Resurrecting Low-Mass Axion Dark Matter via a Dynamical QCD Scale

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in collaboration with

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Strong CP problem

In QCD, the θ-term

$$\theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \in \mathcal{L}_{QCD}$$

violates CP and induces a neutron electric dipole moment

$$\bar{\theta} = \theta + \arg(\det(\mathcal{M}))$$

$\mathcal{M}$: quark mass matrix

$$d_n \simeq 5\times10^{-16}\,\bar{\theta}\ \mathrm{e}\cdot\mathrm{cm}$$

However, experimental constraints show $d_n \lesssim \mathrm{few}\times10^{-26}\mathrm{e}\cdot\mathrm{cm}$, which suggests

$$\bar{\theta} < 10^{-10}$$

STRONG CP PROBLEM

QCD axion

One good solution to the strong CP problem is the QCD axion:

$$C_a \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

- a: axion field, Nambu-Goldstone boson for spontaneously broken $U(1)_{PQ}$, acquires small mass because $U(1)_{PQ}$ is explicitly broken by chiral anomaly
- f_a: scale for spontaneous breaking of $U(1)_{PQ}$
- the potential of a is minimized by

$$a = -\frac{\bar{\theta} f_a}{C_a}$$

thus, the effective $\bar{\theta}$ angle is dynamically driven to zero

Production

Apart from solving the strong CP problem, the QCD axion is also a good dark-matter candidate, typically produced via three different mechanisms:

- Thermal production
- Misalignment production
- Decay of Axionic strings

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For misalignment production, the axion energy density can be obtained by solving its equation of motion

$$\ddot{a} + 3H\dot{a} + \frac{\partial V_{PQ}}{\partial a} = 0 \quad \Rightarrow \quad \rho_a = \frac{1}{2}\dot{a}^2 + V_{PQ}, \qquad P_a = \frac{1}{2}\dot{a}^2 - V_{PQ}$$

$$V_{PQ} \approx \frac{1}{2} m_a^2 a^2$$

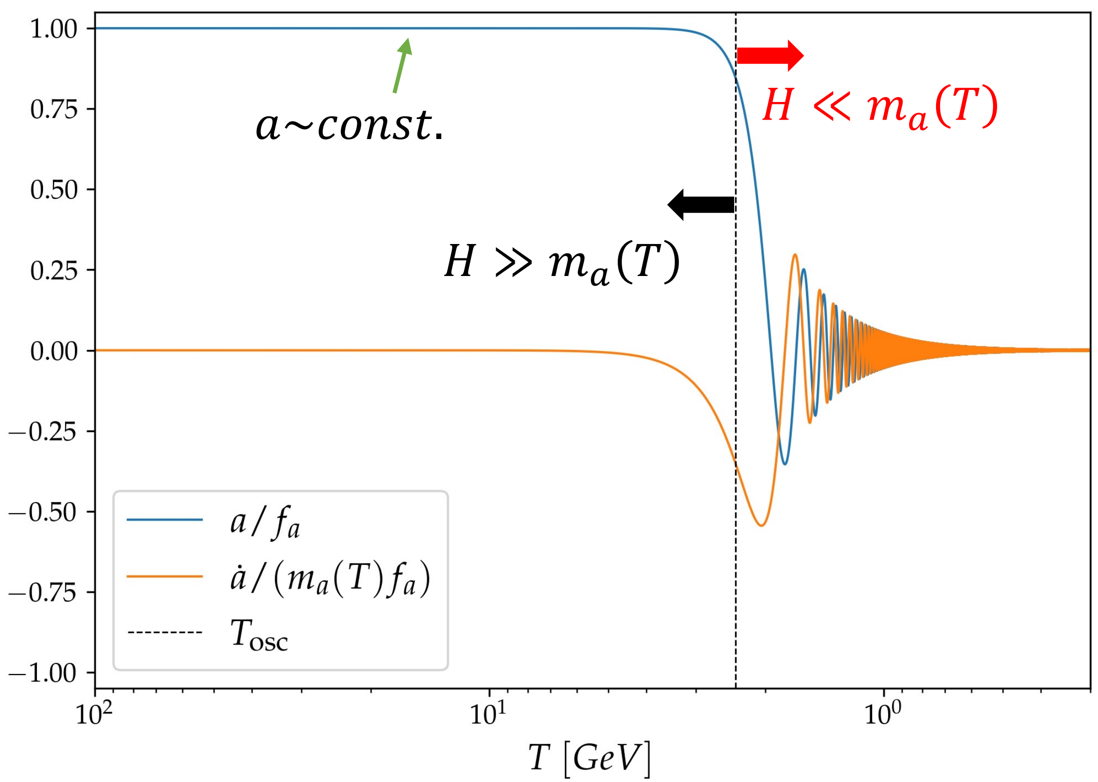
for $\theta = \frac{a}{f_a} \lesssim \mathcal{O}(1)$

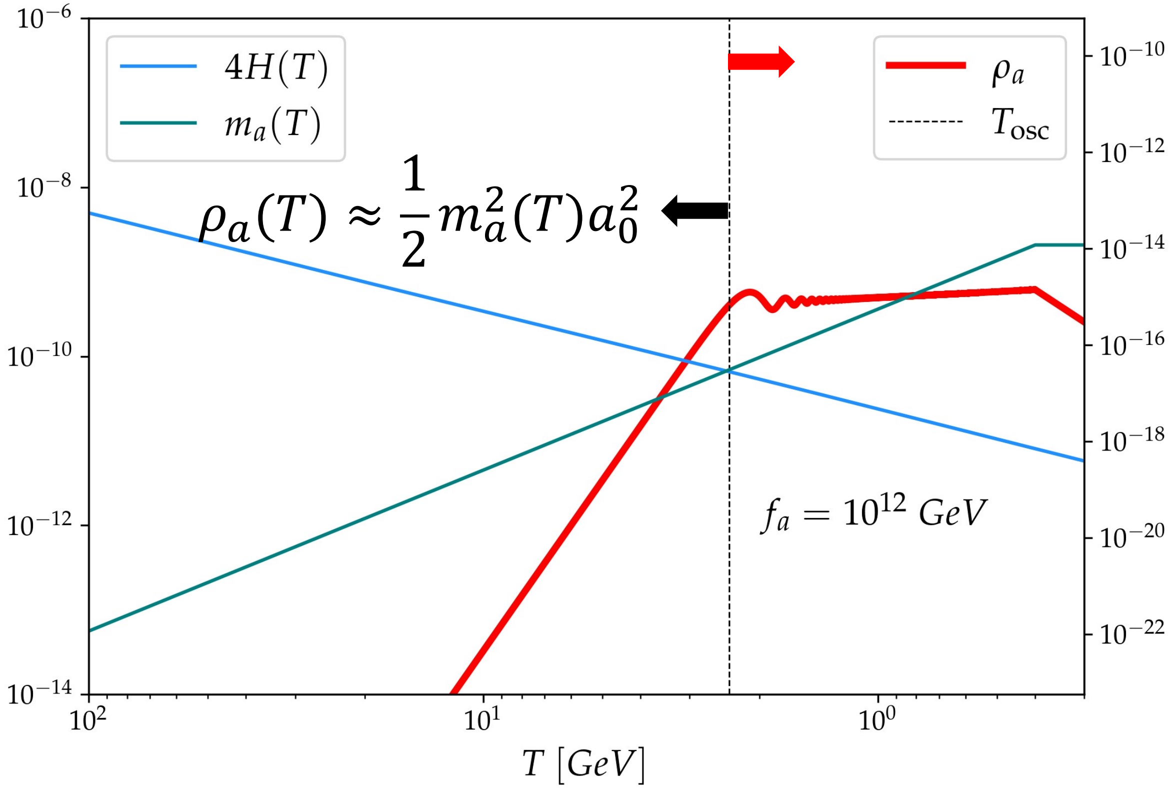
Misalignment production

General behavior:

- $H \gg m_a$, a is overdamped, $\rho_a \approx -P_a \approx V_{PQ} \sim m_a^2$, scales like vacuum energy
- $H \ll m_a$, a rapidly oscillating, virial theorem $\Rightarrow \langle P_a \rangle \approx 0$, scales like pressureless matter

$$\rho_a \approx \frac{\rho_a(T')}{m_a(T')}\left(\frac{R(T')}{R(T)}\right)^3 m_a(T)$$





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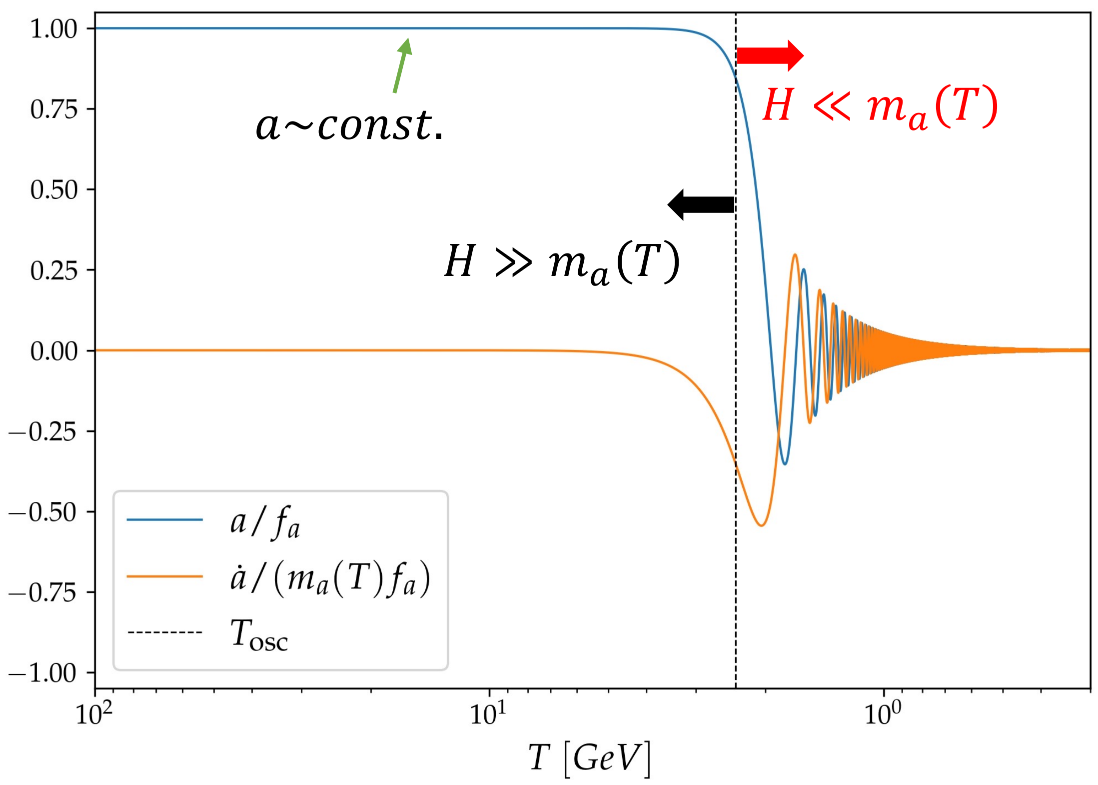

