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# The Radiative SUSY Seesaw Mechanism

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*Based on P. Candia, A. Pilaftsis, Phys. Rev. D 102, 095013 (2020)*

# Seesaw mechanism and neutrino masses

- Neutrino masses and mixings require physics beyond the SM
- Majorana neutrino masses are appealing
- The lowest order effective operator

$$\mathcal{L}^{d=5} = \frac{c_{ij}}{\Lambda} (\bar{L}_i^c \tilde{\Phi}^*) (\tilde{\Phi}^\dagger L_j) + \text{H. c.}$$

- Type I, II and III seesaws and hybrid proposals

# Seesaw mechanism and neutrino masses

Minkowski '77; Yanagida '79; Gell-Mann, Ramond, Slansky '79; Mohapatra, Senjanovic '80

- Type I seesaw

$$\mathbf{m}_\nu = -\mathbf{m}_D \mathbf{m}_M^{-1} \mathbf{m}_D^T$$

- In principle, requires small Yukawa couplings or very massive neutrinos
- **More optimistic:** Inverse seesaw scenarios. Lepton number is an approximate symmetry, Yukawas can be big and RHN masses lower

# General inverse seesaw

Mohapatra, Valle '86; Dev, Pilaftsis '12

- Lagrangian:

$$-\mathcal{L}_Y = \bar{L}^C Y_\nu \tilde{\Phi} \nu_R^C + \bar{S}_L^C M_N \nu_R^C + \frac{1}{2} \bar{\nu}_R \mu_R \nu_R^C + \frac{1}{2} \bar{S}_L^C \mu_S S_L + \text{H.c.}$$

- The particle content of the SM is extended with the fields  $\nu_R, S_L^C$
- The former has LN +1, while the second -1
- LNV occurs softly via the matrices  $\mu_{R,S}$

# General inverse seesaw

- Mass matrix:

$$-\mathcal{L}_{\text{mass}}^{\nu} = \frac{1}{2} \left( \bar{\nu}_L^C, \bar{\nu}_R, \bar{S}_L^C \right) \begin{pmatrix} \mathbf{0}_3 & \mathbf{M}_D & \mathbf{0}_{3 \times n} \\ \mathbf{M}_D^{\text{T}} & \boldsymbol{\mu}_R & \mathbf{M}_N^{\text{T}} \\ \mathbf{0}_{n \times 3} & \mathbf{M}_N & \boldsymbol{\mu}_S \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^C \\ S_L \end{pmatrix} + \text{H.c.}$$

# General inverse seesaw

- Mass matrix:

$$-\mathcal{L}_{\text{mass}}^{\nu} = \frac{1}{2} \left( \bar{\nu}_L^C, \bar{\nu}_R, \bar{S}_L^C \right) \begin{pmatrix} \mathbf{0}_3 & \mathbf{M}_D & \mathbf{0}_{3 \times n} \\ \mathbf{M}_D^T & \mu_R & \mathbf{M}_N^T \\ \mathbf{0}_{n \times 3} & \mathbf{M}_N & \mu_S \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^C \\ S_L \end{pmatrix} + \text{H.c.}$$

Dev, Pilaftsis '12

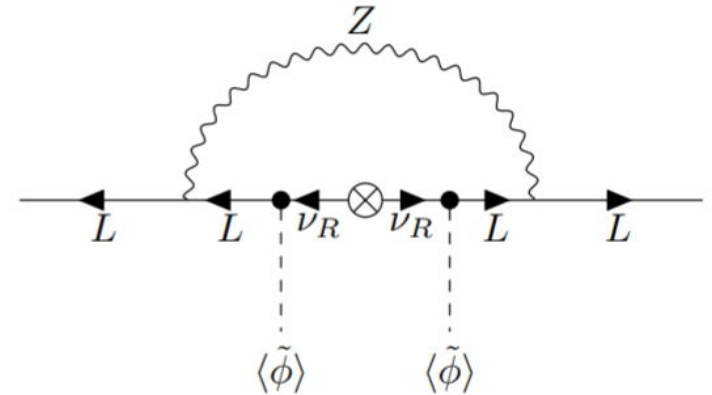
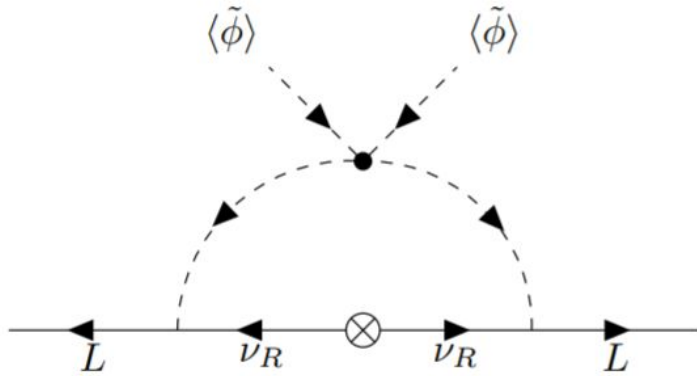
**Tree-level mass (for  $\mu_S = 0$ )  
Vanishes**

Mohapatra, Valle '86

**Tree-level mass (for  $\mu_R = 0$ ) is**

$$\mathbf{m}_{\nu} = -M_D M_N^{-1} \mu_S M_N^{-1T} M_D^T + \mathcal{O}(\mu_S^3)$$

# One-loop neutrino masses



Pilaftsis '92  
 Grimus '02  
 Ma '06  
 Dev, Pilaftsis '12

...

