S-Fold Solutions from D=4 Maximal Supergravity

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Beijing, August 27, 2021

Based on: G.Inverso, H. Samtleben, M.T. **1612.05123**; A. Guarino, C. Sterckx, M.T. **2002.03692**; A. Giambrone, E. Malek, H. Samtleben, M.T. **2103.10797**

Contents

Introduction

- D-dimensional (gauged) supergravities have provided a valuable framework where to study (non-perturbative) string and M-theory compactifications and their duality relations

Captures full non-lin. dynamics of a truncation of the low-lying modes on a b.g.

All non-dyn b.g. quantities (*geometry of M_{int}, fluxes etc.*) are encoded in a single (duality covariant) object called the ***embedding tensor*** Θ

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Gauging:

- minimal couplings,
- Yukawa terms,
- scalar potential
- additional terms in fermion susy variations

G-covariant ***embedding tensor*** Θ

$$\mathcal{G} \hookrightarrow G$$

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<u>Duality covariant formulation of gauged supergravity</u>

See: H. Samtleben **0808.4076**; M.T. **1609.09745** for reviews

Introduction

D=4 maximal supergravity:

- Its ungauged version describes the 0-modes of Type II string on T^6 or M-theory on T^7 and G= $E_{7(7)}$

[Cremmer, Julia '78]

$$\phi^I \in \mathcal{M}_{\text{scal}} = \frac{E_{7(7)}}{SU(8)/\mathbb{Z}_2} \qquad F^M_{\mu\nu} = (F^\Lambda_{\mu\nu},\, F_{\Lambda\,\mu\nu}) \in \mathbf{56}$$

$$\Lambda = 1, \ldots, 28$$

- Most general gauging encoded in $\Theta \in \mathbf{912}$ + quadratic constraints

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Exceptional Field Theory (ExFT) provides a direct embedding of (certain) gauged maximal supergravities in Type II string theories or D=11 SUGRA, through a generalized Scherk-Schwarz ansatz.

[Hohm, Samtleben, **1312.0614, 1312.4542, 1406.3348, 1410.8145**]

Introduction

- In the last 10 years new gaugings were found which involved both electric and magnetic vector fields with respect to a standard symplectic frame (*dyonic gaugings*)

[G. Dall'Agata, G. Inverso, 1112.3345; G. Dall'Agata, G. Inverso, M.T. 1209.0760]

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- The non-semisimple dyonic gaugings (and their vacua) were uplifted to (massive-) Type IIA/Type IIB theories in the context of ExFT $p+q+p'+q' \leq 8$

$$\mathcal{G} = [\underbrace{\mathrm{SO}(p,q)}_{A_\mu^\Lambda} \times \underbrace{\mathrm{SO}(p',q')}_{c\,A_{\Lambda\,\mu}}] \ltimes \underbrace{N^{(p+q,p'+q')}}_{A_\mu^\Lambda + c\,A_{\Lambda\,\mu}}$$

$\mathcal{G} \neq \mathrm{SO}(4)^2 \ltimes N^{(4,4)}$

$c = 0, 1$

(new) dyonic

- Their vacua extensively studied

A.Borghese, A.Guarino, D.Roest, 1209.3003; A.Borghese, G.Dibitetto, A.Guarino, D.Roest, O.Varela, 1211.5335; A.Borghese, A.Guarino, D.Roest, 1302.6057; A.Gallerati, H.Samtleben, M.T., 1410.0711

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(new) dyonic

- Dyonic-ISO(7) N=2 and N=3 vacua uplifted to $\mathrm{AdS}_4 \times \tilde{\mathrm{S}}^6$ solutions to massive Type-IIA $c \propto m$

[A.Guarino, D.Jafferis,O.Varela, 1504.08009; Y. Pang, J. Rong, 1508.05376]

- $p+q\leq 6$ cases uplifted to (massive-) Type IIA/Type IIB theories in

[G. Inverso, H. Samtleben, M.T., 1612.05123]

Type IIB S-Folds from D=4 SUGRA

- Maximal SUGRA with gauge group

$$\mathcal{G} = [\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes N^{(6,2)}$$

embedded in Type IIB through ExFT [G. Inverso, H. Samtleben, M.T., 1612.05123]

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S-fold solutions: non-geometric b.g.s featuring transition functions which involve duality transformation in $SL(2,\mathbf{Z})_{IIB}$ [C.Hull, A. Çatal-Özer, 0308133; C. Hull, 0406102]

- In our case S-folds have topology $\mathrm{AdS}_4 \times \tilde{S}^5 \times S^1$

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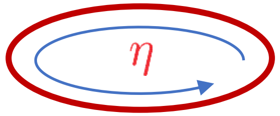

$$\eta \to \eta + T$$
$$\boldsymbol{\Psi} \to \mathfrak{M} \cdot \boldsymbol{\Psi}$$

- The monodromy $\mathfrak{M}$ is a hyperbolic element of SL(2,**Z**)$_{\mathrm{IIB}}$ $\mathfrak{M} = J_n = -ST^n = \begin{pmatrix} n & 1 \\ -1 & 0 \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$

$(n > 2)$ "**J-fold**"

