On the feasibility of Bell Inequality violation at ATLAS experiment with flavor entanglement of B meson pairs from proton-proton collisions

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Hidden variable theory

Einstein's consideration on quantum mechanics

- Quantum Mechanics (QM) is approximation of the complete theory.
- In the complete theory, element of the physical reality (e.g., spin) is a function of hidden variable λ.

→ Hidden Variable Theory (HVT)

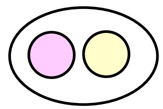
Premise in HVT

- Locality condition: A measurement on one particle does not influence the other.
- Free will: An experimenter has freedom to choose a measurement condition.

Bell developed formula which HVT must satisfy.

Bell inequality (1)

- A(t_a), B(t_b): eigenvalues of two particles (e.g. meson flavors) measured in a certain condition (e.g. measured time t_a, t_b).

A(t_a) $\longleftarrow$  $\longrightarrow$ B(t_b)

- Expectation value of A(t_a)B(t_b): $C(t_a, t_b) = \sum_{A,B} AB\,P_{t_a t_b}(A,B)$

PDF (Probability Density Function) of A(t_a)B(t_b)

- In HVT, (A, B) is a function of hidden variable λ:

$$P_{t_a t_b}(A,B) = \int P_{t_a}(A,\lambda) P_{t_b}(B,\lambda) P(\lambda) d\lambda$$

$$\Rightarrow \quad C(t_a, t_b) = \int \sum_A A P_{t_a}(A,\lambda) \sum_B B P_{t_b}(B,\lambda) P(\lambda)\, d\lambda$$

Free will

Locality condition

Bell inequality (2)

$$\left|\sum_A A P_{t_a}(A,\lambda)\right| = \underbrace{\left|P_{t_a}(A=1,\lambda) - P_{t_a}(A=-1,\lambda)\right|}_{\text{Less than 1 since } P_{t_a}(A,\lambda) \text{ is PDF.}} \leqq 1$$

Flavor: ± 1

$$\Rightarrow \quad |C(t_a,t_b)| = \left|\int \underbrace{\boxed{\textstyle\sum_A A P(t_a,\lambda)}}_{\leqq 1}\ \underbrace{\boxed{\textstyle\sum_B B P(t_b,\lambda)}}_{\leqq 1} P(\lambda)\, d\lambda\right| \leqq 1$$

Bell inequality

$$\boxed{|S| = |C(t_a,t_b) + C(t'_a,t_b) + C(t_a,t'_b) - C(t'_a,t'_b)| \leqq 2}$$

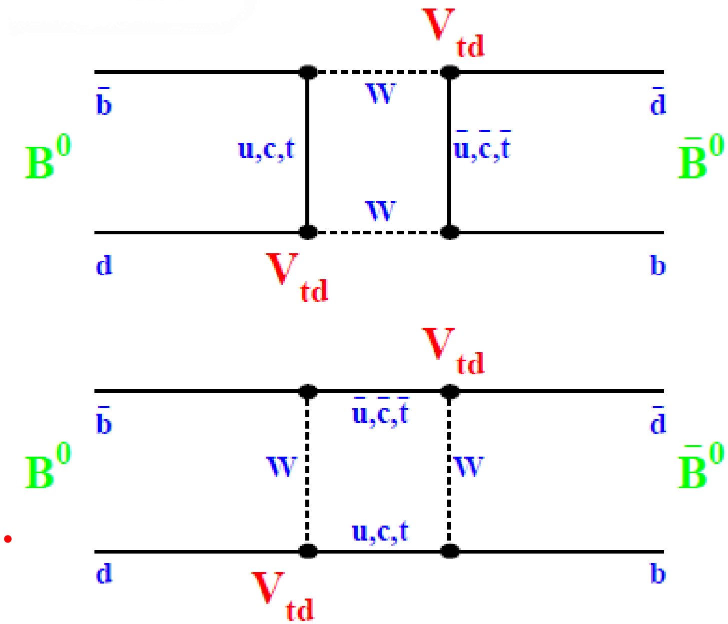
If HVT is correct, $|S| \leqq 2$.

