Realistic neutrino mixing in a scotogenic model using $S3$ symmetry

Soumita Pramanick

National Centre for Nuclear Research (NCBJ, Warsaw), Poland

Phys.Rev. D100 (2019) no.3, 035009

arXiv:1904.07558 [hep-ph]

Three-flavour oscillation

- **Neutrinos are massive and they mix !!**

Three flavor oscillation probability

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U^*_{\alpha j} U^*_{\beta j} \sin^2 \left(\frac{\pi L}{\lambda_{ij}} \right)$$

2 independent Δm^2, 3 mixing angles, 1 phase

$$U^T M_\nu U = \text{diag}(m_1, m_2, m_3)$$

- The Pontecorvo, Maki, Nakagawa, Sakata – PMNS – matrix:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

A measure of CP-violation is given by the basis-independent leptonic Jarlskog(J) parameter:

$$J = \text{Im}[U_{e1} U_{\mu 2} U^*_{e2} U^*_{\mu 1}]$$

Popular Lepton Mixing and oscillation data

The current 3σ global fits of the three mixing angles:

$$\begin{aligned}\theta_{12} &= (31.42 - 36.05)^\circ,\\ \theta_{23} &= (40.3 - 51.5)^\circ\,,\\ \theta_{13} &= (8.09 - 8.98)^\circ.\end{aligned}$$

- $\theta_{13} \neq 0$ but small.
- Best fit $\theta_{23} \neq \pi/4$. Octant not known.

Popular lepton mixings (By construction: $\theta_{13} = 0$, $\theta_{23} = \pi/4$. Differ only in θ_{12}):

$$U^0 = \begin{pmatrix} \cos\theta_{12}^0 & \sin\theta_{12}^0 & 0 \\ -\frac{\sin\theta_{12}^0}{\sqrt{2}} & \frac{\cos\theta_{12}^0}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}^0}{\sqrt{2}} & \frac{\cos\theta_{12}^0}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

Tribimaximal (TBM) [$\theta_{12} = 35.3^\circ$]; Bimaximal (BM) [$\theta_{12} = 45^\circ$]; Golden Ratio (GR) [$\theta_{12} = 31.7^\circ$]

- Corresponding structure of left-handed Majorana neutrino mass matrix:

$$M_{\nu L}^{flavour} = U^0 M_{\nu L}^{mass} U^{0T} = U^0 diag(m_1, m_2, m_3) U^{0T} = \begin{pmatrix} a & c & c \\ c & b & d \\ c & d & b \end{pmatrix}.$$

Here,

$$\begin{aligned} a &= m_1\cos^2\theta_{12}^0 + m_2\sin^2\theta_{12}^0 \\ b &= \frac{1}{2}\left(m_1\sin^2\theta_{12}^0 + m_2\cos^2\theta_{12}^0 + m_3\right) \\ c &= \frac{1}{2\sqrt{2}}\sin 2\theta_{12}^0(m_2 - m_1) \\ d &= \frac{1}{2}\left(m_1\sin^2\theta_{12}^0 + m_2\cos^2\theta_{12}^0 - m_3\right) \end{aligned}$$

$$\Downarrow$$

$$\boxed{\tan 2\theta_{12}^0 = \frac{2\sqrt{2}c}{b+d-a}}$$

Objective

- Note a, b, c and d have to be non-zero for neutrino masses to be non-degenerate and realistic.

Objective

Construct a scotogenic model using $S3 \times Z_2$ with two right-handed neutrinos that at one-loop level can generate:

1. The structure of the left-handed Majorana neutrino mass matrix with $\theta_{13} = 0$, $\theta_{23} = \pi/4$ when the two right-handed neutrinos are maximally mixed.
2. Non-zero θ_{13}, deviations of θ_{23} from maximality and small corrections to solar mixing by a small shift from maximal mixing in the right-handed neutrino sector.

