The Unreasonable Effectiveness of Higher-Derivative Supergravity in AdS$_4$ Holography

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SUSY 2021
ITP-CAS, China

August 24 2021

2006.09390 2011.05971 2106.04581

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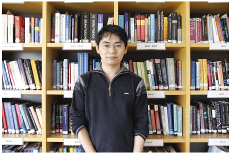
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Motivation

Three easy pieces

1. Higher-derivative corrections to supergravity.
2. Interplay with holography.
3. Lessons for black hole physics and the dual CFT.

Goal: Describe recent progress which relates these topics in the context of 4d supergravity and 3d SCFTs.

Status quo

- Higher-derivative (HD) corrections to 10d and 11d supergravity are a key consequence of string and M-theory and find many applications.
- The coefficients of these HD terms are hard to calculate in general.
- Here: Focus on 11d supergravity and understand HD corrections to asymptotically locally $\mathrm{AdS}_4 \times X^7$ solutions. Important for black hole physics and holography.
- Working with HD terms in 11d supergravity is hard: the leading correction to the 2der action comes at 8der, i.e. $\sim R^4$.

Progress seems hard, with few explicit results...

A different philosophy

- Use 4d $\mathcal{N} = 2$ gauged supergravity to study the leading HD corrections, i.e. $\sim R^2$, directly in 4d.

- Use holography and supersymmetric localization to determine the unknown coefficients in the 4der supergravity action.

- Illustrate the utility of this approach for minimal (i.e. only gravity multiplet) supergravity. This theory captures universal large N dynamics for a large class of 3d $\mathcal{N} = 2$ SCFTs. [NPB-Crichigno]

- The approach is justified by the large N limit in holography, consistent truncations in gauged supergravity, as well as *a posteriori* by showing excellent agreement with field theory results.

Plan

- Introduction and motivation ✓
- Four-derivative terms in 4d supergravity
- The on-shell action and holography
- Black hole thermodynamics
- Lessons for 3d $\mathcal{N} = 2$ SCFTs


KEEP
CALM
AND
DO
SUPERGRAVITY

Conformal supergravity

Use the 4d $\mathcal{N}=2$ conformal supergravity formalism to construct HD supergravity actions, see the review [Lauria-Van Proeyen].

For general matter coupled supergravity there are a number of free functions at 4der level. Here we focus on minimal supergravity (only gravity multiplet on-shell at 2der). The free functions become constants.

The off-shell ingredients are

- Weyl multiplet - metric, $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge field, 2 (anti-)self-dual two-forms, 1 real scalar.
- Vector multiplet - $\mathrm{U}(1)$ gauge field, 1 complex scalar, $\mathrm{SU}(2)$ triplet of scalars.
- Hyper multiplet - 4 real scalars.

1. Go “on-shell” (Conformal $\rightarrow$ Poincaré supergravity) by gauge fixing the conformal and gauge symmetry and solving the EoM for auxiliary fields.

2. Use the hyper and vector multiplet to gauge the Cartan of $\mathrm{SU}(2)$ in the Weyl multiplet (leads to gauged supergravity).

4der supergravity

There is a unique supersymmetric 2der action.

Two different supersymmetric 4der F-terms

- The Weyl^2 invariant. [Cardoso-de Wit-Mohaupt]
- The T-$\log$ invariant. [Butter-de Wit-Kuzenko-Lodato]

Note: The 4der D-terms vanish. [de Wit-Katmadas-van Zalk]

Work mostly in Euclidean signature using the formalism of [de Wit-Reys]

The Lagrangian

After gauge fixing and removing the auxiliary fields using their EoM one finds the "on-shell" Lagrangian

$$\mathcal{L}_{\mathsf{HD}} = \mathcal{L}_{2\partial} + (c_1 - c_2)\,\mathcal{L}_{\mathrm{W}^2} + c_2\,\mathcal{L}_{\mathsf{GB}}\,.$$

There are two undetermined constants c_1 and c_2 corresponding to the two independent 4der susy invariants. They should encode information about the 8der terms in 11d and the internal manifold X^7.