Flavor mixing in B meson

- B^0 meson has mass (B_H, B_L) and CP eigenstates (B^0, $\bar{B}^0$), and they are expressed by each other:

$$|B^0\rangle = \frac{|B_H\rangle + |B_L\rangle}{\sqrt{2}},\ |\bar{B}^0\rangle = \frac{|B_H\rangle - |B_L\rangle}{\sqrt{2}}$$

(Assuming CP conservation)



- B^0 and $\bar{B}^0$ are mixed during time evolution. ($\Delta M = M_H - M_L$, $\Gamma = \Gamma_H = \Gamma_L$)

$$P(B^0 \rightarrow B^0, t) = \frac{e^{-\Gamma t}}{2}(1 + \cos\Delta M t)$$

$$P(B^0 \rightarrow \bar{B}^0, t) = \frac{e^{-\Gamma t}}{2}(1 - \cos\Delta M t)$$

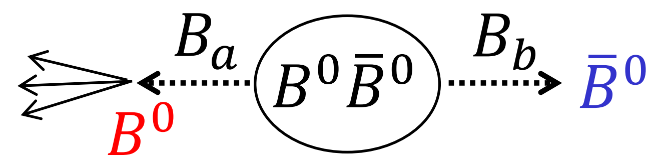
Flavor mixing in entangled state

- In QM, a $B^0\bar{B}^0$ pair (e.g. created from Υ(4S) or gluon) is in entangled state → If one is B^0, the other is $\bar{B}^0$, vice versa.

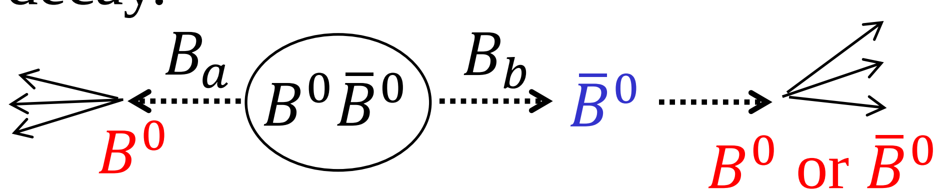
$$\psi(t_a, t_b) = \frac{1}{\sqrt{2}}(|B^0(t_a)\rangle|\bar{B}^0(t_b)\rangle - |\bar{B}^0(t_a)\rangle|B^0(t_b)\rangle)$$

$$B^0 \leftarrow \bar{B}^0 \xleftarrow{B_a} B^0\bar{B}^0 \xrightarrow{B_b} B^0 \rightarrow \bar{B}^0$$

- If B_a decays into B^0 eigenstate, the state of B_b at that timing is $\bar{B}^0$(vice versa)



- The flavor of B_b is mixed during its travel and the flavor is determined at the timing of its decay.



Bell inequality in flavor correlation (1)

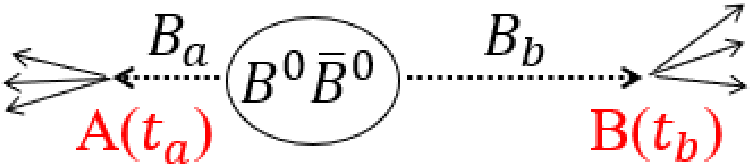
PDF of $B_{a/b}$ with flavor (A, B) at their decay times ($t_{a/b}$):

# of events with flavor (A, B) at the decay time of $B_{a/b}(t_a, t_b)$

$$P^{Q}_{t_a,t_b}(A,B) = \frac{N^{AB}_{(t_a,t_b)}}{N^{B^0B^0}_{(t_a,t_b)} + N^{\bar{B}^0\bar{B}^0}_{(t_a,t_b)} + N^{B^0\bar{B}^0}_{(t_a,t_b)} + N^{\bar{B}^0B^0}_{(t_a,t_b)}}$$

$$= \frac{1}{4}\left(1 - AB\cos(\Delta M \Delta t)\right)$$

$t_a - t_b$



$$C^{Q}(t_a,t_b) = \sum_{A,B} AB P^{Q}_{t_b\,t_b}(A,B)$$

$$= \frac{N^{B^0B^0}_{(t_a,t_b)} + N^{\bar{B}^0\bar{B}^0}_{(t_a,t_b)} - N^{B^0\bar{B}^0}_{(t_a,t_b)} - N^{\bar{B}^0B^0}_{(t_a,t_b)}}{N^{B^0B^0}_{(t_a,t_b)} + N^{\bar{B}^0\bar{B}^0}_{(t_a,t_b)} + N^{B^0\bar{B}^0}_{(t_a,t_b)} + N^{\bar{B}^0B^0}_{(t_a,t_b)}}$$

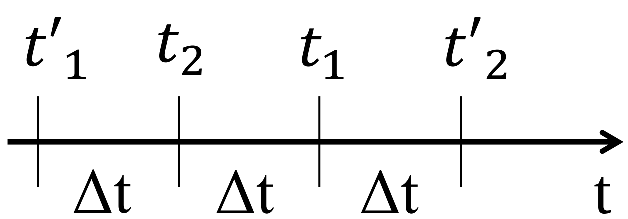
$$= -\cos(\Delta M \Delta t)$$

Bell inequality in flavor correlation (2)