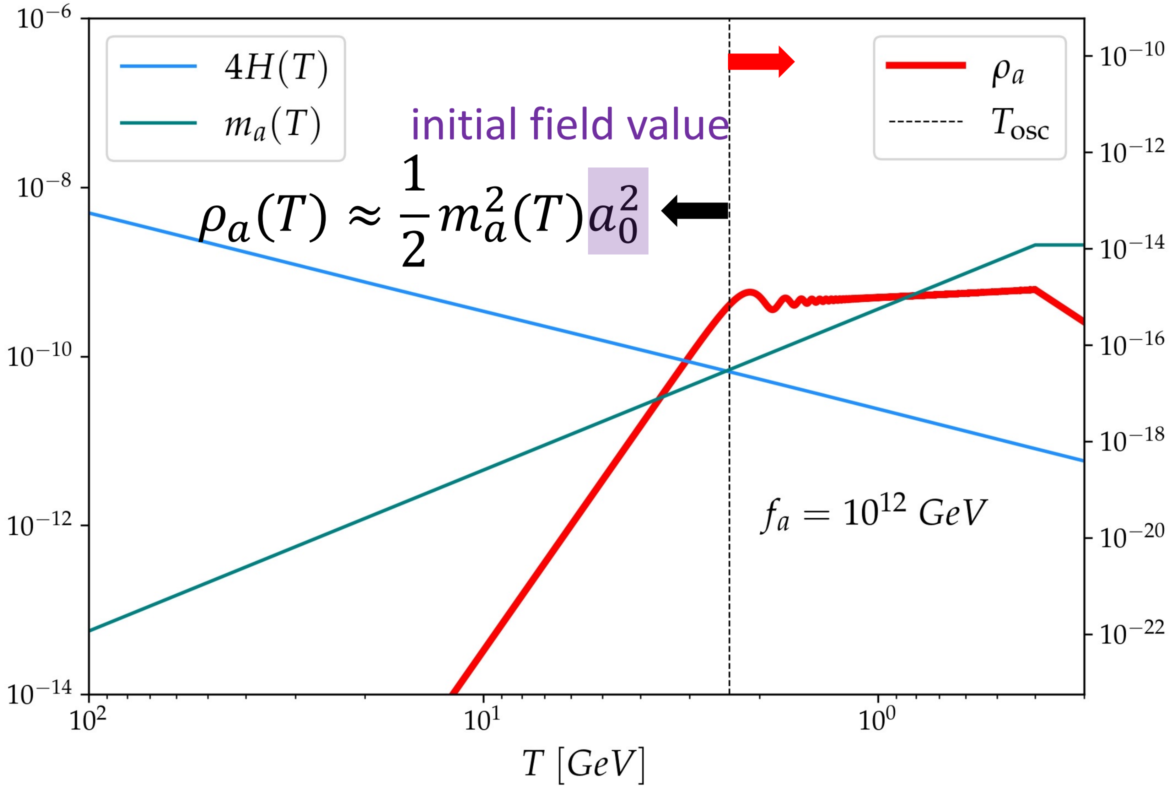

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number density   cosmic expansion

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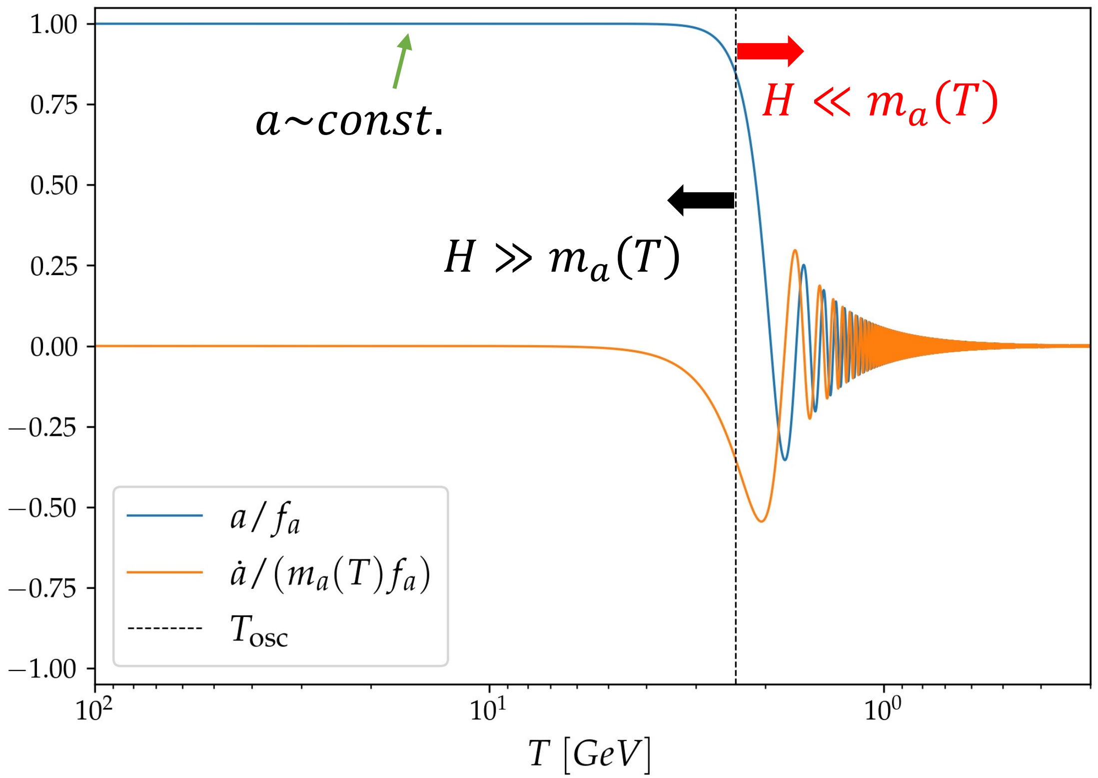

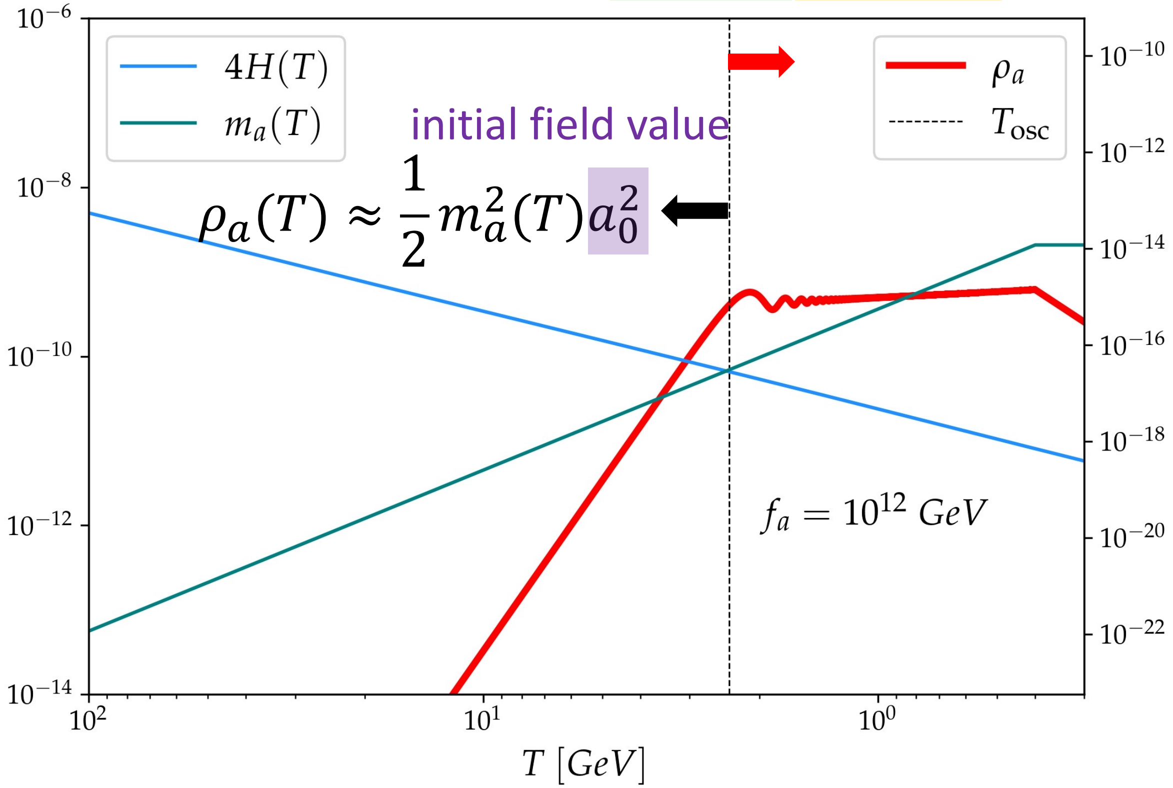

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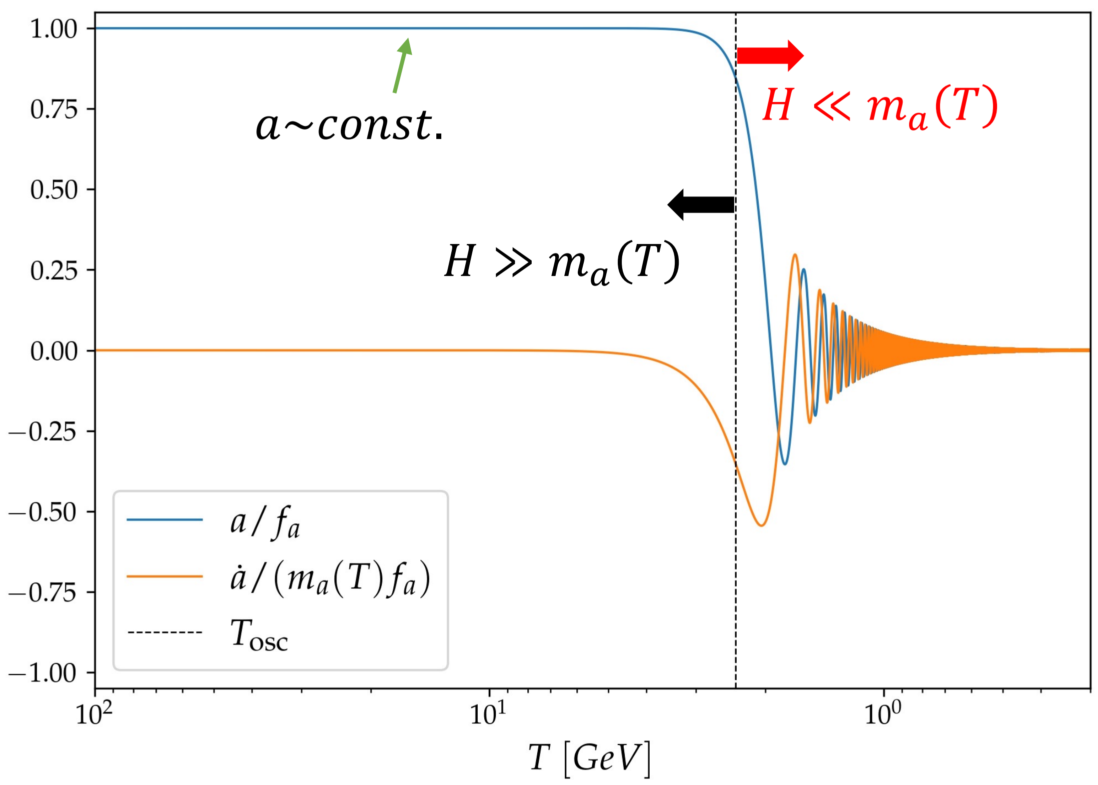