Type IIB S-Folds from D=4 SUGRA

- They locally coincide with (singular) Janus solutions

$$\mathrm{AdS}_4 \times \tilde{S}^5 \times \mathbb{R}$$

[**N=0:** D.Bak, M. Gutperle,S. Hirano, 0304129;
N=1:E. D'Hoker, J. Estes,M. Gutperle, 0603012;
N=4: E. D'Hoker, J. Estes,M. Gutperle, 0705.0022]

- Expected dual SCFT:

Janus sol.s $\longleftrightarrow$ D=3 conformal Janus interfaces in N=4 D=4 SYM;

[**N=0:** A.B.Clark, D.Z.Freedman, A.Karch, M.Schnabl, 0407073;
N=0,1,2,4: D'Hoker, J. Estes, M. Gutperle, 0603013;
N=4: D.Gaiotto, E.Witten, 0804.2907]

J-fold SUGRA sol.s $\longleftrightarrow$ D=3 J-fold SCFT

[D.Gaiotto, E.Witten, 0807.3720;
N=4: B. Assel and A. Tomasiello, 1804.0641;
N=2; N. Bobev, F. Gautason, K. Pilch, M.Suh,J. van Muiden, 2003.09154; E. Beratto, N. Mekareeya, M. Sacchi, 2009.10123; N. Bobev, F. Gautason, J. van Muiden, 2104.00977]

Type IIB S-Folds from D=4 SUGRA

- $\mathcal{N}$=4 with symmetry $SO(4)_R$ J-fold
  [vacuum found in 1410.0711 ; D=10 uplift in: G. Inverso, H. Samtleben, M.T., 1612.05123]
- $\mathcal{N}$=0& SO(6) ; $\mathcal{N}$=1&SU(3) J-fold
  [A. Guarino, C. Sterckx, 1907.04177]
- $\mathcal{N}$=2& SU(2) x $U(1)_R$ J-fold
  [A. Guarino, C. Sterckx, M.T., 2002.03692]
- $\mathcal{N}$=2& U(1) x $U(1)_R$ J-fold 1-parameter, KK spectrum
  [vacua found in 2002.03692 ; D=10 uplift in: A. Giambrone, E.Malek, H. Samtleben, M.T., 2103.10797]
- $\mathcal{N}$=0& $U(1)^3$ (3-param.s) ; $\mathcal{N}$=1&$U(1)^2$ (2- param.s) J-folds and DWs
  [vacua found in 2002.03692 ; D=10 uplift in: A. Guarino, C. Sterckx, 2103.12652]
- $\mathcal{N}$=2& U(1) x $U(1)_R$ J-fold 2-parameters (D=4 vacuum and SUGRA spectrum)
  [N. Bobev, F. Gautason, J. van Muiden, 2104.00977]

Type IIB S-Folds from D=5 SUGRA

- **N=1, N=2&U(2):** Bobev, F. Gautason, K. Pilch, M.Suh,J. van Muiden, 1907.11132, 2003.09154;
- **N=4 and N=2&U(1)2 (1- param.s) J-folds and DWs**: I. Arav, J.Gauntlett, M.Roberts, C.Rosen, 2103.15201

N=2 S-Folds

Solution with U(2)-symmetry: the $SL(2,R)_{IIB}$-invariant sector

Isometry $SU(2) \times U(1)_R$

- *Background geometry:* $\text{AdS}_4 \times \tilde{S}^5 \times S^1 = \text{AdS}_4 \times S^2 \times \tilde{S}^3 \times S^1$

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$$ds^2 = \tfrac{1}{2}\,\Delta^{-1}\,\left(ds^2_{\text{AdS}_4} + ds^2_{\text{S}^2} + \cos^2(\theta)\,ds^2_{\text{S}^3} + d\eta^2\right)$$

$$\Delta \equiv (6 - 2\cos(2\theta))^{-\frac{1}{4}}$$

$$ds^2_{\text{S}^2} = d\theta^2 + \sin^2(\theta)\,d\varphi^2$$

$$ds^2_{\text{S}^3} = \sigma_2^2 + 8\,\Delta^4\,(\sigma_1^2 + \sigma_3^2)$$

$U(1)_R$

$$\sigma^1 = \frac{1}{2}(\mathrm{d}\gamma\cos(\alpha)\sin(\beta) - \mathrm{d}\beta\sin(\alpha))$$

$$\sigma^2 = \frac{1}{2}(\mathrm{d}\beta\cos(\alpha) + \mathrm{d}\gamma\sin(\alpha)\sin(\beta))\,,$$

$$\sigma^3 = \frac{1}{2}(\mathrm{d}\alpha + \mathrm{d}\gamma\cos(\beta))\,.$$

$$d\sigma^x - \epsilon^{xyz}\,\sigma^y \wedge \sigma^z = 0$$

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Internal coord.s $y^a = (y^i, \eta) = (\theta, \phi, \alpha, \beta, \gamma, \eta)$

$$0 \leq \eta < T\,,\quad 0 \leq \theta \leq \frac{\pi}{2}\,,\quad 0 \leq \varphi < 2\pi$$

$$0 \leq \alpha \leq 2\pi\,,\quad 0 \leq \beta \leq \pi\,,\quad 0 \leq \gamma + \frac{\pi}{2} < 4\pi$$

$$\sigma^1 = \frac{1}{2}(\text{d}\gamma\cos(\alpha)\sin(\beta) - \text{d}\beta\sin(\alpha))$$

$$\sigma^2 = \frac{1}{2}(\text{d}\beta\cos(\alpha) + \text{d}\gamma\sin(\alpha)\sin(\beta))\,,$$