Discrete Flavour symmetry $S3$

- Permutation group of three objects. Has two generators A, B and three irreducible representations viz. 2 (dimension 2) and 1, $1'$ (dimension 1).
- **Product rule:** $1' \times 1' = 1$, $\boxed{2 \times 2 = 1 + 1' + 2}$.
- Combining two doublets of $S3$ viz. $\Phi \equiv \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ and $\Psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$, we get:
$$\phi_1\psi_2 + \phi_2\psi_1 \equiv 1 \ , \ \phi_1\psi_2 - \phi_2\psi_1 \equiv 1' \text{ and } \begin{pmatrix} \phi_2\psi_2 \\ \phi_1\psi_1 \end{pmatrix} \equiv 2 \ .$$
- In case one of the two $S3$ doublet fields is a hermitian conjugate we get:
$$\phi_2^\dagger\psi_2 + \phi_1^\dagger\psi_1 \equiv 1 \ , \ \phi_2^\dagger\psi_2 - \phi_1^\dagger\psi_1 \equiv 1' \text{ and } \begin{pmatrix} \phi_1^\dagger\psi_2 \\ \phi_2^\dagger\psi_1 \end{pmatrix} \equiv 2 \ .$$

Fields in the model

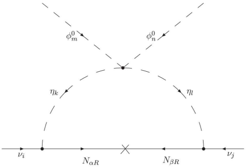
Leptons	$SU(2)_L$	$S3$	Z_2
$L_{e_L} \equiv \begin{pmatrix} \nu_e & e^- \end{pmatrix}_L$	2	1	1
$L_{\zeta_L} \equiv \begin{pmatrix} \nu_\mu & \mu^- \\ \nu_\tau & \tau^- \end{pmatrix}_L$	2	2	1
$N_{\alpha R} \equiv \begin{pmatrix} N_{1R} \\ N_{2R} \end{pmatrix}$	1	2	-1

Scalars	$SU(2)_L$	$S3$	Z_2
$\Phi \equiv \begin{pmatrix} \phi_1^+ & \phi_1^0 \\ \phi_2^+ & \phi_2^0 \end{pmatrix}$	2	2	1
$\eta \equiv \begin{pmatrix} \eta_1^+ & \eta_1^0 \\ \eta_2^+ & \eta_2^0 \end{pmatrix}$	2	2	-1

- Here, $S3$ is acting vertically, $SU(2)_L$ is acting horizontally.
- Inert $SU(2)_L$ doublet scalars $\eta_j \equiv (\eta_j^+, \eta_j^0)^T$, $(j = 1, 2)$ and right-handed neutrinos $N_{\alpha R}$, $(\alpha = 1, 2)$ are odd under Z_2. Thus after spontaneous symmetry breaking (SSB), η_j does not acquire vev. Lightest among η_j can be a potential dark matter candidate.
- After SSB, Φ_i gets *vev* v_i.

The Model

At one-loop level neutrino mass can be radiative generated by the following diagram:



Relevant part of the $S3$ conserving potential at the scalar four-point vertex:

$$\begin{aligned} V_{relevant} &\supset \lambda_1 \left[\left\{ (\eta_2^\dagger \phi_2 + \eta_1^\dagger \phi_1)^2 \right\} + h.c. \right] + \lambda_2 \left[\left\{ (\eta_2^\dagger \phi_2 - \eta_1^\dagger \phi_1)^2 \right\} + h.c. \right] \\ &+ \lambda_3 \left[\left\{ (\eta_1^\dagger \phi_2)(\eta_2^\dagger \phi_1) + (\eta_2^\dagger \phi_1)(\eta_1^\dagger \phi_2) \right\} + h.c. \right] . \end{aligned}$$

From $S3$ conservation we get:

- The Yukawa vertices conserving $S3 \times Z2$ is given by:

$$\mathscr{L}_{Yukawa} = y_1 \left[(\overline{N}_{2R} \eta_2^0 + \overline{N}_{1R} \eta_1^0) \nu_e \right] + y_2 \left[(\overline{N}_{1R} \eta_2^0) \nu_\tau + (\overline{N}_{2R} \eta_1^0) \nu_\mu \right] + h.c.$$

- The direct mass term for the right-handed neutrinos:

$$\mathscr{L}_{right-handed\, neutrinos} = \frac{1}{2} m_{R_{12}} \left[N_{1R}^T C^{-1} N_{2R} + N_{2R}^T C^{-1} N_{1R} \right] .$$

Introduce soft $S3$ breaking terms:

$$\mathscr{L}_{soft} = \frac{1}{2} \left[m_{R_{11}} N_{1R}^T C^{-1} N_{1R} + m_{R_{22}} N_{2R}^T C^{-1} N_{2R} \right]$$