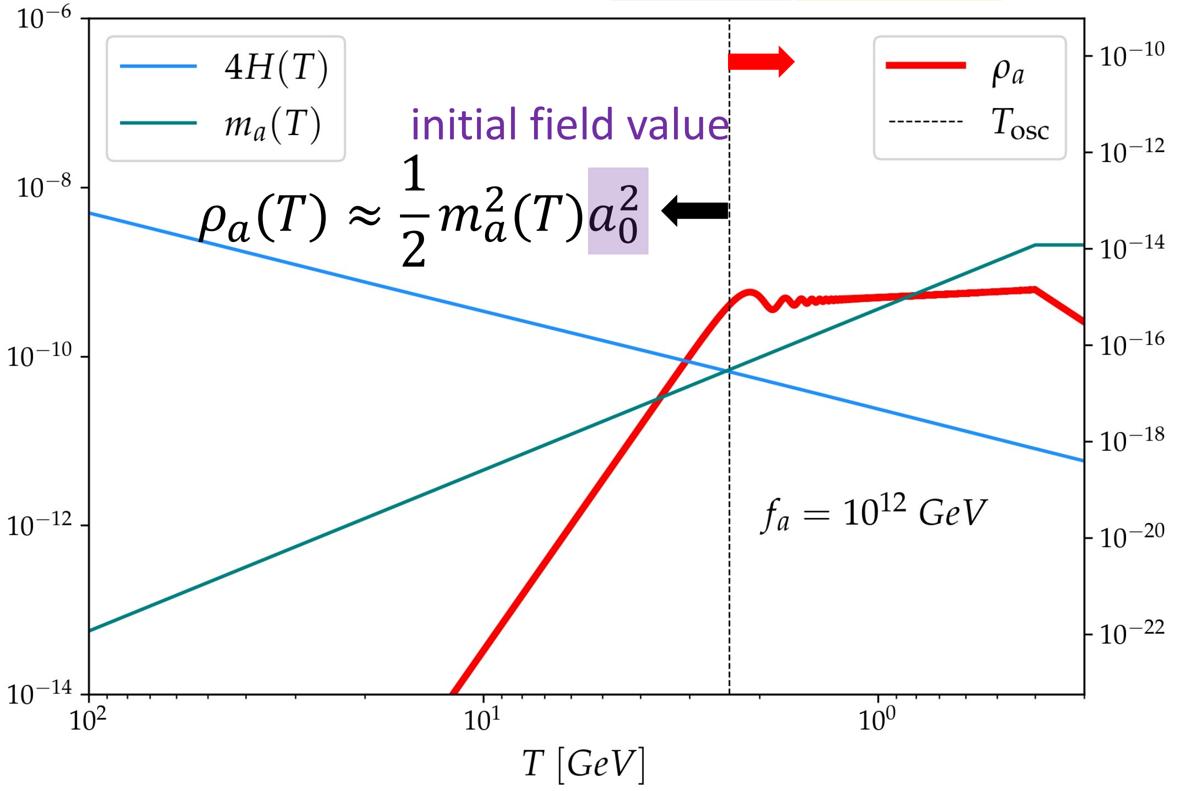

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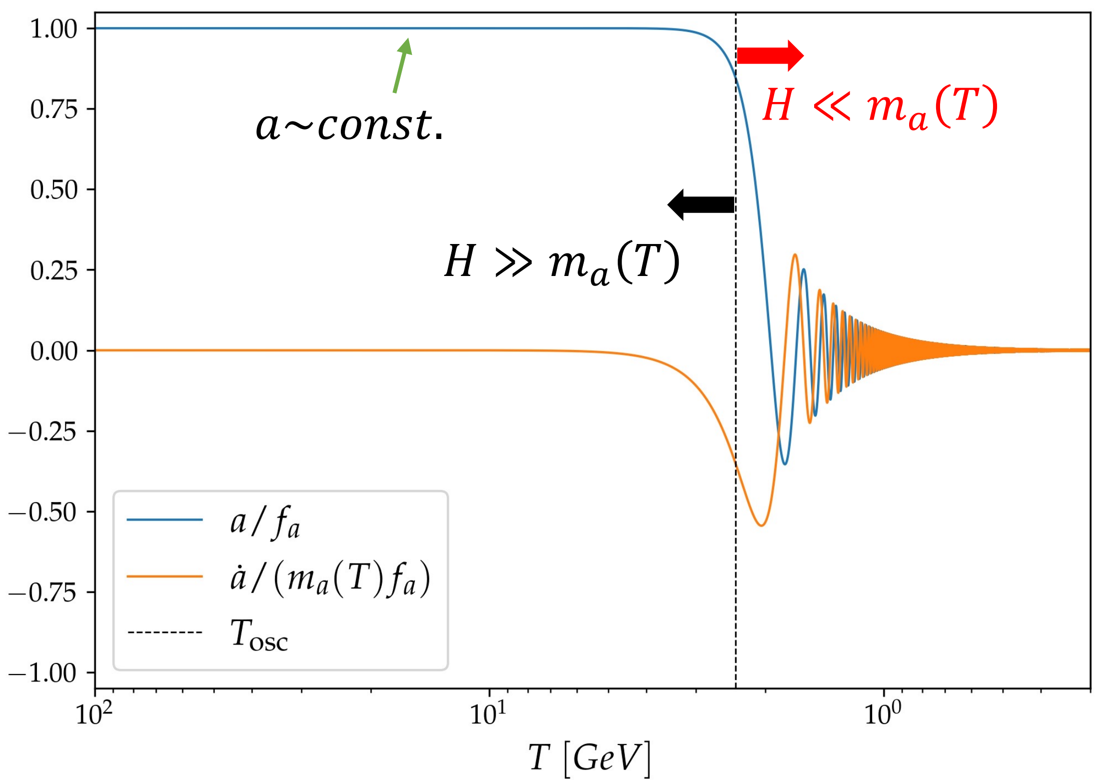
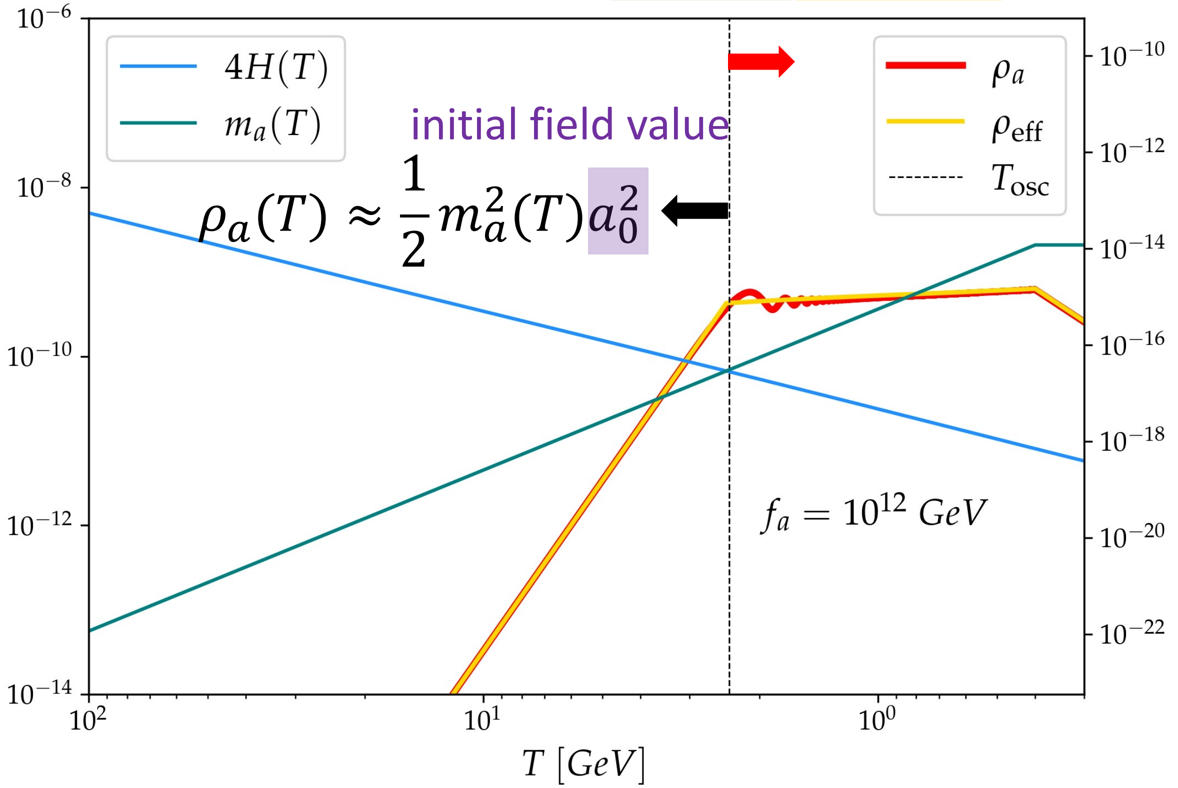




