Leptogenesis from $SU(5)$ GUT with $\mathcal{T}_{13}$ Family Symmetry

Moinul Hossain Rahat

IFT, University of Florida

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C.S. Fong, MHR, S. Saad, arXiv:**2103.14691** [hep-ph]

Outline

- **Baryogenesis via Leptogenesis**
  - out of equilibrium decay of right-handed neutrinos explain baryon asymmetry of the universe
- **The Asymmetric Texture**
  - Yukawa texture based on $SU(5)$ GUT and $\mathcal{T}_{13}$ family symmetry
- **Leptogenesis from the "asymmetric texture"**
  - thermal leptogenesis in both nonresonant and resonant regimes

Baryogenesis via Leptogenesis

Baryon Asymmetry:

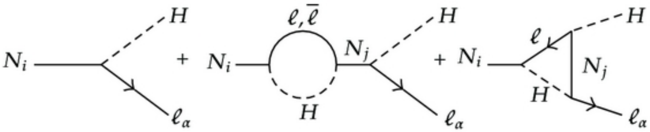
Net baryon to photon ratio of the universe from CMB data:

$$\eta_B = (N_B - N_{\bar{B}})/N_\gamma \simeq (6.12 \pm 0.04) \times 10^{-10}$$

Leptogenesis:

$$\bar{N} \leftrightarrow \ell + H^*, \qquad \bar{N} \leftrightarrow \bar{\ell} + H$$

out of equilibrium decays violate L, C and CP; $B - L$ conserving sphaleron processes convert L violation into B violation



Leptogenesis from the Asymmetric Texture

- $SU(5) \times \mathcal{T}_{13}$ model describes the GUT-scale mass ratios and mixing angles of both quarks and leptons
- Single CP violating phase in the seesaw sector generates both Dirac and Majorana CP violation
- Requires four right-handed neutrinos to generate viable light neutrino mass spectrum
- Can the decay of these right handed neutrinos translate the CP violation into CP asymmetry?

Asymmetric Texture and $\mathcal{T}_{13}$ Family Symmetry

- Lepton mixing is unlike quark mixing!

$$\theta_{12} = 33.65^{\circ}{}^{+2.38^{\circ}}_{-2.47^{\circ}}, \quad \theta_{23} = 47.58^{\circ}{}^{+3.66^{\circ}}_{-3.61^{\circ}}, \quad \theta_{13} = 8.49^{\circ}{}^{+0.40^{\circ}}_{-0.42^{\circ}}$$

- Large angles from Tribimaximal mixing in the seesaw sector

$$\mathcal{U}_{TBM} \equiv \mathcal{R}(\theta_{12} = 35.3^{\circ}),\ \mathcal{R}(\theta_{23} = 45^{\circ}),\ \mathcal{R}(\theta_{13} = 0^{\circ})$$

  and the small reactor angle from EW sector: 'Cabibbo haze'

- The "asymmetric texture": minimal $SU(5)$ texture with a complex TBM mixing does the job! [MHR, Ramond, Xu PRD **98** (2018) 055030]

- The asymmetric term arises naturally from a $\mathcal{T}_{13}$ family symmetry [Pérez, MHR, Ramond, Stuart, Xu PRD **100** (2019) 075008]

Seesaw Parameters for Leptogenesis

- Seesaw matrix in terms of Dirac Yukawa and Majorana mass matrix:

$$\mathcal{S} = v^2 \; Y^{(0)} \; \mathcal{M}^{-1} \; Y^{(0)^T} = \mathcal{U}_{TBM}(\delta_{TBM}) \; \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \; \mathcal{U}_{TBM}(\delta_{TBM})^T$$

- Requires four right-handed neutrinos. $\mathcal{T}_{13}$ determines nonzero entries in $Y^{(0)}(\delta_{TBM})$ and $\mathcal{M}$, expressed in terms of undetermined familon VEV (b_1, b_2, b_3) and a free parameter M
- Prediction: normal ordering of light neutrino masses:
  $|m_{\nu_1}| = 27.6, |m_{\nu_2}| = 28.9, |m_{\nu_3}| = 57.8$ meV
- Cancellation of free parameters between $Y^{(0)}$ and $\mathcal{M}$ implies the seesaw scale is unknown!
- Diagonalizing $\mathcal{M}$ yields right-handed neutrino mass spectrum in terms of b_1, b_2, b_3 and M: masses could be degenerate!
- If leptogenesis is viable, can the undetermined parameters b_1, b_2, b_3 and M be constrained? Is the sign of δ_{TBM} resolved by the sign of baryon asymmetry?

