Non-relativistic Gravity and Supergravity

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work done in collaboration with

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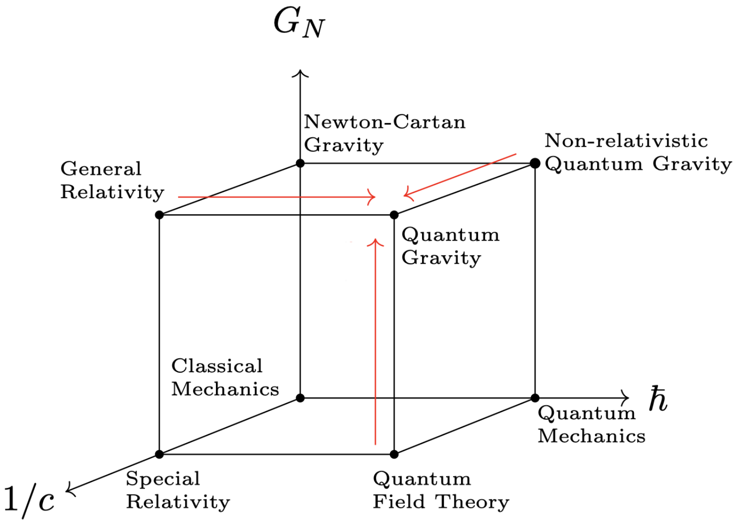
Three Roads to Quantum Gravity



NR Quantum Gravity

Does combining gravity with quantum mechanics require relativity?

Does NR string theory define NR quantum gravity?

Does NR gravity has its own holographic principle?

Some References

NR gravity/string theory involving null-reduction

T. Harmark, J. Hartong and N. A. Obers (2017); Kluson (2018);
T. Harmark, J. Hartong, L. Menculini, N. A. Obers and Z. Yan (2018);
Kluson (2019); Roychowdhury (2019); T. Harmark, J. Hartong, L. Menculini,
N. A. Obers and G. Oling (2019); A.D. Gallegos, U. Gürsoy and N. Zinnato (2019);
L. Bidussi, T. Harmark, J. Hartong, N.A, Obers, G. Oling (2021)

NR strings with NR worldsheet

C. Batlle, J. Gomis and D. Not (2017); C. Batlle, J. Gomis, L. Mezincescu and
P. K. Townsend (2017); T. Harmark, J. Hartong and N. A. Obers (2017); T. Harmark,
J. Hartong, L. Menculini, N. A. Obers and Z. Yan (2018)

Outline

NC Gravity from General Relativity

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Defining a NR limit

STEP 1: decomposing $E_\mu{}^{\hat{A}} = (E_\mu{}^0, E_\mu{}^{A'}) =$ (clock, ruler) and introducing M_μ, perform an invertable field redefinition involving a parameter c:

$$E_\mu{}^0 = c\tau_\mu + c^{-1}m_\mu\,, \qquad E_\mu{}^{A'} = e_\mu{}^{A'}\,, \qquad M_\mu = c\tau_\mu - c^{-1}m_\mu$$

STEP 2: take the limit $c \to \infty$ and take care of possible divergences

red terms in field redefinition cancel when considering action for particle coupled to gravity

The NR limit of the spin-connection fields contains a leading divergence that usually is set to zero by imposing the zero torsion constraint

$$\partial_{[\mu}\tau_{\nu]} = 0$$

Given this constraint the NR limit of the Einstein e.o.m. yields the NC gravity e.o.m. with Newton potential $\Phi \sim \tau^\mu m_\mu$

The Zero Torsion Constraint

$$\partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow \quad \tau_\mu = \partial_\mu \rho \quad \text{with} \quad \tau_\mu \quad \text{clock function}$$

$$\Delta T = \int_{\mathcal{C}} \mathrm{d}x^\mu \tau_\mu = \int_{\mathcal{C}} \mathrm{d}\rho \ \text{ is path-independent} \quad \rightarrow \quad \text{absolute time}$$

$$\text{Torsional NC gravity}: \ \partial_\mu \tau_\nu - \Gamma_{\mu\nu}{}^\rho \tau_\rho = 0 \quad \rightarrow \quad \Gamma_{[\mu\nu]}{}^\rho \tau_\rho = \partial_{[\mu}\tau_{\nu]}$$

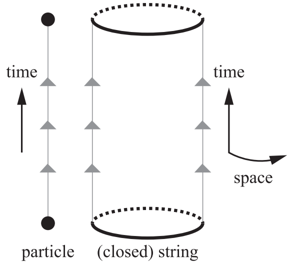
Outline

Geometry with Co-dimension 2 Foliation

The string should be coupled to a 2-form gauge field $B_{\mu\nu}$ with

$$B_{\mu\nu} = -c^2 \epsilon_{AB} {\tau_\mu}^A {\tau_\nu}^B + b_{\mu\nu}$$

defining a geometry with a co-dimension 2 foliation where $\tau_\mu \to {\tau_\mu}^A$ with $\hat{A} = (A, A') = (0, 1, A')$