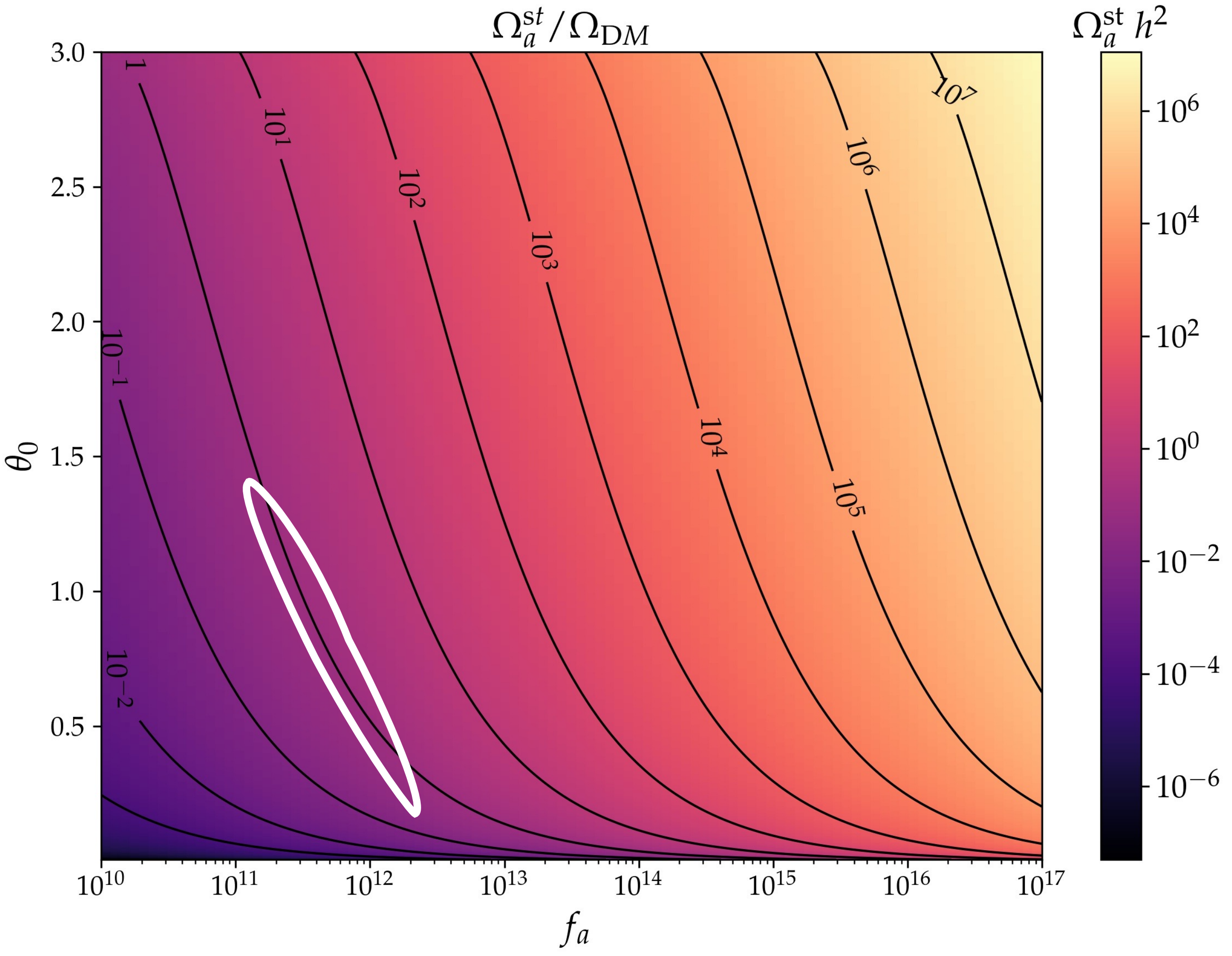
Relic abundance

D. Marsh, Phys.Rept. 643 (2016)

QCD axion relic abundance:

$$\Omega_a h^2 \approx 2\times10^4 \left(\frac{f_a}{10^{16}\ GeV}\right)^{7/6} \theta_0^2 \mathcal{F}(\theta_0)$$

- for $f_a \sim 10^{12}$ GeV, easy to obtain $\Omega_a \sim \Omega_{DM}$ for $\theta_0 \sim \mathcal{O}(1)$
- however, having larger f_a leads to overclosure unless θ_0 is fine-tuned



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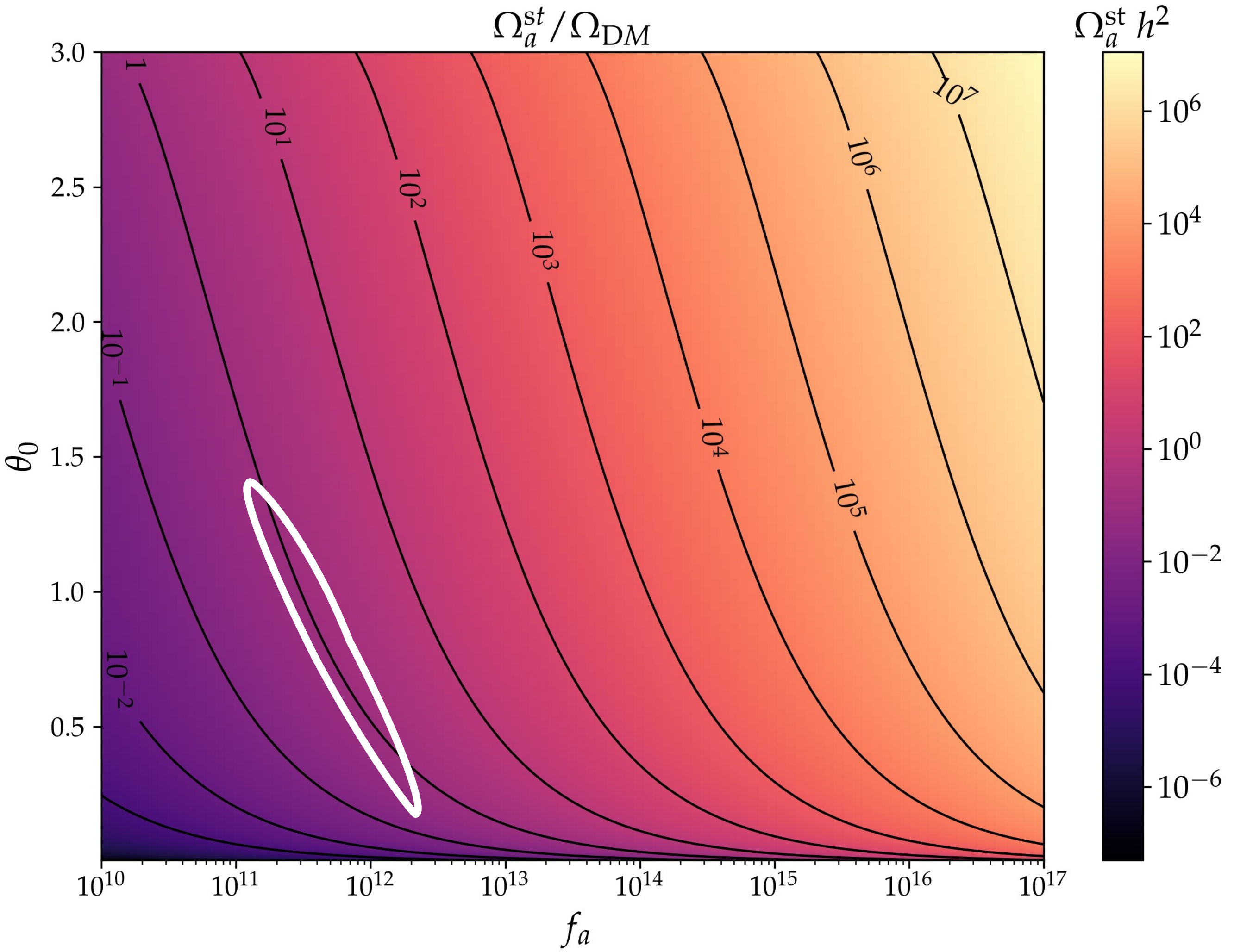

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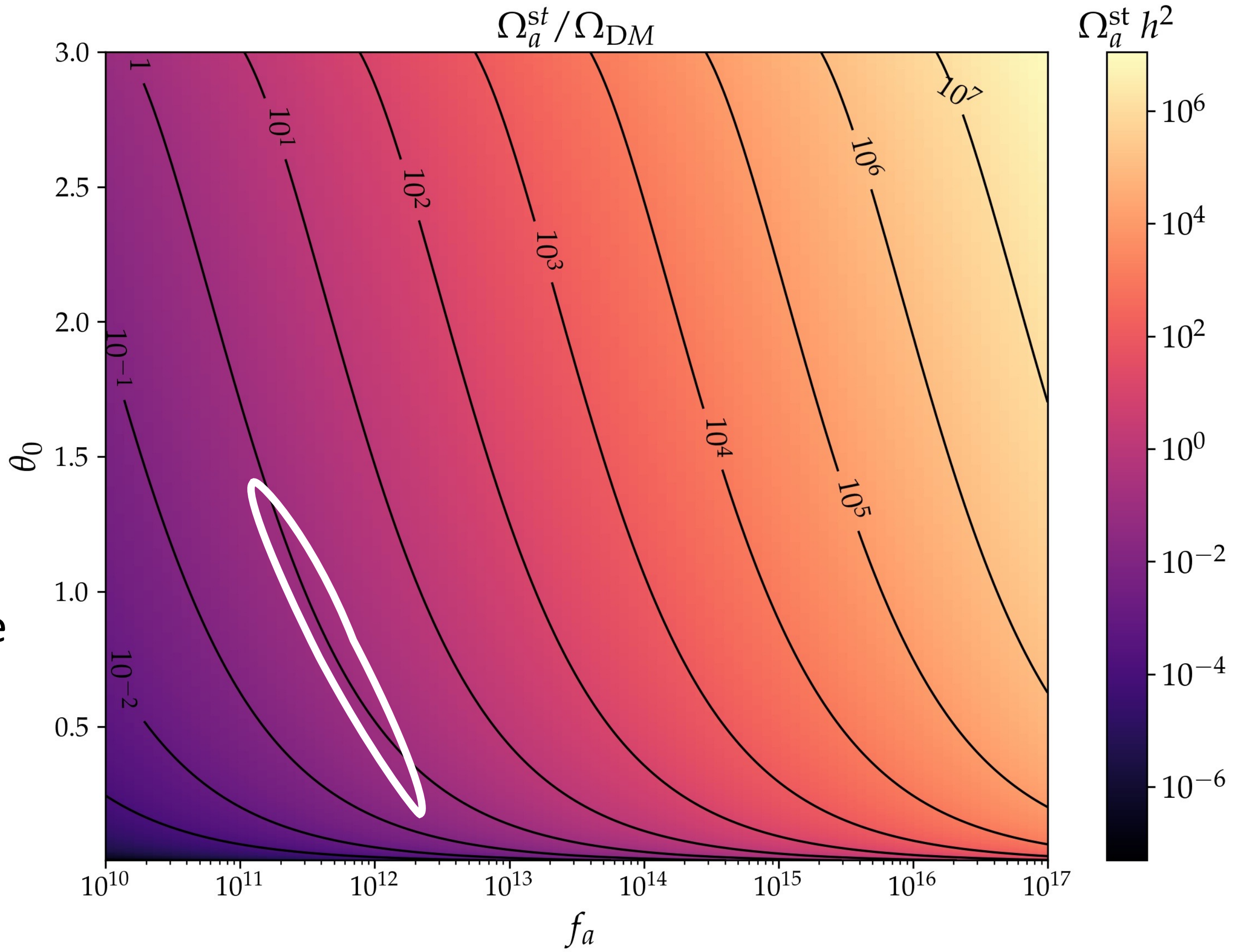
- If $m_a(T)$ evolution modified, a will start/stop oscillating at a different time, thus changing its relic abundance.