High Scale Flavored Leptogenesis [MHR PRD **103** (2021) 035011]

- Classical Boltzmann Equations for three regimes: (i) $T \gg 10^{12}$ GeV: one flavor, (ii) $10^9 \ll T \ll 10^{12}$ GeV: two flavors, (iii) $T \ll 10^9$ GeV: three flavors
- One-flavor leptogenesis fails because of the flavor structure dictated by $\mathcal{T}_{13}$
- What about the transition regions? What if the mass scale is unknown? ~~Classical Boltzmann~~ Density Matrix Equations
- 3×3 CP asymmetry matrix in flavor space with nonzero off-diagonal entries, accounts for flavor transitions
- $B - L$ asymmetries are calculated by solving coupled differential equations
- Final baryon asymmetry is proportional to the trace of the $B - L$ asymmetry matrix

Right-handed Neutrino Mass Spectrum [MHR PRD **103** (2021) 035011]

- The simplest case $(b_1, b_2, b_3) \equiv b(1,1,1)$ yields zero CP asymmetry
- Set $(b_1, b_2, b_3) \equiv b(1, f, 1)$, define $M \equiv ab$
- For a fixed $f \neq 1$, right-handed neutrino masses are determined as a function of a in the non-degenerate regions
- Required right-handed neutrino masses for successful leptogenesis are $\mathcal{O}\left(10^{11-12}\right)$ GeV
- Is something special going on near the $M_3 - M_4$ degeneracy?

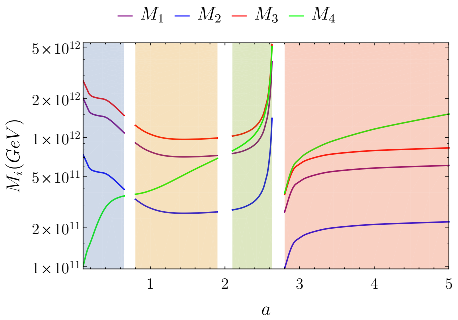


Figure: Right-handed neutrino mass spectrum required for successful leptogenesis setting $f = 2$. Shaded regions represent non-degenerate masses.

- The magnitude of δ_{TBM} is constrained, but the sign remained undetermined
- Final $B - L$ asymmetry is proportional to the diagonal CP asymmetries, which are proportional to $\sin\delta_{TBM}$
- $-85^\circ \leqslant \delta_{TBM} \leqslant -66^\circ$ yields positive baryon asymmetry in all four regions and picks the *correct* sign of the Dirac $\not{CP}$ phase $1.27\pi \leqslant \delta_{CP} \leqslant 1.35\pi$, compared to the 2021 PDG estimate $1.37 \pm 0.17\pi$

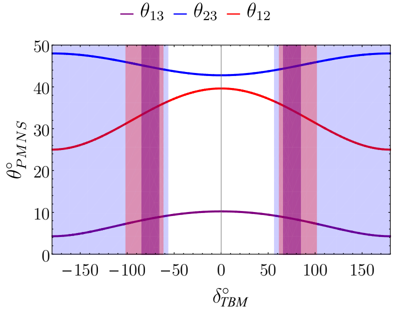


Figure: The asymmetric texture requires $66^\circ \leqslant |\delta_{TBM}| \leqslant 85^\circ$ to yield all PMNS angles within 3σ of their 2021 PDG estimate.

- Exact degeneracy yields zero CP asymmetry, but quasi-degeneracy can enhance the CP asymmetry to nearly $\mathcal{O}(1)$
- Oscillation between sterile neutrinos important, use ~~classical Boltzmann~~ quantum kinetic equations
- CP asymmetry is proportional to Re $\left[\left(Y_\nu^\dagger Y_\nu\right)_{ij}\right]$, which is nonzero only for $N_3 - N_4$ quasi-degeneracy
- Resonance condition $|M_3 - M_4| \simeq \frac{\Gamma_{3,4}}{2}$ fixes a in terms of b and f
- There is a minimum b below which the generated asymmetry is lower than the CMB value

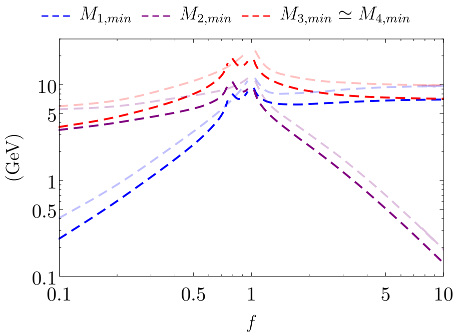


Figure: Minimum right-handed neutrino mass spectrum required for successful leptogenesis at $N_3 - N_4$ resonance.

Maximum Right-handed Neutrino Mass

Fong, MHR, Saad arXiv:2103.14691 [hep-ph]

- Either M_1 or M_2 is smaller than the resonant pair M_3, M_4
- There is a maximum b above which the generated asymmetry is severely washed out by lighter right handed neutrinos
- b sets the mass scale of all right-handed neutrinos
- The upper bound only occurs for $f \gtrsim 2$ below which the washout is not efficient
- There is a discontinuity at $f \simeq 6.2$ due to change of sign of baryon asymmetry for nontrivial flavor interactions

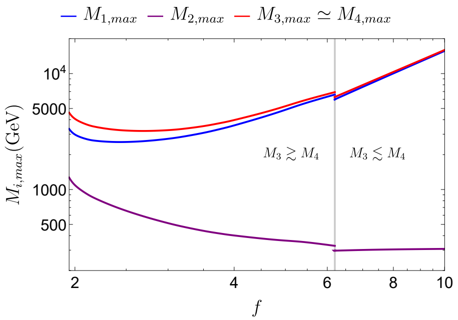
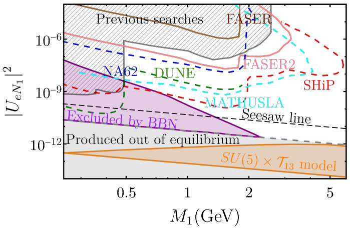


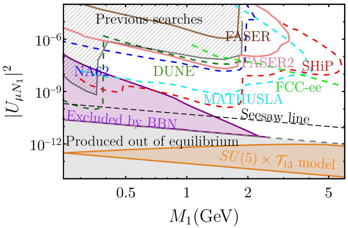
Figure: Maximum right-handed neutrino mass spectrum required for successful leptogenesis at $N_3 - N_4$ resonance.

