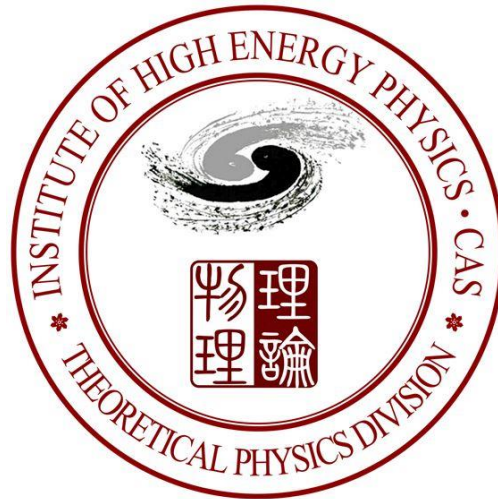


Radiative neutrino masses, lepton flavor mixing and muon $g - 2$ in a leptoquark model



Di Zhang (IHEP, Beijing)

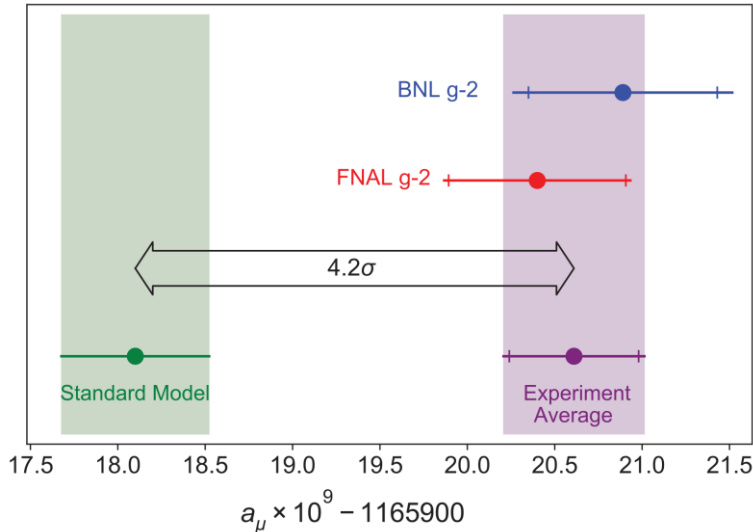
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Motivation

Charged-lepton sector:



SM prediction for muon g-2:

$$a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11} \quad \text{T. Aoyama et al, 2020}$$

The combined experimental average for muon g-2:

$$a_\mu^{\text{exp}} = 116592061(41) \times 10^{-11} \quad \text{G. W. Bennett et al, 2006; B. Abi et al, 2021}$$

The difference between the experimental value and SM prediction:

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251(59) \times 10^{-11} \quad 4.2\sigma$$

Neutrino sector:

Global-fit results

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	0.269 \rightarrow 0.343	$0.304^{+0.013}_{-0.012}$	0.269 \rightarrow 0.343
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	31.27 \rightarrow 35.86	$33.45^{+0.78}_{-0.75}$	31.27 \rightarrow 35.87
$\sin^2 \theta_{23}$	$0.570^{+0.018}_{-0.024}$	0.407 \rightarrow 0.618	$0.575^{+0.017}_{-0.021}$	0.411 \rightarrow 0.621
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	39.6 \rightarrow 51.8	$49.3^{+1.0}_{-1.2}$	39.9 \rightarrow 52.0
$\sin^2 \theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	0.02034 \rightarrow 0.02430	$0.02240^{+0.00062}_{-0.00062}$	0.02053 \rightarrow 0.02436
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	8.20 \rightarrow 8.97	$8.61^{+0.12}_{-0.12}$	8.24 \rightarrow 8.98
$\delta_{\text{CP}}/^\circ$	195^{+51}_{-25}	107 \rightarrow 403	286^{+27}_{-32}	192 \rightarrow 360
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	6.82 \rightarrow 8.04	$7.42^{+0.21}_{-0.20}$	6.82 \rightarrow 8.04
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.514^{+0.028}_{-0.027}$	+2.431 \rightarrow +2.598	$-2.497^{+0.028}_{-0.028}$	-2.583 \rightarrow -2.412

I. Esteban et al, 2020; P. F. de Salas et al, 2020

Beyond the SM

The upper limit on the mass of the lightest anti-neutrino mass.

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2} < 0.8 \text{ eV} \quad \text{M. Aker et al, 2021}$$

Two messages:

- Neutrinos are **massive** but their masses are **tiny**
- **Lepton flavor mixing** exists

Motivation

Anomaly of muon g-2

New interactions for **charged leptons**



Neutrino masses

New interactions for **neutrinos**

Is it possible to connect these two issues?