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N=2 S-Folds

Solution with U(2)-symmetry: the SL(2,R)$_{\text{IIB}}$-invariant sector

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Isometry SU(2) x U(1)$_R$ (of $\tilde{S}^3$)

$$ds^2 = \frac{1}{2}\,\Delta^{-1}\left(ds^2_{\text{AdS}_4} + ds^2_{\text{S}^2} + \cos^2(\theta)\,ds^2_{\text{S}^3} + d\eta^2\right)$$

$$\Delta \equiv (6 - 2\cos(2\theta))^{-\frac{1}{4}}$$

- *5-form field strength:*

$$\tilde{F}_5 \equiv dC_{(4)} + \frac{1}{2}\epsilon_{\alpha\beta}B^{\alpha}_{(2)} \wedge H^{\beta}_{(3)} = (1+\star)4\Delta^4 \sin(\theta)\cos^3(\theta)\,[3\,d\theta \wedge d\phi \wedge \sigma_1 \wedge \sigma_2 \wedge \sigma_3$$
$$-d\eta \wedge \left(\cos(2\theta)\,d\theta - \frac{1}{2}\sin(2\theta)\sin(2\phi)\,d\phi\right) \wedge \sigma_1 \wedge \sigma_2 \wedge \sigma_3]$$

$$B^{\alpha}_{(2)} = (B_{(2)}, C_{(2)})$$

$$H^{\alpha}_{(3)} = \text{d}B^{\alpha}_{(2)}$$

N=2 S-Folds

Solution with U(2)-symmetry: the SL(2,R)$_{\text{IIB}}$-covariant sector

- *2-form fields:*

$$B^{\alpha}_{(2)} = (B_{(2)},\, C_{(2)}) = A(\eta)^{\alpha}{}_{\beta}\, \mathfrak{b}^{\beta}_{(2)}$$

$$\mathfrak{b}^{1}_{(2)} = \frac{1}{\sqrt{2}} \cos(\theta) \left[\left(\cos(\phi)\, d\theta + \frac{1}{2} \sin(2\theta)\, d(\cos(\phi)) \right) \wedge \sigma_2 + \cos(\phi) \frac{4 \sin(2\theta)}{6 - 2\cos(2\theta)} \sigma_1 \wedge \sigma_3 \right]$$

$$\mathfrak{b}^{2}_{(2)} = -\frac{1}{\sqrt{2}} \cos(\theta) \left[\left(\sin(\phi)\, d\theta + \frac{1}{2} \sin(2\theta)\, d(\sin(\phi)) \right) \wedge \sigma_2 + \sin(\phi) \frac{4 \sin(2\theta)}{6 - 2\cos(2\theta)} \sigma_1 \wedge \sigma_3 \right]$$

- *Axion-dilaton system* $\tau = C_{(0)} + i\, e^{-\varphi}$ *:*

$$m_{\alpha\beta} = \frac{1}{\text{Im}(\tau)} \begin{pmatrix} |\tau|^2 & -\text{Re}(\tau) \\ -\text{Re}(\tau) & 1 \end{pmatrix} = (A(\eta)^{-1})^{\sigma}{}_{\alpha}\, (A(\eta)^{-1})^{\gamma}{}_{\beta}\, \mathfrak{m}_{\sigma\gamma}$$

$$\mathfrak{m}_{\sigma\gamma} = 2\, \Delta^2 \begin{pmatrix} \sin^2(\theta)\cos^2(\phi) + 1 & -\frac{1}{2}\sin^2(\theta)\sin(2\phi) \\ -\frac{1}{2}\sin^2(\theta)\sin(2\phi) & \sin^2(\theta)\sin^2(\phi) + 1 \end{pmatrix}$$

N=2 S-Folds

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Dependence on η *through*

$$A(\eta)^{\alpha}{}_{\beta} \equiv \begin{pmatrix} \cosh(\eta) & \sinh(\eta) \\ \sinh(\eta) & \cosh(\eta) \end{pmatrix} \in SL(2,\mathbb{R})_{\mathrm{IIB}}$$

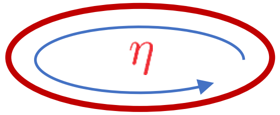
$$\mathfrak{m}_{\sigma\gamma} = 2\,\Delta^2 \begin{pmatrix} \sin^2(\theta)\cos^2(\phi) + 1 & -\frac{1}{2}\sin^2(\theta)\sin(2\phi) \\ -\frac{1}{2}\sin^2(\theta)\sin(2\phi) & \sin^2(\theta)\sin^2(\phi) + 1 \end{pmatrix}$$

N=2 S-Folds

The $SL(2,R)_{IIB}$-twist **A(η)** induces a monodromy

$$\mathfrak{M} = A(\eta)^{-1} \cdot A(\eta + T) = \begin{pmatrix} \cosh(T) & \sinh(T) \\ \sinh(T) & \cosh(T) \end{pmatrix}$$

$$\eta \to \eta + T$$



$$\mathbf{B}_{(2)} \to \mathfrak{M} \cdot \mathbf{B}_{(2)}$$
$$\tau \to \mathfrak{M} \cdot \tau$$

