Minimal SU(5) Unification

Shaikh Saad

University of Basel

The XXVIII International Conference on
Supersymmetry and Unification of Fundamental Interactions
August 24, 2021

Phys.Rev.D 104 (2021) 1, 015023 (Ilja Doršner, Emina Džaferović-Mašić, Saad)

Outline

- Georgi-Glashow model
- Towards Realistic $SU(5)$
- Minimal $SU(5)$ Unification (proposal and predictions)
- Summary

Georgi-Glashow Model

- Gauge group: $SU(5)$

- Fermions:

$$\overline{5}_{\boldsymbol{F}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}, \quad 10_{\boldsymbol{F}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}.$$

- Scalars:

$$24_{\boldsymbol{H}} : \quad SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$5_{\boldsymbol{H}} : \quad SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$$

Georgi, Glashow 1974

Georgi-Glashow Model: Drawbacks

- GG model: $\overline{5}_{\boldsymbol{F}\,i} + 10_{\boldsymbol{F}\,i} + 5_{\boldsymbol{H}} + 24_{\boldsymbol{H}}$

✗ $M_d = M_e^T$ Georgi, Jarlskog 1979

✗ $M_\nu = 0$

✗ Gauge coupling unification

Towards Realistic Model

✓ Renormalizable models

- $\overline{5}_{F\,i} + 10_{F\,i} + 5_H + 24_H + 45_H$ Georgi, Jarlskog 1979

- Fermion masses: Doršner, Perez 2006

$$M_d = v_5\, Y_1 + v_{45}\, Y_2$$
$$M_e^T = v_5\, Y_1 - 3 v_{45}\, Y_2$$
$$M_u = v_5 \left(Y_3 + {Y_3}^T \right) + v_{45} \left(Y_4 - {Y_4}^T \right)$$

Four arbitrary 3×3 matrices. (large number of parameters)

Towards Realistic Model

- $\overline{5}_{\boldsymbol{F}\,i} + 10_{\boldsymbol{F}\,i} + 5_{\boldsymbol{H}} + 24_{\boldsymbol{H}} + 45_{\boldsymbol{H}}$

✓ $M_d \neq M_e^T$

✓ Gauge coupling unification — Doršner, Perez 2006

✓ Proton decay (safe)

✗ $M_\nu = 0$

Purpose of this talk

- Realistic but Minimal theory of $SU(5)$?

Doršner, Saad 2019

Doršner, Emina, Saad 2021

Towards Minimal Model

SU(5) Irreps
Dimension (name)
5
10
15
24
35
40
45
50
70
70′
75
105
126

SU(5) Irreps
Dimension (name)
5
10
15
24
35
40
45
50
70
70′
75
105
126

- why large dim 45?

✗ $M_\nu = 0$

- Minimal possibility! (smaller dim)

✓ $M_\nu \neq 0$

Doršner, Saad 2019

A Novel $SU(5)$ Proposal

- $\overline{5}_{\boldsymbol{F}\,i} + 10_{\boldsymbol{F}\,i} + 5_{\boldsymbol{H}} + 24_{\boldsymbol{H}} + 35_{\boldsymbol{H}} + 15_{\boldsymbol{F}} + \overline{15}_{\boldsymbol{F}}$

- Decompositions

$$35_{\boldsymbol{H}} \equiv \Phi = \Phi_1(1,4,-3/2) + \Phi_3(\overline{3},3,-2/3) + \Phi_6(\overline{6},2,1/6) + \Phi_{10}(\overline{10},1,1)$$

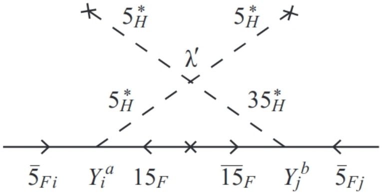
$$15_{\boldsymbol{F}} \equiv \Sigma = \Sigma_1(1,3,1) + \Sigma_3(3,2,1/6) + \Sigma_6(6,1,-2/3)$$