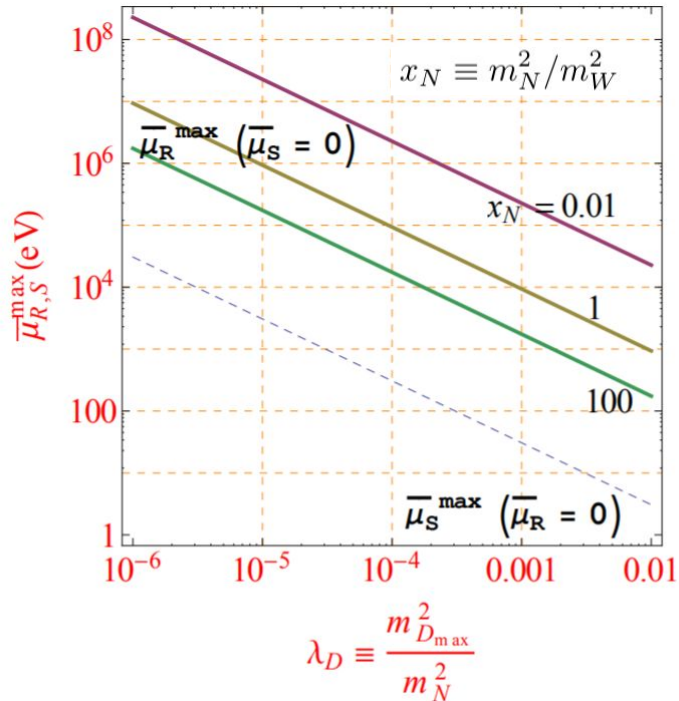
Flavour space calculation leads to the formula:

$$\Delta \mathbf{m}^\nu = \frac{\alpha_W}{16M_W^2} \mathcal{M}_D \mathcal{M}_S \left[ \frac{m_H^2}{\mathcal{M}_S^2 - m_H^2 \mathbf{1}_n} \ln \frac{\mathcal{M}_S^2}{m_H^2} + \frac{3M_Z^2}{\mathcal{M}_S^2 - M_Z^2 \mathbf{1}_n} \ln \frac{\mathcal{M}_S^2}{M_Z^2} \right] \mathcal{M}_D^T$$

$$\mathcal{M}_S = \begin{pmatrix} \mu_R & M_N^T \\ M_N & 0 \end{pmatrix}$$

$$\mathcal{M}_D = (M_D, \mathbf{0})$$

# One-loop neutrino masses



Dev, Pilaftsis 1209.4051

- The results show that LNV is less constrained by neutrino masses in the radiative scenario relative to the standard inverse seesaw
- For the same mixing, bigger values for the LNV parameter can be obtained for the case  $\mu_S = 0$ ,  $\mu_R$  non zero due to loop suppression



# Supersymmetric seesaw

- Superpotential

$$W = W_{\text{MSSM}} + \hat{L} i \sigma_2 \hat{H}_u \mathbf{Y}_\nu \hat{N}^C + \frac{1}{2} \hat{N}^C \mathbf{m}_M \hat{N}^C$$

- The new superfield has RH neutrinos  $\nu_R$  and RH sneutrinos  $\tilde{\nu}_R$
- Extra parameters from soft SUSY breaking

$$-\mathcal{L}_{\text{soft}} = -\mathcal{L}_{\text{soft}}^{\text{MSSM}} + \tilde{\nu}_R^* \mathbf{m}_{\tilde{\nu}}^2 \tilde{\nu}_R + \left( \tilde{\nu}_R^* \mathbf{b}_\nu \mathbf{m}_M \tilde{\nu}_R + \tilde{L}^\top i \sigma_2 H_u \mathbf{Y}_\nu \mathbf{A}_\nu \tilde{\nu}_R^* + \text{H.c.} \right)$$