The right-handed neutrino mass

This leads to right-handed neutrino Majorana mass matrix:

$$M_{\nu_R} = \frac{1}{2}\begin{pmatrix} m_{R_{11}} & m_{R_{12}} \\ m_{R_{12}} & m_{R_{22}} \end{pmatrix}.$$

- $m_{R_{11}} = m_{R_{22}} \Rightarrow$ structure of the left-handed neutrino mass matrix with $\theta_{13} = 0$ and $\theta_{23} = \pi/4$.
- $m_{R_{11}} \neq m_{R_{22}}$ i.e., small shift from $m_{R_{11}} = m_{R_{22}}$ can produce realistic mixings i.e., non-zero θ_{13}, deviation of θ_{23} from maximality and small corrections to the solar mixing.

Let the average mass of the right-handed neutrinos be given by m_R and m_0 is the common mass of the η_i fields.

If $m_R^2 >> m_0^2$, the one-loop diagram gives the following contribution to the left-handed Majorana neutrino mass matrix:

$$(M_{\nu_L}^{flavour})_{22} = \lambda \frac{v_m v_n}{8\pi^2} \frac{y_2^2}{m_{R_{22}}} [\ln z - 1] \text{ and } (M_{\nu_L}^{flavour})_{23} = \lambda \frac{v_m v_n}{8\pi^2} \frac{y_2^2 m_{R_{12}}}{m_{R_{11}} m_{R_{22}}} [\ln z - 1].$$

where $z \equiv \frac{m_R^2}{m_0^2}$ and y_i are the Yukawa couplings. Similarly, one can write the expressions for (1,1), (1,2) and (1,3) entries.

Neglecting M_α dependence of z_α one can absorb everything else in the loop expression other than the vevs and the quartic couplings in right-handed loop contributing factors $r_{\alpha\beta}$ given by:

$$r_{11} \equiv \frac{1}{8\pi^2 m_{R_{11}}} [\ln z - 1], \; r_{22} \equiv \frac{1}{8\pi^2 m_{R_{22}}} [\ln z - 1] \text{ and } r_{12} \equiv \frac{m_{R_{12}}}{8\pi^2 m_{R_{11}} m_{R_{22}}} [\ln z - 1].$$

The left-handed neutrino mass matrix

Thus, $m_{R_{11}} = m_{R_{22}} \Rightarrow r_{11} = r_{22} = r$ and $m_{R_{11}} \neq m_{R_{22}} \Rightarrow r_{11} \neq r_{22}$ i.e., $r_{22} = r_{11} + \epsilon$.
Let us consider $r_{11} \neq r_{22}$ and $v_1 \neq v_2$ first and obtain the most general left-handed Majorana neutrino mass matrix as:

$$M_{\nu_L}^{flavour} = \begin{pmatrix} \chi_1 & \chi_4 & \chi_5 \\ \chi_4 & \chi_2 & \chi_6 \\ \chi_5 & \chi_6 & \chi_3 \end{pmatrix}$$

with,

$$\begin{aligned}
\chi_1 &\equiv y_1^2 \left[4r_{12}v_1v_2(\lambda_3 + \lambda_1 - \lambda_2) + (r_{11}v_1^2 + r_{22}v_2^2)(\lambda_1 + \lambda_2)\right] \\
\chi_2 &\equiv y_2^2 \left[r_{22}(\lambda_1 + \lambda_2)v_1^2\right] \\
\chi_3 &\equiv y_2^2 \left[r_{11}(\lambda_1 + \lambda_2)v_2^2\right] \\
\chi_4 &\equiv y_1y_2 \left[r_{12}(\lambda_1 + \lambda_2)v_1^2 + 2r_{22}(\lambda_3 + \lambda_1 - \lambda_2)v_1v_2\right] \\
\chi_5 &\equiv y_1y_2 \left[r_{12}(\lambda_1 + \lambda_2)v_2^2 + 2r_{11}(\lambda_3 + \lambda_1 - \lambda_2)v_1v_2\right] \\
\chi_6 &\equiv y_2^2 \left[2r_{12}(\lambda_3 + \lambda_1 - \lambda_2)v_1v_2\right].
\end{aligned}$$