S-fold solution to Type IIB superstring theory:

[G. Inverso, H. Samtleben, M.T., 1612.05123;
B. Assel and A. Tomasiello, 1804.0641]

$$A(\eta) \to A(\eta) \cdot g_n \qquad e^T = \frac{1}{2}\left(n + \sqrt{n^2 - 4}\right) \qquad g_n \equiv \begin{pmatrix} \frac{(n^2-4)^{\frac{1}{4}}}{\sqrt{2}} & 0 \\ \frac{n}{\sqrt{2}(n^2-4)^{\frac{1}{4}}} & \frac{\sqrt{2}}{(n^2-4)^{\frac{1}{4}}} \end{pmatrix} \qquad (n > 2)$$

$$\mathfrak{M} \to g_n^{-1} \cdot \mathfrak{M} \cdot g_n = J_n = -S \cdot T^n \in SL(2, \mathbb{Z})_{IIB}$$

N=2 S-Folds

Solution with U(1) x U(1)$_R$ symmetry and 1 parameter χ

Vacua found in [A. Guarino, C. Sterckx, M.T., 2002.03692] within an N=1 truncation of the maximal theory

Uplifted to Type IIB solution in [A. Giambrone, E.Malek, H. Samtleben, M.T., 2103.10797] where the KK spectrum was computed and the global properties of the parameter χ studied

χ is a flat direction of the scalar potential at the extremum

$$V_0 = -\frac{3}{|c|} = -\frac{3}{(L_{\mathrm{AdS}})^2}$$

χ expected to be exactly marginal deformation of dual SCFT, coordinate of its **Conformal Manifold**

N=2 S-Folds

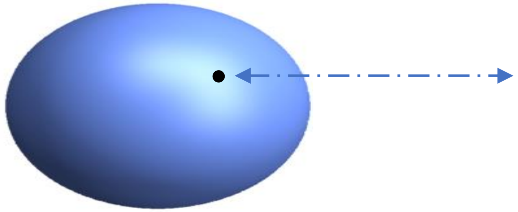
Solution with U(1) x $U(1)_R$ symmetry and 1 parameter χ

$\tilde{S}^3$ isometry

The parameter χ induces a second twist: $\mathbf{h}(\eta) = e^{2\chi \mathbf{H}\eta} \in \mathrm{U}(1) \in \mathrm{SU}(2)$

defining a fibration of the 3-sphere over S^1

$\chi = 0$

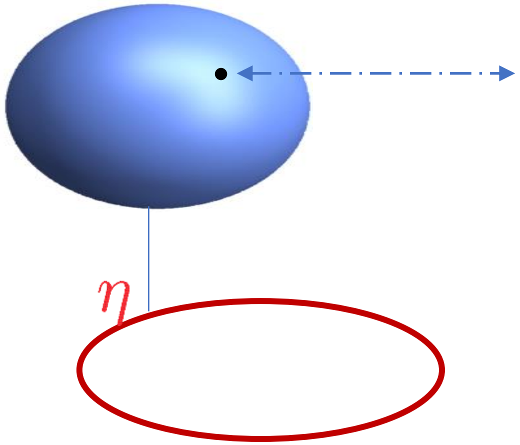


$\tilde{S}^3$

$g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$

$\mathrm{SU}(2) \rightarrow g[\alpha, \beta, \gamma] \leftarrow \mathrm{U}(1)_\mathrm{R}$

$\chi \neq 0$



$\tilde{S}^3$

η

S^1

$\hat{g}[\alpha, \beta, \gamma, \eta] = h(\eta) \cdot g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$

Locally: $\hat{g}[\alpha, \beta, \gamma, \eta] = g[\alpha', \beta', \gamma']$

χ affects the *global geometry* of the background

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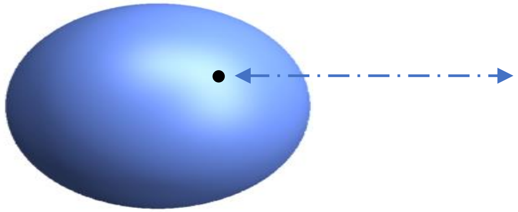

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$\mathrm{SU}(2) \rightarrow g[\alpha, \beta, \gamma] \leftarrow \mathrm{U}(1)_R$

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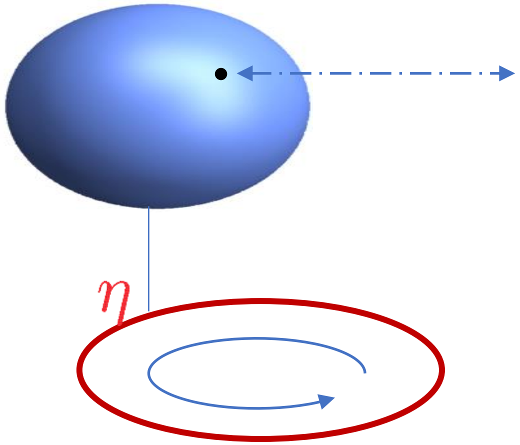


$\tilde{S}^3$, η, S^1

$$\hat{g}[\alpha, \beta, \gamma, \eta] = h(\eta) \cdot g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$$

$$\tilde{S}^3 \ltimes S^1 \equiv \tilde{S}^3 \times [0, T]/\sim$$

$$[g\,(\alpha, \beta, \gamma)\,,\, \eta = 0] \sim [\underbrace{h(T)}_{\text{new monodromy}} \cdot g\,(\alpha, \beta, \gamma)\,,\, \eta = T]$$