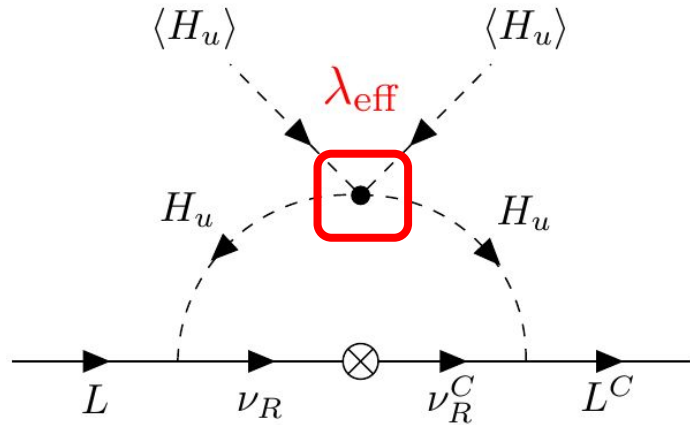
- We assume:  $\mathbf{m}_{\tilde{\nu}}^2 = m_{\tilde{\nu}}^2 \mathbf{1}_{n_R}$ ,  $\mathbf{A}_\nu = A_\nu \mathbf{1}_{n_R}$ ,  $\mathbf{b}_\nu = b_\nu \mathbf{1}_{n_R}$

# Supersymmetric seesaw

- The tree-level seesaw neutrino mass matrix is not modified by SUSY
- Radiative corrections **must vanish in the supersymmetric limit**  
•  $\tan \beta = 1$ ,  $\mu = 0$  and  $M_{\text{SUSY}} \rightarrow 0$  M. Grisar, W. Siegel, M. Rocek '79
- This is due to the non-renormalisation of the superpotential
- Radiative neutrino mass scenarios are of particular interest since **LNV observables could be less constrained by neutrino masses**

# Supersymmetric seesaw

- The SM-like diagrams now involve the Z and the up-type Higgs  $H_u$
- Its quartic coupling receives sizeable corrections from stop loops to raise the predicted Higgs mass from  $m_h \leq M_Z$  to  $m_h \simeq 125$  GeV

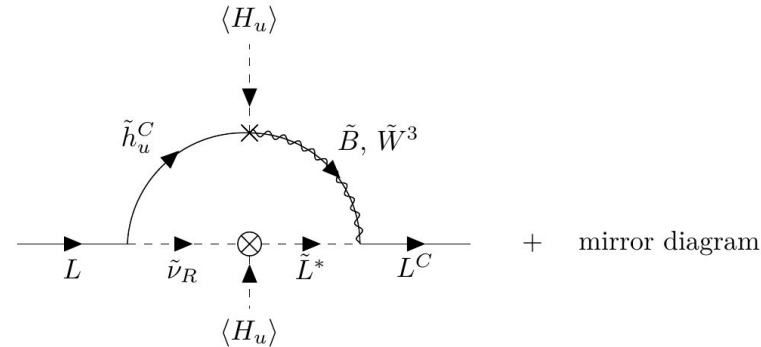
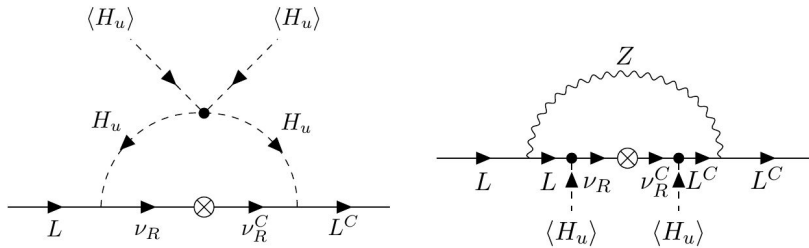


Ellis, Ridolfi, Zwirner '91  
Haber, Hempfling '91  
Okada, Yamaguchi, Yanagida '91

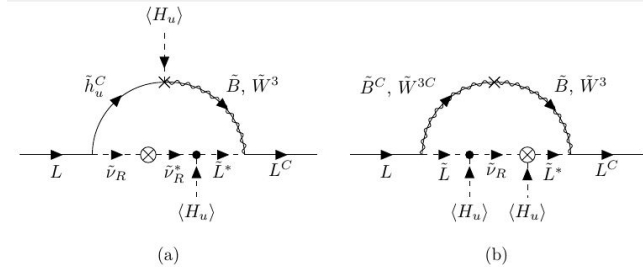
- We assume a simplified approach where we promote the tree-level quartic coupling to an effective one

# One-loop corrections in SUSY

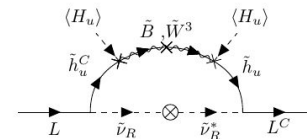
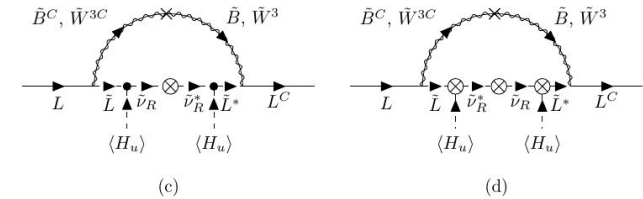
SUSY diagrams