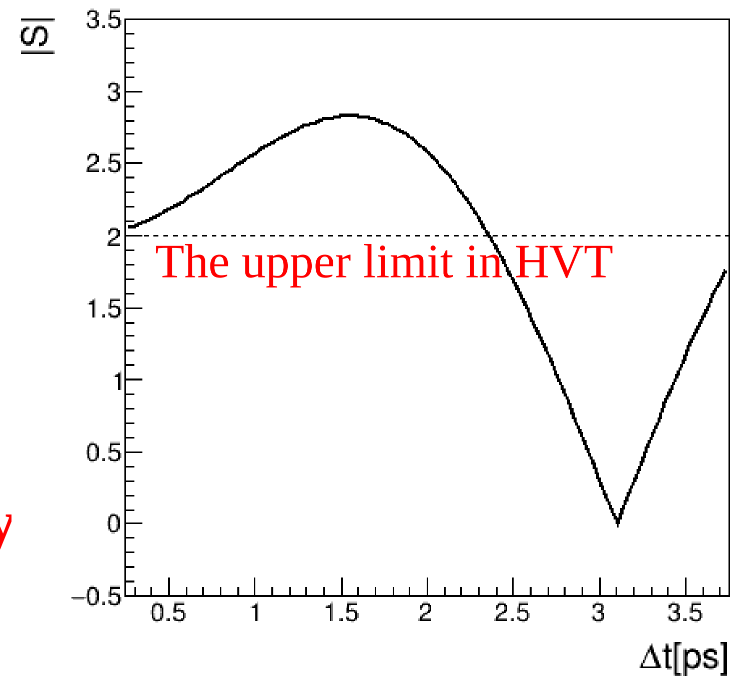
Defining $t_2 - t'_1 = t_1 - t_2 = t'_2 - t_1 = \Delta t$,

$$|S^Q(\Delta t)| = |\underbrace{C^Q(t_1,t_2)}_{-\cos(\Delta M \Delta t)} + \underbrace{C^Q(t'_1,t_2)}_{-\cos(\Delta M \Delta t)} + \underbrace{C^Q(t_1,t'_2)}_{-\cos(\Delta M \Delta t)} - \underbrace{C^Q(t'_1,t'_2)}_{-\cos(3\Delta M \Delta t)}|$$

$$= |-3\cos(\Delta M \Delta t) + \cos(3\Delta M \Delta t)|$$



|S| as a function of Δt in QM



In QM, |S| has the maximum value of $2\sqrt{2}$ at Δt = 1.55 ps. → **QM violates Bell inequality**

- $\Delta M = 3.334 \times 10^{-10}$ MeV

Previous experiments

[PLA373 (2008) 39-44]

B meson (Belle) [arXiv:0310192 (2003)]

- A measurement only for Δt, where (t_a, t_b) was not measured separately.
- $|S|$ has the maximum value of 4 and includes that of QM.

K meson (CPLEAR) [PLB422, 339-348]

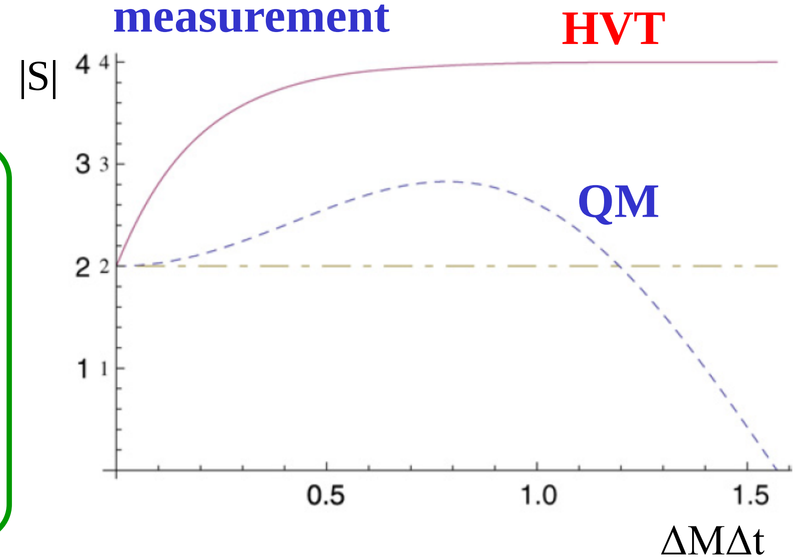
A measurement only for $C^Q(t_a, t_b)$ and is not Bell test.