For $v_1 = v_2 = v$ and $r_{11} = r_{22} = r$ i.e., $m_{R_{11}} = m_{R_{22}}$ we get:

$$M_{\nu_L}^{flavour} = v^2 \begin{pmatrix} y_1^2[4r_{12}\lambda_{123} + 2r\lambda_{12}] & y_1y_2[r_{12}\lambda_{12} + 2r\lambda_{123}] & y_1y_2[r_{12}\lambda_{12} + 2r\lambda_{123}] \\ y_1y_2[r_{12}\lambda_{12} + 2r\lambda_{123}] & y_2^2 r\lambda_{12} & y_2^2(2r_{12}\lambda_{123}) \\ y_1y_2[r_{12}\lambda_{12} + 2r\lambda_{123}] & y_2^2(2r_{12}\lambda_{123}) & y_2^2 r\lambda_{12} \end{pmatrix}$$

This corresponds to $\theta_{13} = 0$ and $\theta_{23} = \pi/4$

Recall, for $\theta_{13} = 0$ and $\theta_{23} = \pi/4$:

$$M_{\nu L}^{flavour} = \begin{pmatrix} a & c & c \\ c & b & d \\ c & d & b \end{pmatrix}.$$

Here, $\lambda_{12} = \lambda_1 + \lambda_2$ and $\lambda_{123} = \lambda_3 + \lambda_1 - \lambda_2$.

Results

For $v_1 = v_2 = v$ **and** $r_{22} = r_{11} + \epsilon$ **we get:** $M_{\nu_L}^{flavour} = M^0 + M'$ **where,**

$$M^0 = \begin{pmatrix} y_1^2[4r_{12}\lambda_{123} + 2r_{11}\lambda_{12}] & y_1y_2[r_{12}\lambda_{12} + 2r_{11}\lambda_{123}] & y_1y_2[r_{12}\lambda_{12} + 2r_{11}\lambda_{123}] \\ y_1y_2[r_{12}\lambda_{12} + 2r_{11}\lambda_{123}] & y_2^2 r_{11}\lambda_{12} & y_2^2(2r_{12}\lambda_{123}) \\ y_1y_2[r_{12}\lambda_{12} + 2r_{11}\lambda_{123}] & y_2^2(2r_{12}\lambda_{123}) & y_2^2 r_{11}\lambda_{12} \end{pmatrix} \text{ and}$$

$$M' = \epsilon \begin{pmatrix} x & y & 0 \\ y & x' & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here, $x = y_1^2 v^2$, $x' = y_2^2 v^2$ and $y = y_1 y_2 v^2 \lambda_{123}$. Note, M^0 and M' are symmetric owing to the Majorana nature and we define $M^0_{11} \equiv a'$, $M^0_{22} = M^0_{33} \equiv b'$, $M^0_{12} = M^0_{13} \equiv c'$ and $M^0_{23} \equiv d'$.

Thus M^0 corresponds to $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. M' gives $\theta_{13} \neq 0$ and $\theta_{23} \neq \pi/4$ and small corrections to solar mixing angle.

The third first-order corrected ket:

$$|\psi_3\rangle = \begin{pmatrix} \frac{\epsilon}{\gamma^2 - \rho^2}\left[\rho(\sqrt{2}y \cos 2\theta^0_{12} - x' \sin 2\theta^0_{12}) - \gamma\sqrt{2}y\right] \\ -\frac{1}{\sqrt{2}}[1 + \xi\epsilon] \\ \frac{1}{\sqrt{2}}[1 - \xi\epsilon] \end{pmatrix}.$$