Experimental Constraints

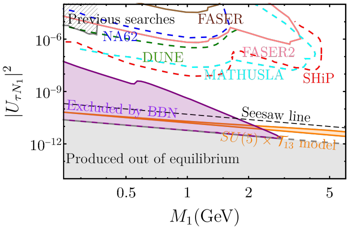
Fong, MHR, Saad arXiv:2103.14691 [hep-ph]



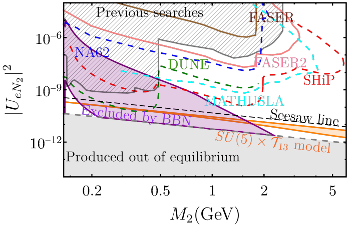
(a) $0.1 < f < 1$



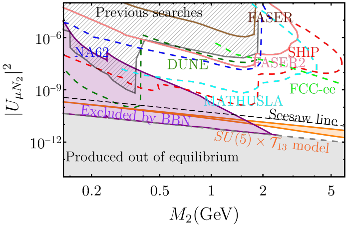
(b) $0.1 < f < 1$



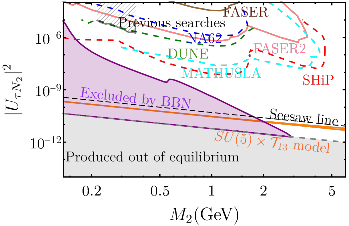
(c) $0.1 < f < 1$



(d) $1 < f < 10$



(e) $1 < f < 10$



(f) $1 < f < 10$

Key Findings

- Nontrivial relation between seesaw familon VEVs generate light neutrino masses and lepton mixings without fixing the seesaw scale!

Nonresonant Leptogenesis

- No unflavored leptogenesis, flavored leptogenesis requires right-handed neutrino masses $\mathcal{O}\left(10^{11-12}\right)$ GeV
- TBM phase δ_{TBM} must be negative, yields positive baryon asymmetry and Dirac CP phase consistent with global fits

Resonant Leptogenesis

- Nontrivial upper bound on right-handed neutrino mass because of washout by lighter neutrinos
- Minimum right-handed neutrino mass $\mathcal{O}(1)$ GeV, mixing parameters close to the sensitivity of DUNE

Backup Slides

Hunting the 'Minimal' Texture

What is the 'minimal' Yukawa texture that

- reproduces the GUT-scale mass relations, the CKM mixing angles, and
- generates *enough* 'Cabibbo haze' for the reactor angle?

The Asymmetric Texture arXiv:1805.10684

$$Y^{(\frac{2}{3})} \sim \text{ diag }(\lambda^8, \lambda^4, 1),$$

$$Y^{(-\frac{1}{3})} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & b\lambda^3 \\ a\lambda^3 & c\lambda^2 & g\lambda^2 \\ d\lambda & g\lambda^2 & 1 \end{pmatrix}, \quad Y^{(-1)} \sim \begin{pmatrix} bd\lambda^4 & a\lambda^3 & d\lambda \\ a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\ b\lambda^3 & g\lambda^2 & 1 \end{pmatrix}$$

$$\mathcal{U}_{Seesaw} = \text{diag }(1, 1, e^{i\delta})\ \mathcal{U}_{TBM}$$

$$a = c = \frac{1}{3}, \quad g = A, \quad b = A\sqrt{\rho^2 + \eta^2}, \quad d = \frac{2a}{g} = \frac{2}{3A}, \quad \cos\delta = 0.2$$

TBM Mixing with a Phase

$$\mathcal{U}_{PMNS} = \mathcal{U}^{(-1)^\dagger}\; \mathcal{U}_{TBM}(\delta)$$

$$\mathcal{U}_{TBM}(\delta) = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{e^{i\delta}}{\sqrt{6}} & -\frac{e^{i\delta}}{\sqrt{3}} & \frac{e^{i\delta}}{\sqrt{2}} \end{pmatrix}$$

With real TBM mixing $\delta = 0$,

$$|\sin\theta_{13}| = \frac{1}{\sqrt{2}} \left| \mathcal{U}_{21}^{(-1)} + \mathcal{U}_{31}^{(-1)} \right|$$

$\theta_{13} \simeq 10.59^\circ$ (2.26° above PDG), $\theta_{12} \simeq 39.81^\circ$ (6.16° above PDG),

