The Standard Model and Dark Matter in Structures of Generalised Proper Time

(or, ‘how extra dimensions minus an assumption lead to the properties of ordinary matter and a dark sector’)

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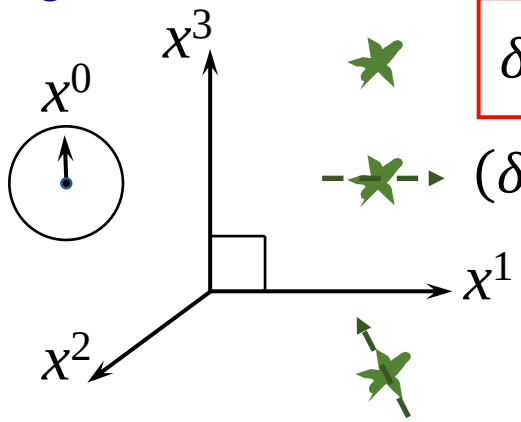
Begin with local coordinate frame in 4D spacetime:



$$\boxed{\delta s = \delta x^0}$$ measured time interval for object at rest

$$(\delta s)^2 = (\delta x^0)^2 - (\delta x^1)^2$$ proper time s, motion in x^1

proper time interval invariant under $\mathrm{SO}^+(1,3)$ Lorentz transformations:

$$\boxed{(\delta s)^2} = (\delta x^0)^2 - (\delta x^1)^2 - (\delta x^2)^2 - (\delta x^3)^2 \boxed{= \eta_{ab}\delta x^a \delta x^b} \quad a, b = 0,1,2,3$$

can't picture extra space dimensions, but can embed in local structure of $(n > 4)$D spacetime with $\mathrm{SO}^+(1, n-1)$ symmetry:

$$(\delta s)^2 = \eta_{ab}\delta x^a \delta x^b - \underbrace{(\delta x^4)^2 \ldots\ldots - (\delta x^{n-1})^2}$$ to describe <u>matter</u>?

with residual $\mathrm{SO}(n-4)$ symmetry, looks nothing like known matter.

However, don't need more <u>space</u> and can hence <u>drop quadratic assumption</u>: generalise $\delta s \to (\delta s)^2 \to (\delta s)^p, \quad p > 2$