Both experiments

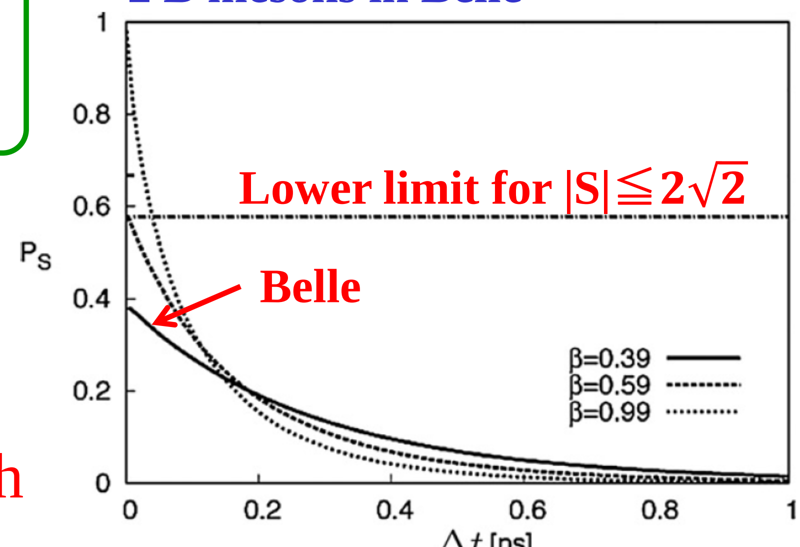
Most of 2 mesons are not space-like and the locality is not ensured.

The previous experiments were not enough for Bell test on the flavor entanglement.

|S| in QM and HVT only with Δt measurement



Fraction of space-like events of 2 B mesons in Belle

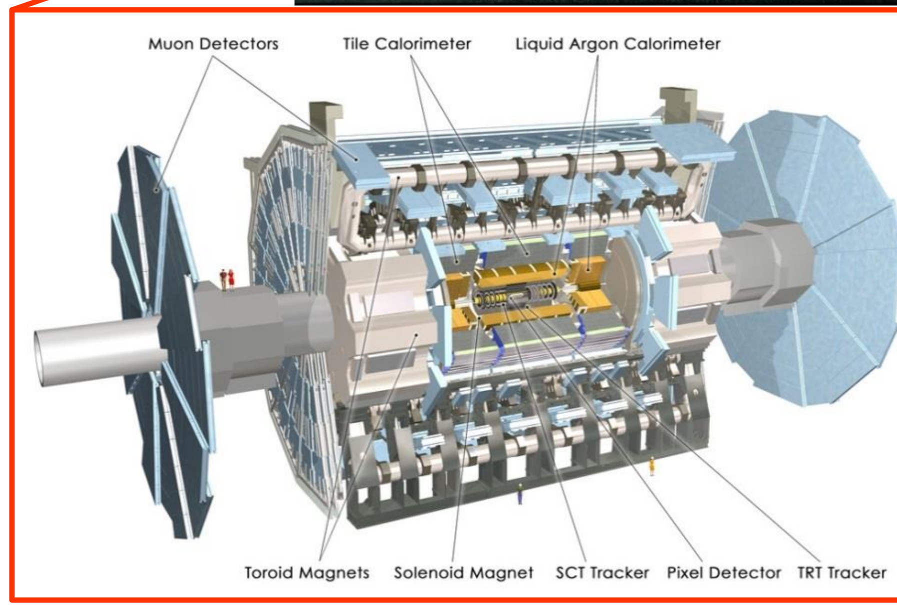


ATLAS experiment

- High energy experiment using proton-proton collisions at LHC (Large Hadron Collider).

- The experiment started in 2010 and discovered Higgs boson in 2012.