The Basic Variables

The decomposition leading to NC gravity

$$\{E_\mu{}^{\hat{A}}, M_\mu\} \quad \to \quad \{\tau_\mu, e_\mu{}^{A'}, m_\mu\}$$

gets replaced by the following redefinition:

$$\{E_\mu{}^{\hat{A}}, B_{\mu\nu}, \Phi\} \quad \to \quad \{\tau_\mu{}^{A}, e_\mu{}^{A'}, b_{\mu\nu}, \phi\}$$

The Newton potential Φ can be identified with the time-space component $\epsilon^{AB}\tau^\mu{}_A\tau^\nu{}_B b_{\mu\nu}$ of the 2-form gauge field $b_{\mu\nu}$

The NR String Sigma Model

J. Gomis, Z. Yan + E.B. (2018); J. Gomis, J. Rosseel, C. Şimşek, Z. Yan + E.B. (2019)

$$S_{\mathrm{NR}\sigma} = -\frac{T}{2}\int d^2\sigma \left[\sqrt{-h}\, h^{\alpha\beta}\, \partial_\alpha x^\mu \partial_\beta x^\nu\, e_\mu{}^{A'} e_\nu{}^{B'} \delta_{A'B'} + \epsilon^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu\, b_{\mu\nu}\right] + S_{\mathrm{dilaton}}$$

with world-sheet metric $\quad h_{\alpha\beta} \;\sim\; \tau_{\alpha\beta} \equiv \partial_\alpha x^\mu \partial_\beta x^\nu \tau_\mu{}^A \tau_\nu{}^B \eta_{AB}$

This is the generalization of flat spacetime to a string NC background

Gomis, Ooguri (2001); Danielsson, Guijosa, Kruczenski (2000)

Note: we have not imposed any geometric constraint sofar.

Special Features

- The KR 2-form field $b_{\mu\nu}$ transforms under string-Galilean boost transformations:

$$\delta b_{\mu\nu} = \partial_{[\mu}\lambda_{\nu]} + 2\epsilon_{AB}\lambda_{A'}{}^{A}\tau_{[\mu}{}^{B}e_{\nu]}{}^{A'}$$

A relativistic matter field $B_{\mu\nu}$ becomes a NR geometric field $b_{\mu\nu}$

- There is an emergent dilatation symmetry:

$$\delta\tau_{\mu}{}^{A} = \lambda_D\tau_{\mu}{}^{A}\,, \qquad \delta\phi = \lambda_D$$

This means that the # of NR background fields is one less than the # of relativistic background fields

Outline

Canceling the Divergences

$$S_{\mathrm{rel}} = \frac{1}{2\kappa^2}\int d^{10}x\, E\left(\mathcal{R} - \frac{1}{12}\mathcal{H}_{\mu\nu\rho}\mathcal{H}^{\mu\nu\rho}\right)$$

with $\mathcal{H}_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]}$. We redefine

$$E_\mu{}^A = c\tau_\mu{}^A\,, \qquad E_\mu{}^{A'} = e_\mu{}^{A'}\,, \qquad B_{\mu\nu} = -c^2\epsilon_{AB}\tau_\mu{}^A\tau_\nu{}^B + b_{\mu\nu}$$

and find

$$S = c^2 \overset{(2)}{S} + \overset{(0)}{S} + c^{-2}\overset{(-2)}{S} + c^{-4}\overset{(-4)}{S}$$

where $\overset{(2)}{S}$ consists of two terms that are both proportional to the torsion tensor

$$\tau_{\mu\nu}{}^A \equiv \partial_{[\mu}\tau_{\nu]}{}^A$$

Miracle: the metric and 2-form contributions to $\overset{(2)}{S}$ precisely cancel!