➤ (Type-I, II, III) Seesaw mechanisms: simple and natural

A. Abada et al, 2007; Z. Z. Xing, S. Zhou, 2010

$$\frac{\mathcal{L}_{d=5}}{\Lambda} = \begin{cases} \frac{1}{2} (Y_\nu M_R^{-1} Y_\nu^T)_{\alpha\beta} \bar{\ell}_{\alpha L} \tilde{H} \tilde{H}^T \ell_{\beta L}^c + \text{h.c.} & \text{(Type I)}, \\ -\frac{\lambda_\Delta}{M_\Delta} (Y_\Delta)_{\alpha\beta} \bar{\ell}_{\alpha L} \tilde{H} \tilde{H}^T \ell_{\beta L}^c + \text{h.c.} & \text{(Type II)}, \\ \frac{1}{2} (Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T)_{\alpha\beta} \bar{\ell}_{\alpha L} \tilde{H} \tilde{H}^T \ell_{\beta L}^c + \text{h.c.} & \text{(Type III)}. \end{cases}$$



$$M_\nu = \begin{cases} -\frac{1}{2} Y_\nu \frac{v^2}{M_R} Y_\nu^T & \text{(Type I)}, \\ \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta} & \text{(Type II)}, \\ -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T & \text{(Type III)}. \end{cases}$$

But they lead to a **small and/or negative** Δa_μ

C. Biggio, 2008; S. Zhou, 2021



Anomaly of muon g-2

➤ Radiative neutrino masses: generating neutrino masses at loop level

Y. Cai et al, 2017;
K. S. Babu et al, 2019

One kind of radiative neutrino mass models: extending the SM with **leptoquarks (LQs)**

- Scalar LQ $\tilde{R}_2(3, 2, 1/6)$ with either $S_1(\bar{3}, 1, 1/3)$ or $S_3(\bar{3}, 3, 1/3)$ can generate neutrino masses at the **one-loop level**
C. K. Chua, X. G. He, W.-Y. P. Hwang, 1999; U. Mahanta, 2000; D. A. Sierra, M. Hirsch, S. G. Kovalenko, 2007; I. Dorsner, S. Fajfer, O Sumensari, 2019
- Single scalar LQ $S_1(\bar{3}, 1, 1/3)$ or $R_2(3, 2, 7/6)$ with both the left-handed and right-handed chiral couplings can account for Δa_μ
K. M. Cheung, 2001; S. Saad, 2020

$\tilde{R}_2(3, 2, 1/6)$ and $S_1(\bar{3}, 1, 1/3)$ may give a **combined explanation** of **neutrino masses**, **lepton flavor mixing** and the anomaly of **muon anomalous magnetic moment**

The Leptoquark Model

Enforcing baryon number conservation, the Lagrangian with two **scalar LQs** S_1 and \tilde{R}_2 is

$$\mathcal{L}_{\text{LQ}} = \lambda_{i\alpha}^L \overline{Q_{iL}^c} \epsilon \ell_{\alpha L} S_1 + \lambda_{i\alpha}^R \overline{u_{iR}^c} E_{\alpha R} S_1 + \lambda_{i\alpha} \overline{d_{iR}} \tilde{R}_2^T \epsilon \ell_{\alpha L} + \text{h.c.} \\ + (D_\mu S_1)^\dagger (D^\mu S_1) + (D_\mu \tilde{R}_2)^\dagger (D^\mu \tilde{R}_2) - V_{\text{LQ}},$$

$$\tilde{R}_2 = \begin{pmatrix} \tilde{R}_2^{+\frac{2}{3}} \\ \tilde{R}_2^{-\frac{1}{3}} \end{pmatrix}$$

where the scalar potential is given by

$$V_{\text{LQ}} = \mu_S^2 S_1^\dagger S_1 + \mu_R^2 \tilde{R}_2^\dagger \tilde{R}_2 + \left(\lambda_{\text{mix}} S_1^* \tilde{R}_2^\dagger H + \text{h.c.} \right) + \lambda_{HS} (H^\dagger H) (S_1^\dagger S_1) + \lambda_{HR}^{(1)} (H^\dagger H) (\tilde{R}_2^\dagger \tilde{R}_2) \\ + \lambda_{HR}^{(3)} (H^\dagger \tau^I H) (\tilde{R}_2^\dagger \tau^I \tilde{R}_2) + \lambda_S (S_1^\dagger S_1)^2 + \lambda_R^{(1)} (\tilde{R}_2^\dagger \tilde{R}_2)^2 + \lambda_R^{(8)} (\tilde{R}_2^\dagger T^A \tilde{R}_2)^2 \\ + \lambda_{SR}^{(1)} (S_1^\dagger S_1) (\tilde{R}_2^\dagger \tilde{R}_2) + \lambda_{SR}^{(8)} (S_1^\dagger T^A S_1) (\tilde{R}_2^\dagger T^A \tilde{R}_2),$$

- The scalar LQ S_1 has both the **left-handed and right-handed** couplings with quarks and leptons, which can **chirally enhance** the contributions to muon g-2
- The LQ-Higgs interaction $\lambda_{\text{mix}} S_1^* \tilde{R}_2^\dagger H$ plays an important role in **radiatively generating** neutrino masses, and has **some effects** on the contributions from LQs to muon g-2