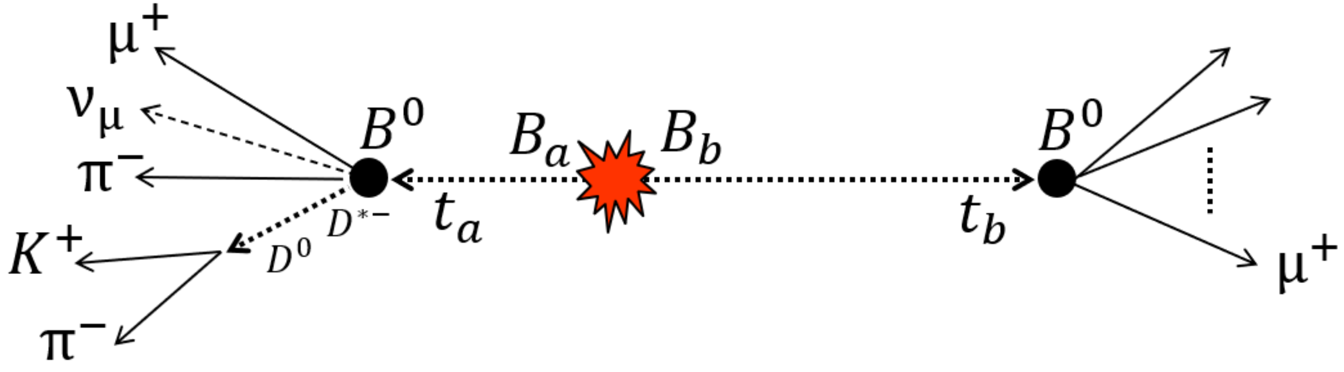
	Period	Integrated luminosity
Run1	2010-12	5.1fb-1 @7TeV 21.3 fb-1 @8 TeV
Run2	2015-18	149 fb-1 @13 TeV
Run3	2021-24	180 fb-1 @~14 TeV



- The instantaneous luminosity reached more than twice of LHC design value (2.2×10^{34} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$) and # of pile-up (μ) became <60.
- Except for usual pp collisions, data are taken in low-μ operation (μ~1) and heavy ion collisions.

Simulation study

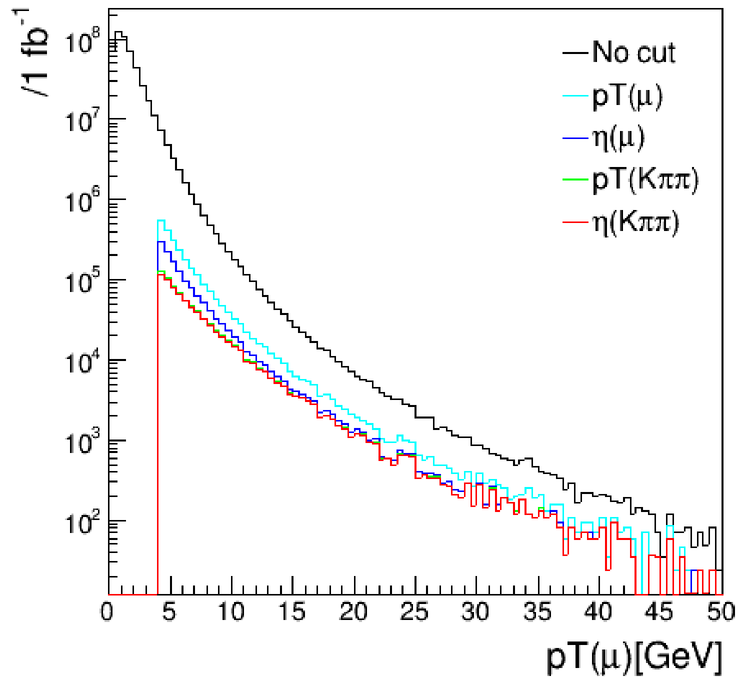
- The simulation was performed for truth level study with PYTHIA8.245.
- The low-μ (μ~1) operation with 1 fb^{-1} of data was assumed.
- The sensitivity to Bell inequality violation was evaluated with $B^0\bar{B}^0$ events from 「gg/qq→$b\bar{b}$」(69μb) and 「gg/qq→jj」(319μb).
- 「$B^0 \to D^{*-}\mu^+\nu$ ($D^{*-} \to D^0\pi^-$, $D^0 \to K^+\pi^-$)」 events were selected.
  - B meson flavor can be identified with charge sign of decay objects.
  - The decay time (t_a, t_b) can be measured, reconstructing vertex with μ^+ and π^- from D^{*-} decay (σ_t~0.11 ps)



Acceptance cut

- Selection criteria used in 「$D^{*+}\mu^{-}X$」 analysis at ATLAS was assumed [Nucl. Phys. B 864 (2012) 341-381].
- 「pT(μ)>6 GeV」 was modified to 「> 4 GeV」
- 「pT($K^{\pm}\pi^{\mp}\pi^{\mp}$) > 4.5 GeV」 was modified to 「>3 GeV」

μ-pT distribution in 「$B^0 \to D^{*-}\mu^{+}\nu$」



Cut	Total acceptance (A)	$\sigma \times A$ (pb)
No cut	1.0	247,611
pT(μ) > 4 GeV	5.03×10^{-3}	1,246
$\lvert\eta(\mu)\rvert < 2.4$	2.79×10^{-3}	690
pT($K^{\pm}\pi^{\mp}\pi^{\mp}$) > 3 GeV	1.56×10^{-3}	385
$\lvert\eta(K^{-}\pi^{+}\pi^{+})\rvert < 2.5$	1.49×10^{-3}	369

Event selection

	Efficiency	Comment
Track reconstruction (ε_{reco})	0.483	From「$D^{*+}\mu^{-}X$」analysis
Trigger ($\varepsilon_{trigger}$)	0.429	$(0.819 \times 0.8)^2$ was assumed. • 0.819 is efficiency for single-μ trigger with pT>6GeV.

