Implications of $g_\mu - 2$ for 3-3-1 Models

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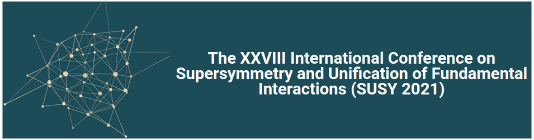

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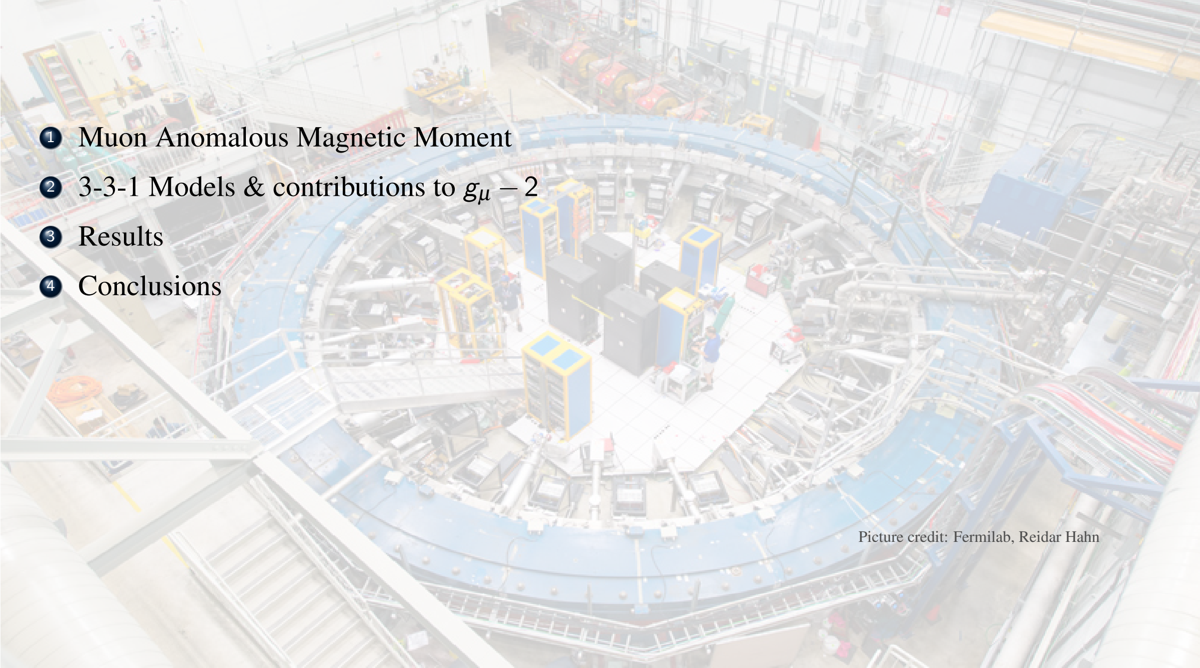
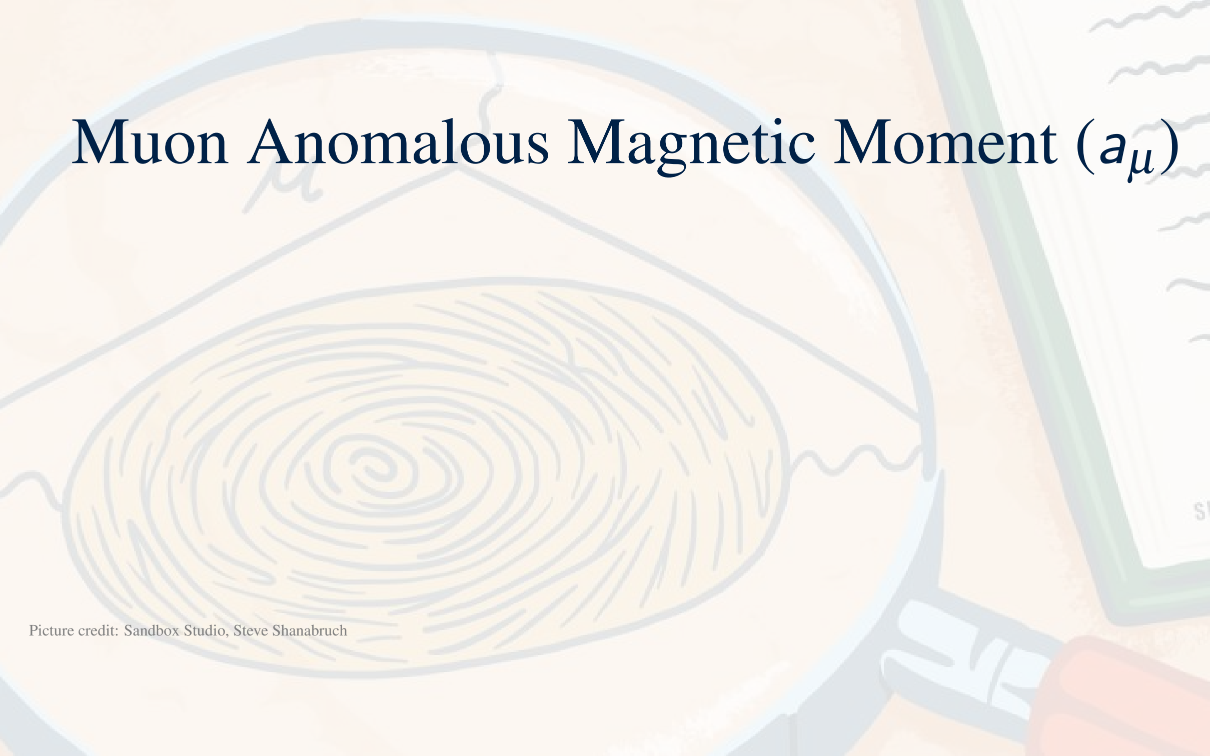
Overview