The (bosonic) 2der Lagrangian in Euclidean signature is

$$\mathcal{L}_{2\partial} = \; -(16\pi\, G_N)^{-1}\Big[R + 6\,L^{-2} - \tfrac{1}{4}\,F_{ab}F^{ab}\Big]\,.$$

The (bosonic) 4der Lagrangians in Euclidean signature are

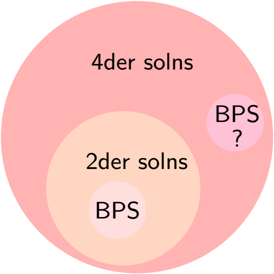
$$\begin{aligned}\mathcal{L}_{\mathrm{W}^2} &= \big(C_{ab}{}^{cd}\big)^2 - L^{-2}F_{ab}F^{ab} + \tfrac{1}{2}\big(F^{+}_{ab}\big)^2\big(F^{-}_{cd}\big)^2 \\ &\quad - 4\,F^{-}_{ab}R^{ac}F_c^{+b} + 8\,\big(\nabla^a F^{-}_{ab}\big)\big(\nabla^c F_c^{+b}\big)\,, \\ \mathcal{L}_{\mathsf{GB}} &= R^{abcd}\,R_{abcd} - 4\,R^{ab}R_{ab} + R^2\,.\end{aligned}$$

G_N: Newton constant; $C_{ab}{}^{cd}$: Weyl tensor; F_{ab}: graviphoton field strength; L: determines the cosmological constant.

Solutions

One can show that all solutions of the 2der equations of motion solve also the 4der equations. Similar statements hold for 4der Einstein gravity with a cc [Smolic-Taylor] as well as 4der ungauged minimal supergravity [Charles-Larsen].

In addition one can show that the amount of supersymmetry preserved at 2der level persists at 4 derivatives. see also [Benetti Genolini-Richmond]



Note: There could be interesting genuinely new 4-derivative solutions that we have not explored.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On-shell action

An important observable in holography is the on-shell action. It corresponds to the partition function in the dual field theory.

One needs to perform holographic renormalization to cancel UV divergences arising from the boundary of AdS_4.

This is somewhat non-trivial in a gravitational theory with higher-derivative terms.

The fact that the 2der solutions remain intact is an important simplification in the technical analysis.

Another important relation is

$$I_{\mathrm{W}^2} = I_{\mathsf{GB}} - \frac{64\pi G_N}{L^2}\, I_{2\partial}\,.$$

Holographic renormalization

The divergences in the on-shell action can be removed via holographic renormalization using the following counterterms [Myers], [Emparan-Johnson-Myers]:

$$I_{2\partial}^{\mathsf{CT}} = (8\pi\, G_N)^{-1} \int d^3x\sqrt{h}\left(-K + \tfrac{1}{2}\, L\,\mathcal{R} + 2\, L^{-1}\right),$$

$$I_{\mathsf{GB}}^{\mathsf{CT}} = 4 \int d^3x\sqrt{h}\left(\mathcal{J} - 2\,\mathcal{G}_{ab}\, K^{ab}\right),$$

h_{ab}: induced metric on the boundary; K_{ab}: extrinsic curvature; $\mathcal{R}$ and $\mathcal{G}_{ab}$: boundary Ricci scalar and Einstein tensor, and

$$\mathcal{J} = \tfrac{1}{3}\left(3K(K_{ab})^2 - 2(K_{ab})^3 - K^3\right).$$

The end result is the simple expression (valid for **all** 2der solutions including non-susy ones!)

$$\boxed{I_{\mathsf{HD}} = \left[1 + \frac{64\pi G_N}{L^2}(c_2 - c_1)\right]\frac{\pi L^2}{2G_N}\mathcal{F} + 32\pi^2 c_1\,\chi\,.}$$

$\mathcal{F} = \frac{2G_N}{\pi L^2}(I_{2\partial} + I_{2\partial}^{\mathsf{CT}})$: regularized on-shell action of the 2der theory.
$\chi = \frac{1}{32\pi^2}(I_{\mathsf{GB}} + I_{\mathsf{GB}}^{\mathsf{CT}})$: Euler characteristic of the 4-manifold.

Results

I_{HD} can be computed explicitly for all known 2der solutions of 4d minimal gauged supergravity.

Supersymmetric solutions are rare and are of particular holographic interest.

Solution $\mathcal{M}_4$	Susy	$\mathcal{F}$	χ
AdS$_4$ w. S^3 bdry	1	1	1
$\mathrm{U}(1)\times\mathrm{U}(1)$ sq.	1/2	$\frac{1}{4}(b+\frac{1}{b})^2$	1
$\mathrm{SU}(2)\times\mathrm{U}(1)$ sq.	1/2	s^2	1
$\mathrm{SU}(2)\times\mathrm{U}(1)$ sq.	1/4	1	1
KN-AdS	1/4	$\frac{(\omega+1)^2}{2\omega}$	2
AdS$_2\times\Sigma_{\mathfrak{g}}$	1/2	$(1-\mathfrak{g})$	$2(1-\mathfrak{g})$
Romans	1/4	$(1-\mathfrak{g})$	$2(1-\mathfrak{g})$
Bolt$_\pm$	1/4	$(1-\mathfrak{g})\mp\frac{p}{4}$	$2(1-\mathfrak{g})$

Table: I_{HD} for various supersymmetric Euclidean solutions of holographic interest. The double line separates solutions with NUT ($\mathbb{R}^4$) and Bolt ($\mathbb{R}^2\times\Sigma_{\mathfrak{g}}$) topology.