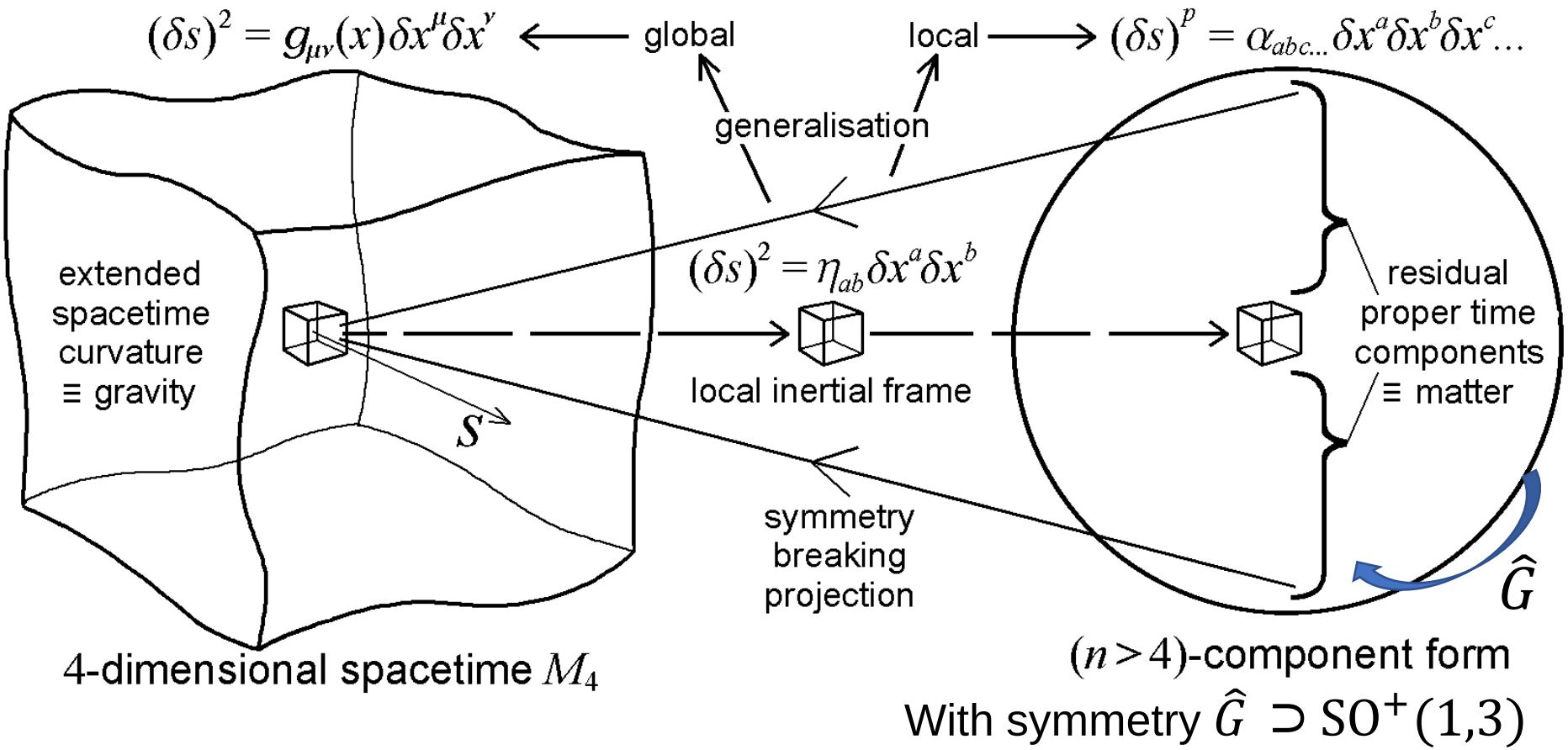
$$\boxed{(\delta s)^p} = \underbrace{\left(\eta_{ab}\delta x^a \delta x^b\right)}_{\text{4D spacetime}} \times \underbrace{(\delta x \ldots)^{p-2} + (\delta x \ldots)^p}_{\text{basis for matter here, will connect with SM}} \boxed{= \alpha_{abc\ldots}\delta x^a \delta x^b \delta x^c \ldots}$$

$a, b, c, \ldots = 0, \ldots, n-1$

each $\alpha_{abc\ldots} \in \{-1, 0\ \ 1\}$

Compared with general relativity:

complementary metric generalisation, but invariance of proper time central



With symmetry $\hat{G} \supset SO^+(1,3)$

Generalising from <u>local</u> 4D spacetime form for proper time reasonable since aiming to account for properties of <u>local</u> particle interactions

What are maths possibilities for $p > 2$ extensions from 4D form?

$p > 2$ generalisation from $4\mathrm{D}$ spacetime naturally leads to unique sequence of maths structures involving the octonions $\mathbb{O}$ and exceptional Lie groups.

In context of generalized proper time, begin with <u>$4\mathrm{D}$ quadratic</u> form:

$$(\delta s)^2 = (\delta x^0)^2 - (\delta x^1)^2 - (\delta x^2)^2 - (\delta x^3)^2 = \det\begin{pmatrix} \delta x^0 + \delta x^3 & \delta x^1 - \delta x^2 \\ \delta x^1 + \delta x^2 & \delta x^0 - \delta x^3 \end{pmatrix} = \det(\boldsymbol{\delta x}_4)$$

with Lorentz group double cover $\mathrm{SL}(2,\mathbb{C})$ on $\boldsymbol{\delta x}_4 \in \mathrm{h}_2\mathbb{C}$

$2 \times 2 \to 3 \times 3$ matrix

$(\delta s)^3 = \det(\boldsymbol{\delta x}_9)$ <u>$9\mathrm{D}$ cubic</u> form with $\mathrm{SL}(3,\mathbb{C})$ on $\boldsymbol{\delta x}_9 \in \mathrm{h}_3\mathbb{C}$

$\mathbb{C} \to \mathbb{O}$ algebra

$(\delta s)^3 = \det(\boldsymbol{\delta x}_{27})$ <u>$27\mathrm{D}$ cubic</u> with $\mathrm{SL}(3,\mathbb{O}) \equiv \mathrm{E}_6$ on $\boldsymbol{\delta x}_{27} \in \mathrm{h}_3\mathbb{O}$