Fermion masses

- Quarks, charged leptons, neutrino masses- all connected !
- $M_u = \left(\mathbb{I}_{3\times 3} + \delta^2\, Y^c Y^{c\dagger}\right)^{-\frac{1}{2}} v_H Y^u$
- $M_d = \left(\mathbb{I}_{3\times 3} + \delta^2\, Y^c Y^{c\dagger}\right)^{-\frac{1}{2}} v_H \left(Y^d + \delta\, Y^c Y^a\right)$
- $M_e = v_H {Y^d}^T$
- $(\mathcal{M}_\nu)_{ij} = \frac{\lambda' v_5^2}{8\pi^2} \left(Y_i^a Y_j^b + Y_j^b Y_i^a\right) \left[\frac{M_{\Sigma_1}}{M_{\Sigma_1}^2 - M_{\Phi_1}^2} \ln\left(\frac{M_{\Sigma_1}^2}{M_{\Phi_1}^2}\right)\right]$
- Minimum parameters: (18 real Yukawas)
  Y^a, Y^b, Y^c: vectors of length 3;
  Y^d_{diag}; Y^u: symmetric 3×3 matrix

Neutrino sector: highly predictive

◇ Majorana particles



$35_H \supset (1, 4, -3/2)$ $\qquad$ $15_F \supset (1, 3, 1)$

◇ $m_1^\nu = 0$

◇ Inextricable link: $M_d \neq M_e^T \Longleftrightarrow M_\nu \neq 0$

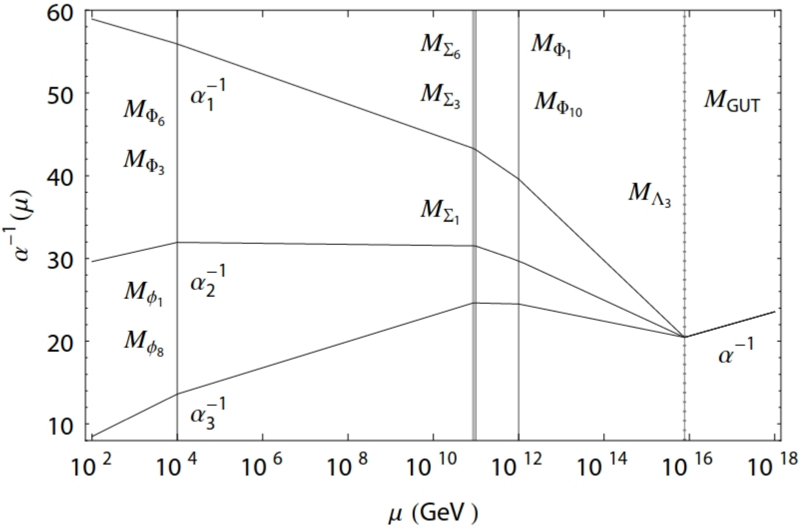
→ Normal mass ordering

Gauge Coupling Unification

Restrictions:

- $M_{\Sigma_6} = 2M_{\Sigma_3} - M_{\Sigma_1}$
- $M^2_{\Phi_{10}} = M^2_{\Phi_1} - 3M^2_{\Phi_3} + 3M^2_{\Phi_6}$
- $M_{\Lambda_3} \geq 3 \times 10^{11}$ GeV (proton decay)
- $M_{\mathrm{GUT}} \geq 6 \times 10^{15}$ GeV (proton decay)
- $M_k \geq 1$ TeV ($k =$ any BSM state)
- ν-mass requires specific M_{Σ_1} and M_{Φ_1}
  (rules out most of the parameter space consistent with unification)

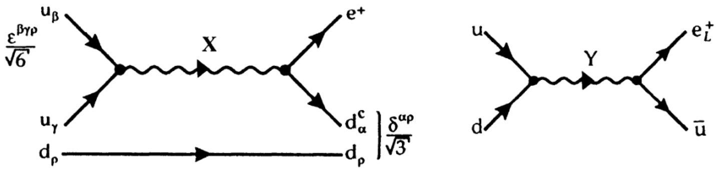
Gauge Coupling Unification



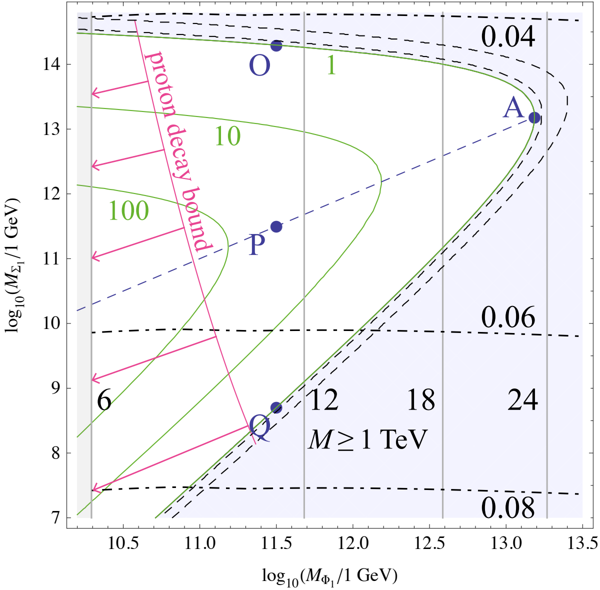
prediction $\sim$ three scales in the theory