N=2 S-Folds

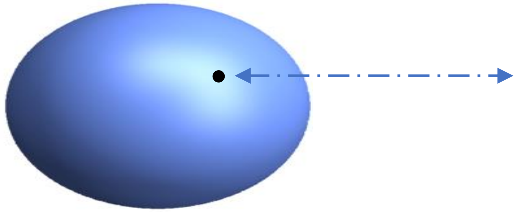
Solution with U(1) x U(1)$_R$ symmetry and 1 parameter χ

The parameter χ induces a second twist: $\mathbf{h}(\eta) = e^{2\chi \mathbf{H} \eta} \in \mathrm{U}(1) \in \mathrm{SU}(2)$

defining a fibration of the 3-sphere over S^1

$\tilde{S}^3$ isometry

$\chi = 0$

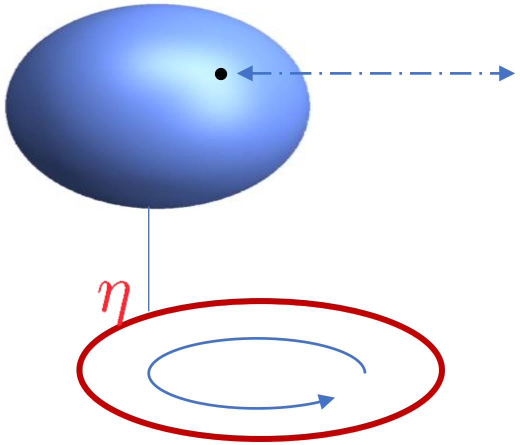


$\tilde{S}^3$

$g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$

$\mathrm{SU}(2) \rightarrow g[\alpha, \beta, \gamma] \leftarrow \mathrm{U}(1)_R$

$\chi \neq 0$



$\tilde{S}^3$

η

S^1

$\hat{g}[\alpha, \beta, \gamma, \eta] = h(\eta) \cdot g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$

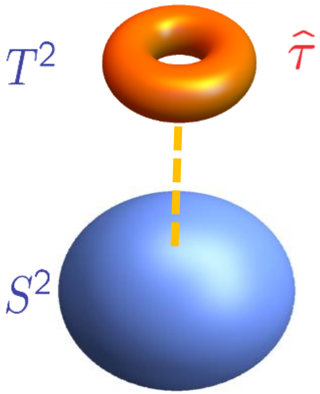
SU(2) broken to U(1) commuting with $h(T) = \begin{pmatrix} \cos(\chi T) & \sin(\chi T) \\ -\sin(\chi T) & \cos(\chi T) \end{pmatrix}$

$\mathrm{U}(1) \rightarrow \mathrm{SU}(2)$ enhancement: $\chi = \frac{\pi}{T}, \frac{2\pi}{T}$

N=2 S-Folds

Global properties of the parameter χ: $\chi \in \left[0, \frac{2\pi}{T}\right)$

- Write the 3-sphere as an Hopf fibration over S^2.The Hopf-fiber and S^1 combine in a torus so that $\tilde{S}^3 \ltimes S^1$ can be written as an elliptic fibration over S^2



$\hat{\tau} = \frac{i}{4\pi} - \frac{1}{2\pi}\chi T$ (χ part of a complex structure parameter) $z = \frac{\psi}{4\pi} + \hat{\tau}\frac{\eta}{T}\,, \quad 0 \leq \psi < 4\pi$

$\chi \to \chi + \frac{2\pi}{T} \;\Rightarrow\; \hat{\tau} \to \hat{\tau} - 1$ (Dehn twist)

N=2 S-Folds

Global properties of the parameter χ: $\chi \in \left[0, \frac{2\pi}{T}\right)$

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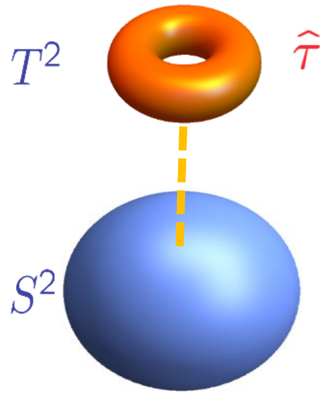

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- Solution at $\chi \neq 0$ obtained from that at $\chi = 0$ by: $\sigma^x \;\to\; \hat{\sigma}^x$

$$g^{-1}dg = \sum_{x=1}^{3} \sigma^x (i\,\boldsymbol{\sigma}^x)$$
$$\hat{g}^{-1}d\hat{g} = \sum_{x=1}^{3} \hat{\sigma}^x (i\,\boldsymbol{\sigma}^x)$$

$$\hat{\sigma}^1 \equiv \sigma^1 + \chi\,(-\cos(\alpha)\cos(\beta)\cos(\gamma) + \sin(\alpha)\sin(\gamma))d\eta\,,$$
$$\hat{\sigma}^2 \equiv \sigma^2 - \chi\,(\sin(\alpha)\cos(\beta)\cos(\gamma) + \cos(\alpha)\sin(\gamma))d\eta\,,$$
$$\hat{\sigma}^3 \equiv \sigma^3 + \chi\,\cos(\gamma)\sin(\beta)d\eta\,.$$

The two solutions are locally related by a reparametrization, though globally different

N=2 S-Folds

KK spectrum

- Within the framework of ExFT we computed the KK spectrum on the solution and the OSp(2|4) supermultiplet structure (*see Henning's talk for a review of the general approach*).