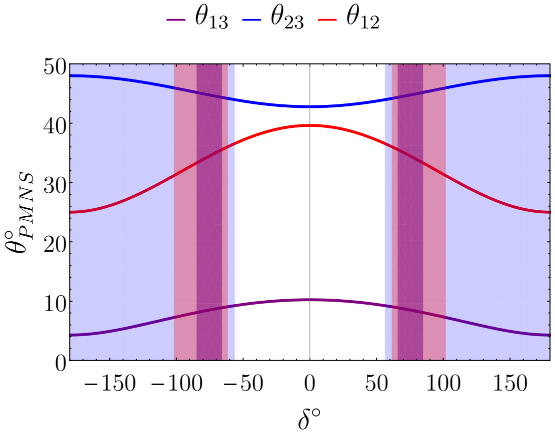
$\theta_{23} \simeq 42.67^\circ$ (2.90° below PDG)

With complex TBM mixing,

$$|\sin\theta_{13}| = \frac{1}{\sqrt{2}} \left| \mathcal{U}_{21}^{(-1)} + e^{i\delta}\; \mathcal{U}_{31}^{(-1)} \right|$$

TBM Mixing with a Phase

One phase to rule them all: $66^\circ < |\delta| < 85^\circ$ brings all three angles within 3σ of PDG 2020 value



Family Symmetry arXiv:1907.10698

Suppose $SU(5)$ matter fields $F \sim \bar{\mathbf{5}}$ and $T \sim \mathbf{10}$ are triplets of some discrete family symmetry group, G_f.

F and T must be different triplets of G_f

- $Y^{(-1/3)}$ and $Y^{(-1)}$ comes from $F \otimes T = (\bar{\mathbf{5}}, \mathbf{r}) \otimes (\mathbf{10}, \mathbf{s})$
- $3 \times 3 \Rightarrow$ either symmetric or antisymmetric
  We need a group with at least two different triplets!
- Candidates: $\mathcal{S}_4$ (order 24), $\Delta(27)$ (order 27), $\mathcal{T}_{13}$ (order 39)

$\mathrm{s} \otimes \mathrm{s}$ must distinguish diagonal from off-diagonal

- $Y^{(2/3)}$ comes from $T \otimes T = (\mathbf{10}, \mathbf{s}) \otimes (\mathbf{10}, \mathbf{s})$

Only $\mathcal{T}_{13}$ survives!

Generating the Asymmetric Term

$Y^{(-\frac{1}{3})} \leftarrow FTH_{\bar{\mathbf{5}}}\varphi^{(-\frac{1}{3})}$

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}_{\mathbf{3}_1} \otimes \begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{\mathbf{3}_2} = \begin{pmatrix} F_3T_2 \\ F_1T_1 \\ F_2T_3 \end{pmatrix}_{\mathbf{3}_1} \oplus \begin{pmatrix} F_3T_1 \\ F_1T_3 \\ F_2T_2 \end{pmatrix}_{\bar{\mathbf{3}}_2} \oplus \begin{pmatrix} F_3T_3 \\ F_1T_2 \\ F_2T_1 \end{pmatrix}_{\mathbf{3}_2}$$

$\mathcal{T}_{13}$ can dial *individual* matrix elements!

$Y^{(\frac{2}{3})} \leftarrow TT\bar{H}_{\mathbf{5}}\varphi^{(\frac{2}{3})}$

$$\begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{\mathbf{3}_2} \otimes \begin{pmatrix} T_1 \\ T_3 \\ T_2 \end{pmatrix}_{\mathbf{3}_2} = \begin{pmatrix} T_3T_3 \\ T_2T_2 \\ T_1T_1 \end{pmatrix}_{\bar{\mathbf{3}}_1} \oplus \begin{pmatrix} T_3T_2 \\ T_2T_1 \\ T_1T_3 \end{pmatrix}_{\bar{\mathbf{3}}_2} \oplus \begin{pmatrix} T_2T_3 \\ T_1T_2 \\ T_3T_1 \end{pmatrix}_{\bar{\mathbf{3}}_2}$$

Diagonals are distinguished from off-diagonals!

Lepton Sector of the $SU(5) \times \mathcal{T}_{13} \times \mathcal{Z}_{12}$ Model

arXiv:2001.04019

	F	$\bar{N}$	$\bar{N}_4$	$\bar{H}_{\mathbf{5}}$	Λ	$\varphi_{\mathcal{A}}$	$\varphi_{\mathcal{B}}$	φ_{v}
$SU(5)$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathcal{T}_{13}$	$\mathbf{3}_1$	$\mathbf{3}_2$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{3}}_1$	$\bar{\mathbf{3}}_2$	$\mathbf{3}_2$	$\bar{\mathbf{3}}_1$
$\mathcal{Z}_{12}$	ω	$\omega^{\mathbf{3}}$	$\mathbf{1}$	$\omega^{\mathbf{9}}$	$\omega^{\mathbf{2}}$	$\omega^{\mathbf{11}}$	$\omega^{\mathbf{6}}$	$\omega^{\mathbf{2}}$

Table: Transformation properties of matter, Higgs, messenger and familon fields in the seesaw sector. Here $\omega^{12} = 1$. The $\mathcal{Z}_{12}$ 'shaping' symmetry prevents unwanted tree-level operators.

