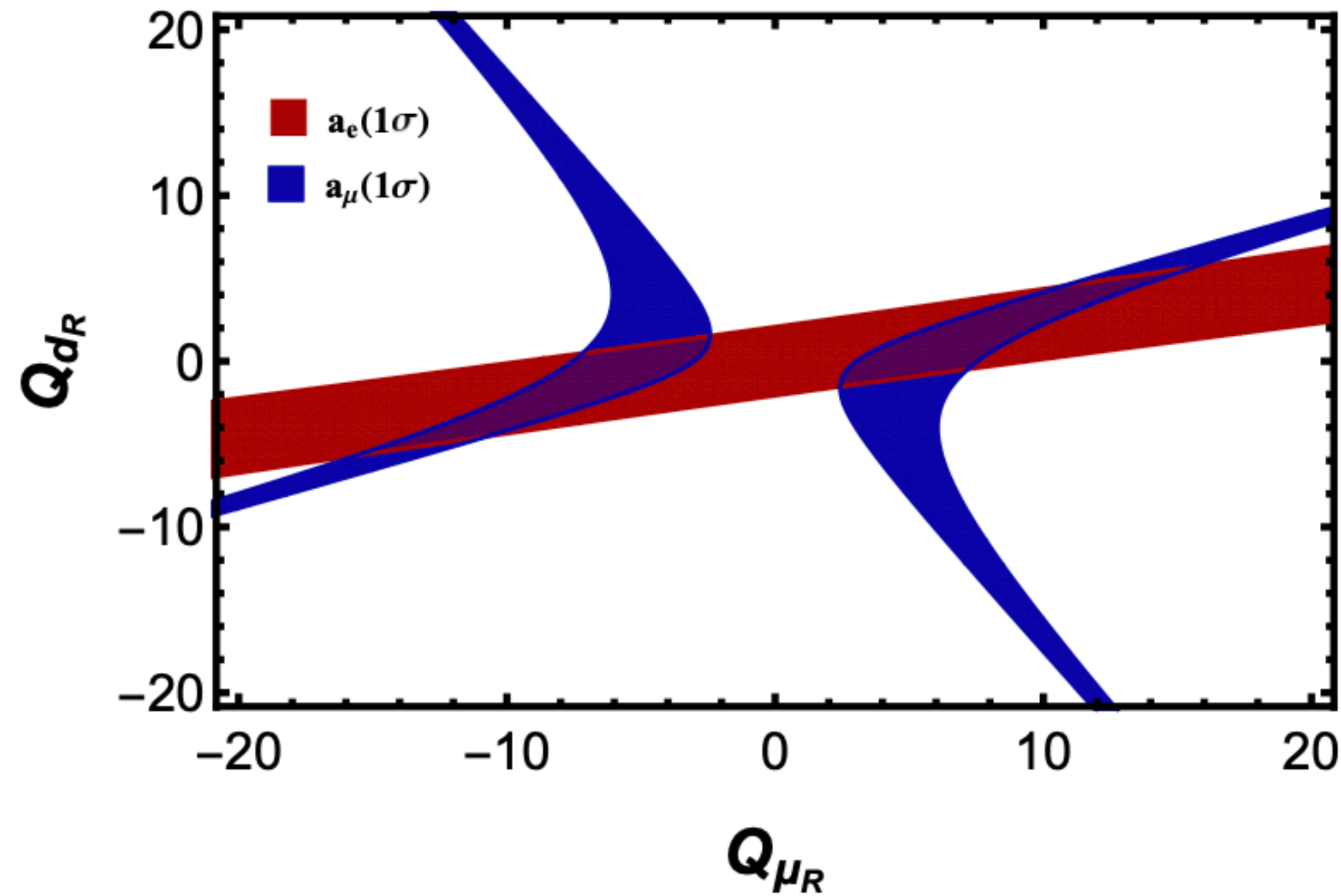


Solution II

$R_{K^{(*)}}$ & $g - 2$



$$\Delta a_\ell = -\frac{m_\ell^2}{8\pi^2} \frac{g'^2}{m_{Z'}^2} \frac{2}{3} [(Q_{\ell_L}^2 + Q_{\ell_R}^2) - 3Q_{\ell_L}Q_{\ell_R}]$$



Summary and Outlook

- ◆ FG2HDM, a type of flavor gauged 2HDM-III, is proposed.
- ◆ Only two exotic Higgs bosons and one gauge boson are added into particle spectrum.
- ◆ In an extreme case of FG2HDM,
 - ◆ FCNC processes only occur in down-type quark sector;
 - ◆ Plenty of room for $R_{D^{(*)}}$;
 - ◆ $R_{K^{(*)}}$ & $\Delta a_\mu, \Delta a_e$ can be explained at 2σ level;
- ◆ More general cases to be explored...

Backup

$$\begin{array}{c}
 \ell \\
 \nearrow \\
 H \text{ ---} \\
 \searrow \\
 \bar{\ell}
 \end{array}
 = \frac{-i [\cos(\beta - \alpha)N_\ell + \sin(\beta - \alpha)M_\ell]}{v}$$

$$\begin{array}{c}
 b \\
 \nearrow \\
 H \text{ ---} \\
 \searrow \\
 \bar{s}
 \end{array}
 = \frac{-i [\cos(\beta - \alpha)(N_d)_{23} + \sin(\beta - \alpha)(M_d)_{23}]}{v}$$

$$\mathcal{O}_S = m_b(\bar{s}\mathbb{P}_R b)(\bar{\ell}\ell),$$

$$\mathcal{O}'_S = m_b(\bar{s}\mathbb{P}_L b)(\bar{\ell}\ell),$$

$$\begin{aligned}
 C_S^\ell &= C_S^{\prime\ell} \\
 &= -\frac{1}{N} \frac{G_F V_{tb} V_{ts}^*}{m_H^2 \sin(2\beta)} [\cos^2(\beta - \alpha)N_\ell + \cos(\beta - \alpha) \sin(\beta - \alpha)M_\ell]
 \end{aligned}$$

$$C_9^\ell = \frac{1}{N} \frac{g'^2}{m_{Z'}^2} \frac{1}{4} V_{tb} V_{ts}^* (Q_{tR} - Q_{uR})(Q_{lR} + Q_{lL}),$$

$$C_{10}^\ell = \frac{1}{N} \frac{g'^2}{m_{Z'}^2} \frac{1}{4} V_{tb} V_{ts}^* (Q_{tR} - Q_{uR})(Q_{lR} - Q_{lL}),$$

negligible scalar operator contributions due to a Fermi constant suppression!