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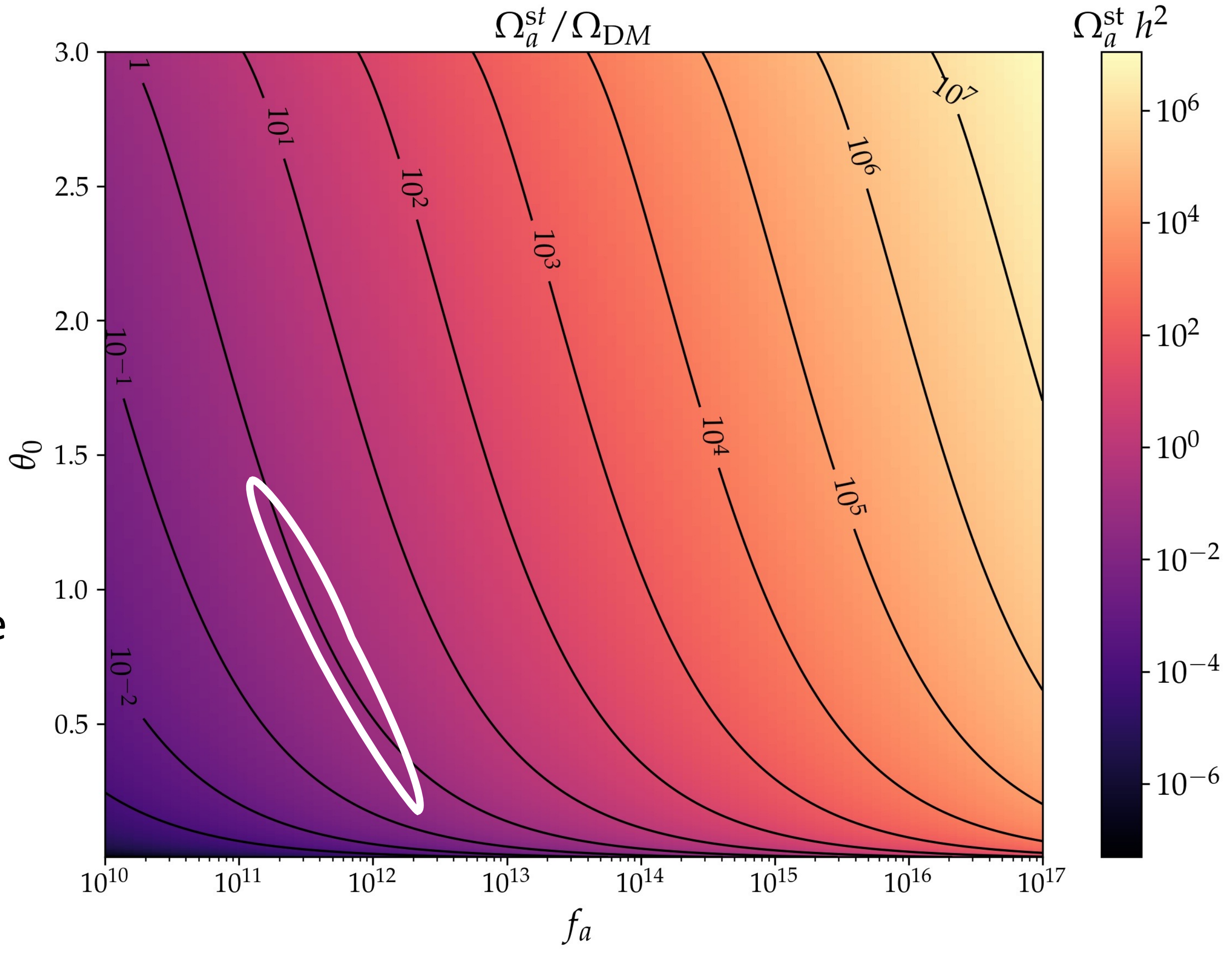

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How to realize such a scenario?

Mass evolution

Take a closer look at $m_a(T)$

$$m_a^2(T) f_a^2 \sim \begin{cases} m_\pi^2 f_\pi^2 \,, & T > \Lambda_{QCD} \\ m_\pi^2 f_\pi^2 \left(\frac{\Lambda_{QCD}}{T}\right)^n, & T < \Lambda_{QCD} \end{cases}$$

← $n \approx 6.68$

$m_a(T)$ evolution would be modified if Λ_{QCD} is different from its SM value in the early universe.

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$$m_\pi^2 f_\pi^2 \simeq m_{\pi 0}^2 f_{\pi 0}^2 \left(\frac{v_h}{v_h^0}\right)\left(\frac{\Lambda_{QCD}}{\Lambda_{QCD}^0}\right)^3$$

Obviously, the axion mass could receive large enhancement from having a larger Λ_{QCD}.

How to modify Λ_{QCD}?

Dynamical QCD scale

Promote the strong coupling to a dynamical quantity

$$-\frac{1}{4}\left(\frac{1}{g_{s0}^2}+\frac{\phi}{M_\star}\right)G_{\mu\nu}G^{\mu\nu}$$

S. Ipek, T. Tait, *PRL 122, 112001 (2019)*
D. Croon, J. Howard, S. Ipek, T. Tait *PRD 101, 055042 (2020)*

- ϕ: scalar, SM singlet
- $M_\star$: UV scale
- Strong coupling larger if $\langle\phi\rangle < 0$; restores to SM value g_{s0} if $\langle\phi\rangle = 0$

With this setup, the strong coupling runs as

$$\frac{1}{\alpha_s(\mu,\langle\phi\rangle)}=\frac{33-2n_f}{6\pi}\ln\frac{\mu}{\Lambda_{QCD}^0}+4\pi\frac{\langle\phi\rangle}{M_\star}$$

Λ_{QCD}^0: SM confinement scale
n_f : # of massless flavors at μ

Asking $\alpha_s^{-1}\to 0$ ⇒ $$\Lambda_{QCD}=\Lambda_{QCD}^0\exp\left(\frac{24\pi^2}{2n_f-33}\frac{\langle\phi\rangle}{M_\star}\right)$$

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Negative for SM particle content

$\Lambda_{QCD}\gg\Lambda_{QCD}^0$, if $\langle\phi\rangle<0$

Confinement scale truly dynamical!

Mass evolution

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QCD confine/deconfine at different times!

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$m_a(T)$ evolution would be modified if Λ_{QCD} is different from its SM value in the early universe. In fact

$$m_\pi^2 f_\pi^2 \simeq m_{\pi 0}^2 f_{\pi 0}^2 \left(\frac{v_h}{v_h^0}\right)\left(\frac{\Lambda_{QCD}}{\Lambda_{QCD}^0}\right)^3$$

Obviously, the axion mass could receive large enhancement from having a larger Λ_{QCD}.

How to modify Λ_{QCD}?

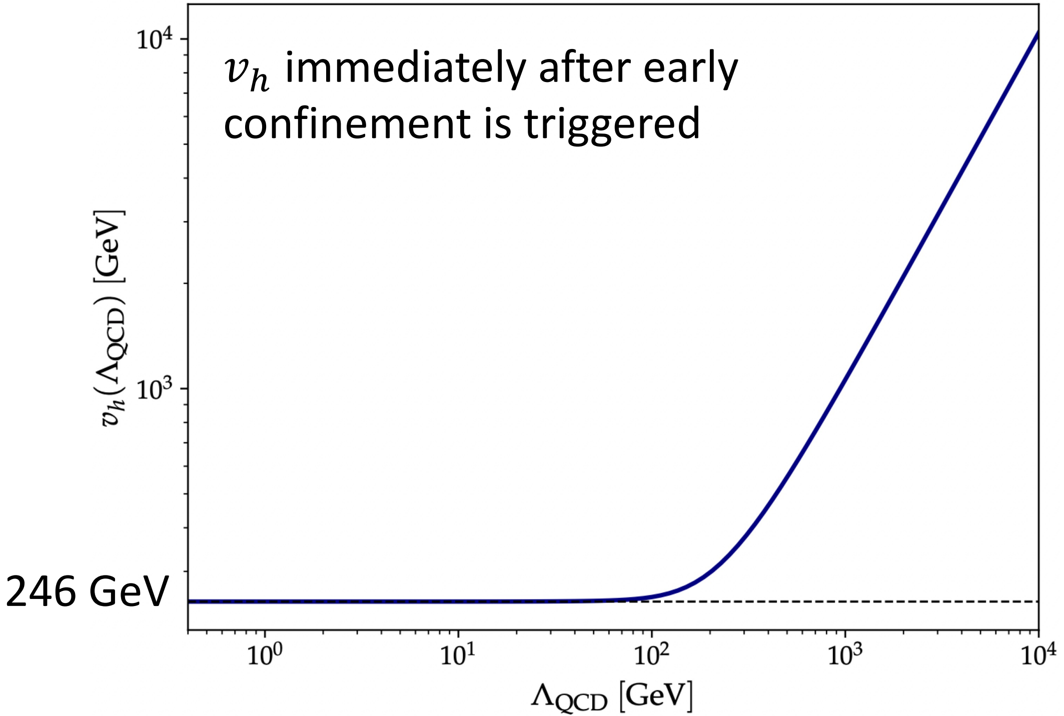
How about the Higgs VEV?

Higgs VEV

The evolution of Higgs potential will also be affected

$$V(h,T) = \begin{cases} V_0(h) + \frac{T^4}{2\pi^2} \sum_{i=h,W,Z,t} (-1)^F n_i J_{B/F}(m_i^2/T^2), & T > \Lambda_{QCD} \text{ or } T < T_d \\ V_0(h) - \sqrt{2}\kappa y_t h + \frac{T^4}{2\pi^2} \sum_{i=h,W,Z,\pi^a} (-1)^F n_i J_{B/F}\left(\frac{m_i^2}{T^2}\right), & \Lambda_{QCD} > T > T_d \end{cases}$$

During early confinement, Higgs VEV is shifted from zero, causing EWSB. (prepare explanations)



approaches SM value as Λ_{QCD} decreases.

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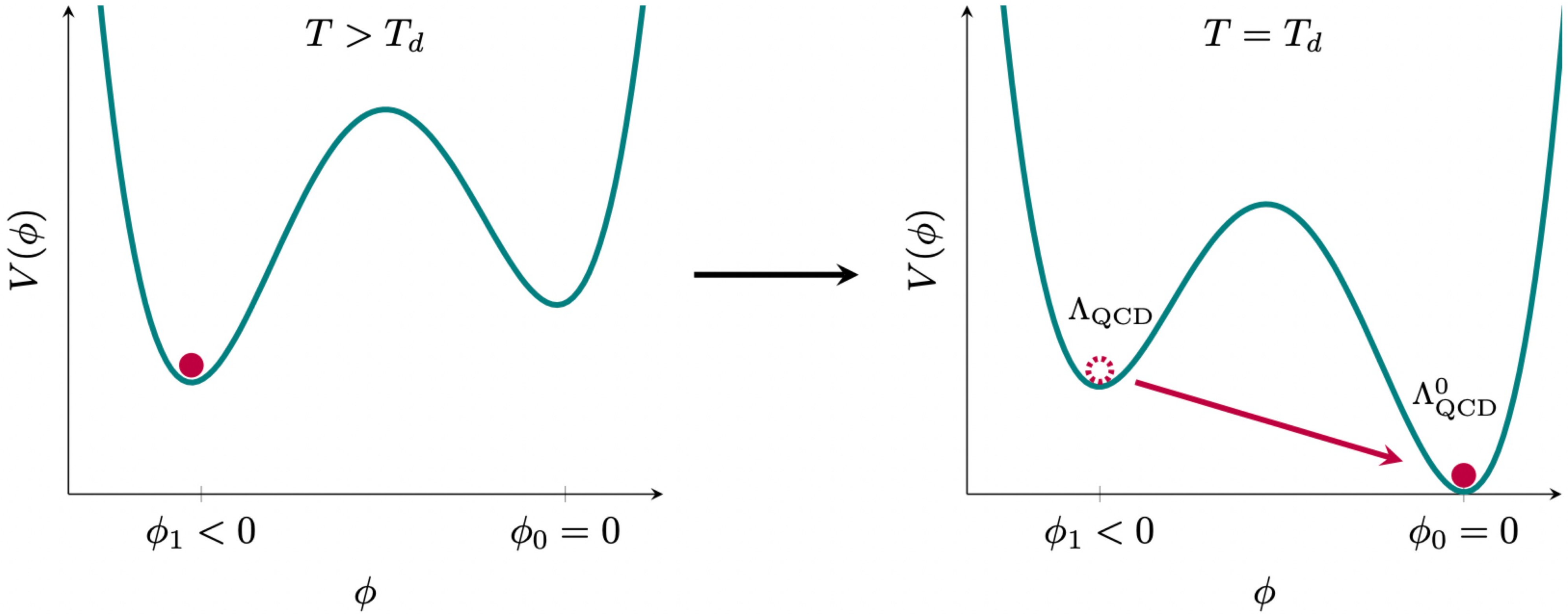
Enhancement from having both a larger Λ_{QCD} and a larger v_h!