Selection criteria ($\varepsilon_{selection}$)	Total eff.	Comment
pT > 1 GeV for π^{+}/K^{-} in D^{0} candidates	0.510	
pT > 250 MeV for π^{+} from D^{*+}	0.452	
•|m($K^{-}\pi^{+}$) – m(D^{0})| < 64 MeV (pT($K^{-}\pi^{+}\pi^{+}$) > 12 GeV, |η($K^{-}\pi^{+}\pi^{+}$)| > 1.3) •|m($K^{-}\pi^{+}$) – m(D^{0})| < 40 MeV elsewhere	0.209	Assume σ^2 cut (0.46)
2.5 GeV < m($D^{*+}\mu^{-}$) < 5.4 GeV	0.097	Assume σ^2 cut (0.46)

$\varepsilon_{total}\ (\varepsilon_{reco} \times \varepsilon_{trigger} \times \varepsilon_{selection}) = 0.020$

$(\sigma \times A) \times \varepsilon_{total} \times L(pb^{-1})$=7.4L($pb^{-1}$) events → 7.4k events with 1fb^{-1}.

Background & systematic errors

Background

- Contamination of $B^0\bar{B}^0$ originated from different gluons is less than 0.1% (negligible)
- BG contamination in 「$D^{*+}\mu^-$X」 analysis was 「6.8±0.26%」, which is taken into account in this study.
  - 6.2% is combinatorial BG (e.g. : 「c → D^{*+}X」 and 「$\bar{c}$ → μ^-X'」)

Systematic errors on |S|

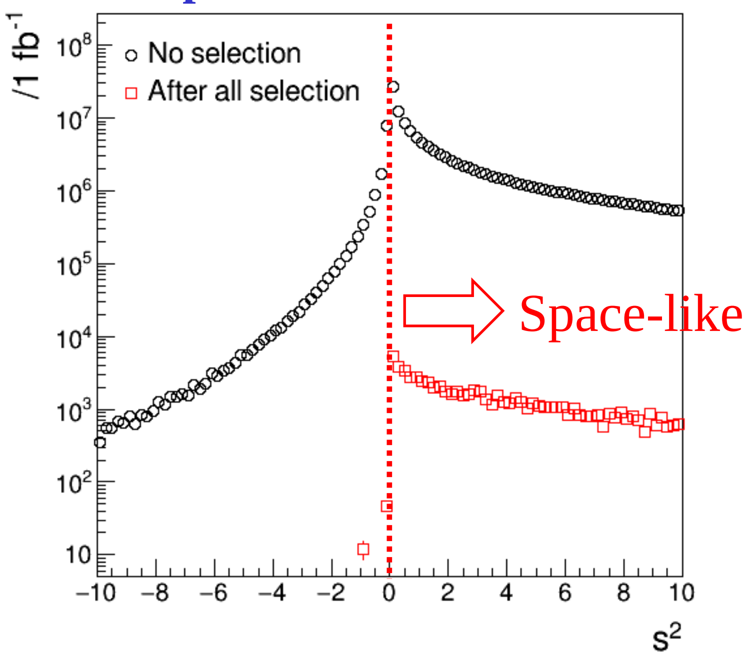
- BG contamination
  - Shifting the entries in one Δt bin by 0.26%, the maximum shift in |S| was adopted as the systematic error of BG contamination (most conservative evaluation).
  - BG should be smaller for μ~1 since 「$D^{*+}\mu^-$X」 analysis used data with μ>2.
- Δt resolution
  - Evaluated by fluctuating Δt with the resolution ($0.11\sqrt{2}$ps) 1000 times.