SUSY breaking diagrams

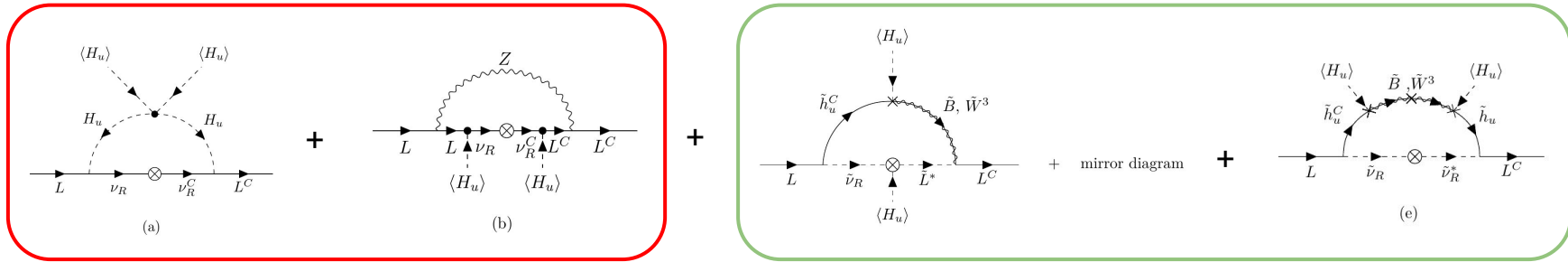


+ mirror diagrams



Candia da Silva, Pilaftsis 2008.05450

# Cancellation scenarios: bilinear case

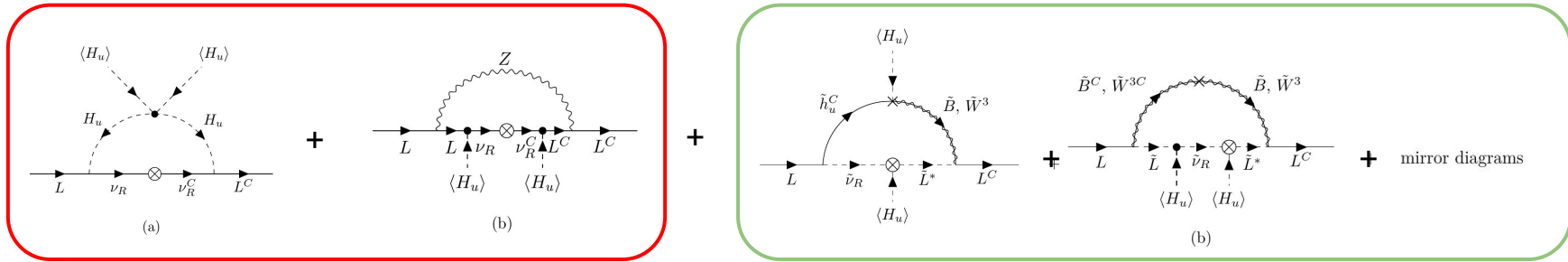


These corrections dominate when the singlet sector is nearly supersymmetric.  
Assuming  $b_\nu \neq 0$ ,

$$\Sigma = \mathbf{m}_D \mathbf{m}_M^\dagger f_1 (\mathbf{m}_M \mathbf{m}_M^\dagger) \mathbf{m}_D^\top + b_\nu^* \mathbf{m}_D \mathbf{m}_M^\dagger f_2 (\mathbf{m}_M \mathbf{m}_M^\dagger) \mathbf{m}_D^\top$$

Some values of the Majorana sneutrino mass screen the corrections to the neutrino mass!

# Cancellation scenarios: trilinear case



Another interesting scenario is found when Assuming  $b_\nu = 0$ ,  $\mathbf{A}_\nu \neq 0$

$$\Sigma = \mathbf{m}_D \mathbf{m}_M^\dagger f_1 (\mathbf{m}_M \mathbf{m}_M^\dagger) \mathbf{m}_D^\top + \mathbf{m}_D \left[ (\mathbf{A}_\nu - \mu^* \cot \beta \mathbf{1}_{n_R}) \mathbf{m}_M^\dagger f_3 (\mathbf{m}_M \mathbf{m}_M^\dagger) + \text{transpose} \right] \mathbf{m}_D^\top$$