Two examples

Two solutions of special interest are

- AdS-Taub-NUT ($\mathrm{U}(1) \times \mathrm{U}(1)$ squashed S^3)

$$
\begin{aligned}
ds^2 &= f_1^2 dx^2 + f_2^2 dy^2 + \frac{1}{f_1^2}(d\psi + y^2 d\phi)^2 + \frac{1}{f_2^2}(d\psi + x^2 d\phi)^2 \,, \\
f_1^2 &= \frac{L^2(y^2 - x^2)}{(x^2-1)(b^4 - x^2)} \,, \qquad f_2^2 = \frac{L^2(y^2 - x^2)}{(y^2-1)(y^2 - b^4)} \,, \\
F &= d\left[\frac{b^4 - 1}{L(x+y)}(d\psi - xyd\phi) \right] \,, \\
\mathcal{F} &= \frac{1}{4}\left(b + \frac{1}{b}\right)^2 \,, \qquad \chi = 1 \,.
\end{aligned}
$$

- Euclidean Romans solution (Euclidean supersymmetric RN in AdS$_4$)

$$
\begin{aligned}
ds^2 &= \left[\left(\frac{r}{L} + \frac{\kappa L}{2r} \right)^2 - \frac{q^2}{4r^2} \right] d\tau^2 + \left[\left(\frac{r}{L} + \frac{\kappa L}{2r} \right)^2 - \frac{q^2}{4r^2} \right]^{-1} dr^2 + r^2 ds^2_{\Sigma_{\mathfrak{g}}} \,, \\
F &= \frac{q}{r^2} d\tau \wedge dr - \kappa L \mathrm{vol}_{\Sigma_{\mathfrak{g}}} \,, \\
\mathcal{F} &= (1 - \mathfrak{g}) \,, \qquad \chi = 2(1 - \mathfrak{g}) \,.
\end{aligned}
$$

C_T

An important observable in AdS$_4$ holography is the coefficient, C_T, of the two-point function of the energy momentum tensor in the dual SCFT. Using our 4der gravitational action we find [Sen-Sinha]

$$C_T = \frac{32L^2}{\pi G_N} + 2048(c_2 - c_1)\,.$$

This result is valid for all 3d holographic SCFTs captured by our minimal supergravity setup.

Consistency check

In 3d $\mathcal{N} = 2$ SCFTs C_T can also be computed from the free energy, $F = -\log Z$, on the squashed S^3 [Closset-Dumitrescu-Festuccia-Komargodski]

$$C_T = \frac{32}{\pi^2}\, \frac{\partial^2 I_{S^3_b}}{\partial b^2}\bigg|_{b=1}\,.$$

Indeed, this relation holds for our 4der on-shell action!


IF YOU AREN'T IMPRESSED WITH THIS PICTURE OF THE BLACK HOLE...
...YOU CLEARLY DON'T UNDERSTAND THE GRAVITY OF THE SITUATION
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Black hole entropy

Consider a general stationary black hole solution of the 2der theory, i.e. the AdS-Kerr-Newman black hole (work in Lorentzian signature here).

The 4der terms in the action affect the black hole entropy which is given by [Wald]...

$$S = -2\pi \int_H E^{abcd}\, \varepsilon_{ab}\, \varepsilon_{cd}\,,$$

E^{abcd}: variation of the HD Lagrangian with respect to the Riemann tensor; ε_{ab}: unit binormal to the horizon.

Applying this to our model yields

$$S = (1+\alpha)\,\frac{A_H}{4G_N} - 32\pi^2 c_1\,\chi(H)\,, \qquad \alpha := \frac{64\pi G_N}{L^2}(c_2 - c_1)\,.$$

A_H: horizon area; $\chi(H)$: horizon Euler characteristic.

Note: This result is independent of supersymmetry and applies to all AdS_4 black holes in the Einstein-Maxwell theory.

Conserved charges

The 4der terms in the action modify conserved charges associated to conserved currents and Killing vectors.