1. Muon Anomalous Magnetic Moment
2. 3-3-1 Models & contributions to $g_\mu - 2$
3. Results
4. Conclusions



Picture credit: Fermilab, Reidar Hahn

Muon Anomalous Magnetic Moment (a_μ)



Picture credit: Sandbox Studio, Steve Shanabruch

The Dirac equation predicts at tree level, $\overrightarrow{\mu_\mu} = g_\mu \frac{q}{2m_\mu} \overrightarrow{S}$. Where $g_\mu = 2$ is the gyromagnetic ratio, m_μ, q and S are the muon mass, the electric charge and the spin respectively. However, through quantum corrections at the loop $g_\mu \neq 2$, letting us define the Muon Anomalous Magnetic Moment as

$$a_\mu \equiv \frac{g_\mu - 2}{2} = 116591802(2)(42)(26) \times 10^{-11}.$$

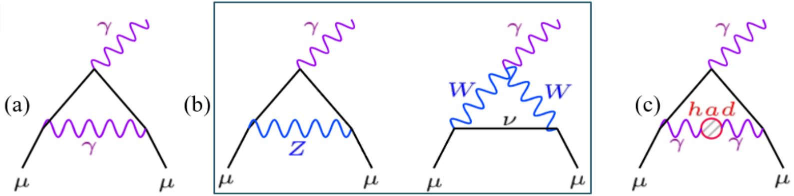


Figure 1: Feynman diagram of the corrections to a_μ on SM interactions: (a) first order QED, (b) lowest-order weak, and (c) lowest-order hadronic effects. $a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{QCD}}$

Comparing the SM prediction with the measurements from Brookhaven National Lab,we get Δa_μ [1]:

$$\Delta a_\mu = (261 \pm 78) \times 10^{-11}\ (3.3\sigma) - (2009)^a$$
$$\Delta a_\mu = (325 \pm 80) \times 10^{-11}\ (4.05\sigma) - (2012)^b$$
$$\Delta a_\mu = (287 \pm 80) \times 10^{-11}\ (3.6\sigma) - (2013)^c$$
$$\Delta a_\mu = (377 \pm 75) \times 10^{-11}\ (5.02\sigma) - (2015)^d$$
$$\Delta a_\mu = (313 \pm 77) \times 10^{-11}\ (4.1\sigma) - (2017)^e$$
$$\Delta a_\mu = (270 \pm 36) \times 10^{-11}\ (3.7\sigma) - (2018)^f$$

$$\text{FERMILAB: } \Delta a_\mu = (251 \pm 59) \times 10^{-11}\ (4.2\sigma) - (2021)^g$$

We will explore new physics contributions to a_μ on the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry and will use the following a_μ discrepancies,

$$\Delta a_{\mu \text{Current}} = (261 \pm 78) \times 10^{-11}\ (3.3\sigma)$$
$$\Delta a_{\mu \text{Projected}} = (261 \pm 34) \times 10^{-11}\ (5\sigma)$$

[1] Refs: [a]Prades, Joaquim, Eduardo De Rafael, and Arkady Vainshtein., Tanabashi, Masaharu, et al.; [b]Benayoun, M., et al.; [c]Blum, Thomas, et al. ; [d] Benayoun, M., et al.; [e]Jegerlehner, Fred.; [f]Keshavarzi, Alexander, Daisuke Nomura, and Thomas Teubner.; [g]B. Abi, et al. (Muon g-2 Collaboration)

$SU(3)_C \times SU(3)_L \times U(1)_X$

(3-3-1) Models

Models based on 3-3-1 gauge symmetry[2]:

1. Minimal 3-3-1 Model[a]
2. 3-3-1 with right-handed neutrinos, (r.h.n)[b]
3. 3-3-1 with neutral lepton (3-3-1 LHN)[c],
4. Economical 3-3-1[d]
5. 3-3-1 with exotic leptons[e],

The electric charge operator for 3-3-1 Models is,

$$\frac{Q}{e} = \frac{1}{2}(\lambda_3 + \alpha\lambda_8) + XI, \qquad \alpha = -\sqrt{3},\ \pm\frac{1}{\sqrt{3}}$$

These models are quite popular because they can explain:

- neutrino masses,
- dark matter,
- meson oscillations,
- flavor violation,
- collider physics,
- among others.

where $\lambda_{3,8}$ and I are the generators of $SU(3)_C$ and $U(1)_X$, respectively.

[2] Refs: Pisano, F., and Vicente Pleitez.[a]; Hoang Ngoc Long[b]; Martinez, R., and F. Ochoa., Mizukoshi, J. K., et al. [c]; Model, Dong, P. V., et al. , R. Martínez and F. Ochoa, Dong, P. V., and H. N. Long.[d]; Ponce, William A., Juan B. Florez, and Luis A. Sanchez., Anderson, David L., and Marc Sher., Cabarcas, J. M., J. Duarte, and J-Alexis Rodriguez.[e].