Thus,

$$\sin\theta_{13} = \frac{\epsilon}{\gamma^2 - \rho^2}\left[\rho(\sqrt{2}y \cos 2\theta^0_{12} - x' \sin 2\theta^0_{12}) - \gamma\sqrt{2}y\right],$$

$$\tan\varphi \equiv \tan(\theta_{23} - \pi/4) = \xi\epsilon,$$

$$\tan\theta_{12} = \frac{\sin\theta^0_{12} + \epsilon\zeta\cos\theta^0_{12}}{\cos\theta^0_{12} - \epsilon\zeta\sin\theta^0_{12}}.$$

with,

$$\gamma \equiv (b' - 3d' - a') \text{ and } \rho \equiv \sqrt{a'^2 + b'^2 + 8c'^2 + d'^2 - 2a'b' - 2a'd' + 2b'd'},$$

$$\xi \equiv [\gamma x' + \rho(x' \cos 2\theta^0_{12} + \sqrt{2}y \sin 2\theta^0_{12})]/(\gamma^2 - \rho^2),$$

$$\zeta \equiv \frac{\left[\frac{y}{\sqrt{2}}\cos 2\theta^0_{12} + \frac{1}{2}(x - \frac{x'}{2})\sin 2\theta^0_{12}\right]}{\rho}.$$

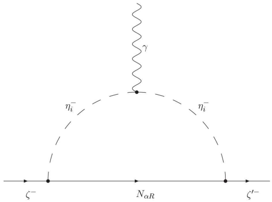
Conclusions

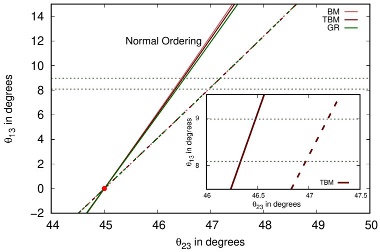
- Neutrino masses and mixings in general.
- A short discussion on discrete flavour symmetry: $S3$.
- Realistic neutrino mixing radiatively at one-loop level using $S3 \times Z_2$ symmetry.
- Two right-handed neutrinos, maximally mixed to produce the structure of the left-handed Majorana neutrino mass matrix characterized by $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and any value of θ_{12}^0 particular to the Tribimaximal (TBM), Bimaximal (BM) and Golden Ratio (GR) or other mixings.
- Small deviation from this maximal mixing between the two right-handed neutrinos could generate non-zero θ_{13}, shifts of the atmospheric mixing angle θ_{23} from $\pi/4$ and correct the solar mixing angle θ_{12} by a small amount.
- Two Z_2 odd inert scalar $SU(2)_L$ doublets were used, the lightest of which can serve as a dark matter candidate.

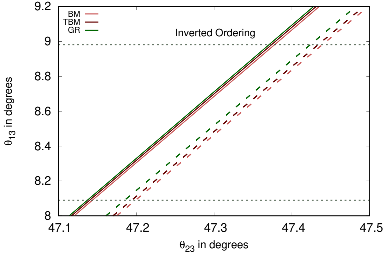
Thank you

Backup Slides

Miscellaneous







Scalar Potential

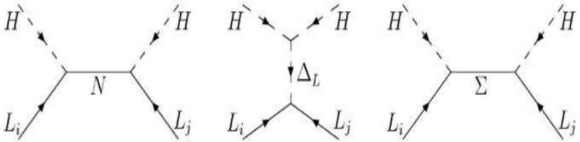
The scalar sector of the model as can be seen from Table. 2, comprises of two inert $SU(2)_L$ doublets, $\eta_i \equiv (\eta_i^+ \eta_i^0)^T$, $(i = 1, 2)$, forming a doublet under $S3$ denoted by η and two other $SU(2)_L$ doublet scalar fields $\Phi_j \equiv (\phi_j^+ \phi_j^0)^T$, $(j = 1, 2)$, represented by Φ, transforming as a doublet under $S3$. Under the unbroken Z_2, η is odd whereas Φ is even. Thus after SSB, ϕ_j^0 can acquire vevs v_j,$(j = 1, 2)$, but the η_i^0 cannot. The complete scalar potential consisting of all the terms allowed by the SM gauge symmetry and $S3 \times Z_2$ is given by:

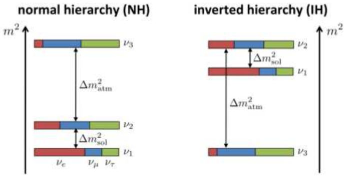
$$
\begin{aligned}
V_{total} &= m_\eta^2 \left(\eta_2^\dagger \eta_2 + \eta_1^\dagger \eta_1\right) + m_\phi^2 \left(\phi_2^\dagger \phi_2 + \phi_1^\dagger \phi_1\right) \\
&+ \widetilde{\lambda}_1 \left(\eta_2^\dagger \eta_2 + \eta_1^\dagger \eta_1\right)^2 + \widetilde{\lambda}_2 \left(\eta_2^\dagger \eta_2 - \eta_1^\dagger \eta_1\right)^2 + \widetilde{\lambda}_3 \left(\phi_2^\dagger \phi_2 + \phi_1^\dagger \phi_1\right)^2 + \widetilde{\lambda}_4 \left(\phi_2^\dagger \phi_2 - \phi_1^\dagger \phi_1\right)^2 \\
&+ \widetilde{\lambda}_5 \left[\left(\eta_2^\dagger \eta_2 + \eta_1^\dagger \eta_1\right)\left(\phi_2^\dagger \phi_2 + \phi_1^\dagger \phi_1\right)\right] + \widetilde{\lambda}_6 \left[\left(\eta_2^\dagger \eta_2 - \eta_1^\dagger \eta_1\right)\left(\phi_2^\dagger \phi_2 - \phi_1^\dagger \phi_1\right)\right] \\
&+ \widetilde{\lambda}_7 \left[\left(\phi_1^\dagger \phi_2\right)\left(\phi_2^\dagger \phi_1\right)\right] + \widetilde{\lambda}_8 \left[\left(\eta_1^\dagger \eta_2\right)\left(\eta_2^\dagger \eta_1\right)\right] \\
&+ \widetilde{\lambda}_9 \left[\left\{\left(\phi_1^\dagger \phi_2\right)\left(\eta_2^\dagger \eta_1\right)\right\} + \left\{\left(\phi_2^\dagger \phi_1\right)\left(\eta_1^\dagger \eta_2\right)\right\}\right] + V_{relevant}
\end{aligned}
\qquad (B.1)
$$

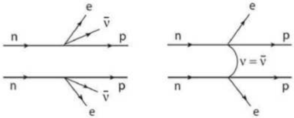
where,

$$
\begin{aligned}
V_{relevant} &= \lambda_1 \left[\left\{(\eta_2^\dagger \phi_2 + \eta_1^\dagger \phi_1)^2\right\} + h.c.\right] + \lambda_2 \left[\left\{(\eta_2^\dagger \phi_2 - \eta_1^\dagger \phi_1)^2\right\} + h.c.\right] \\
&+ \lambda_3 \left[\left\{(\eta_1^\dagger \phi_2)(\eta_2^\dagger \phi_1) + (\eta_2^\dagger \phi_1)(\eta_1^\dagger \phi_2)\right\} + h.c.\right].
\end{aligned}
\qquad (B.2)
$$

Since at the four-point scalar vertex in Fig. 1, two ϕ are destroyed and two η are created, the terms only of $(\eta^\dagger \phi)(\eta^\dagger \phi)$ type play a crucial role in determining the neutrino mass matrix. Thus we call these terms as the relevant part of the scalar potential, represented by $V_{relevant}$ in Eq. (B.2). The quartic couplings λ_j $(j = 1, 2, 3)$ appearing in Eq. (B.2) were taken to be real for the analysis.







Golden Ratio:

$$\frac{a+b}{a} = \frac{a}{b} \stackrel{\text{def}}{=} \varphi, \qquad \varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887$$

$\theta_{12} = 31.7^\circ$ for GR mixing $\Rightarrow \frac{\cos 31.7^\circ}{\sin 31.7^\circ} = 1.618..$

Seesaw in brief

Extend the SM by a singlet RH neutrino N_R per family.

Neutrino Majorana mass term: $m\psi^T_{L(R)}C^{-1}\psi_{L(R)}$

$$\Downarrow$$

$$\mathscr{L}_{mass} = \frac{1}{2}\alpha_L^T C^{-1}\mathscr{M}_{D+M}\alpha_L + h.c.$$

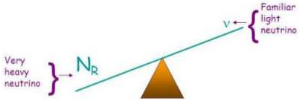
where, $\mathscr{M}_{D+M} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix} \Rightarrow 6 \times 6$ matrix and $\alpha_L = \begin{pmatrix} \nu_L \\ C(\bar{N}_R)^T \end{pmatrix}$

$\searrow$

Diagonalize: $W^T\mathscr{M}_{D+M}W = \begin{pmatrix} M_{light} & 0 \\ 0 & M_{heavy} \end{pmatrix}$