General setup

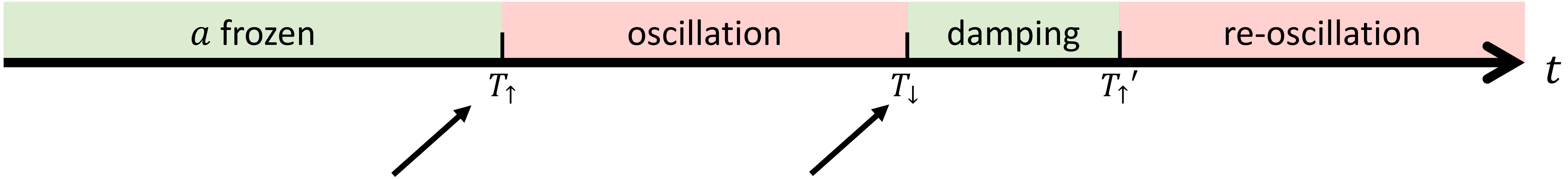


- At earlier times, ϕ in true vacuum with $\langle\phi\rangle < 0$, both Λ_{QCD}, v_h and thus m_a enhanced
- Later, $V(\phi)$ evolves and $\phi = 0$ becomes the true vacuum. ϕ transitions to the true vacuum, m_a decreases and its evolution back to normal
- The transition must occur before BBN

Cosmological evolution of the axion field

Standard picture

Dynamical QCD



- Could be triggered by early confinement if Λ_{QCD} large enough
- If Λ_{QCD} relatively small, could also be between T_{EW}^{0} and Λ_{QCD}

When de-confinement occurs $\sim T_d$

relic abundance suppressed by

$$S = \frac{\Omega_a}{\Omega_a^{st}} \approx \sqrt{\frac{g_\star(T_\downarrow)g_\star^{st}(T_{osc})}{g_\star(T_\uparrow)g_\star(T_\uparrow')}}\frac{T_\uparrow T_{osc}}{T_\uparrow T_\uparrow'}$$

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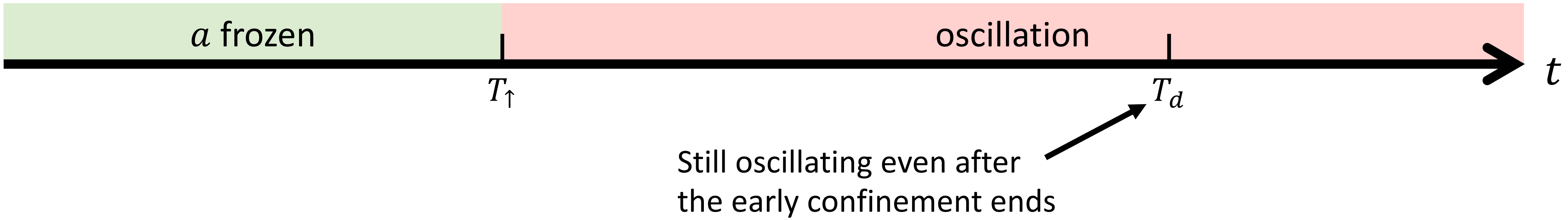

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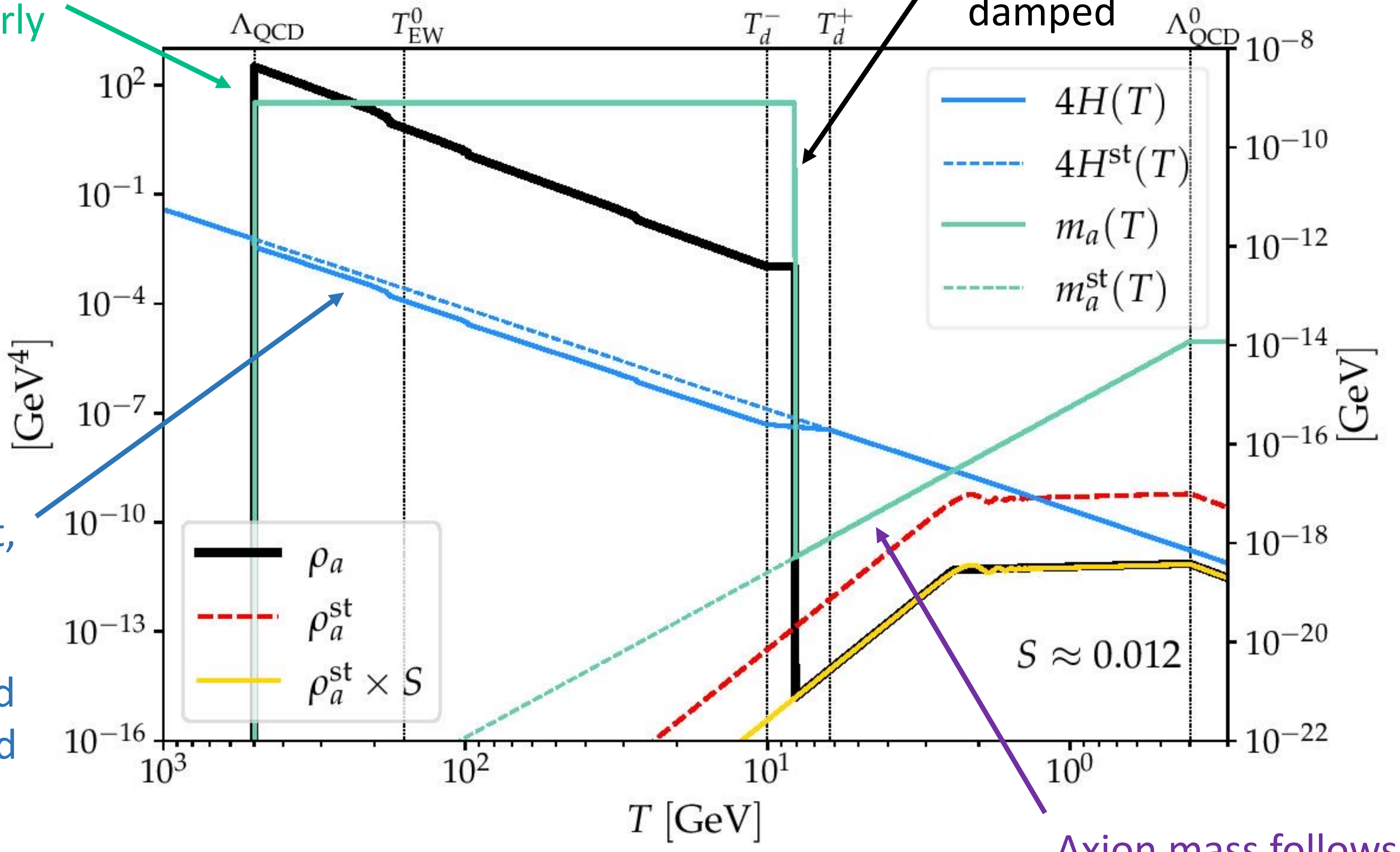
Evolution of energy density: Example 1

Axion mass suddenly increases due to early confinement and oscillation starts

Vacuum decay occurs and QCD de-confines. Axion mass falls below H and axion field becomes damped



During early confinement, d.o.f in strong sector are bound state hadrons instead of free quarks and gluons, thus H is modified since

$$H^2 \approx \frac{8\pi G}{3}\rho_R$$

$$\rho_R = \frac{\pi^2}{30} g_\star(T) T^4$$

Axion mass follows the standard evolution and crosses H later, field oscillation starts again

Evolution of energy density: Example 2

Axion mass suddenly increases due to early confinement and oscillation starts

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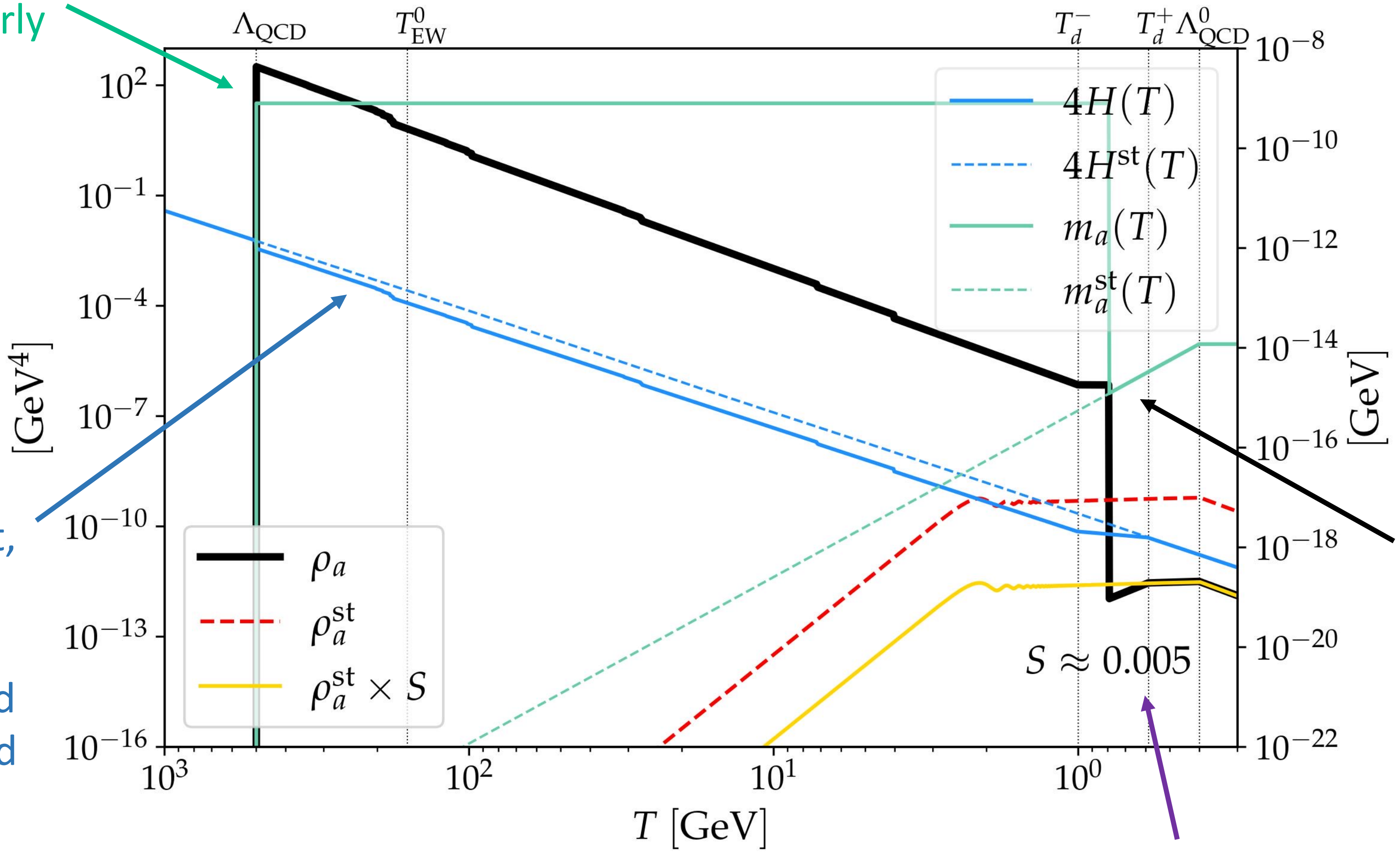