[E. Malek, H. Samtleben, 1911.12640; M. Cesàro, O.Varela, 2012.05249]

- Only use S^5 x S^1 scalar harmonics.

- Generic pattern: at each level the KK states gather in long vector, gravitino and graviton multiplets

[see C. Cordova, T. Dumitrescu, K, Intriligator, 1602.01217 for notation]

$$L\bar{L}[J]^{R}_{\Delta}$$

Lorentz spin of HWS $J = 0,\ \frac{1}{2},\ 1$

vector, gravitino, graviton

$R = \mathrm{U}(1)_R\text{-charge}$

- Shortening if the unitarity bound $\Delta \geq 1 + |R| + J$ is saturated

N=2 S-Folds

KK spectrum

- Conformal dimension for a spin-J state in a representation [**k**] of SU(2)

$$\Delta = \frac{1}{2} + \sqrt{\frac{17}{4} + \frac{1}{2}R^2 - J(J+1) - 2k(k+1) + \ell(\ell+4) + 4\left(\frac{\pi n}{T} - j\chi\right)^2}$$

$\ell = \tilde{S}^5$-level $\quad n = S^1$-level

$j = -k, -k+1, \ldots, k-1, k$: U(1) $\subset$ SU(2)-charge

N=2 S-Folds

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$j = -k, -k+1, \dots, k-1, k$: U(1) $\subset$ SU(2)-charge

- New χ –twist **h**(η) introduces extra dependence of the KK states on η

$$\Phi^{(k,j)}_{(n)}(y^{\mathsf{i}}, \eta) = \hat{\Phi}^{(k,j)}_{(n)}(y^{\mathsf{i}}) \underbrace{e^{-2ij\chi\eta}}_{\mathbf{h}(\eta)} e^{\frac{2i\pi n\eta}{T}}$$

$$\frac{\partial^2}{\partial\eta^2}\Phi^{(k,j)}_{(n)} = -4\left(j\chi - \frac{\pi n}{T}\right)^2 \Phi^{(k,j)}_{(n)}$$

N=2 S-Folds

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- $\chi = \frac{p\,\pi}{T}$ two real states with $n = pj$ become massless when $pj \in \mathbb{Z}$

SPACE INVADERS

[Duff,Nilsson,Pope, '86]

N=2 S-Folds

KK spectrum

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- $\chi = \frac{p\,\pi}{T}$ two vectors in Adj(SU(2)) (| j |=1) become massless

$\chi = \frac{\pi}{T}$ at $n = 1$

$\chi = \frac{2\pi}{T}$ at $n = 2$

N=2 S-Folds

KK spectrum

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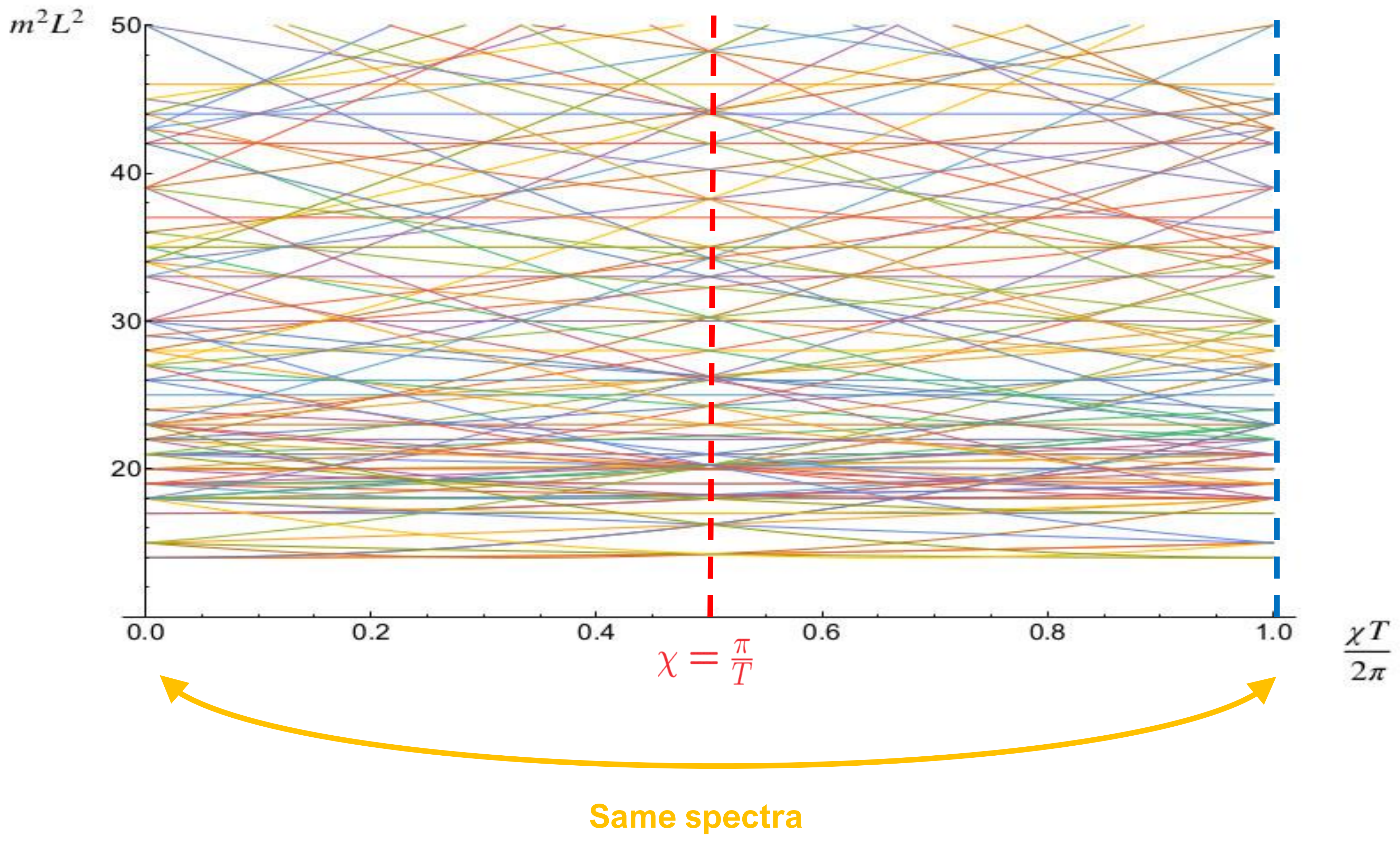
$$U(1) \to SU(2)$$