Freudenthal triple system

$(\delta s)^4 = q(\boldsymbol{\delta x}_{56})$ <u>$56\mathrm{D}$ quartic</u> form with E_7 on $\boldsymbol{\delta x}_{56} \in F(\mathrm{h}_3\mathbb{O})$

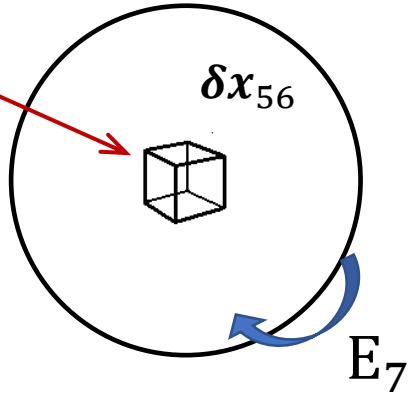
contains $\mathrm{E}_7 \supset \mathrm{E}_6$ $\mathbf{27}$ and $\overline{\mathbf{27}}$ reps.

main source of explicit maths:

A. Wangberg and T. Dray, 'E_6, the Group: The Structure of $\mathrm{SL}(3,\mathbb{O})$', arXiv:1212.3182

and references therein. Plus many further papers on above structures.

From 56 components of $(\delta s)^4 = q(\boldsymbol{\delta x}_{56})$ project 4D $\boldsymbol{\delta x}_4$ part onto local coordinates of external 4D spacetime



For all physics in 4D spacetime full E_7 symmetry broken to external $\times$ internal form :

$56 \backslash E_7 \supset$	Lorentz	$\times$	$SU(3)_c$	$\times$	$U(1)_Q$	matter
4	<u>vector</u>		**1**		0	'ν_L'
8	Dirac		**1**		1	$\begin{pmatrix} e_L \\ e_R \end{pmatrix}$
12	<u>scalar</u>		**3**		$\frac{2}{3}$	$\begin{pmatrix} \text{'}u_L\text{'} \\ \text{'}u_R\text{'} \end{pmatrix}$
24	Dirac		**3**		$\frac{1}{3}$	$\begin{pmatrix} d_L \\ d_R \end{pmatrix}$
4	<u>vector</u>		**1**		0	'Higgs' $\boldsymbol{\delta x}_4$
4	scalar		**1**		0	Yukawa

Rich in SM properties:

Lorentz spinor structures

Colour $SU(3)_c$ singlets and triplets

Electromagnetic $U(1)_Q$ fractional charges

elements of Electroweak theory

DJ, arXiv:1709.03877, section 4

While incomplete, and discrepancies seen as <u>underlined</u>, obtain structures of matter far closer to SM than with extra spatial dimensions ($(\delta s)^2$ case)

Possible to extend further to account for full Standard Model?

Relation of E_6 and E_7 to E_8 exhibits a "threeness", with branching pattern:

$$\mathrm{E}_8 \supset \mathrm{E}_6 \times \mathrm{SU}(3): \quad \mathbf{248} \rightarrow (\mathbf{27},\mathbf{3}) + (\overline{\mathbf{27}},\mathbf{3}) + (\mathbf{78},\mathbf{1}) + (\mathbf{1},\mathbf{8})$$

Known that $\mathbf{248}$ rep. of E_8 is in the 'ballpark' to accommodate 3 generation SM

Here looking for E_8 sym. of $(\delta s)^{p>4} = Q(\boldsymbol{\delta x}_{248})$ form for proper time ($p = 8$?)

- Explicit (<u>non-associative</u>) octonion construction of an "E_8" symmetry action
- $Q(\boldsymbol{\delta x}_{n\approx 248})$ must be <u>homogeneous</u> polynomial form for proper time $(\delta s)^{p>4}$

<u>Mathematical Prediction</u>: such a structure exists to complete the full SM pattern

DJ, `Time, E_8, and the Standard Model', arXiv:1709.03877

<u>Physical Predictions</u>: extrapolating from E_6 and E_7 to potential full E_8 level suggests Higgs may be composite and embedded in right-handed neutrino sector

DJ, `Generalised Proper Time as Unifying Basis for Models with Two Right-Handed Neutrinos', arXiv:1905.12419

Compare e.g.:

S. Antusch *et al.*, 'Dynamical Electroweak Symmetry Breaking by a Neutrino Condensate', (2003)

J. Krog and C. T. Hill, 'Is the Higgs Boson Composed of Neutrinos?', (2015)

Here any BSM predictions robust as via SM and from simple (unique) basis

Extra space dimensions motivated by the progression in relations between space, time and matter towards more unifying framework