For example, consider EM charges. The Maxwell equations are $dG = dF = 0$, where

$$(\star G)_{\mu\nu} = 32\pi\, G_N\, \frac{\delta \mathcal{L}_{\mathsf{HD}}}{\delta F^{\mu\nu}}\,.$$

The electric and magnetic charges, Q and P, are defined by integrating G and F over the boundary, $\partial\Sigma$, of a surface, Σ, at spatial infinity:

$$Q = \int_{\partial\Sigma} G\ , \qquad P = \int_{\partial\Sigma} F\,.$$

The field strength F, and therefore P, is unaffected by the HD terms. The electric charge is modified to

$$Q = (1+\alpha)Q_{2\partial}\,.$$

We can also compute the Komar integrals for mass and angular momentum to find

$$M = (1+\alpha)M_{2\partial}\,, \qquad J = (1+\alpha)J_{2\partial}\,.$$

Black hole thermodynamics

As a consistency check of our results we consider the quantum statistical relation (QSR) [Gibbons-Perry-Pope]

$$I = \beta \left(M - TS - \Phi Q - \omega J \right) \ ,$$

Here, $T = \beta^{-1}$ is the temperature, Φ is the electric potential, and ω is the angular velocity. These intensive quantities are determined by the 2der solution and are therefore not modified.

The on-shell action I, as well as S, M, Q, and J are extensive quantities and are modified by the HD terms. If the QSR is satisfied in the 2der theory then it is also satisfied in the 4der theory (provided that $\chi(\mathcal{M}_4) = \chi(H)$, which one can prove).

Comments:

1. The ratio Q/M for extremal black holes is not affected by the 4der terms and therefore the corrections to the black hole entropy have no relation to the extremality bound.

2. The black hole entropy corrections do not have a definite sign and therefore do not necessarily lead to an increase in the entropy for all black holes.

scft

means

Super Conformal Field Theory

M2-branes at large N

Apply the on-shell action results above to the class of 3d SCFTs arising from N M2-branes at the tip of a CY_4 conical singularity in M-theory. Motivated by the 2der consistent truncation from 11d to 4d minimal supergravity. [Gauntlett-Varela]

General arguments about HD terms in holography combined with 2der results in 11d supergravity lead to the following large N behavior of the constants in our supergravity model. [Camanho-Edelstein-Maldacena-Zhiboedov]

$$\frac{L^2}{2G_N} = A\,N^{\frac{3}{2}} + a\,N^{\frac{1}{2}}\,, \qquad c_i = v_i\,\frac{N^{\frac{1}{2}}}{32\pi}\,.$$

With this at hand the 4der on-shell action becomes

$$I_{\text{HD}} = \pi\,\mathcal{F}\,\left[A\,N^{\frac{3}{2}} + (a+v_2)\,N^{\frac{1}{2}}\right] - \pi\,(\mathcal{F}-\chi)\,v_1\,N^{\frac{1}{2}}\,.$$

Simple idea: Fix the unknown constants $(A, a+v_2, v_1)$ by using supersymmetric localization results for C_T and the S^3 free energy.

Fixing the constants

Consider two classes of SCFTs.

1. $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ $\mathcal{N} = 6$ ABJM theory.
2. $\mathrm{U}(N)$ 3d $\mathcal{N} = 4$ SYM coupled to 1 adjoint and N_f fundamental hypers.

Using localization results for the round S^3 free energy we can calculate A and the linear combination $a + v_1 + v_2$. [Mariño-Putrov], [Fuji-Hirano-Moriyama], [Mezei-Pufu]

Computing C_T via localization allows us to determine v_1 independently. [Agmon-Chester-Pufu], [Chester-Kalloor-Sharon]

This allows us to fix the full QFT answer to order $N^{\frac{1}{2}}$!

Theory	A	$a + v_2$	v_1
ABJM at level k	$\frac{\sqrt{2k}}{3}$	$-\frac{k^2+8}{24\sqrt{2k}}$	$-\frac{1}{\sqrt{2k}}$
$\mathcal{N} = 4$ SYM w. N_f fund.	$\frac{\sqrt{2N_f}}{3}$	$\frac{N_f^2-4}{8\sqrt{2N_f}}$	$-\frac{N_f^2+5}{6\sqrt{2N_f}}$

Note: The theory with $N_f = 1$ is dual to the ABJM theory with $k = 1$.