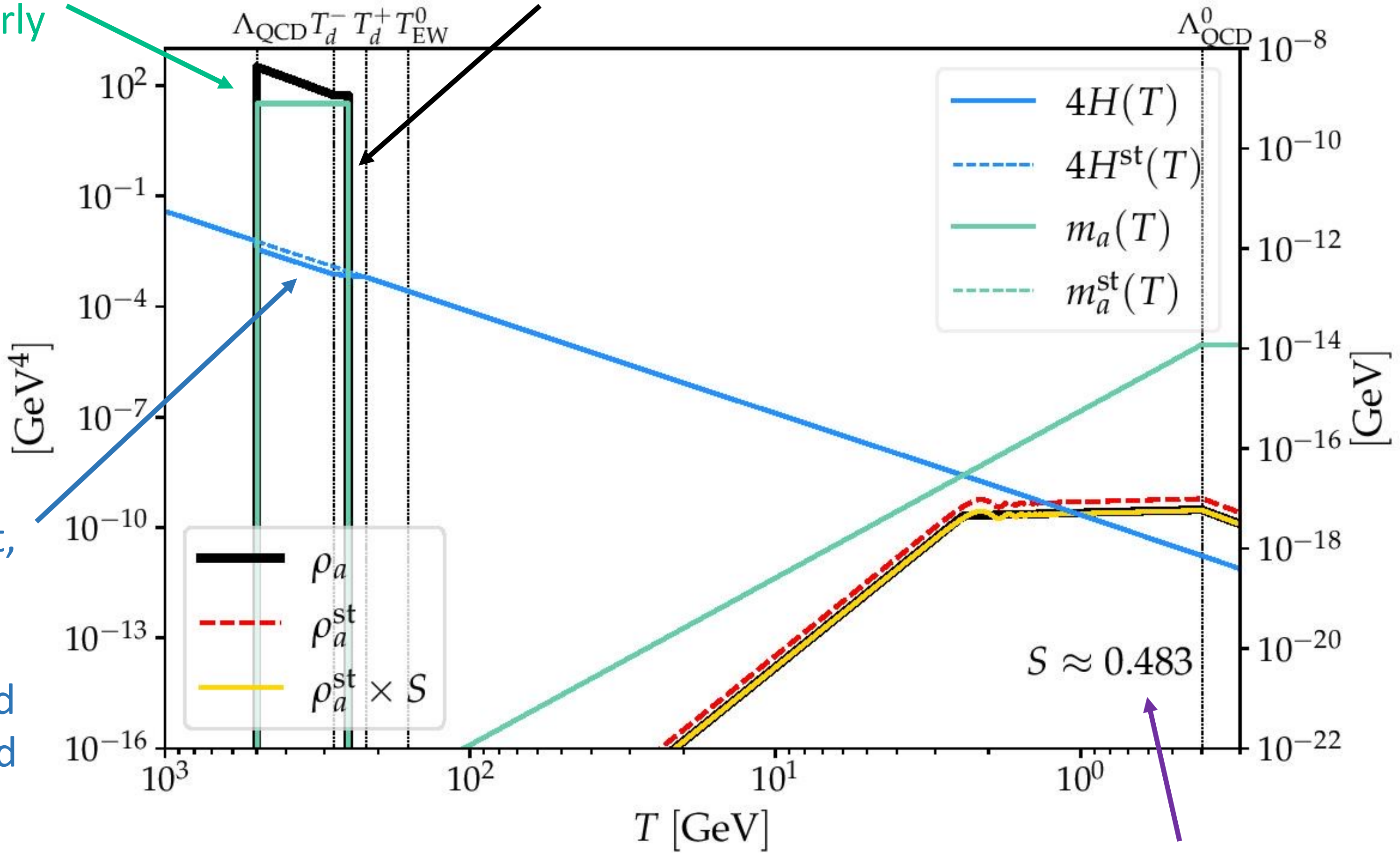
Vacuum decay occurs and QCD de-confines. However, axion mass still above H, axion field keeps oscillating

Suppression relatively strong due to long period of early oscillation

Evolution of energy density: Example 3

Axion mass suddenly increases due to early confinement and oscillation starts

Vacuum decay occurs before SM EWPT, axion mass falls below H. EWPT occurs twice in this case



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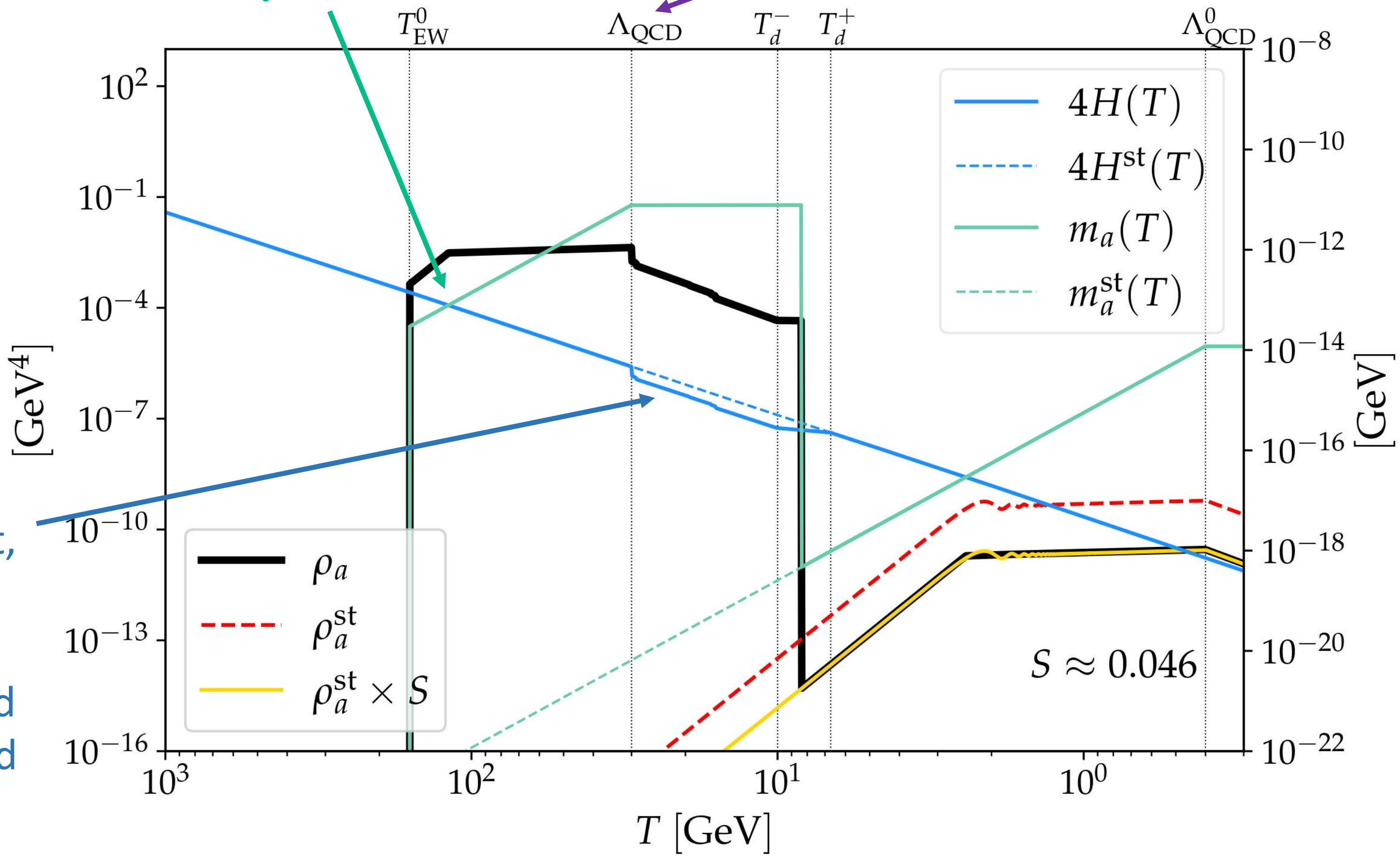
Suppression relatively weak due to short period of early oscillation

Evolution of energy density: Example 4

Axion mass jumps above H after SM EWPT but before early confinement due to large Λ_{QCD}