After spontaneous gauge symmetry breaking, the mass matrix for S_1 and $\tilde{R}_2^{-\frac{1}{3}*}$ is found to be

$$M_{\text{mix}}^2 = \begin{pmatrix} m_S^2 & \frac{v}{\sqrt{2}} \lambda_{\text{mix}} \\ \frac{v}{\sqrt{2}} \lambda_{\text{mix}}^* & m_R^2 \end{pmatrix} \quad \text{with} \quad m_S^2 = \mu_S^2 + \frac{v^2}{2} \lambda_{HS}, \quad m_R^2 = \mu_R^2 + \frac{v^2}{2} (\lambda_{HR}^{(1)} + \lambda_{HR}^{(3)})$$

which is not diagonal leading to a **mixing** between S_1 and $\tilde{R}_2^{-\frac{1}{3}*}$, and we assume that all coupling constants involved in above mass matrix are **real**

The Leptoquark Model

To work in the basis of the physical LQs, one can make following transformation

$$\begin{pmatrix} S_1 \\ \tilde{R}_2^{-\frac{1}{3}*} \end{pmatrix} \rightarrow \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} S_1 \\ \tilde{R}_2^{-\frac{1}{3}*} \end{pmatrix} \quad \text{with} \quad \tan 2\theta = \frac{\sqrt{2}\lambda_{\text{mix}}v}{m_R^2 - m_S^2}$$

Mixing angle

➤ The masses of the physical LQs are given by

$$M_{1,2}^2 = \frac{1}{2} \left[m_S^2 + m_R^2 \pm \sqrt{(m_S^2 - m_R^2)^2 + 2\lambda_{\text{mix}}^2 v^2} \right], \quad M_3^2 = \mu_R^2 + \frac{v^2}{2} \left(\lambda_{HR}^{(1)} - \lambda_{HR}^{(3)} \right)$$

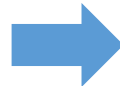
where M_1 , M_2 and M_3 are the masses of physical S_1 , $\tilde{R}_2^{-\frac{1}{3}}$ and $\tilde{R}_2^{+\frac{2}{3}}$, respectively

➤ The Yukawa coupling interactions involving the physical LQs after SSB are

$$\begin{aligned} \mathcal{L}_Y = & \bar{\nu}_\alpha \left(\lambda_{i\alpha}^* \sin \theta P_R - \lambda_{i\alpha}^L \cos \theta P_L \right) d_i S_1 + \bar{l}_\alpha \left(\lambda_{i\alpha}^L P_L + \lambda_{i\alpha}^R P_R \right) \cos \theta u_i S_1 \\ & - \bar{\nu}_\alpha \left(\lambda_{i\alpha}^* \cos \theta P_R + \lambda_{i\alpha}^L \sin \theta P_L \right) d_i \tilde{R}_2^{-\frac{1}{3}*} + \bar{l}_\alpha \left(\lambda_{i\alpha}^L P_L + \lambda_{i\alpha}^R P_R \right) \sin \theta u_i \tilde{R}_2^{-\frac{1}{3}*} \\ & + \lambda_{i\alpha} \bar{d}_i P_L l_\alpha \tilde{R}_2^{+\frac{2}{3}} + \text{h.c.}, \end{aligned}$$

in which $\lambda^{L} = V^T \lambda^L$ with V being the CKM matrix

Similar interactions for S_1 and $\tilde{R}_2^{-\frac{1}{3}}$



Similar contributions from S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ to neutrino masses and muon g-2

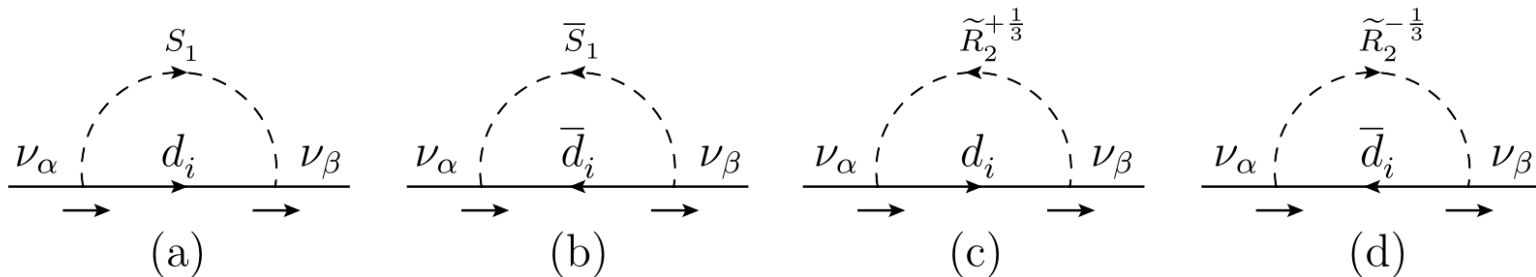
Radiative Neutrino Masses

No right-handed neutrinos in this LQ model



Neutrinos are massless at tree level

Neutrino masses are generated at **one-loop level** in the present LQ model



Calculating above self-energy diagrams, one can obtain the radiative neutrino mass matrix:

$$(M_\nu)_{\alpha\beta} = -\frac{N_c}{2(4\pi)^2} \left(\lambda_{i\alpha}^* \lambda_{i\beta}^{L*} + \lambda_{i\alpha}^{L*} \lambda_{i\beta}^* \right) \hat{m}_i \sin 2\theta \begin{bmatrix} M_2^2 \ln \frac{\hat{m}_i^2}{M_2^2} & -M_1^2 \ln \frac{\hat{m}_i^2}{M_1^2} \\ M_2^2 - \hat{m}_i^2 & -M_1^2 - \hat{m}_i^2 \end{bmatrix} \quad N_c = 3$$

$$\hat{m} = (m_d, m_s, m_b)$$

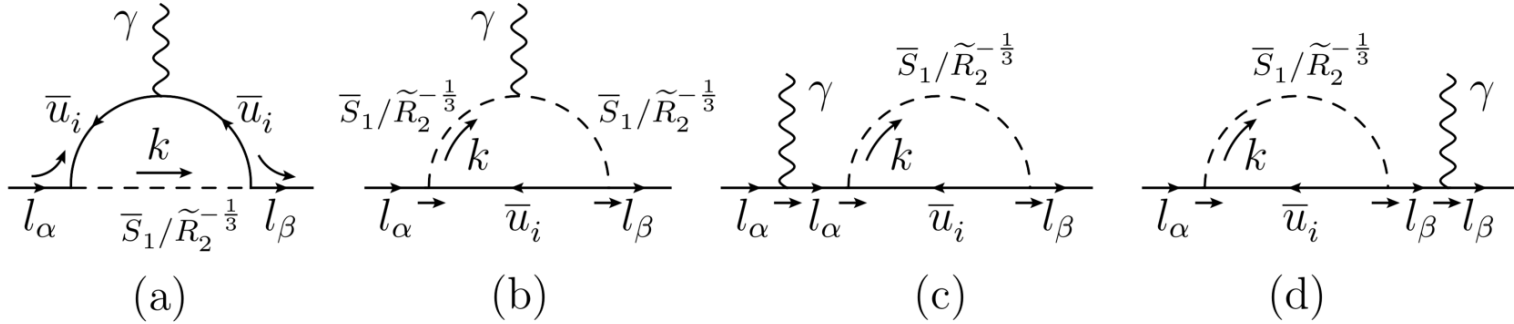
$$\simeq \frac{3 \sin 2\theta}{32\pi^2} \ln \frac{M_2^2}{M_1^2} \left[(\lambda^\dagger)_{\alpha i} \hat{m}_i (\lambda^{L*})_{i\beta} + (\lambda^{L\dagger})_{\alpha i} \hat{m}_i (\lambda^*)_{i\beta} \right], \quad \hat{m}_i \ll M_{1,2}$$

- Only coupling matrices λ and λ^L are involved, λ^R does not contribute
- The **mixing** between S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ is important to radiatively generate neutrino masses
- If the masses of the physical S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ are exactly degenerate, neutrinos will remain massless at one-loop level

$$\theta = 0 \text{ or } M_1 = M_2 \quad \longrightarrow \quad M_\nu = 0$$

Muon anomalous magnetic moment

Contributions from S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ to muon g-2 or radiative decays of charged leptons:



Calculating above diagrams, the total amplitude mediated by S_1 is:

$$i\mathcal{M}^{S_1} = -\frac{ie}{(4\pi)^2} \epsilon_\mu^*(q) \bar{u}(p-q) \left(\mathcal{A}_L^{S_1} P_L + \mathcal{A}_R^{S_1} P_R \right) i\sigma^{\mu\nu} q_\nu u(p) \quad \begin{array}{l} \tilde{m} = (m_u, m_c, m_t) \\ m = (m_e, m_\mu, m_\tau) \end{array}$$

$$\mathcal{A}_L^{S_1} = \frac{N_c \cos^2 \theta}{12M_1^2} \left[2\tilde{m}_i \lambda_{i\alpha}^{L*} \lambda_{i\beta}^{R*} \mathcal{F} \left(\frac{\tilde{m}_i^2}{M_1^2} \right) - (m_\beta \lambda_{i\alpha}^{L*} \lambda_{i\beta}^{L*} + m_\alpha \lambda_{i\alpha}^{R*} \lambda_{i\beta}^{R*}) \mathcal{G} \left(\frac{\tilde{m}_i^2}{M_1^2} \right) \right] \quad \mathcal{F}(x) = \frac{7 - 8x + x^2 + 2(2+x) \ln x}{(1-x)^3}$$

$$\mathcal{A}_R^{S_1} = \frac{N_c \cos^2 \theta}{12M_1^2} \left[2\tilde{m}_i \lambda_{i\alpha}^R \lambda_{i\beta}^{L*} \mathcal{F} \left(\frac{\tilde{m}_i^2}{M_1^2} \right) - (m_\alpha \lambda_{i\alpha}^{L*} \lambda_{i\beta}^{L*} + m_\beta \lambda_{i\alpha}^R \lambda_{i\beta}^{R*}) \mathcal{G} \left(\frac{\tilde{m}_i^2}{M_1^2} \right) \right] \quad \mathcal{G}(x) = \frac{1 + 4x - 5x^2 + 2x(2+x) \ln x}{(1-x)^4}$$