$$\begin{aligned}
\mathcal{L}_{ss} &\supset y_{\mathcal{A}} F \Lambda \bar{H}_{\mathbf{5}} + y'_{\mathcal{A}} \bar{N} \overline{\Lambda} \varphi_{\mathcal{A}} + y_{\mathcal{B}} \bar{N} \bar{N} \varphi_{\mathcal{B}} + M_{\Lambda} \overline{\Lambda} \Lambda + y'_{v} \bar{N}_4 \overline{\Lambda} \varphi_{v} + M \bar{N}_4 \bar{N}_4 \\
&\supset \frac{1}{M_{\Lambda}} y_{\mathcal{A}} y'_{\mathcal{A}} F \bar{N} \bar{H}_{\mathbf{5}} \varphi_{\mathcal{A}} + \frac{1}{M_{\Lambda}} y_{\mathcal{A}} y'_{v} F \bar{N}_4 \bar{H}_{\mathbf{5}} \varphi_{v} + y_{\mathcal{B}} \bar{N} \bar{N} \varphi_{\mathcal{B}} + M \bar{N}_4 \bar{N}_4 .
\end{aligned}$$

Familon Structure

- Vacuum Expectation Value (VEV) of the familons:

$$y_{\mathcal{A}} y'_{\mathcal{A}} \langle \varphi_{\mathcal{A}} \rangle = \frac{M_\Lambda}{v} \sqrt{m_\nu b_1 b_2 b_3}\, (-b_2^{-1} e^{i\delta}, b_1^{-1}, b_3^{-1}),$$
$$y_{\mathcal{B}} \langle \varphi_{\mathcal{B}} \rangle = (b_1, b_2, b_3),$$
$$y_{\mathcal{A}} y'_v \langle \varphi_v \rangle = \frac{M_\Lambda}{v} \sqrt{M m'_v}\, (2, -1, e^{i\delta}),$$

- Majorana matrix from $y_{\mathcal{B}} \bar{N} \bar{N} \varphi_{\mathcal{B}} + M \bar{N}_4 \bar{N}_4$:

$$\mathcal{M} \equiv \begin{pmatrix} 0 & b_2 & b_3 & 0 \\ b_2 & 0 & b_1 & 0 \\ b_3 & b_1 & 0 & 0 \\ 0 & 0 & 0 & M \end{pmatrix}$$

Familon Structure (contd.)

- Vacuum Expectation Value (VEV) of the familons:

$$y_{\mathcal{A}} y'_{\mathcal{A}} \langle \varphi_{\mathcal{A}} \rangle = \frac{M_\Lambda}{v} \sqrt{m_\nu b_1 b_2 b_3}\, (-b_2^{-1} e^{i\delta}, b_1^{-1}, b_3^{-1}),$$
$$y_{\mathcal{B}} \langle \varphi_{\mathcal{B}} \rangle = (b_1, b_2, b_3),$$
$$y_{\mathcal{A}} y'_{v} \langle \varphi_{v} \rangle = \frac{M_\Lambda}{v} \sqrt{M m'_v}\, (2, -1, e^{i\delta}),$$

- Yukawa matrix from $\frac{1}{M_\Lambda} y_{\mathcal{A}} y'_{\mathcal{A}} F \bar{N} \bar{H}_{\mathbf{5}} \varphi_{\mathcal{A}} + \frac{1}{M_\Lambda} y_{\mathcal{A}} y'_{v} F \bar{N}_4 \bar{H}_{\mathbf{5}} \varphi_{v}$:

$$Y^{(0)} \equiv \frac{\sqrt{b_1 b_2 b_3 m_\nu}}{v} \begin{pmatrix} 0 & b_3^{-1} & 0 & 2\sqrt{\frac{M m'_v}{b_1 b_2 b_3 m_\nu}} \\ b_1^{-1} & 0 & 0 & -\sqrt{\frac{M m'_v}{b_1 b_2 b_3 m_\nu}} \\ 0 & 0 & -e^{i\delta} b_2^{-1} & e^{i\delta}\sqrt{\frac{M m'_v}{b_1 b_2 b_3 m_\nu}} \end{pmatrix}$$

Light Neutrino Masses

- $\sum_i |m_{\nu_i}| = 114.3$ meV compared to Planck: $\sum_i |m_{\nu_i}| < 120$ meV [arXiv:1807.06209].
- Combining data from Euclid and LSST to DESI and WFIRST, the error bound on $\sum_i |m_{\nu_i}|$ will be constrained to $8 - 11$ meV.
- Normal ordering is preferred above 3σ by Super-K, T2K and NOvA [arXiv:1710.09126].
- DUNE and Hyper-K will resolve the correct mass ordering beyond 5σ in $5 - 7$ yrs [arXiv:1807.10334, 1805.04163].

Neutrinoless Double Beta Decay

- Dirac $\not{C}\not{P}$ Jarlskog-Greenberg Invariant, $|\mathcal{J}| = 0.028$
- Majorana Invariants, $|\mathcal{I}_1| = 0.106, \quad |\mathcal{I}_2| = 0.011$

Prediction for $0\nu\beta\beta$

$$|m_{\beta\beta}| = 13.02 \text{ or } 25.21 \text{ meV}$$

compared to $|m_{\beta\beta}| \leqslant 61 - 165$ meV by KamLAND-Zen [arXiv:1605.02889]

- Our predictions are expected to be tested in several next generation experiments [J.Phys.Conf.Ser. 1390 (2019) 1, 012048]:

Experiment	Sensitivity (meV)	Experiment	Sensitivity (meV)
LEGEND	11 - 28	SNO+-II	20 - 70
nEXO	8 - 22	AMoRE-II	15 - 30
CUPID	6 - 17	PandaX-III	20 - 55

Seesaw Parameters

- Define $\beta = \sqrt{\frac{a}{11.5f}}$. Dirac and Majorana matrices:

$$Y^{(0)} = \frac{\sqrt{bfm_\nu}}{v} \begin{pmatrix} 0 & 1 & 0 & 2\beta \\ 1 & 0 & 0 & -\beta \\ 0 & 0 & -f^{-1}e^{i\delta} & \beta e^{i\delta} \end{pmatrix}, \quad \mathcal{M} = b \begin{pmatrix} 0 & f & 1 & 0 \\ f & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & a \end{pmatrix}$$

- Takagi factorization: $\mathcal{M} = \mathcal{U}_m \, \mathcal{D}_m \, \mathcal{U}_m^T$

$$M_1 = bf, M_2 = \frac{b}{2}\left(\sqrt{f^2+8} - f\right), M_3 = \frac{b}{2}\left(\sqrt{f^2+8} + f\right), M_4 = ab$$

$$\mathcal{U}_m = \begin{pmatrix} -\frac{i}{\sqrt{2}} & \frac{-i}{2}\sqrt{1 - \frac{f}{\sqrt{f^2+8}}} & \frac{1}{2}\sqrt{1 + \frac{f}{\sqrt{f^2+8}}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{2}\sqrt{1 - \frac{f}{\sqrt{f^2+8}}} & \frac{1}{2}\sqrt{1 + \frac{f}{\sqrt{f^2+8}}} & 0 \\ 0 & \frac{i}{\sqrt{2}}\sqrt{1 + \frac{f}{\sqrt{f^2+8}}} & \frac{1}{\sqrt{2}}\sqrt{1 - \frac{f}{\sqrt{f^2+8}}} & 0 \end{pmatrix}$$

Viability of Leptogenesis

- Neutrino Yukawa matrix $Y_\nu = \mathcal{U}^{(-1)}\ Y^{(0)}\ \mathcal{U}_m^*$

$$Y_\nu^\dagger Y_\nu = \frac{bfm_\nu}{v^2}$$

$$\times \begin{pmatrix} 1 & 0 & 0 & \frac{3i\beta}{\sqrt{2}} \\ * & \frac{1}{2}\left(1 - \frac{f^3 - f - \sqrt{f^2+8}}{f^2\sqrt{f^2+8}}\right) & -\frac{i\sqrt{2}\left(f^2-1\right)}{f^2\sqrt{f^2+8}} & -\frac{i\beta}{2f}\left(f\sqrt{1 - \frac{f}{\sqrt{f^2+8}}} + \sqrt{2 + \frac{2f}{\sqrt{f^2+8}}}\right) \\ * & * & \frac{1}{2}\left(1 + \frac{f^3 - f + \sqrt{f^2+8}}{f^2\sqrt{f^2+8}}\right) & \frac{\beta}{2f}\left(f\sqrt{1 + \frac{f}{\sqrt{f^2+8}}} - \sqrt{2 - \frac{2f}{\sqrt{f^2+8}}}\right) \\ * & * & * & 6\beta^2 \end{pmatrix}$$

- Unflavored leptogenesis is ruled out

$$\varepsilon_i \equiv \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i} = \frac{1}{8\pi}\sum_{j\neq i}\frac{\mathrm{Im}\left[\left((Y_\nu^\dagger Y_\nu)_{ij}\right)^2\right]}{(Y_\nu^\dagger Y_\nu)_{ii}}\ \xi\left(\frac{M_j^2}{M_i^2}\right)$$

Unflavored Leptogenesis

- Structure of $Y^{(0)}$ and $\mathcal{M} = \mathcal{U}_m \, \mathcal{D}_m \, \mathcal{U}_m^T$ determined by $\mathcal{T}_{13}$
- Change basis to *weak* basis: $Y^{(0)} \to Y_\nu = \mathcal{U}^{(-1)} \, Y^{(0)} \, \mathcal{U}_m^*$
- Charged-lepton flavors can be neglected for $T \gg 10^{12}$ GeV. Unflavored CP asymmetry depends on

$$\text{Im}\left[\left(Y_\nu^\dagger Y_\nu\right)^2\right] = \text{Im}\left[\left(\mathcal{U}_m^T \, Y^{(0)^\dagger} Y^{(0)} \, \mathcal{U}_m^*\right)^2\right]$$

- $Y^{(0)} = \text{diag}(1, 1, e^{i\delta}) Y_{real}^{(0)}$ implies $Y^{(0)^\dagger} Y^{(0)}$ is real
- Real, symmetric $\mathcal{M}$ implies $\mathcal{U}_m = \mathcal{U}_{m,real} \, \mathcal{P}$ where $\mathcal{P}$ is a diagonal phase matrix with entries ± 1 or $\pm i$
- $\text{Im}\left[\left(Y_\nu^\dagger Y_\nu\right)^2\right] = \text{Im}\left[\left(\mathcal{P}^T \ \text{Real} \ \mathcal{P}^*\right)^2\right]$ vanishes, unflavored leptogenesis fails!