$M_{light} = M_D^T M_R^{-1} M_D$ and $M_{heavy} = M_R$

$\Rightarrow$



Can be done in three ways:

Type I	Type II	Type III
Fermion Singlet	Scalar Triplet	Fermion Triplet

Popular lepton mixings

Recall:

$$U_{PMNS} \equiv V_l^\dagger U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

$\downarrow$

θ_{13}=0, θ_{23}=$\pi/4$

$\downarrow$

$$U^0 = \begin{pmatrix} \cos\theta_{12}^0 & \sin\theta_{12}^0 & 0 \\ -\frac{\sin\theta_{12}^0}{\sqrt{2}} & \frac{\cos\theta_{12}^0}{\sqrt{2}} & \sqrt{\frac{1}{2}} \\ \frac{\sin\theta_{12}^0}{\sqrt{2}} & -\frac{\cos\theta_{12}^0}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \rightarrow \theta_{12}^0 = 0^\circ (\text{NSM}) \rightarrow U^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$\Downarrow$

Studied in $A4$ case

θ_{12}^0 : 35.26° [TriBimaximal(TBM)], 45° [Bimaximal (BM)], 31.7° [Golden Ratio(GR)].

$$\begin{aligned} \Delta m_{21}^2 &= (7.02 - 8.08) \times 10^{-5}\,\text{eV}^2, \;\; \theta_{12} = (31.52 - 36.18)^\circ, \\ |\Delta m_{31}^2| &= (2.351 - 2.618) \times 10^{-3}\,\text{eV}^2, \;\; \theta_{23} = (38.6 - 53.1)^\circ, \\ \theta_{13} &= (7.86 - 9.11)^\circ, \;\; \delta = (0 - 360)^\circ \;\; . \end{aligned}$$

Amendment Required!!

Neutrino Oscillations

A Purely Quantum Mechanical Phenomenon.

Oscillation conserves probability hence *Hamiltonian* is *Hermitian*.$\Rightarrow$ Diagonalized by Unitary transformation (U).

Consider two flavors oscillation first:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \overset{U_{2\times 2}}{\Longrightarrow} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Flavor Eigenstates Stationary States

Flavor Eigenstates: Participate in weak interactions

Stationary States: Mass eigenstates and are admixtures of the flavor states.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$P_{\nu_e \to \nu_e}(t)$: **Probability of ν_e emitted from the source to remain an ν_e after time t.**

Calculate Time Evolution :

$$|\nu_i(t)\rangle = e^{-iE_i t}|\nu_i(0)\rangle \ (i = 1, 2) \text{ in Natural units.}$$

Oscillation continued ...

$$P_{\nu_e \to \nu_e}(t) \equiv |\langle \nu_e(t)|\nu_e(0)\rangle|^2 = 1 - \sin^2 2\theta \sin(\frac{\Delta m^2 L}{4E})$$

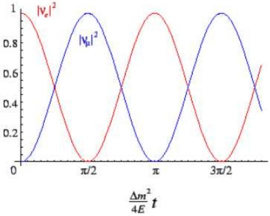
$L \to$ distance between source and detector.

In ultra relativistic limit: $E_i - E_j = (m_i^2 - m_j^2)/2E = \frac{\Delta m^2}{2E}$

$$P_{\nu_e \to \nu_\mu}(t) = 1 - P_{\nu_e \to \nu_e}(t) = \sin^2 2\theta \sin(\frac{\Delta m^2 L}{4E})$$

Necessary Requirements: $\Delta m^2 \neq 0$ and $\sin^2 2\theta \neq 0$.

Maximal mixing when $\theta = \frac{\pi}{4}$.



Three flavour Oscillation

Oscillation with three flavors: ν_e, ν_μ, ν_τ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\Downarrow$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

The Pontecorvo, Maki, Nakagawa, Sakata – PMNS – matrix

$$U^T M_\nu U = \mathrm{diag}(m_1, m_2, m_3)$$

Oscillation probability

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U^*_{\alpha j} U^*_{\beta j} \sin^2\left(\frac{\pi L}{\lambda_{ij}}\right)$$

2 independent Δm^2, 3 mixing angles, 1 phase