Sensitivity to Bell inequality violation

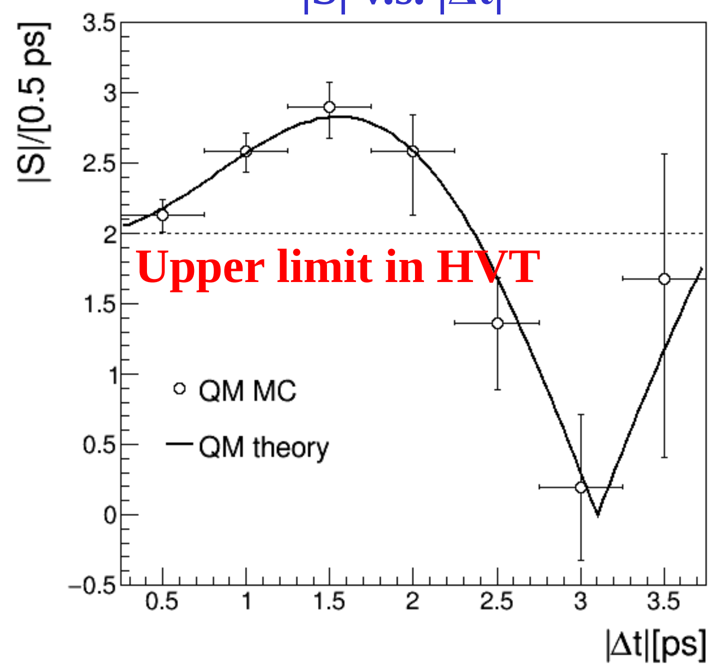
- More than 99% B meson pairs are space-like in ATLAS.
- |S(1.5±0.25 ps)| = 2.89 ± 0.17 (stat) ± -0.13 (syst)
  - |S| has the maximum value at 1.55ps and deviation from 2 can be detected with 4.2σ significance.

The first Bell test on B meson flavor is possible!

Proper distance distribution



|S| v.s. |Δt|



Summary & conclusions

- Bell inequality provides the upper limit of correlation between two particles that HVT should satisfy.
- Entangled state of two meson flavors in QM violates Bell inequality.
- The previous experiments on meson flavor entanglement were inconclusive for Bell inequality violation only with Δt measurement without satisfying the locality condition.
- Our simulation study concluded that Bell test on B meson flavor is possible at ATLAS experiment.
  - Bell inequality violation can be tested with 4.2 σ precision.
- We organized analysis group in ATLAS and aim the measurement during Run 3 (2022-24).
- The paper was accepted and will be published in PRD ([arXiv:2106.07399]).

Backup

Q&A in Bell test with meson flavor

Q1 |S| has the maximum value at 1.55 ps. Is the oscillation frequency is too long with respect to lifetime of B_d^0 (1.5 ps)?

A1 No problem since $C^Q(t_a, t_b)$ is normalized by # of events at (t_a, t_b).

Q2 HVT assumes free will of experimenter in a measurement but decay of B_d^0 is determined by nature. Can it be assumed as free will?

A2 We assumed that B_d^0 decay randomly and it corresponds to free will of the particle. Also in Aspect experiment, a random generator is used to operate the detector and it is assumed as free will.

Possible loophole in Bell test

- Free will loophole

  This study is assumed that the decay of a particle happened randomly with particle's will.