The scalar sector contains between 2 or 3 scalar triplets (χ, η, ρ) to give the masses of the fermions and one scalar sextet to generate neutrino masses via a type II seesaw mechanism. The 3-3-1 gauge symmetry experiences the following spontaneous symmetry breaking:
$\mathrm{SU(3)_L \times U(1)_X} \xrightarrow{\langle\chi\rangle} \mathrm{SU(2)_L \times U(1)_Y} \xrightarrow{\langle\eta\rangle,\ \langle\rho\rangle} \mathrm{U(1)_Q}$, with VEV different scales: $v_\chi \gg v_\eta, v_\rho$.

The fermionic sector of each 3-3-1 model contains leptonic triplets,

1. Minimal 3-3-1 Model
2. 3-3-1 with right-handed neutrinos, (r.h.n)
3. 3-3-1 with neutral lepton (3-3-1 LHN),
4. Economical 3-3-1 Model,
5. 3-3-1 with exotic leptons,

$$f_L^a = \begin{pmatrix} \nu^a \\ \ell^a \\ (\ell^c)^a \end{pmatrix};$$

where $a = 1, 2, 3$ is the generation index and ν and ℓ are the SM particles.

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The fermionic sector of each 3-3-1 model contains leptonic triplets,

$$f_L^a = \begin{pmatrix} \nu^a \\ \ell^a \\ (\nu^c)^a \end{pmatrix} ; \ell_R^a$$

where $a = 1, 2, 3$ is the generation index and ν^c is the r.h.n.

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5. 3-3-1 with exotic leptons,

The fermionic sector of each 3-3-1 model contains leptonic triplets,

$$f_L^a = \begin{pmatrix} \nu^a \\ \ell^a \\ N^a \end{pmatrix} ; N_R^a \ \ell_R^a$$

where $a = 1, 2, 3$ is the generation index and N is the heavy neutral lepton.

The fermionic sector of each 3-3-1 model contains leptonic triplets,

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2. 3-3-1 with right-handed neutrinos, (r.h.n)
3. 3-3-1 with neutral lepton (3-3-1 LHN),
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5. 3-3-1 with exotic leptons,

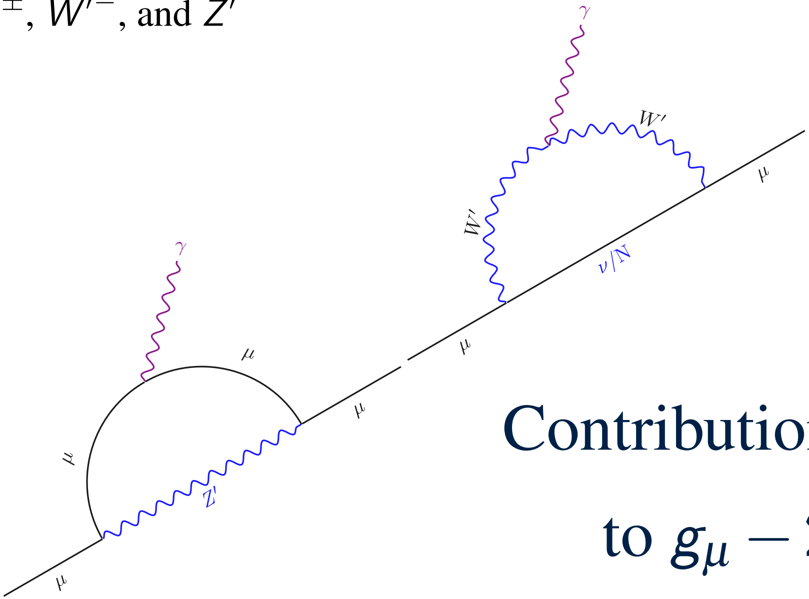
$$f_{1L} = \begin{pmatrix} \nu_1 \\ \ell_1 \\ E_1^- \end{pmatrix}; \quad f_{2,3L} = \begin{pmatrix} \nu_{2,3} \\ \ell_{2,3} \\ N_{2,3} \end{pmatrix};$$

$$f_{4L} = \begin{pmatrix} E_2^- \\ N_3 \\ N_4 \end{pmatrix}; \quad f_{5L} = \begin{pmatrix} N_5 \\ E_3^+ \\ \ell_3^+ \end{pmatrix};$$

$$\ell_1^c; \; \ell_{2,3}^c; \; E_2^c; \; E_3^c$$

where N and E are the exotic neutral and charged leptons, respectively.

- Besides, new known gauge bosons appear, such as $U^{\pm\pm}$, W'^{-}, and Z'



Contributions to $g_\mu - 2$

We make our Mathematica numerical codes of the analytical expressions to Muon Anomalous Magnetic Moment(Δa_μ) corresponding to the 3-3-1 Models available at `https://bit.ly/2vFZLnG`

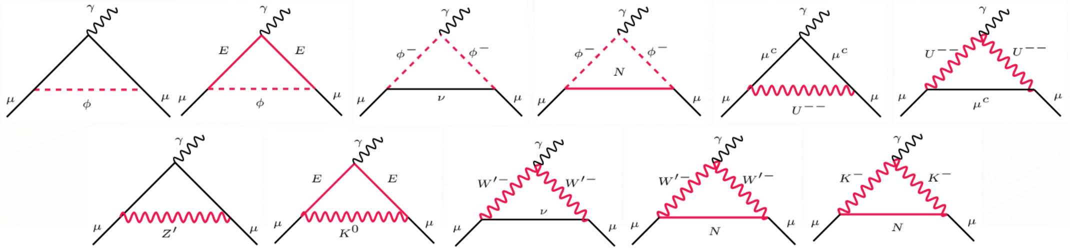


Figure 2: Feynman diagrams that contribute to the $g_\mu - 2$ in the 3-3-1 models investigated in this work. Where $U^{\pm\pm}$, W'^{-}, K^{-}, K^{0} and Z' are new gauge bosons. With ϕ and ϕ^{-} are the neutral and singlet charged scalars fields, and correspond to the scalars χ^{0}, S_2, η_1^{+}, h_1^{+}, h_2^{+}, and χ^{+}