[MHR PRD **103** (2021) 035011]

Flavored Leptogenesis

- Density Matrix Equations

$$
\begin{aligned}
\frac{dN_{N_i}}{dz} &= -(D_i + S_i)(N_{N_i} - N_{N_i}^{eq}) \\
\frac{dN_{\alpha\beta}}{dz} &= \sum_i \varepsilon_{\alpha\beta}^{(i)} D_i (N_{N_i} - N_{N_i}^{eq}) - \frac{1}{2}\sum_i W_i \{P^{0(i)}, N\}_{\alpha\beta} \\
&\quad - \frac{\mathrm{Im}(\Lambda_\tau)}{Hz}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N\right]\right]_{\alpha\beta} \\
&\quad - \frac{\mathrm{Im}(\Lambda_\mu)}{Hz}\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N\right]\right]_{\alpha\beta}
\end{aligned}
$$

- Final value of the $B - L$ asymmetry: $N_{B-L}^f \equiv \sum_{\alpha=e,\mu,\tau} N_{\alpha\alpha}^f$

- Baryon asymmetry $\eta_B \simeq 1.28 \times 10^{-2} N_{B-L}^f$ to be compared with $\eta_B^{CMB} = (6.12 \pm 0.04) \times 10^{-10}$

The Simplest Case

- All seesaw parameters are described in terms of the familon VEV $\langle \varphi_B \rangle \equiv (b_1, b_2, b_3)$ and mass parameter M

- The simplest case $(b_1, b_2, b_3) \equiv b(1, 1, 1)$ fails

- Degeneracy in the mass spectrum

$$M_1 = M_2 = b, \quad M_3 = 2b, \quad M_4 = M \equiv ab$$

- CP asymmetries

$$\varepsilon^{(1)} = -\varepsilon^{(2)}, \qquad \varepsilon^{(3)} = \varepsilon^{(4)} = 0$$

- Final $B - L$ asymmetry

$$N^f_{B-L} \propto \sum_i \varepsilon^{(i)} \times \text{ Rate of Number density}$$

A Simpler Case

- Set $(b_1, b_2, b_3) \equiv b(1, f, 1)$, define $M \equiv ab$
- $f \neq 1$ breaks degeneracy in the mass spectrum

$$M_1 = bf, M_2 = \frac{1}{2}\left(\sqrt{f^2+8} - f\right)$$
$$M_4 = ab, M_3 = \frac{1}{2}\left(\sqrt{f^2+8} + f\right)$$

- Non-hierarchical mass spectrum: asymmetry generated by heavier neutrinos are not entirely washed out

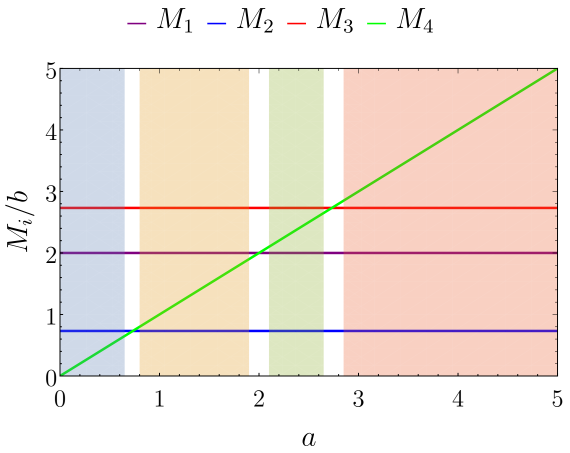
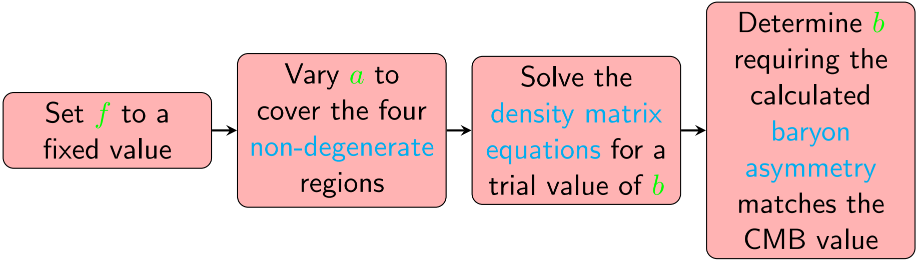
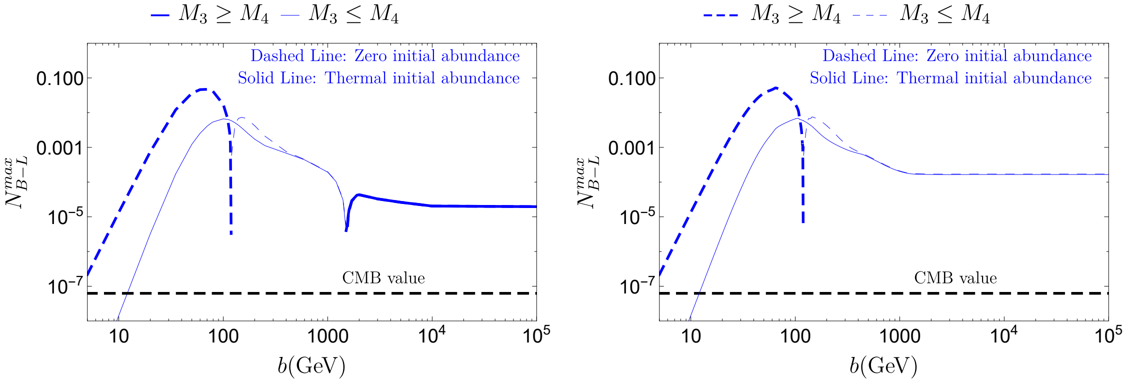