Special Features of Non-relativistic Action

$$S_{\rm NR} = \frac{1}{2\,\kappa^2}\int \mathrm{d}^{10}x\, e \Big(\mathrm{R}(J) - \frac{1}{12}\, h_{A'B'C'}\, h^{A'B'C'} - 4\,\mathcal{D}_{A'}\, b^{A'} - 4\, b_{A'}\, b^{A'} - 4\,\tau_{A'\{AB\}}\tau^{A'\{AB\}} \Big)\,.$$

- the action has an emergent dilatation symmetry and therefore has one 'missing field' and one 'missing e.o.m.'
- The 'missing' e.o.m. follows from taking the NR limit of the e.o.m. and is precisely the Poisson equation of the Newton potential
- Furthermore, the e.o.m. of the Newton potential itself is given by a non-linear equation

  $$\tau_{B'C'A}\tau^{B'C'A} = 0$$

  giving a constraint on the geometry

The full set of e.o.m. form a reducible, but indecomposable representation

Action, E.O.M. and β-Functions

The e.o.m. of non-relativistic string theory are determined by calculating the β-functions

Gomis, Oh, Yan (2019), Yan, Yu (2019), Gomis, Yan, Yu (2020); see also Gallegos, Gürsoy and Zinnatos (2019)

The emergent dilatation symmetry has the following effect:

$$\text{NR NS-NS action} \quad \rightarrow \quad \text{common equations} \; + \text{Non-linear}$$

$$\text{NR } \beta\text{-functions} \quad \rightarrow \quad \text{common equations} \; + \text{Poisson}$$

$$\text{NR e.o.m.} \quad \rightarrow \quad \text{common equations} \; + \text{Poisson} + \text{Non-linear}$$

The nonlinear equation is required in order that the NR string σ model does not flow towards a relativistic string σ model

Z. Yan (2021)

Torsional String Newton-Cartan (TSNC) geometry

basic variables: $\{\tau_\mu{}^A, e_\mu{}^{A'}, b_{\mu\nu}, \phi\}$

$\{\omega_\mu, \omega_\mu{}^{AA'}, \omega_\mu{}^{A'B'}, b_\mu\}$ are dependent, e.g., $b_\mu = e_\mu{}^{A'}\tau_{A'A}{}^A + \tau_\mu{}^A\partial_A\phi$

$$\nabla_\mu\tau_\nu{}^A \equiv \partial_\mu\tau_\nu{}^A - \omega_\mu\,\epsilon^{AB}\tau_{\nu B} - b_\mu\,\tau_\nu{}^A - \Gamma^\rho_{\mu\nu}\tau_\rho{}^A = 0\,,$$

$$\nabla_\mu e_\nu{}^{A'} \equiv \partial_\mu e_\nu{}^{A'} - \omega_\mu{}^{A'B'}e_{\nu B'} + \omega_\mu{}^{AA'}\tau_{\nu A} - \Gamma^\rho_{\mu\nu}e_\rho{}^{A'} = 0$$

non-zero torsion: $T^\rho_{\mu\nu} = 2\,\Gamma^\rho_{[\mu\nu]} = 2D_{[\mu}(\omega, b)\tau_{\nu]}{}^A\tau_A{}^\rho$

Outline

Two new features

- There is no direct connection between a two-dimensional sigma model description and the NR target space effective action
- Taking the naive NR limit leads to divergent terms in the supersymmetry rules

These divergences can be controlled by

- the occurrence of 2 'superconformal' Stueckelberg symmetries beyond dilatations
- imposing by hand the following twistless torsional constraint:

$$T_{\mu\nu}^{-} = 0 \quad \text{or} \quad \tau_{A'B'}{}^{-} = \tau_{A'+}{}^{-} = 0 \quad \text{or} \quad \tau_{[\mu}{}^{-}\partial_\nu \tau_{\rho]}{}^{-} = 0$$

Christensen, Hartong, Obers, Rollier (2013)

defining a 'self-dual' DSNC geometry (invariant under SUSY!)

Minimal Supergravity Action versus E.O.M.

- the action has one emergent dilatation and two emergent superconformal symmetries. It therefore has one 'missing' bosonic and two 'missing' fermionic fields plus corresponding 'missing' e.o.m.
- The 'missing' e.o.m. follow from taking the NR limit of the e.o.m. and are precisely the Poisson equation of the Newton potential plus two fermionic partner equations
- The NR action is a pseudo action in the sense that it is only invariant under supersymmetry if one uses the twistless torsional constraint after varying the action. Due to this, the e.o.m. that follow from the action transform under supersymmetry to the 'missing' e.o.m.: they belong to the same supermultiplet

Outline

Outlook

- we need confirmation from β-function calculations
- including Yang-Mills to obtain heterotic supergravity
- T-duality: taking a NR limit followed by spatial reduction is dual to a null-reduction but the null-reduction constraint imposed on the relativistic supergravity multiplet is not supersymmetric!
- connection to Double Field Theory

  Ko, Melby-Thompson, Meyer and Park (2015); Gallegos, Gürsoy, Verma and Zinnato (2020)
- extension to IIA/IIB supergravity and M-theory

  for bosonic sector of M-theory, see Blair, Gallegos, Zinnato (2021)

Take-Home Message

Our results pave the way for a target space approach to NR string theory:

supersymmetric brane solutions, compactifications, NR holography etc.