New Physics contributions to g-2

Lindner, Manfred, Moritz Platscher, and Farinaldo S. Queiroz. "A call for new physics: the muon anomalous magnetic moment and lepton flavor violation." Physics Reports 731 (2018): 1-82. & arXiv:1403.2309

The corrections to $g_\mu - 2$ rise from the presence of new gauge bosons, and charged and neutral scalars. The contributions for heavy bosons are given as:

$$\Delta a_\mu\left(U^{++}\right) \simeq -2\frac{1}{\pi^2}\frac{m_\mu^2}{M_U^2}\left|\frac{g}{2\sqrt{2}}\right|^2, \text{ with } M_U \gg m_\mu \quad \Rightarrow \textbf{Minimal 3-3-1}$$

$$\Delta a_\mu\left(\nu, W'\right) \simeq \frac{1}{4\pi^2}\frac{m_\mu^2}{M_{W'}^2}\left|\frac{g}{2\sqrt{2}}\right|^2\left(\frac{5}{3}\right), \text{ with } M_{W'} \gg m_\nu \quad \Rightarrow \textbf{Minimal/Eco. 3-3-1, 3-3-1 R.H.N}$$

$$\Delta a_\mu\left(\mu, Z'\right) \simeq \frac{-1}{4\pi^2}\frac{m_\mu^2}{M_{Z'}^2}\left|\frac{g}{2c_W}\frac{\sqrt{3}\sqrt{1-4s_W^2}}{2}\right|^2\left(-\frac{4}{27}\right), \text{ with } M_{Z'} \gg m_\mu \quad \Rightarrow \textbf{Minimal 3-3-1}$$

$$\Delta a_\mu\left(\phi^\pm\right) \simeq \frac{-1}{4\pi^2}\frac{m_\mu^2}{M_{\phi^\pm}^2}\left|\frac{m_\mu\sqrt{2}}{2v_\eta}\right|^2\left(\frac{1}{6}\right), \text{ with } M_{\phi^\pm} \gg m_\mu, m_{\nu_L}$$

$$\Delta a_\mu\left(\phi\right) \simeq \frac{1}{4\pi^2}\frac{m_\mu^2}{M_\phi^2}\left(\frac{m_\mu\sqrt{2}}{2v_\eta}\right)^2\left[\frac{1}{6} - \left(\frac{3}{4} + \log\left(\frac{m_\mu}{M_\phi}\right)\right)\right]$$

$$\Delta a_\mu\left(N, W'\right) \simeq \frac{1}{4\pi^2}\frac{m_\mu^2}{M_{W'}^2}\left|\frac{g}{2\sqrt{2}}\right|^2\frac{5}{3}$$ $\Rightarrow$ **3-3-1 LHN**

$$\Delta a_\mu\left(\mu, Z'\right) \simeq \frac{-1}{4\pi^2}\frac{m_\mu^2}{M_{Z'}^2}\frac{1}{3}\left|-\frac{g}{4c_W\sqrt{3-4s_W^2}}\right|\left[-\left|1-4s_W^2\right|^2+5\right]$$ $\Rightarrow$ **3-3-1 R.H.N, Eco. & LHN**

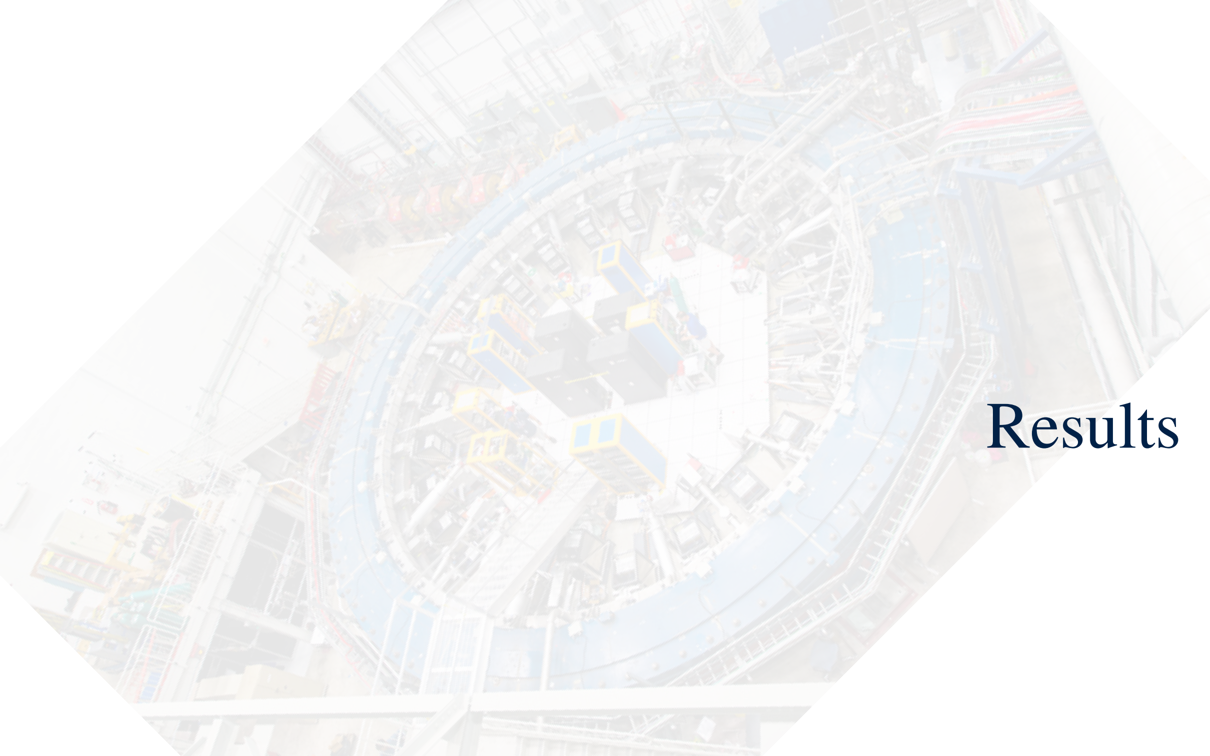
$$\Delta a_\mu\left(N, K^+\right) \simeq \frac{1}{4\pi^2}\frac{m_\mu^2}{M_{K^+}^2}\left|\frac{g}{\sqrt{2}}\right|^2\frac{5}{3}$$ $\Rightarrow$ **3-3-1 model with exotic leptons**