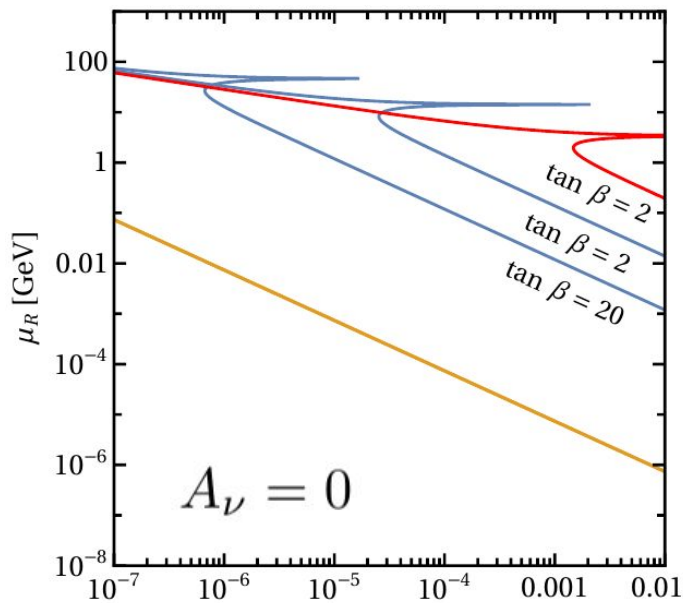
In this case, some values of the soft trilinear parameter could screen the neutrino masses

# Numerical results

- We evaluate the loop corrections for the  $n = 2$  radiative inverse seesaw
- The free parameters are adjusted to enforce agreement with low energy neutrino data
- The quartic Higgs coupling is adjusted to agree with the observed value of the Higgs mass

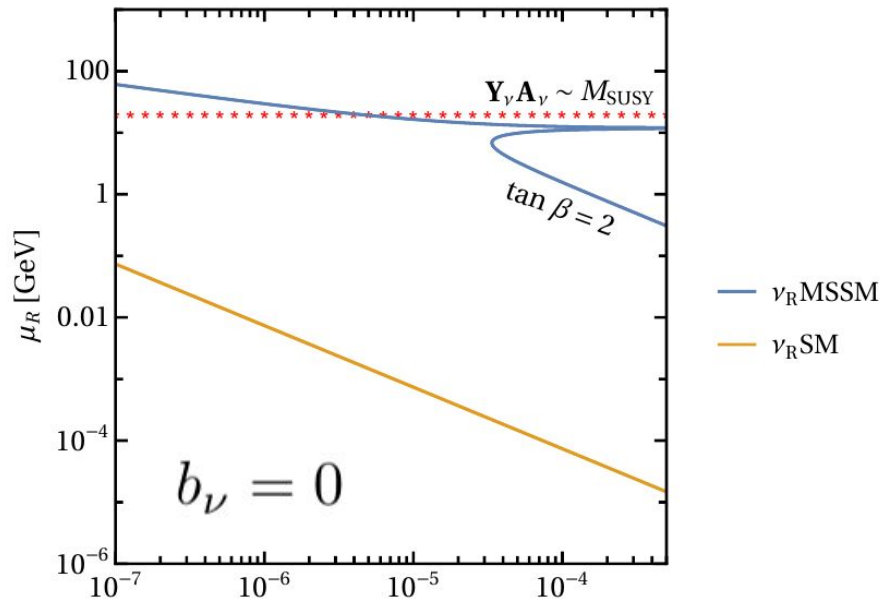
Parameter	Numerical value/interval
$\tan \beta$	2, 20
$\mu$	1200 GeV
$M_1$	1500 GeV
$M_2$	1500 GeV
$m_h$	$125.38 \pm 0.14$ GeV
$m_A$	5000 GeV
$m_H$	5002.8 GeV
$m_N$	500 GeV
$m_{\tilde{\ell}}^2$	$(3500 \text{ GeV})^2$
$\mu_R$	$[10^{-6}, 10^2]$ GeV