Figure: Right-handed neutrino mass spectrum setting $f = 2$. The four shaded regions represent

Constraining Model Parameters from Leptogenesis

- Three undetermined parameters: a, b and f
- b is the overall mass scale, $f \neq 1$ lifts the degeneracy between M_1 and M_2, a determines how heavy M_4 is w.r.t. others

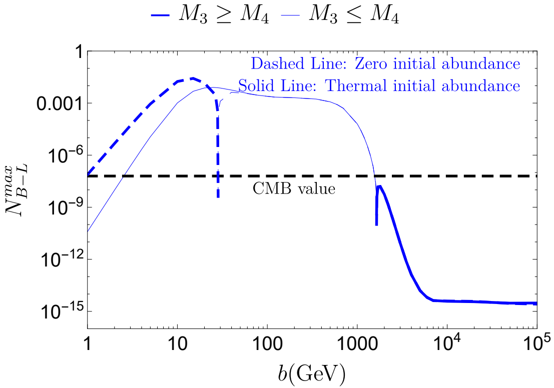




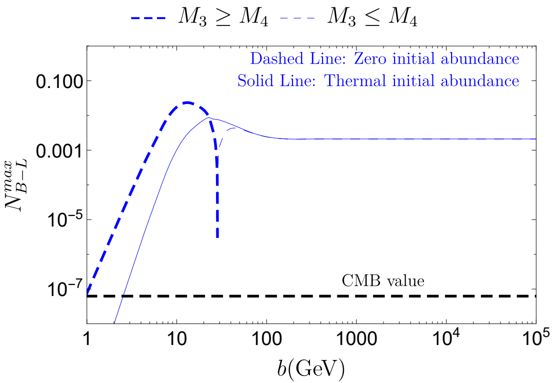
(a) $f = 0.1$ (b) $f = 0.1$, without N_1 washout

Figure: Maximum $B - L$ asymmetry at the resonance $M_3 \simeq M_4$ for (a) $f = 0.1$ and (b) $f = 0.1$ without considering N_1 washout. For large b, the $B - L$ asymmetry saturates at a value higher than the CMB value in case (a) and (b), thus indicating that there is no upper limit on b. N_1 washout decreases the final asymmetry by only a factor of 10, and is not very efficient.

Upper Bound on Right-Handed Neutrino Mass



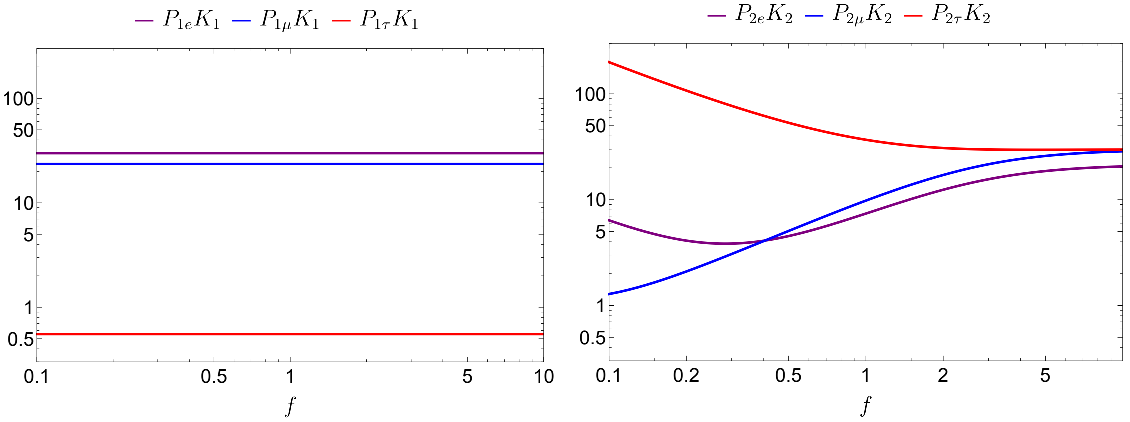
(a) $f = 10$



(b) $f = 10$, without N_2 washout

Figure: Maximum $B - L$ asymmetry at the resonance $M_3 \simeq M_4$ for (a) $f = 10$, and (b) $f = 10$ without considering N_2 washout. For large b, the $B - L$ asymmetry saturates below the CMB value for large b, thus setting an upper limit above which successful resonant leptogenesis is not feasible. If the N_2 washout is disregarded, the final asymmetry is $\mathcal{O}(10^{12})$ times large, as shown in case (b), thus

Washout Factors



(a) (b)

Figure: Decay parameter times branching ratio as a function of f at resonance. The asymmetry generated by N_3 and N_4 at resonance is partially washed out by N_1 and N_2, and is proportional to $e^{-P_{i\alpha}K_i}$.