$$\Delta a_\mu\left(E, K^0\right) \simeq \frac{-1}{4\pi^2}\frac{m_\mu^2}{M_{K^0}^2}\left|\frac{g}{\sqrt{2}}\right|^2\left(\frac{4}{3}\right)$$ $\Rightarrow$ **3-3-1 model with exotic leptons**

3-3-1 model with exotic leptons

$$\Delta a_\mu\left(\mu, Z'\right) \simeq \frac{-1}{4\pi^2}\frac{m_\mu^2}{M_{Z'}^2}\left|\frac{g'}{2\sqrt{3}s_W c_W}\right|^2\frac{1}{12}\left[-\left|\left(-c_{2W}+2s_W^2\right)\right|^2+5\left|\left(c_{2W}+2s_W^2\right)\right|^2\right].$$

$s_W = \mathrm{sen}\left(\theta_W\right), c_W = \cos\left(\theta_W\right)$, θ_W is the Weinberg angle and g is the $SU(2)_L$ coupling constant.



Results

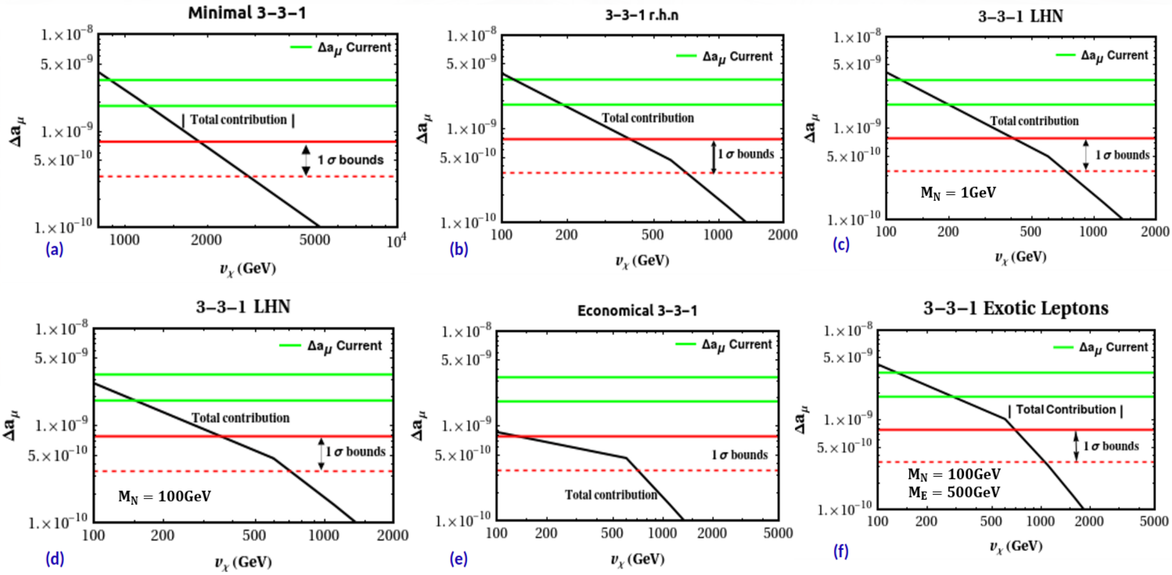


Figure 3: Overall contribution to Δa_μ from the 3-3-1 models. The green bands are delimited by $\Delta a_\mu = (261 \pm 78) \times 10^{-11}$ (3.3σ). The projected 1σ bound: $\Delta a_\mu < 78 \times 10^{-11}$ - $\Delta a_\mu < 34 \times 10^{-11}$.