Conceptual Relations	Theoretical Context	Role of Time s
space time matter	Newtonian physics	$s \in \mathbb{R}$ absolute
spacetime matter	special relativity	$(\Delta s)^2 = \eta_{ab}\Delta x^a \Delta x^b$ globally
spacetime matter	general relativity	$(\delta s)^2 = \eta_{ab}\delta x^a \delta x^b$ locally
spacetime matter	extra space dimensions	$(\delta s)^2 = \hat{\eta}_{ab}\delta x^a \delta x^b \quad n > 4$
space time matter	generalised proper time	$(\delta s)^p = \alpha_{abc\ldots}\delta x^a \delta x^b \delta x^c \ldots \quad p > 2$

Generalised proper time can then be seen as a natural further step along this trajectory

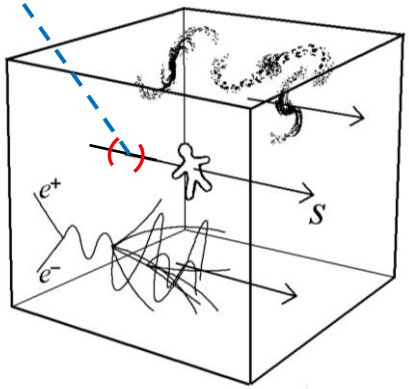
DJ, `Unification through Generalised Proper Time: The Short Story', arXiv:2011.01765

Extra space dimensions | 4D spacetime | **Generalised proper time**

$(\delta s)^2 = \hat{\eta}_{ab}\delta x^a \delta x^b$ ← $(\delta s)^2 = \eta_{ab}\delta x^a \delta x^b$ → $(\delta s)^p = \alpha_{abc\ldots}\delta x^a \delta x^b \delta x^c \ldots$

$a, b = 0, \ldots, n-1; \quad n > 4$

$\hat{\eta} = \mathrm{diag}(1, -1, -1, \ldots, -1)$



$n > 4$ and $p > 2$

each $\alpha_{abc\ldots} \in \{-1, 0\ \ 1\}$

'matter FROM spacetime' ← matter IN spacetime → 'matter AND space FROM time'

✓ unifying + comprehensive

✓ simple + unique basis

✓ conservative (time very familiar)

(extra space dims unseen, 100 years after Kaluza)

(sounds) plausible ✓

✓ (sounds) implausible ('we need a new crazy idea')

✓ maths beauty

✓ E_6, E_7, (E_8 ?), $\mathbb{O}$ algebra

✓ Works far better for SM

Extra space dimensions 4D spacetime **Generalised proper time**

$$(\delta s)^2 = \hat{\eta}_{ab}\delta x^a \delta x^b \leftarrow (\delta s)^2 = \eta_{ab}\delta x^a \delta x^b \rightarrow (\delta s)^p = \alpha_{abc\ldots}\delta x^a \delta x^b \delta x^c \ldots$$

$(n > 4)$D quadratic 'spacetime' form

full $\mathrm{SO}^+(1, n-1)$ symmetry broken:

$n \backslash \mathrm{SO}^+(1,n-1) \supset$	Lorentz $\times$	$\mathrm{SO}(m)$	matter
$m\ (= n-4)$	scalar	m-vector	'dark quarks' $\boldsymbol{\delta x}_m$
4	4-vector	invariant	'Higgs' $\boldsymbol{\delta x}_4$

$\mathrm{SO}(m)$ internal symmetry and scalar matter field, in parallel with, but 'hidden' from, the SM sector. Candidate for Dark Matter.

This sector, with m arbitrarily large, highly self-interacting; form massive 'dark glueballs' or 'scalar dark quark nuggets'.

With no Coulomb repulsion and no Fermi pressure extremely heavy states prone to collapse into 'primordial black holes'.

$(\delta s)^4 = q(\boldsymbol{\delta x}_{56})$ 56D quartic form

full E_7 symmetry broken:

$56 \backslash \mathrm{E}_7 \supset$	Lorentz $\times$	$\mathrm{SU}(3)_c$ $\times$	$\mathrm{U}(1)_Q$	matter
4	vector	**1**	0	'ν_L'
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Standard Model-like structures directly obtained

$\rightarrow \sim\mathrm{E}_8$ octic form for full SM?