The total amplitude mediated by $\tilde{R}_2^{-\frac{1}{3}}$ can be obtained by making following replacements:

$$\cos^2 \theta \rightarrow \sin^2 \theta \quad \text{and} \quad M_1 \rightarrow M_2$$

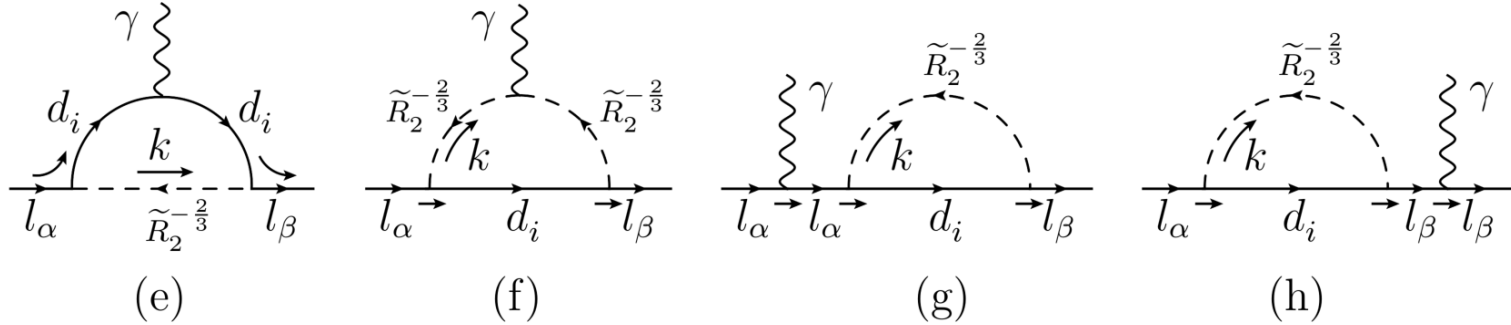


$$i\mathcal{M}^{\tilde{R}_2^{-\frac{1}{3}}} = i\mathcal{M}^{S_1}(M_1 \rightarrow M_2) \tan^2 \theta$$

All divergences are canceled out

Muon anomalous magnetic moment

Contributions from $\tilde{R}_2^{+\frac{2}{3}}$ to muon g-2 or radiative decays of charged leptons :



Calculating above diagrams, the total amplitude mediated by $\tilde{R}_2^{+\frac{2}{3}}$ is:

$$i\mathcal{M}^{\tilde{R}_2^{+\frac{2}{3}}} = -\frac{ie}{(4\pi)^2} \epsilon_\mu^*(q) \bar{u}(p-q) \left(\mathcal{A}_L^{\tilde{R}_2^{+\frac{2}{3}}} P_L + \mathcal{A}_R^{\tilde{R}_2^{+\frac{2}{3}}} P_R \right) i\sigma^{\mu\nu} q_\nu u(p)$$

$$\mathcal{A}_L^{\tilde{R}_2^{+\frac{2}{3}}} = -\frac{N_c m_\beta}{12M_3^2} \lambda_{i\alpha} \lambda_{i\beta}^* \mathcal{I} \left(\frac{\hat{m}_i^2}{M_3^2} \right), \quad \mathcal{A}_R^{\tilde{R}_2^{+\frac{2}{3}}} = -\frac{N_c m_\alpha}{12M_3^2} \lambda_{i\alpha} \lambda_{i\beta}^* \mathcal{I} \left(\frac{\hat{m}_i^2}{M_3^2} \right) \quad \mathcal{I}(x) = \frac{x [5 - 4x - x^2 + (2 + 4x) \ln x]}{(1-x)^4}$$

All divergences are canceled out

The branching ratios of the radiative decays of charged leptons:

$$\mathcal{B}(l_\alpha^- \rightarrow l_\beta^- + \gamma) = \frac{\alpha_{\text{em}} (m_\alpha^2 - m_\beta^2)^3}{4(4\pi)^4 m_\alpha^3 \Gamma_\alpha} \left(\left| \mathcal{A}_L^{S_1} + \mathcal{A}_L^{\tilde{R}_2^{-\frac{1}{3}}} + \mathcal{A}_L^{\tilde{R}_2^{+\frac{2}{3}}} \right|^2 + \left| \mathcal{A}_R^{S_1} + \mathcal{A}_R^{\tilde{R}_2^{-\frac{1}{3}}} + \mathcal{A}_R^{\tilde{R}_2^{+\frac{2}{3}}} \right|^2 \right)$$

➔ Strong constraints on the Yukawa couplings, especially that from $\tau^- \rightarrow \mu^- + \gamma$