Model	LHC-13TeV	g-2 current	g-2 projected
Minimal 3-3-1	$M_{Z'} > 3.7$ TeV [1] $M_{W'} > 3.2$ TeV [1]	$M_{Z'} > 434.5$ GeV $M_{W'} > 646$ GeV	$M_{Z'} > 632$ GeV $M_{W'} > 996.1$ GeV
3-3-1 r.h.n	$^*M_{Z'} > 2.64$ TeV [2] ——-	$M_{Z'} > 158$ GeV $M_{W'} > 133$ GeV	$M_{Z'} > 276.5$ GeV $M_{W'} > 239$ GeV
3-3-1 LHN for $M_N = 1$ GeV	$^*M_{Z'} > 2$ TeV [2] —-	$M_{Z'} > 160$ GeV $M_{W'} > 134.3$ GeV	$M_{Z'} > 285$ GeV $M_{W'} > 238.3$ GeV
3-3-1 LHN for $M_N = 100$ GeV	$^*M_{Z'} > 2$ TeV [2] ——	$M_{Z'} > 136.7$ GeV $M_{W'} > 114.2$ GeV	$M_{Z'} > 276.5$ GeV $M_{W'} > 231$ GeV
Economical 3-3-1	$^*M_{Z'} > 2.64$ TeV [2] ——	$M_{Z'} > 59.3$ GeV $M_{W'} > 49.5$ GeV	$M_{Z'} > 271.4$ GeV $M_{W'} > 226.7$ GeV
3-3-1 exotic leptons for $M_N(M_E) = 10(150)$ GeV	$^*M_{Z'} > 2.91$ TeV [3] ——	$M_{Z'} > 429$ GeV $M_{W'} > 359$ GeV	$M_{Z'} > 693$ GeV $M_{W'} > 579.6$ GeV
3-3-1 exotic leptons for $M_N(M_E) = 100(150)$ GeV	$^*M_{Z'} > 2.91$ TeV [3] ——	$M_{Z'} > 369$ GeV $M_{W'} > 309.1$ GeV	$M_{Z'} > 600$ GeV $M_{W'} > 501.4$ GeV

Table 1: Summary of the lower bounds based on our calculations. For comparison we include the LHC bounds at 13 TeV center-of-mass energy.

[1] Nepomuceno, A. A., and Bernhard Meirose, [2] Lindner, Manfred, Moritz Platscher, and Farinaldo S. Queiroz., [3] Salazar, Camilo, et al.

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Table 1: Summary of the lower bounds based on our calculations. For comparison we include the LHC bounds at 13 TeV center-of-mass energy.

None of the five models investigated here can accommodate the anomaly in agreement with existing bounds.

The inert scalar triplet allows us to include $\mathscr{L} \supset y_{ab}\overline{f_a}\phi e_{bR}$, taking $y_{22} = 1$. Such scalar triplet gets a mass from the quartic coupling in the scalar potential $(\lambda\phi^\dagger\phi\chi^\dagger\chi)$, after the scalar triplet χ acquires a *vev*.

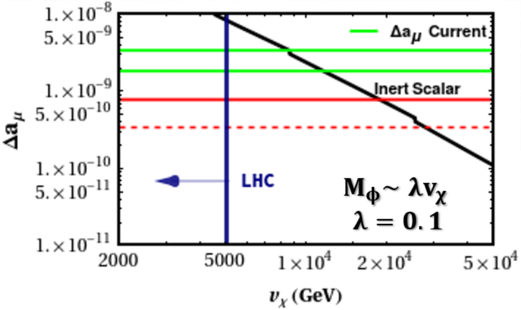

Figure 4: Overall contribution of the 3-3-1 LHN Model augmented by an inert scalar triplet ϕ.

$$\Delta a_\mu(\phi) = \frac{1}{8\pi^2}\frac{m_\mu^2}{M_\phi^2}\int_0^1 dx \left[\frac{(2-x)x^2}{\frac{m_\mu^2}{M_\phi^2}x + (1-x)(1-\frac{m_\mu^2}{M_\phi^2}x)}\right]$$

We have presented a solution to $g_\mu - 2$ in the context of 3-3-1 models.

Conclusions

1. We concluded that none of the five models investigated here can accommodate the anomaly.
2. We derived robust and complementary 1σ lower mass bounds on the masses of the new gauge bosons, namely the Z' and W' bosons, that contribute to muon anomalous magnetic moment assuming the anomaly is otherwise resolved.
3. The 3-3-1 models must be extended to explain the anomaly observed in the muon anomalous magnetic moment.
4. We presented a plausible extension to the 3-3-1 LHN model, which features an inert scalar triplet.

The XXVIII International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY 2021)

Thank you so much for your attention!

