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## Odd-dimensional analogue of the Euler characteristic

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When compact manifolds  $X$  and  $Y$  are both even dimensional, their Euler characteristics obey the Kunnet formula  $\chi(X \times Y) = \chi(X)\chi(Y)$ . In terms of the Betti numbers of  $b_p(X)$ ,  $\chi(X) = \sum_p (-1)^p b_p(X)$ , implying that  $\chi(X) = 0$  when  $X$  is odd dimensional. We seek a linear combination of Betti numbers, called  $\rho$ , that obeys an analogous formula  $\rho(X \times Y) = \chi(X)\rho(Y)$  when  $Y$  is odd dimensional. The unique solution is  $\rho(Y) = -\sum_p (-1)^p b_p(Y)$ . Physical applications include: (1)  $\rho \rightarrow (-1)^m \rho$  under a generalized mirror map in  $d = 2m+1$  dimensions, in analogy with  $\chi \rightarrow (-1)^m \chi$  in  $d = 2m$ ; (2)  $\rho$  appears naturally in compactifications of M-theory. For example, the 4-dimensional Weyl anomaly for M-theory on  $X^4 \times Y^7$  is given by  $\chi(X^4)\rho(Y^7) = \rho(X^4 \times Y^7)$  and hence vanishes when  $Y^7$  is self-mirror. Since, in particular,  $\rho(Y \times S^1) = \chi(Y)$ , this is consistent with the corresponding anomaly for Type IIA on  $X^4 \times Y^6$ , given by  $\chi(X^4)\chi(Y^6) = \chi(X^4 \times Y^6)$ , which vanishes when  $Y^6$  is self-mirror; (3) In the partition function of p-form gauge fields,  $\rho$  appears in odd dimensions as  $\chi$  does in even.

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