Proton decay

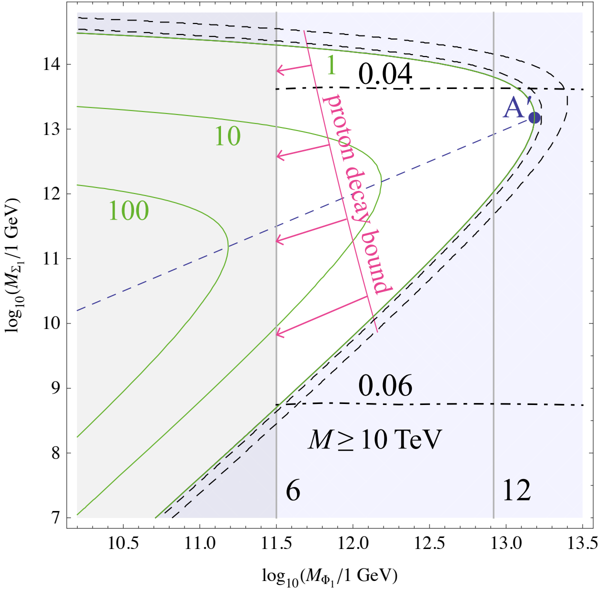
- Dominant mode: $p \to \pi^0 e^+$ (gauge boson mediated)
- Partial decay width: $\Gamma_p \sim \alpha_{GUT}^2 m_p^5 / M_V^4$
- Super-Kamiokande: $\tau^{\text{exp}}_{p \to \pi^0 e^+} > 2.4 \times 10^{34}$ years
- Naively: $M_{GUT} \geq 6 \times 10^{15}$ GeV



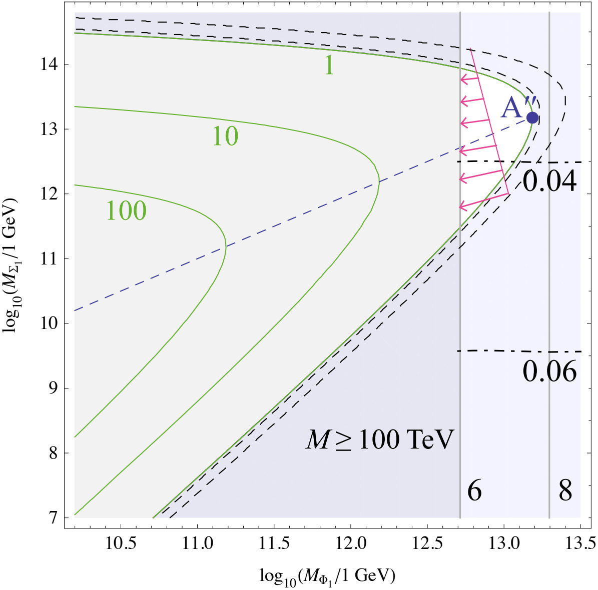
Full Parameter Space Exploration



Full Parameter Space Exploration



Full Parameter Space Exploration



Proton decay and Future prospects

- Dominant mode: $p \to \pi^0 e^+$
- An improvement by a factor of 2, 15, and 96 would completely rule out the $M \geq 100\,\text{TeV}$, $M \geq 10\,\text{TeV}$, and $M \geq 1\,\text{TeV}$ scenarios, respectively
- Hyper-Kamiokande: 10 years (20 years) of operation is 7.8×10^{34} (1.3×10^{35}) years

Minimal $SU(5)$ Unification: Summary

- ✓ First five lowest dimensional representations
- ✓ Least number of Yukawa parameters
- ✓ $M_d \neq M_e \Longleftrightarrow M_\nu \neq 0$ (all fermions are non-trivially connected)
- ✓ Neutrinos: Majorana, Normal ordering, $m_1 = 0$
- ✓ Gauge coupling unification $\Longleftrightarrow M_\nu$ (restricted parameter space)
- ✓ Rest of the allowed parameter space fixed by Proton Decay
- ✓ $(1,3,0), (8,1,0), (3,3,\frac{2}{3}), (6,2,\frac{-1}{6}) \sim$ 1-100 TeV (scalars)

THANK YOU!