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Questions & Comments

Backup

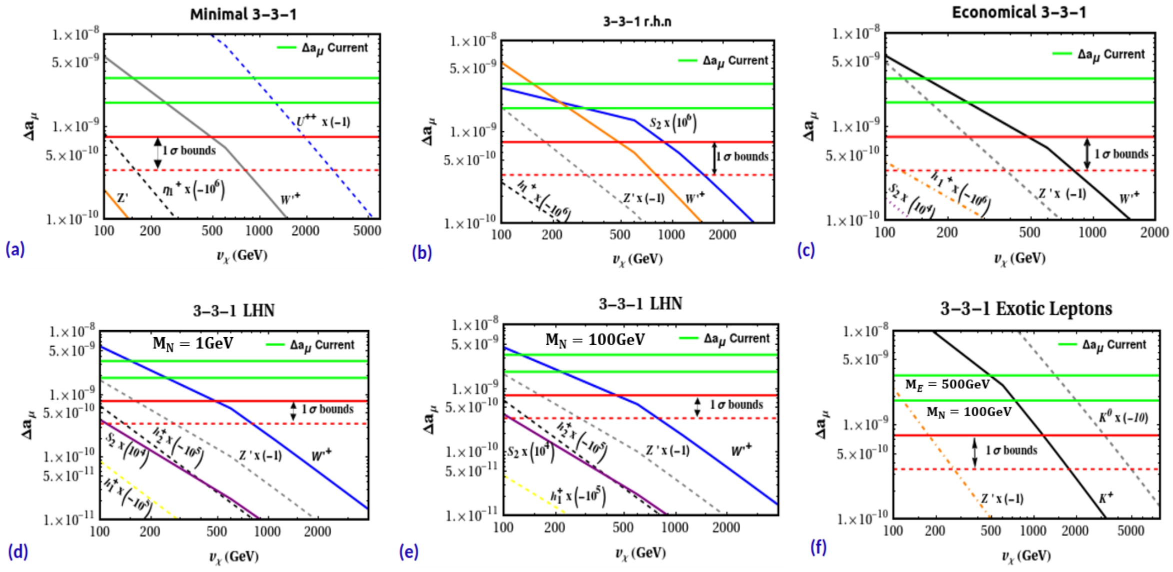

Figure 5: Individual contributions to Δa_μ from the 3-3-1 models. The green bands are delimited by $\Delta a_\mu = (261 \pm 78) \times 10^{-11}$ (3.3σ). The projected 1σ bound is found by requiring $\Delta a_\mu < 78 \times 10^{-11}$ while the bound is obtained for $\Delta a_\mu < 34 \times 10^{-11}$.

The relevant interactions to a_μ are,

Minimal 3-3-1: $\mathscr{L}^{CC} \supset -\frac{g}{2\sqrt{2}}\left[\overline{\nu}\gamma^\mu(1-\gamma_5)\,C\overline{\ell}^T W'^{-}_\mu - \overline{\ell}\gamma^\mu\gamma_5 C\overline{\ell}^T U^{--}_\mu\right],$

$\mathscr{L}^{NC} \supset \overline{f}\gamma^\mu\left[g_V(\ell)+g_A(\ell)\gamma_5\right] f Z'_\mu,\quad \mathscr{L}_{Yukawa} \supset G_\ell\left[\overline{\ell_R}\nu_L\eta_1^- + \overline{\ell^c_R}\nu_L h_1^+ + \overline{\ell_R}\nu_L h_2^+ + \overline{\ell_R}\ell_L R_{\sigma_2}\right] + h.c.$

Where $\mathscr{L}^{CC}$ and $\mathscr{L}^{NC}$ are the charged and neutral currents Lagrangians, $g_A(\ell) = \frac{g}{2c_W}\frac{\sqrt{3}\sqrt{1-4s_W^2}}{6}$, $g_V(\ell) = 3g_A(\ell)$ are the vector and axial coupling constants, $s_W = \mathrm{sen}(\theta_W)$, $c_W = \cos(\theta_W)$, g and $G_\ell = m_\ell\sqrt{2}/v_\eta$ are coupling constants and η_1^-, h_1^+, h_2^+, and R_{σ_2} are the scalars fields.

3-3-1 r.h.n: $\mathscr{L}^{CC} \supset -\frac{g}{2\sqrt{2}}\left[\overline{\nu^c_R}\gamma^\mu(1-\gamma_5)\,\overline{\ell}W'^{-}_\mu\right],\quad \mathscr{L}^{NC} \supset \overline{f}\gamma^\mu\left[g'_V(\ell)+g'_A(\ell)\gamma_5\right] f Z'_\mu,$

$\mathscr{L}_{Yukawa} \supset G_s\overline{\mu}\mu S_2,$ with $G_s = m_\mu\sqrt{2}/(2v)$.

$\mathscr{L}_{Yukawa}$ involving the charged scalars is essentially the same as Minimal 3-3-1 Model. G_s is a coupling contant. $g'_V(\ell) = \frac{g}{4c_W}\frac{(1-4s_W^2)}{\sqrt{3-4s_W^2}}$, $g'_A(\ell) = -\frac{g}{4c_W\sqrt{3-4s_W^2}}$ are the vector and axial coupling constants.

Gauge boson and scalar fields interactions with leptons in the 3-3-1 Models

The relevant interactions to a_μ are,

Economical: $\mathscr{L}_{Yukawa} \supset G_s \overline{\mu} \mu S_2 + G_\ell \ell_R^- \nu_L \eta_1^+,$

$\mathscr{L}^{NC}$ and $\mathscr{L}^{CC}$ are the same as in model 3-3-1 r.h.n. .

3-3-1 L.H.N: $\mathscr{L}^{CC} \supset -\frac{g}{\sqrt{2}} \left[\overline{N_L} \gamma^\mu \overline{\ell}_L W'^{-}_\mu \right], \quad \mathscr{L}_{Yukawa} \supset G_\ell \overline{\ell_R} N_L h_1^- + G_\ell \overline{\ell_R} \nu_L h_2^+ + G_s \overline{\mu} \mu S_2$

$\mathscr{L}^{NC}$ is the same as in model 3-3-1 r.h.n.