# Numerical results



**Red:** Fully supersymmetric singlet sector. Screening is generated by mu parameter

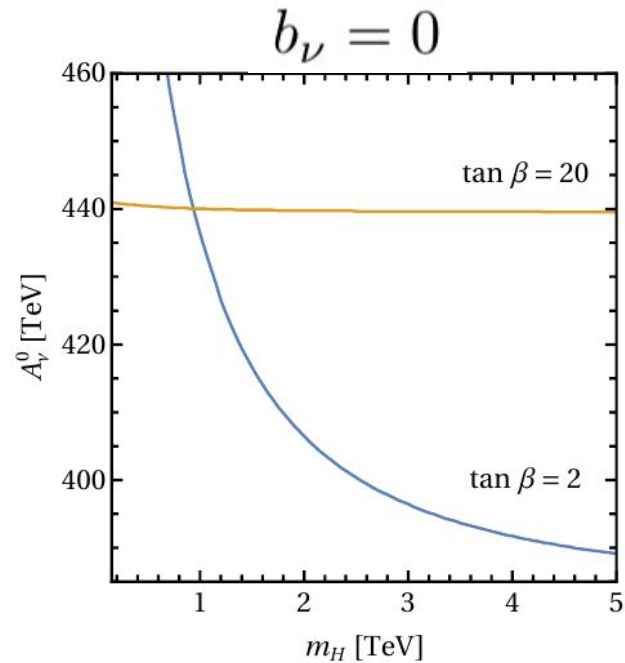
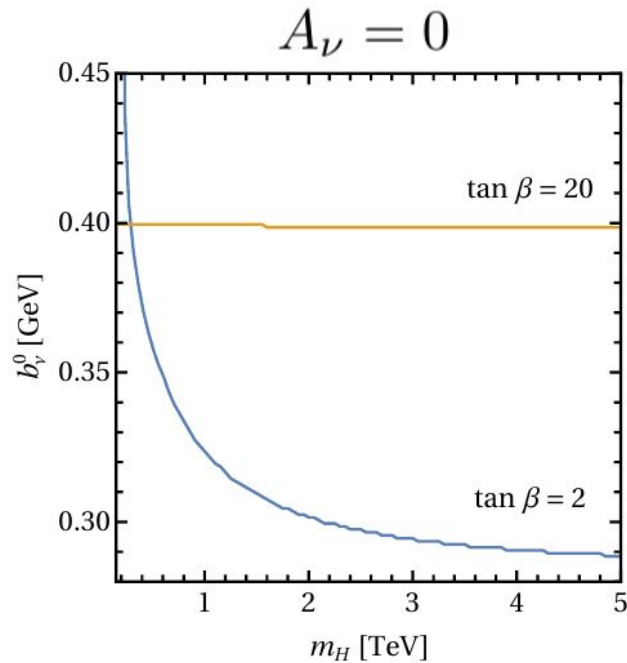
**Blue:** Screening is generated by sneutrino Majorana mass term



**Blue:** Screening is generated by trilinear parameter



# Numerical results



As  $\tan \beta$  increases, the heavy Higgs loop is less significant, allowing smaller  $b_\nu$

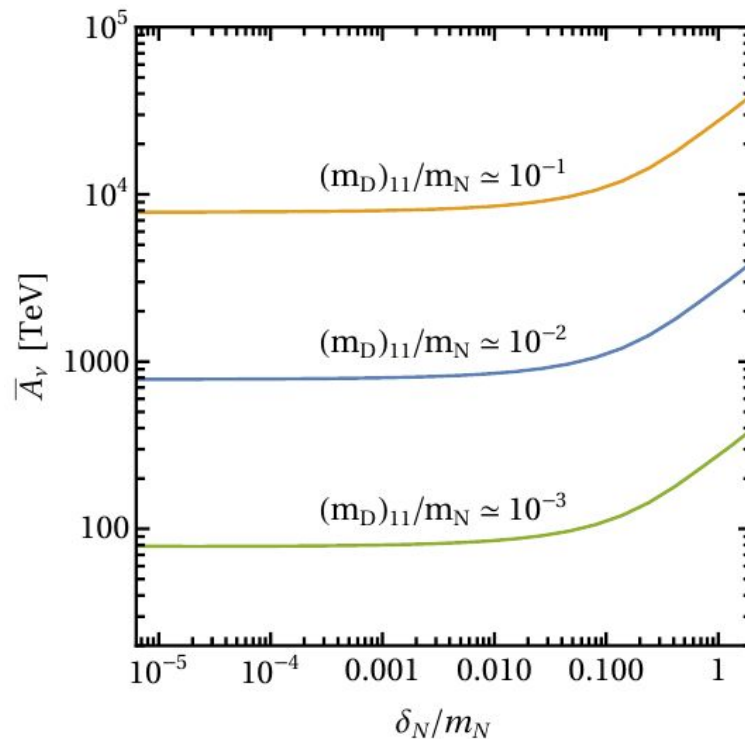
# Other radiative neutrino models

$$\mathbf{m}_D = \frac{v_u}{\sqrt{2}} \begin{pmatrix} a & b e^{2i\pi/3} & c e^{-2i\pi/3} \\ a & b e^{2i\pi/3} & c e^{-2i\pi/3} \\ a & b e^{2i\pi/3} & c e^{-2i\pi/3} \end{pmatrix}$$

$$\mathbf{m}_M = \text{diag} \left( m_N, m_N + \delta_N, m_N + 2\delta_N \right)$$

$$b = a \sqrt{1 + \frac{\delta_N}{m_N}}, \quad c = a \sqrt{1 + \frac{2\delta_N}{m_N}},$$

- This scenario leads to large corrections in the non-SUSY case (Pilaftsis '92) unless  $\delta_N/m_N \lesssim 10^{-5}$ , but these can be compensated by soft SUSY cancellations



# Conclusions

- We performed a **complete flavour space calculation of one-loop corrections** to neutrino masses in the  $\nu_R$ MSSM
- In the radiative SUSY seesaw, thanks to screening effects from sneutrino loops, **LNV is significantly less constrained by low energy neutrino data**
- In the  $\nu_R$ MSSM **both the visible and DM sectors of the theory contribute to the generation of neutrino masses**
- The RH sneutrinos can be the DM when R parity is preserved