Predictions for localization

Three concrete predictions for supersymmetric localization in ABJM

- S_b^3 free energy ($F := -\log Z$) for general b

$$F_{S_b^3} = \frac{\pi\sqrt{2k}}{12}\left[\left(b+\frac{1}{b}\right)^2\left(N^{\frac{3}{2}}+\frac{16-k^2}{16k}N^{\frac{1}{2}}\right)-\frac{6}{k}N^{\frac{1}{2}}\right].$$

  Note: Agrees with localization results for $k=1$ and $b^2=3$ obtained using topological strings. [Hatsuda] Also with very recent results for $b \approx 1$. [Chester-Kalloor-Sharon]

- Topologically twisted index on $S^1 \times \Sigma_{\mathfrak{g}}$ with the so-called universal twist [Benini-Hristov-Zaffaroni], [Azzurli-NPB-Crichigno-Min-Zaffaroni]

$$-\log Z_{S^1\times\Sigma_{\mathfrak{g}}} = (1-\mathfrak{g})\frac{\pi\sqrt{2k}}{3}\left[N^{\frac{3}{2}}-\frac{32+k^2}{16k}N^{\frac{1}{2}}\right].$$

  Note: Agrees with localization for $\mathfrak{g}=0$. [Liu-Pando Zayas-Rathee-Zhao]

- Superconformal index

$$-\log Z_{S^1\times S^2} = \frac{\pi\sqrt{2k}}{3}\left[\frac{(\omega+1)^2}{2\omega}\left(N^{\frac{3}{2}}+\frac{16-k^2}{16k}N^{\frac{1}{2}}\right)-\frac{3}{k}N^{\frac{1}{2}}\right].$$

Corrections to the BH entropy

Using these explicit results we can also compute the leading correction to the entropy of any asymptotically $\mathrm{AdS}_4 \times S^7$ black hole.

Example: AdS-Schwarzschild black hole

$$ds^2 = \left(\frac{r^2}{L^2} + 1 - \frac{m}{r}\right) d\tau^2 + \left(\frac{r^2}{L^2} + 1 - \frac{m}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2 \,,$$

where $d\Omega_2^2$ is the metric on S^2. The location of the outer horizon r_+ is related to the mass parameter m as

$$m = \frac{r_+^3}{L^2} + r_+ \,.$$

The area and Euler number of the horizon are given by

$$A_H = 4\pi\, r_+^2 \,, \qquad \chi(H) = 2 \,.$$

Applying the results above we find

$$\boxed{S_{\mathrm{Sch}}^{\mathrm{ABJM}} = \frac{2\pi r_+^2}{L^2}\frac{\sqrt{2k}}{3}\left(N^{\frac{3}{2}} + \frac{16-k^2}{16k}N^{\frac{1}{2}}\right) + 2\pi\frac{1}{\sqrt{2k}}N^{\frac{1}{2}}} \,.$$

Summary

- Discussed two 4der supersymmetry invariants in minimal gauged supergravity.
- Studied their implications for holography: a simple formula for the on-shell action and a compact expression for C_T.
- Analyzed the effects of the 4der terms on black hole physics: corrections to the BH entropy; conserved charges are modified by the 4der terms; the quantum statistical relation still holds non-trivially.
- Confronted these results with supersymmetric localization to arrive at new explicit results for the $N^{\frac{1}{2}}$ contributions to the partition function on compact 3-manifolds of the ABJM theory and 3d $\mathcal{N} = 4$ $\mathrm{U}(N)$ SYM with 1 adjoint and N_f fundamental hypers.
- Similar explicit results for 3d $\mathcal{N} = 2$ theories of class $\mathcal{R}$ arising from M5-branes wrapped on hyperbolic 3-manifolds. [NPB-Charles-Gang-Hristov-Reys]

Outlook

- Extend to matter coupled 4d supergravity. [in progress]
- Extend to 5d, 6d, and 7d supergravity. [Baggio-Halmagyi-Mayerson-Robbins-Wecht], [in progress]
- Use 4d $\mathcal{N} = 4$ conformal supergravity to constrain c_1 and c_2. [in progress]
- Revisit supersymmetric localization on general three-manifolds and for more general classes of $\mathcal{N} = 2$ SCFTs to compute the $N^{\frac{1}{2}}$ terms in the partition function.
- Generalize to 3d $\mathcal{N} = 2$ large N SCFTs arising from other branes: D2-branes in mIIA ($N^{\frac{5}{3}}$) and wrapped D4-D8 branes ($N^{\frac{5}{2}}$).
- Establish a direct relation between our results and 10d and 11d supergravity. [Cheseter-Pufu-Yin], [Binder-Chester-Pufu], ...
- Study the $\log N$ and $N^{-\frac{n}{2}}$ terms in the large N expansion of the ABJM free energy using 4d supergravity? [Hristov-Reys], [in progress]


thank you
danke
謝謝
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Баярлалаа
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