3-3-1 with exotic leptons: $$\mathscr{L} \supset \frac{g'}{2\sqrt{3}_W c_W} \overline{\mu} \gamma_\mu (g_V + g_A) \mu Z' - \frac{g}{\sqrt{2}} \left(\overline{N_{1L}} \gamma_\mu \mu_L + \overline{\mu}_L \gamma_\mu N_{4L} \right) K_\mu^+$$
$$- \frac{g}{\sqrt{2}} \left(\overline{\mu}_L \gamma_\mu E_L \right) K_\mu^0 + h_1 \overline{\mu} (1 - \gamma_5) N \chi^+ + h_2 \overline{\mu} E^- \chi^0 + h_3 \overline{\mu} E_2^- \chi^0 + \text{H.c.}$$

where χ^+ and χ^0 are scalars coming from the scalar triplets, and K_μ^+ and K_μ^0 are new gauge bosons.
$g_V = \frac{-c_{2W} + 2s_W^2}{2}$, and $g_A = \frac{c_{2W} + 2s_W^2}{2}$ are the vector and vector-axial couplings.

Neutral Gauge Boson Mediator:

$$\Delta a_\mu\left(f, Z'\right) = \frac{1}{8\pi^2}\frac{m_\mu^2}{M_{Z'}^2}\int_0^1 \mathrm{d}x \sum_f \left[\frac{\left|g_{v1}^{f\mu}\right|^2 P_1^+(x) + \left|g_{a1}^{f\mu}\right|^2 P_1^-(x)}{(1-x)\left(1-\lambda_1^2 x\right) + \varepsilon_f^2\lambda_1^2 x}\right],$$

$P_1^\pm = 2x(1-x)(x-2\pm 2\varepsilon_f) + \lambda_1^2 x^2 (1\mp\varepsilon_f)^2(1-x\pm\varepsilon_f)$, $\varepsilon_f \equiv \dfrac{m_f}{m_\mu}$, $\lambda_1 \equiv \dfrac{m_\mu}{M_{Z'}}$. $g_{v1}^{f\mu}$ and $g_{a1}^{f\mu}$ are the vector and vector-axial coupling constants. m_f is the fermion mass in the loop.

Charged Gauge Boson Mediator:

$$\Delta a_\mu\left(f, W'\right) = \frac{-1}{8\pi^2}\frac{m_\mu^2}{M_{W'}^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{v2}^{f\mu}\right|^2 P_2^+(x) + \left|g_{a2}^{f\mu}\right|^2 P_2^-(x)}{\varepsilon_f^2\lambda_2^2(1-x)\left(1-\varepsilon_f^{-2}x\right) + x},$$

with $P_2^\pm = -2x^2(1+x\mp 2\varepsilon_f) + \lambda_2^2 x(1-x)(1\mp\varepsilon_f)^2\left(x\pm\varepsilon_f\right)$, where and $\varepsilon_f \equiv \dfrac{m_f}{m_\mu}$, $\lambda_2 \equiv \dfrac{m_\mu}{M_{W'}}$.

Doubly Charged Vector Boson Mediator:

$$\Delta a_\mu\left(U^{++}\right) = \frac{8}{8\pi^2}\frac{m_\mu^2}{M_U^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{v3}^{f\mu}\right|^2 P_2^+(x) + \left|g_{a3}^{f\mu}\right|^2 P_2^-(x)}{\varepsilon_f^2\lambda_4^2(1-x)\left(1-\varepsilon_f^{-2}x\right)+x} - \frac{4}{8\pi^2}\frac{m_\mu^2}{M_U^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{v3}^{f\mu}\right|^2 P_1^+(x) + \left|g_{a3}^{f\mu}\right|^2 P_1^-(x)}{(1-x)\left(1-\lambda_4^2 x\right)+\varepsilon_f^2\lambda_4^2 x},$$

where $\varepsilon_f \equiv \dfrac{m_f}{m_\mu}$, $\lambda_4 \equiv \dfrac{m_\mu}{M_U}$, and $g_{a3}^{f\mu}$ ($g_{v3}^{f\mu}$) are symmetric and anti-symmetric couplings in flavor space.

General expressions for Δa_μ

Neutral Scalar Mediator:

$$\Delta a_\mu(\phi) = \frac{1}{8\pi^2}\frac{m_\mu^2}{M_\phi^2}\int_0^1 \mathrm{d}x \sum_f \left[\frac{\left|g_{s1}^{f\mu}\right|^2 P_3^+(x) + \left|g_{p1}^{f\mu}\right|^2 P_3^-(x)}{(1-x)(1-x\lambda_3^2) + x\,\varepsilon_f^2\lambda_3^2}\right], \text{ with } P_3^\pm(x) = x^2\,(1-x\pm\varepsilon_f)\,,$$

with $g_{s1}^{f\mu}$ and $g_{p1}^{f\mu}$ being the scalar (s) and pseudo-scalar (p) matrices in flavor space, $\varepsilon_f \equiv \dfrac{m_f}{m_\mu}$ and $\lambda_3 \equiv \dfrac{m_\mu}{M_\phi}$.

Charged Scalar Mediator:

$$\Delta a_\mu\left(\phi^+\right) = \frac{-1}{8\pi^2}\frac{m_\mu^2}{M_{\phi^+}^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{s2}^{f\mu}\right|^2 P_4^+(x) + \left|g_{p2}^{f\mu}\right|^2 P_4^-(x)}{\varepsilon_f^2\lambda^2(1-x)\left(1-\varepsilon_f^{-2}x\right) + x},$$

where $P_4^\pm(x) = x(1-x)\,(x\pm\varepsilon_f)$, with $g_{s2}^{f\mu}$ and $g_{p2}^{f\mu}$ being the scalar (s) and pseudo-scalar (p) matrices in flavor space, $\varepsilon_f \equiv \dfrac{m_\nu}{m_\mu}$ and $\lambda \equiv \dfrac{m_\mu}{M_{\phi^+}}$.