# BACK-UP SLIDES

# SUSY limit, effective coupling and Higgs mass

- Minimization conditions:

$$m_{H_d}^2 + |\mu|^2 = -B\mu \frac{v_u}{v_d} + \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)$$

$$m_{H_u}^2 + |\mu|^2 = -B\mu \frac{v_u}{v_d} + \frac{1}{8}(g^2 + g'^2)(v_d^2 - v_u^2)$$

SUSY limit:

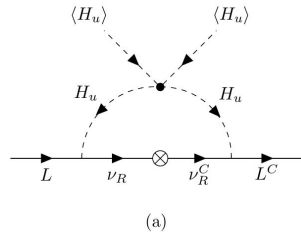
$$\tan \beta = 1, \mu = 0 \text{ and } M_{\text{SUSY}} \rightarrow 0$$

- We promote the tree-level quartic coupling of  $H_u$  from  $\lambda_{\text{tree}} = (g^2 + g'^2)/8$  to an effective coupling that renders the lightest mass eigenstate of the mass matrix

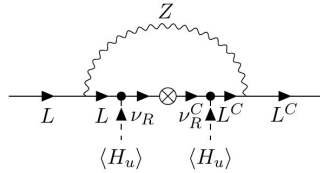
$$\mathcal{L}_{\text{CP-even}}^{\text{mass}} = -\frac{1}{2} \begin{pmatrix} \phi_u & \phi_d \end{pmatrix} \begin{pmatrix} B\mu \cot \beta + \widetilde{M}_Z^2 \sin^2 \beta & -B\mu - M_Z^2 \cos \beta \sin \beta \\ -B\mu - M_Z^2 \cos \beta \sin \beta & B\mu \tan \beta + M_Z^2 \cos^2 \beta \end{pmatrix} \begin{pmatrix} \phi_u \\ \phi_d \end{pmatrix}$$

equal to 125 GeV. Above, we defined  $\widetilde{M}_Z^2 \equiv 2\lambda_{\text{eff}}(v_u^2 + v_d^2)$ .

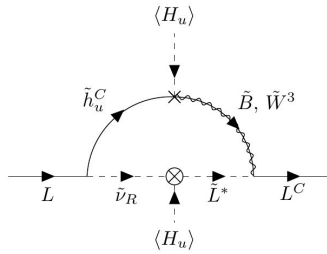
# Analytical expressions



$$\begin{aligned}
 i\Sigma_M^{[1(a)]} = & 2\lambda_{\text{eff}} \mathbf{m}_D \mathbf{m}_M^\dagger \left[ I_3(m_h^2, m_H^2, \mathbf{m}_M \mathbf{m}_M^\dagger) \right. \\
 & + m_A^2 (1 + 2 \cos 2\beta) I_4(m_A^2, m_h^2, m_H^2, \mathbf{m}_M \mathbf{m}_M^\dagger) \\
 & \left. + m_A^4 \cos^2 2\beta I_5(0, m_A^2, m_h^2, m_H^2, \mathbf{m}_M \mathbf{m}_M^\dagger) \right] \mathbf{m}_D^\top
 \end{aligned}$$



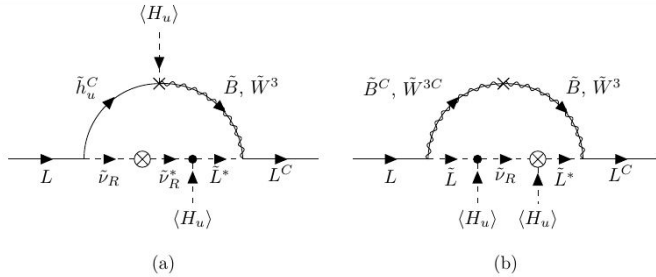
$$i\Sigma_M^{[1(b)]} = \frac{3}{4} (g^2 + g'^2) \mathbf{m}_D \mathbf{m}_M^\dagger I_3(0, M_Z^2, \mathbf{m}_M \mathbf{m}_M^\dagger) \mathbf{m}_D^\top$$



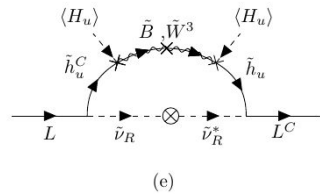
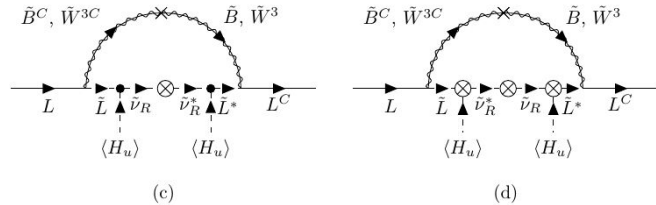
$$\begin{aligned}
 i\Sigma_M^{[2]} = & \mathbf{m}_D \mathbf{m}_M^\dagger \left[ A I_3(\tilde{H}_1^*, \tilde{\mathbf{H}}_2, m_{\chi_4^0}^2) + \mathcal{B}_{A,B} I_4(\tilde{H}_1^*, \tilde{\mathbf{H}}_2, m_{\chi_3^0}^2, m_{\chi_4^0}^2) \right. \\
 & \left. + \mathcal{C}_{A,B,C} I_5(\tilde{H}_1^*, \tilde{\mathbf{H}}_2, m_{\chi_2^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2) + \mathcal{D}_{A,B,C,D} I_6(\tilde{H}_1^*, \tilde{\mathbf{H}}_2, m_{\chi_1^0}^2, m_{\chi_2^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2) \right] \mathbf{m}_D^\top
 \end{aligned}$$