From simple basis can generate Dark Matter sector with range of properties

Different branches of generalised proper time

$(\delta s)^2 = \eta_{ab}\delta x^a \delta x^b$ 4D root

$(\delta s)^2 = \hat{\eta}_{ab}\delta x^a \delta x^b \leftarrow \boxed{(\delta s)^p = \alpha_{abc\ldots}\delta x^a \delta x^b \delta x^c \ldots} \rightarrow (\delta s)^8 = Q(\boldsymbol{\delta x}_{248})$?

$\mathrm{SO}^+(1, n-1) \rightarrow$ Lorentz $\times\ \mathrm{SO}(m)$ $\qquad$ $\mathrm{E}_8 \rightarrow$ Lor. $\times\ \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

Dark Matter Sector $\qquad$ **Standard Model Sector**

Full gravitational interaction between DM and SM Sectors; plus potential for (very) weak interaction analogous to Higgs portal models.

Symmetric picture with parallel generation of DM and SM sectors, with most mass density associated with confining gauge forces of 'dark QCD' and QCD.

Since DM dominated by $\mathrm{SO}(m)$ with large m, while $\mathrm{SU}(3)_c$ is only part of SM, possible tilt in mass density balance towards DM by observed factor of five.

May be further mathematically permitted sectors of generalised proper time as further dark sector contributions, for example…

$(\delta s)^2 = \det(\boldsymbol{\delta x}_4 \in \mathrm{h}_2\mathbb{C}) \rightarrow (\delta s)^3 = \det(\boldsymbol{\delta x}_9 \in \mathrm{h}_3\mathbb{C}) \rightarrow (\delta s)^p = \det\left(\boldsymbol{\delta x}_{p^2} \in \mathrm{h}_p\mathbb{C}\right)$

→ non-compact internal gauge groups and negative kinetic energy in '$\mathcal{L} \sim FF$'

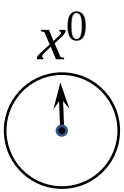
hence non-physical <u>or</u> a potential source of '**Phantom Dark Energy**'?

Conclusions

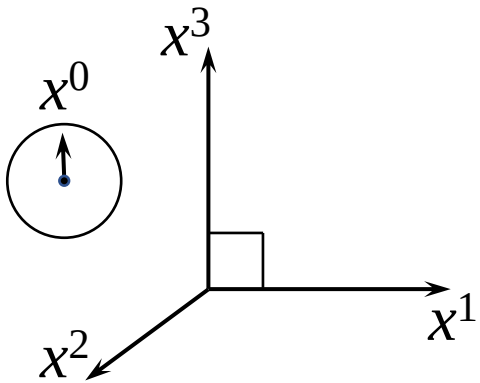
If postulate extra <u>space</u> dimensions, need to <u>add</u> further structure to get ~SM

Here use forms of proper <u>time</u>, and <u>drop assumption</u> of quadratic form → ~SM

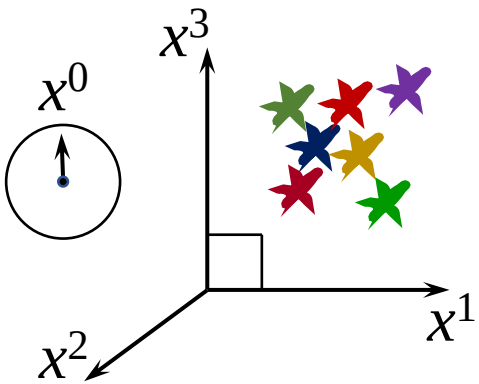
$$\delta s = \delta x^0 \qquad (\delta s)^2 = \eta_{ab}\delta x^a \delta x^b \qquad (\delta s)^p = \alpha_{abc\ldots}\delta x^a \delta x^b \delta x^c \ldots$$



time alone



spacetime



spacetime and matter

While unique maths structures of $p > 2$ extensions connect with SM, also have possible sectors of 'generalised proper time' for dark matter and dark energy.

'Occam's razor' type arguments suggest this <u>time</u>-based approach preferred.

Unified framework also needed for general relativity and quantum theory:

DJ, `Quantum Gravity from the Composition of Spacetime Constructed through Generalised Proper Time', arXiv:2010.02703