Early confinement occurs after SM EWPT



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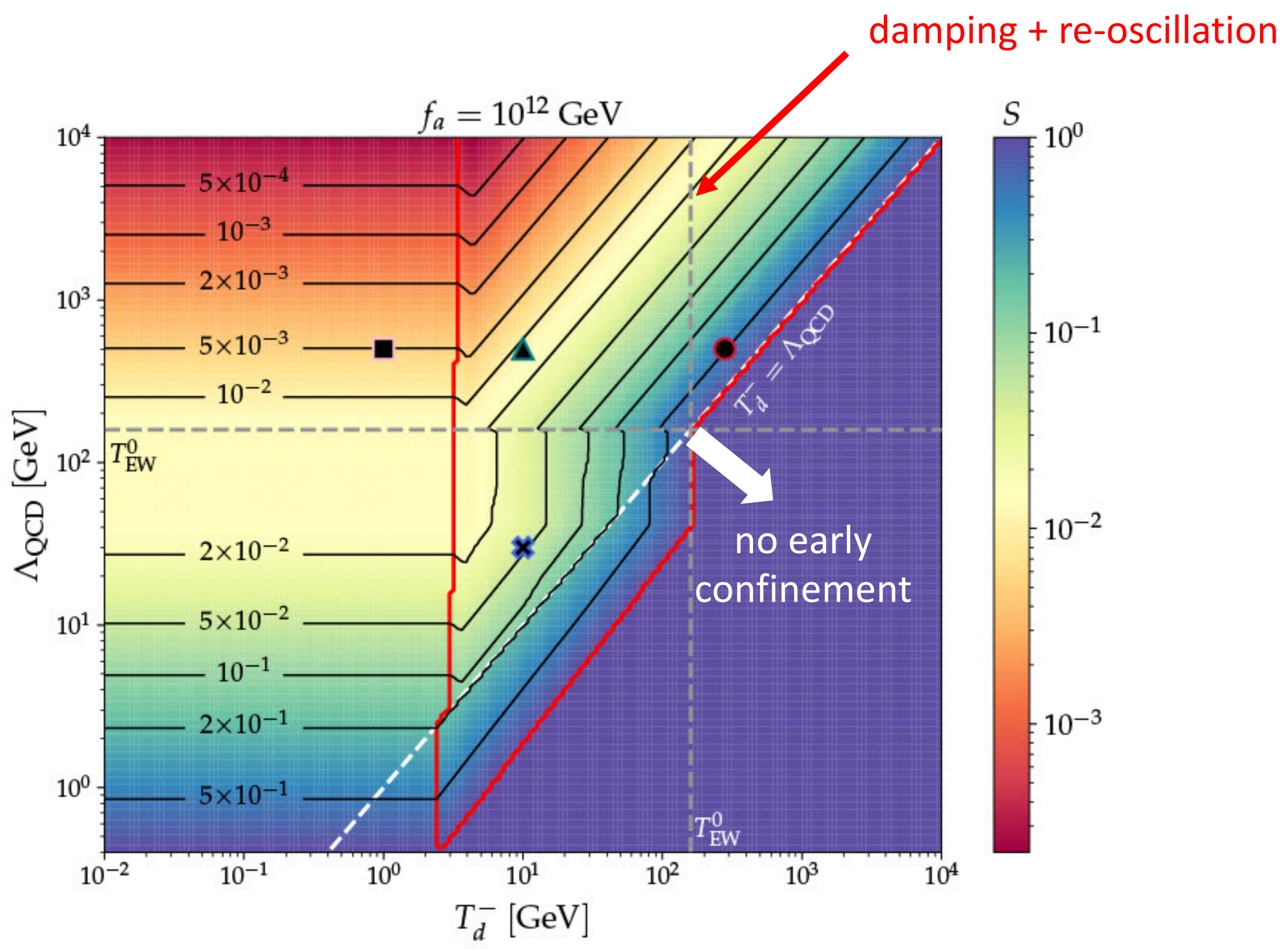
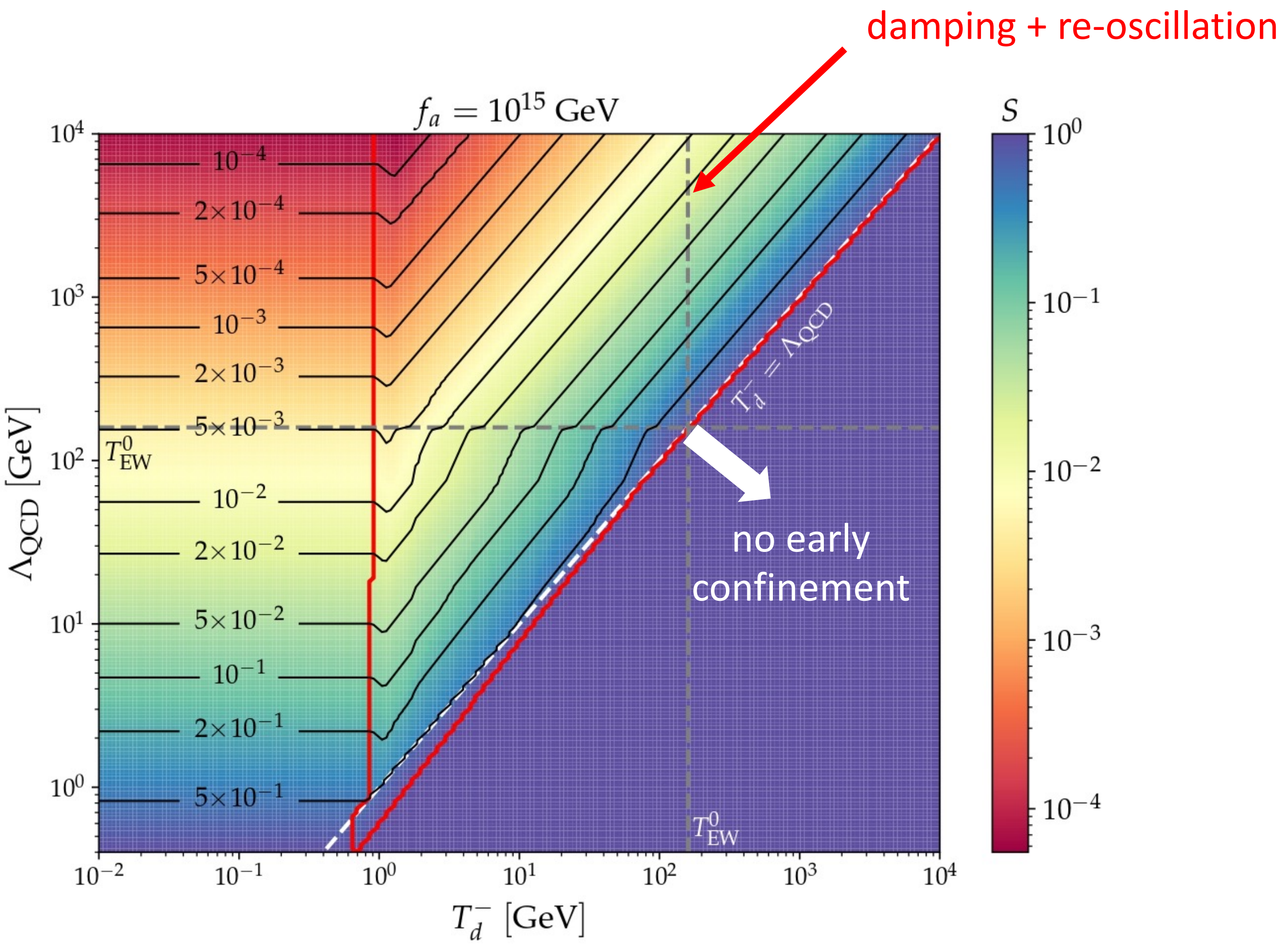
$$\rho_R = \frac{\pi^2}{30} g_\star(T) T^4$$

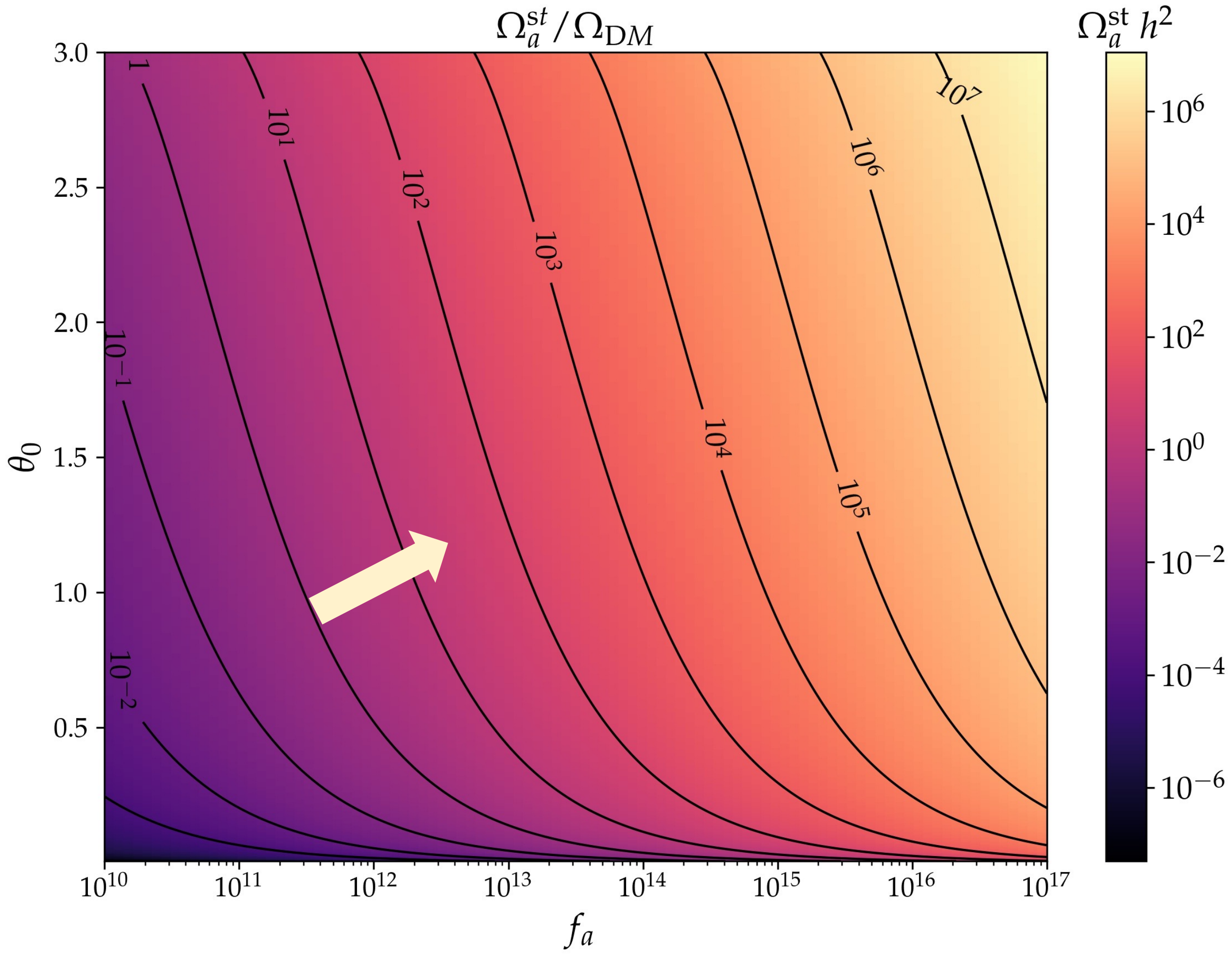
Parameter scan



Parameter scan



Enlarged parameter space



Larger values of f_a (or smaller axion mass) accessible if suppression of relic abundance is realized by having a dynamical QCD scale.

Conclusion and outlook

- QCD scale can be made dynamical by coupling the gluon field strength term to a scalar field.
- The dynamical QCD scale modifies the evolution of the temperature-dependent axion mass which makes it possible for the axion field to start oscillating earlier than in the standard scenario.
- Axion relic abundance suppressed since axion field spend more time being matter-like.
- With the suppression factor, the correct DM relic abundance can be obtained for larger PQ scale without fine-tuning the initial misalignment angle.

➢ If sufficiently large mass and long period of early oscillation, axions may create an early epoch of matter domination

➢ Early confinement (and de-confinement if it exists), together with the early EWPT may lead to interesting GW signals

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