Muon anomalous magnetic moment

Muon anomalous magnetic moment in the LQ model:

$$\Delta a_\mu = \Delta a_\mu^{S_1} + \Delta a_\mu^{\tilde{R}_2^{-\frac{1}{3}}} + \Delta a_\mu^{\tilde{R}_2^{+\frac{2}{3}}}$$

$$\Delta a_\mu^{S_1} = -\frac{N_c m_\mu \cos^2 \theta}{6 (4\pi)^2 M_1^2} \left[\frac{2\tilde{m}_i \text{Re}(\lambda_{i\mu}^L \lambda_{i\mu}^{R*})}{M_1^2} \mathcal{F}\left(\frac{\tilde{m}_i^2}{M_1^2}\right) - m_\mu \left(|\lambda_{i\mu}^L|^2 + |\lambda_{i\mu}^R|^2 \right) \mathcal{G}\left(\frac{\tilde{m}_i^2}{M_1^2}\right) \right],$$

$$\Delta a_\mu^{\tilde{R}_2^{-\frac{1}{3}}} = -\frac{N_c m_\mu \sin^2 \theta}{6 (4\pi)^2 M_2^2} \left[\frac{2\tilde{m}_i \text{Re}(\lambda_{i\mu}^L \lambda_{i\mu}^{R*})}{M_2^2} \mathcal{F}\left(\frac{\tilde{m}_i^2}{M_2^2}\right) - m_\mu \left(|\lambda_{i\mu}^L|^2 + |\lambda_{i\mu}^R|^2 \right) \mathcal{G}\left(\frac{\tilde{m}_i^2}{M_2^2}\right) \right],$$

$$\Delta a_\mu^{\tilde{R}_2^{+\frac{2}{3}}} = \frac{N_c m_\mu^2 |\lambda_{i\mu}|^2}{6 (4\pi)^2 M_3^2} \mathcal{I}\left(\frac{\hat{m}_i^2}{M_3^2}\right).$$

$$\tilde{m} = (m_u, m_c, m_t) \quad \hat{m} = (m_d, m_s, m_b)$$

$$\tilde{m}_i, \hat{m}_j \ll M_k \quad \longrightarrow \quad \mathcal{F}\left(\frac{\tilde{m}_i^2}{M_{1,2}^2}\right) \simeq 7 + 4 \ln \frac{\tilde{m}_i^2}{M_{1,2}^2}, \quad \mathcal{G}\left(\frac{\tilde{m}_i^2}{M_{1,2}^2}\right) \simeq 1, \quad \mathcal{I}\left(\frac{\hat{m}_i^2}{M_3^2}\right) \simeq \frac{\hat{m}_i^2}{M_3^2} \left(5 + 2 \ln \frac{\hat{m}_i^2}{M_3^2} \right)$$

$$\Delta a_\mu \simeq \frac{4m_\mu \tilde{m}_i}{(4\pi)^2} \text{Re}(\lambda_{i\mu}^L \lambda_{i\mu}^{R*}) \left[\frac{\cos^2 \theta}{M_1^2} \left(\ln \frac{M_1^2}{\tilde{m}_i^2} - \frac{7}{4} \right) + \frac{\sin^2 \theta}{M_2^2} \left(\ln \frac{M_2^2}{\tilde{m}_i^2} - \frac{7}{4} \right) \right]$$

- Contributions from S_1 and $\tilde{R}_2^{-\frac{1}{3}}$ are **enhanced by masses of up-type quarks**
- Contribution from $\tilde{R}_2^{+\frac{2}{3}}$ is **strongly suppressed by M_3^{-4}** , thus is usually omitted
- Coupling matrices λ^L , λ^R and λ are involved, but effects of λ are suppressed
- If $\theta = 0$ or $M_1 = M_2$, this result for muon g-2 will recover the results caused by S_1 all alone

Textures for Yukawa Coupling Matrices

Assuming following textures for the Yukawa coupling matrices:

$$\lambda = \begin{pmatrix} 0 & 0 & 0 \\ b(1+\varepsilon) & 0 & c(1+\varepsilon) \\ \frac{m_s}{m_b} b^* & \frac{m_s}{m_b} c^* & d \end{pmatrix}, \quad \lambda^L = a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda^R = a \begin{pmatrix} 0 & 0 & 0 \\ 0 & x & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Breaks μ - τ symmetry

Neutrino masses

Muon g-2

Suppresses $\tau^- \rightarrow \mu^- \gamma$

● Neutrino masses and flavor mixing

$$M_\nu = \frac{3 \sin 2\theta}{32\pi^2} \ln \left(\frac{M_2^2}{M_1^2} \right) a m_b \begin{pmatrix} 0 & \frac{m_s}{m_b} b & \frac{m_s}{m_b} b^* (1+\varepsilon) \\ \frac{m_s}{m_b} b & 2 \frac{m_s}{m_b} c & d \\ \frac{m_s}{m_b} b^* (1+\varepsilon) & d & 2 \frac{m_s}{m_b} c^* (1+\varepsilon) \end{pmatrix}$$