$$\text{with } I_n(m_1^2, m_2^2, \dots, m_n^2) \equiv \int \frac{d^d k}{(2\pi)^d} \prod_{j=1}^n \frac{1}{k^2 - m_j^2}$$

# Analytical expressions 2



+ mirror diagrams



$$i\Sigma_M^{[3(a)]} = b_\nu^* \mathbf{m}_D \left( \mathbf{A}_\nu - \mu^* \cot \beta \mathbf{1}_{n_R} \right) \mathbf{m}_M^\dagger \left[ A I_4 \left( \tilde{H}_1^*, \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_3^0}^2 \right) \right. \\ \left. + \mathcal{B}_{A,B} I_5 \left( \tilde{H}_1^*, \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) + \mathcal{C}_{A,B,C} I_6 \left( \tilde{H}_1^*, \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_2^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right. \\ \left. + \mathcal{D}_{A,B,C,D} I_7 \left( \tilde{H}_1^*, \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_1^0}^2, m_{\chi_2^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right] \mathbf{m}_D^\top$$

$$i\Sigma_M^{[3(b)]} = \mathbf{m}_D \left( \mathbf{A}_\nu - \mu^* \cot \beta \mathbf{1}_{n_R} \right) \mathbf{m}_M^\dagger \left[ A' I_4 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2, m_{\chi_4^0}^2 \right) \right. \\ \left. + \mathcal{B}_{A',B'} I_5 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) + \mathcal{C}_{A',B',C'} I_6 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2, m_{\chi_1^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right. \\ \left. + \mathcal{D}_{A',B',C',D'} I_7 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2, m_{\chi_1^0}^2, m_{\chi_2^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right] \mathbf{m}_D^\top$$

$$i\Sigma_M^{[3(c)]} = b_\nu^* \mathbf{m}_D \left( \mathbf{A}_\nu - \mu^* \cot \beta \mathbf{1}_{n_R} \right) \mathbf{m}_M^\dagger \left[ A' I_5 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_4^0}^2 \right) \right. \\ \left. + \mathcal{B}_{A',B'} I_6 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) + \mathcal{C}_{A',B',C'} I_7 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_1^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right. \\ \left. + \mathcal{D}_{A',B',C',D'} I_8 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_1^0}^2, m_{\chi_2^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right] \left( \mathbf{A}_\nu^\top - \mu^* \cot \beta \mathbf{1}_{n_R} \right) \mathbf{m}_D^\top$$

$$i\Sigma_M^{[3(d)]} = b_\nu \mathbf{m}_D \mathbf{m}_M^\dagger \mathbf{m}_M \left[ A' I_5 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2^*, \tilde{\mathbf{H}}_2^*, m_{\chi_4^0}^2 \right) \right. \\ \left. + \mathcal{B}_{A',B'} I_6 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2^*, \tilde{\mathbf{H}}_2^*, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) + \mathcal{C}_{A',B',C'} I_7 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2^*, \tilde{\mathbf{H}}_2^*, m_{\chi_1^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right. \\ \left. + \mathcal{D}_{A',B',C',D'} I_8 \left( \tilde{H}_1, \tilde{H}_1^*, \tilde{\mathbf{H}}_2^*, \tilde{\mathbf{H}}_2^*, m_{\chi_1^0}^2, m_{\chi_2^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right] \mathbf{m}_M^\dagger \mathbf{m}_D^\top$$

$$i\Sigma_M^{[3(e)]} = b_\nu^* \mathbf{m}_D \mathbf{m}_M^\dagger \left[ A'' I_3 \left( \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_4^0}^2 \right) + \mathcal{B}_{A'',B''} I_4 \left( \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right. \\ \left. + \mathcal{C}_{A'',B'',C''} I_5 \left( \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_1^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right. \\ \left. + \mathcal{D}_{A'',B'',C'',D''} I_6 \left( \tilde{\mathbf{H}}_2, \tilde{\mathbf{H}}_2, m_{\chi_1^0}^2, m_{\chi_2^0}^2, m_{\chi_3^0}^2, m_{\chi_4^0}^2 \right) \right] \mathbf{m}_D^\top$$