Example: spin-2 at level $\ell = 3$



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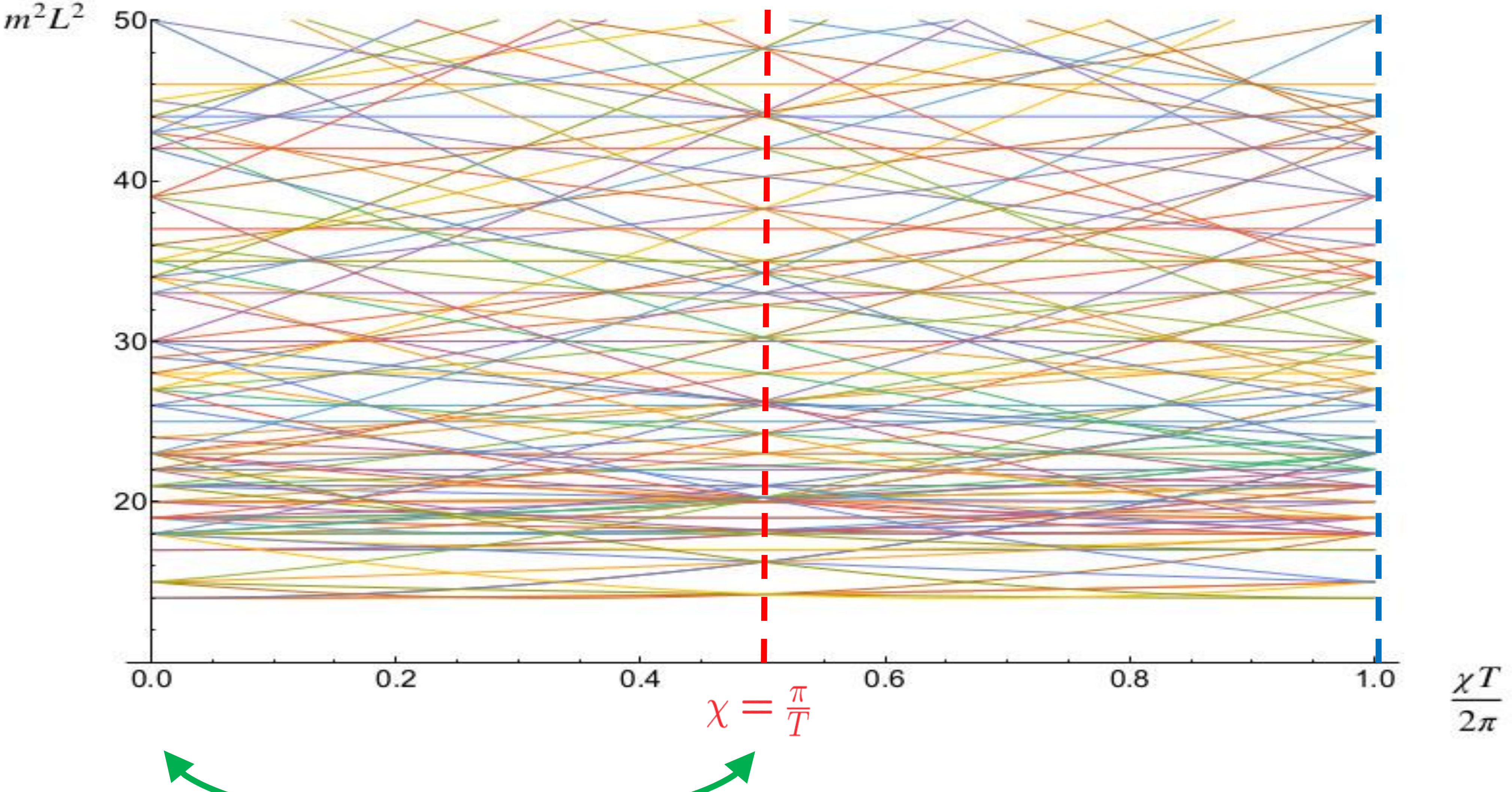
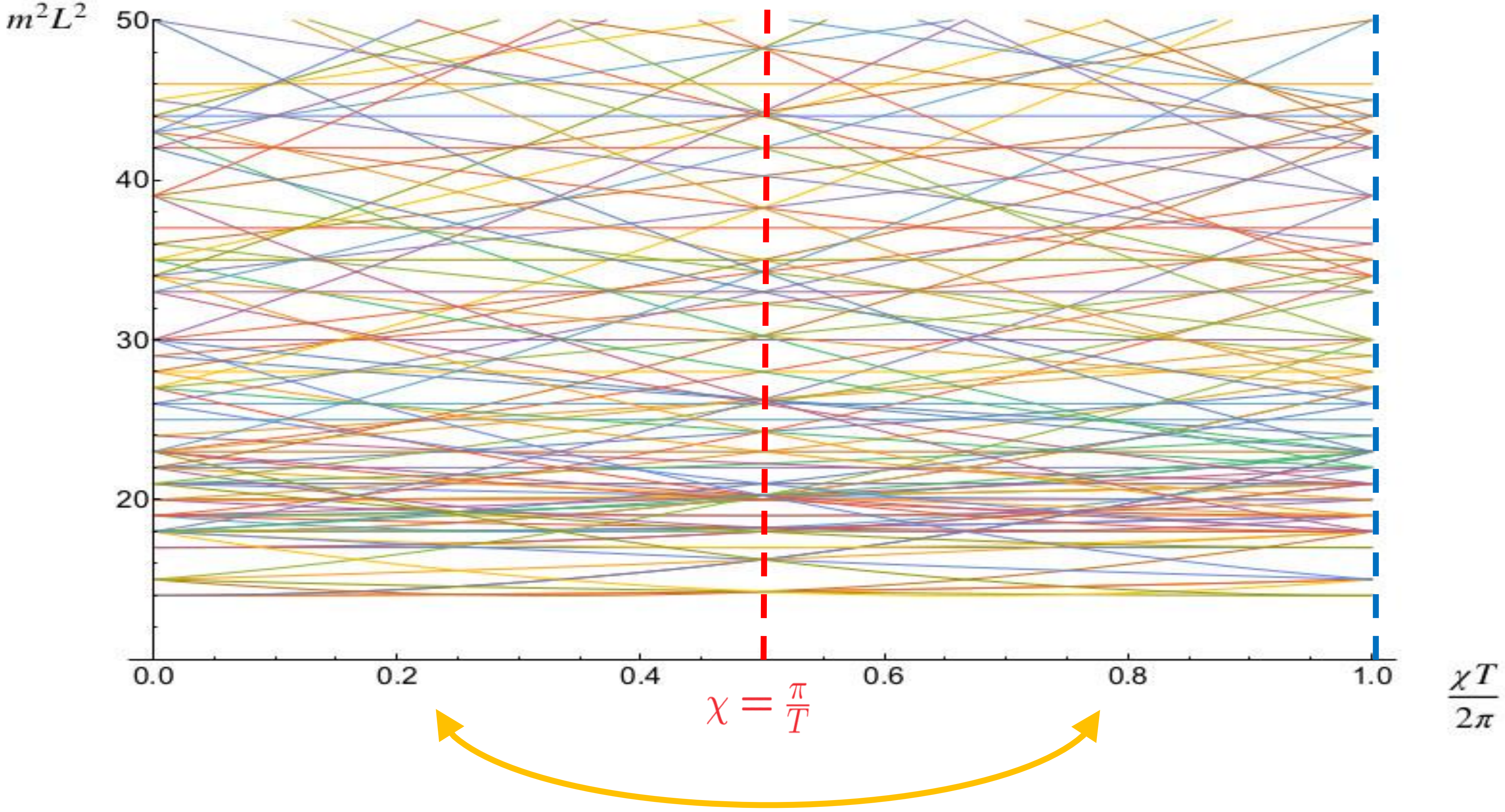


Different spectra
same only for integer j

Example: spin-2 at level $\ell = 3$



Symmetry from periodicity of χ and parity: $\chi \to -\chi$

Conclusions and outlook

- Constructed N=2 J-fold solutions of Type IIB with symmetry $U(1)^2$ depending on 1 parameter χ from D=4 gauged maximal supergravity and using ExFT

- Computed the full KK spectrum on the same class of solutions. Gave geometric interpretation of the parameter and determined its global properties: $\chi \in \left[0, \frac{2\pi}{T}\right)$

- Conformal manifold of dual SCFT is Kaehler: χ part of a complex modulus

- Found N=2 solutions in gauged maximal D=4 and D=5 SUGRAS depending on a new modulus δ

I. Arav, J.Gauntlett, M.Roberts, C.Rosen, 2103.15201 ;N. Bobev, F. Gautason, J. van Muiden, 2104.00977

- Uplifting the (χ, δ) –solutions to D=10. Global properties of the new modulus

- Understand the two moduli (χ, δ) by generalizing the analysis of E. D'Hoker, J. Estes,M. Gutperle, 0603013

Thank You !