$\triangleright (M_\nu)_{ee} = 0 \rightarrow$ **NMO**

$\triangleright \begin{cases} m_1 \sim 2.2 \text{ meV for } (\rho, \sigma) \sim (\pi/2, 0) \\ m_1 \sim 6.3 \text{ meV for } (\rho, \sigma) \sim (0, \pi/2) \end{cases}$

$\triangleright \theta_{23} \sim \frac{\pi}{4} - \frac{\varepsilon}{4}$,

$\triangleright \delta_{\text{CP}} \sim \pm \frac{\pi}{2} \pm \frac{\varepsilon [1 - r_{12} - (1 + r_{12}) \cos 2\theta_{12}] \cot 2\theta_{12}}{4(1 - r_{12})\theta_{13}}$

Approximate μ - τ reflection symmetry ($\varepsilon \ll 1$)

$\triangleright \text{Tr}(M_\nu M_\nu^\dagger) \rightarrow r \sim \frac{16\pi^2}{3am_b} \sqrt{\frac{1}{2d^2} (3m_1^2 + \Delta m_{21}^2 + \Delta m_{31}^2)} \sim 8 \times 10^{-7} \leftarrow a \sim 0.085 \quad d \sim \mathcal{O}(0.01) \quad m_1 \sim \mathcal{O}(0.01 \text{ eV})$

● Muon g-2

$M_1 = M_2/(1+r) = M_3 = 2 \text{ TeV}, \quad \theta = \pi/4 \text{ with } r \equiv (M_2 - M_1)/M_1$

$$\Delta a_\mu \simeq \frac{m_\mu m_t a^2}{4\pi^2 M_1^2} \text{Re} \left(V_{tb} - \frac{V_{cb} V_{ts}}{V_{cs}} \right) \left(\ln \frac{M_1^2}{m_t^2} - \frac{7}{4} \right) \sim 251 \times 10^{-11} \left(\frac{a}{0.085} \right)^2 \leftarrow x = - \frac{m_t V_{cb}^* \left(7 + \ln \frac{m_t^2}{M_1^2} \right)}{m_c V_{cs}^* \left(7 + \ln \frac{m_c^2}{M_1^2} \right)}$$

$a \sim 0.085$ can accommodate the experimental value of Δa_μ

Numerical Calculations

Assuming $M_1 = M_2/(1+r) = M_3 = 2 \text{ TeV}$, $\theta = \pi/4$ with $r \equiv (M_2 - M_1)/M_1$

➔ **Eight parameters are left:** $\{r, a, \text{Re}(b), \text{Im}(b), \text{Re}(c), \text{Im}(c), d, \varepsilon\}$

- Accommodate the experimental values of $\{\Delta m_{21}^2, \Delta m_{31}^2, \sin^2 \theta_{12}, \sin^2 \theta_{13}, \sin^2 \theta_{23}, \Delta a_\mu\}$
- Predict the values of $\{m_1, \delta_{\text{CP}}, \rho, \sigma\}$
- Satisfy the constraints from the radiative decays of charged leptons

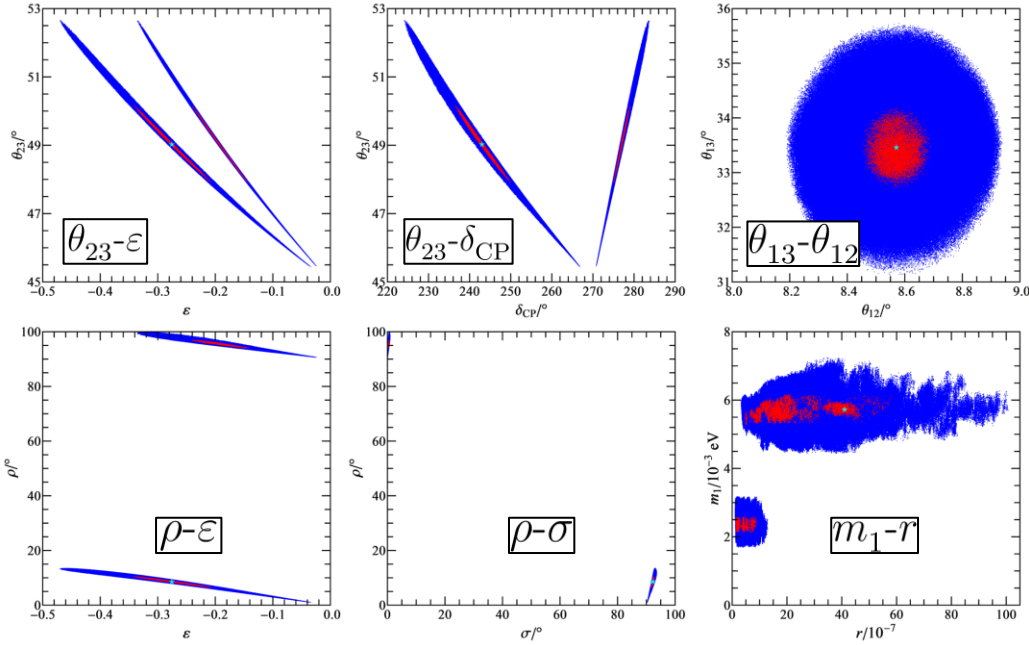
Best-fit values of the model parameters and predicted observables:

$$a = 8.37 \times 10^{-2}, d = 1.47 \times 10^{-3}, \text{Re}(b) = -4.73 \times 10^{-4}, \text{Im}(b) = 2.46 \times 10^{-2}$$
$$\text{Re}(c) = 2.80 \times 10^{-4}, \text{Im}(c) = 3.67 \times 10^{-2}, r = 4.10 \times 10^{-6}, \varepsilon = -0.276$$

$$m_1 = 5.73 \text{ meV}, \Delta m_{21}^2 = 7.42 \times 10^{-5} \text{ eV}^2, \Delta m_{31}^2 = 2.51 \times 10^{-3} \text{ eV}^2$$
$$\theta_{12} = 33.46^\circ, \theta_{13} = 8.57^\circ, \theta_{23} = 49.03^\circ$$
$$\delta_{\text{CP}} = 242.99^\circ, \rho = 8.58^\circ, \sigma = 92.27^\circ$$
$$\Delta a_\mu = 251.10 \times 10^{-11}, \mathcal{B}(\tau^- \rightarrow \mu^- + \gamma) = 1.17 \times 10^{-18}$$

Successfully give a combined explanation of neutrino masses, lepton flavor mixing and the muon anomalous magnetic moment

Numerical Calculations

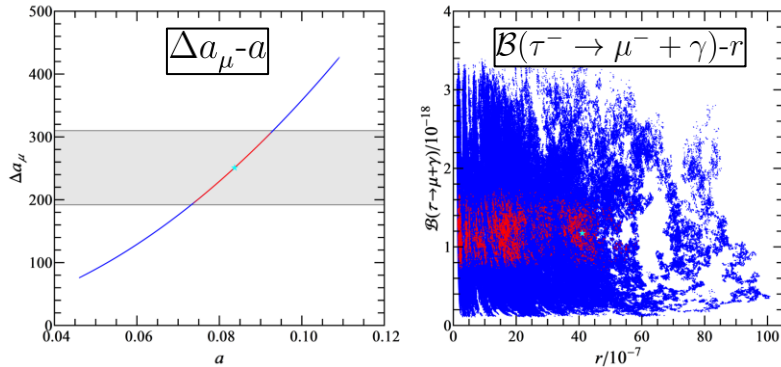


$$\theta_{23} \sim \frac{\pi}{4} - \frac{\epsilon}{4}$$

$$\delta_{\text{CP}} \sim -\frac{\pi}{2} - \frac{\epsilon [1 - r_{12} - (1 + r_{12}) \cos 2\theta_{12}] \cot 2\theta_{12}}{4(1 - r_{12})\theta_{13}}$$

$$\begin{cases} m_1 \sim 2.2 \text{ meV for } (\rho, \sigma) \sim (\pi/2, 0) \\ m_1 \sim 6.3 \text{ meV for } (\rho, \sigma) \sim (0, \pi/2) \end{cases}$$

$$r \sim \frac{16\pi^2}{3am_b} \sqrt{\frac{1}{2d^2} (3m_1^2 + \Delta m_{21}^2 + \Delta m_{31}^2)}$$



$$\Delta a_\mu \simeq \frac{m_\mu m_t a^2}{4\pi^2 M_1^2} \text{Re} \left(V_{\text{tb}} - \frac{V_{\text{cb}} V_{\text{ts}}}{V_{\text{cs}}} \right) \left(\ln \frac{M_1^2}{m_t^2} - \frac{7}{4} \right) \sim 251 \times 10^{-11} \left(\frac{a}{0.085} \right)^2$$

$$\mathcal{B}(\tau^- \rightarrow \mu^- + \gamma) \simeq \frac{\alpha_{\text{em}} m_\tau^3 m_t^2 |V_{\text{cb}}|^2 a^4 r^2}{16\pi^4 M_1^4 \Gamma_\tau} \left(\frac{\ln \frac{m_t^2}{m_c^2}}{7 + 4 \ln \frac{m_c^2}{M_1^2}} \right)^2 \sim 1 \times 10^{-6} a^4 r^2$$

See **DZ, JHEP 07 (2021) 069** for more details

Summary

- The LQ-Higgs interaction plays a greatly important role in generating neutrino masses at one-loop level
- The mixing between S_1 and $\tilde{R}_2^{-\frac{1}{3}*}$ has significant effects on the contributions from the LQs to muon g-2
- With specific textures for the LQ Yukawa coupling matrices, the neutrino mass matrix possesses an approximate μ - τ reflection symmetry with $(M_\nu)_{ee} = 0$
- The present LQ model can successfully explain neutrino masses, lepton flavor mixing and the anomaly of muon g-2

THANKS FOR YOUR ATTENTION