- Efficiency loophole

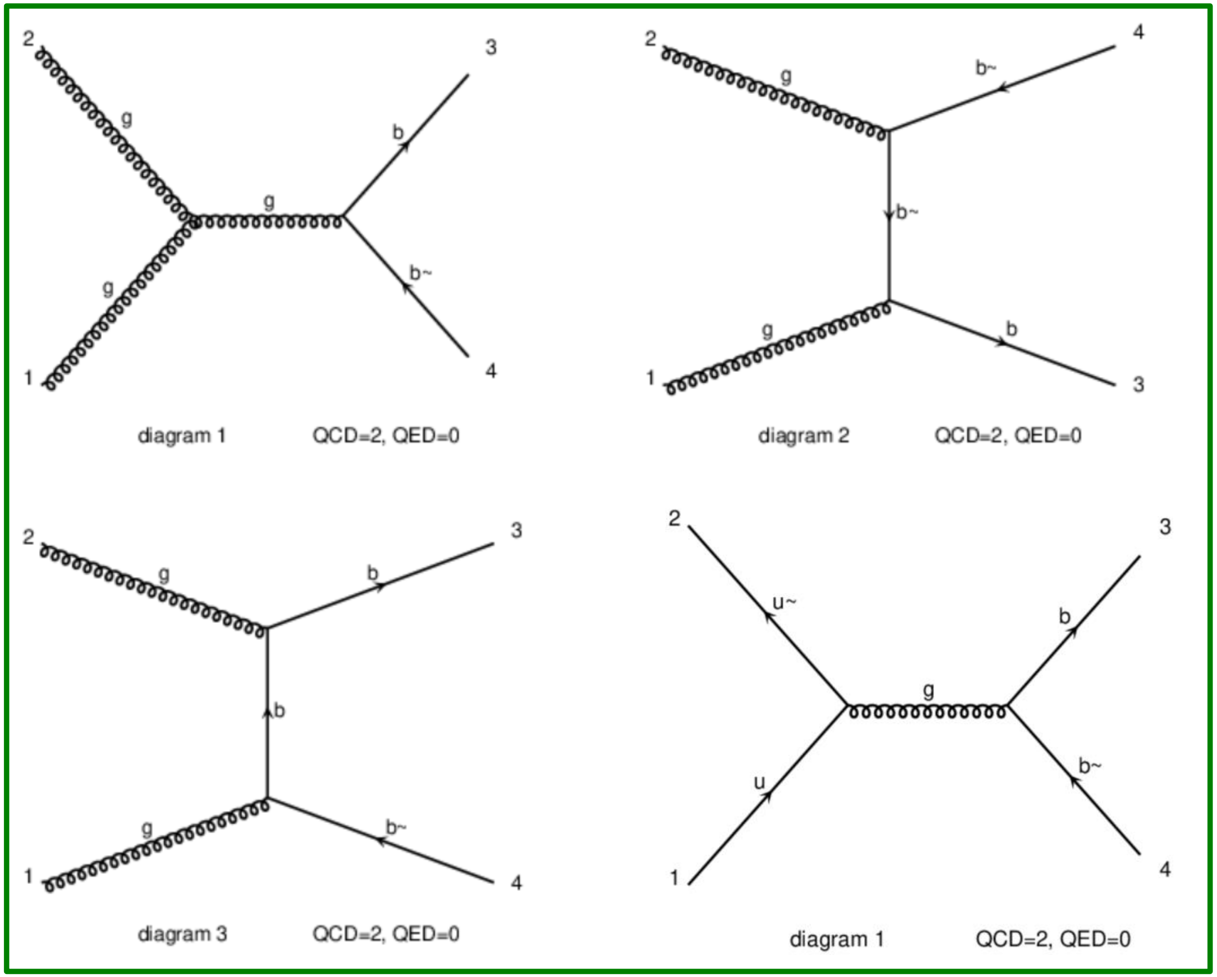
  About 82.8% $(2\sqrt{2}-2)$ of efficiency is necessary to close the efficiency loophole. Since the efficiency in this study is only 2%, fair sampling assumption is assumed.

- Locality loophole

  2 particles decay randomly and they are space-like, therefore, locality loop hole is closed successfully.

$b\bar{b}$ events in proton-proton collision

① $gg/q\bar{q} \to b\bar{b}$



Cross-section of "$gg/q\bar{q} \to b\bar{b}$" events decaying into "$D^*\mu\nu$"

	Cross-sec. [nb]
$B^0\bar{B}^0$	4.5
$B^{*0}\bar{B}^{*0}$	21.8
$B^0\bar{B}^{*0}$	19.8
Total	46.2

② $b\bar{b}$ from「$gg/q\bar{q} \to jj$」

Decay time resolution in ATLAS

- Refer to ΔΓ measurement "$B^0 \to J/\psi K_s$, $B^0 \to J/\psi K^*$" in ATLAS
  - [JEP06(2016)081](more detail is in[ATL-COM-PHYS-2015-170])
- Decay length ($L^B_{prop} = \mathrm{ct}$) is calculated as follows:
  - $x^{J/\psi}$: Decay position of "J/ψ→μμ"

$$L^B_{\mathrm{prop}} = \frac{(x^{J/\psi} - x^{\mathrm{PV}})p^B_x + (y^{J/\psi} - y^{\mathrm{PV}})p^B_y}{(p^B_{\mathrm{T}})^2} m_{B^0} \quad = \frac{\vec{d} \cdot \vec{k}_\perp}{\beta_\perp \gamma_\perp}$$

$\sigma(L^B_{prop}) = 34\ \mu\mathrm{m}$

$\Rightarrow$ $\sigma(t) = \frac{\sigma(L^B_{prop})}{c} = 0.11\ \mathrm{ps}$

ATLAS v.s. Belle

Decay length/time measurement

- 2 B^0 mesons travel along the z-axis in Belle.

  → The production position of B^0 is unknown due to beam size in the z-direction. Only Δz and Δt can be measured.

- 2 B^0 mesons can go to xy direction in ATLAS, production and decay position of an individual B^0 mesons can be measured.

Resolution of decay length/time

- Belle : σ(Δz) ~ 100 um, σ(Δt) ~ 1.2 ps
- ATLAS : σ(L_{prop}^{B} = ct) ~ 34 um, σ(t) = 0.11 ps