Towards the minimal non-supersymmetric E6 GUT

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Babu, BB, Susič, work in progress

Introduction

The most studied GUTs are SU(5) and SO(10)

E_6 proposed long ago

Gur̃sey, Ramond, Sikivie, '76

its susy version studied only in the last decade

BB, Susič, '13

Babu, BB, Susič, '15

In this talk we will try to construct the minimal realistic non-supersymmetric E_6

How to construct invariants in E_6?

- In SU(N):
  1. Tensors with upper and/or lower indices:
     $T_{\alpha\ldots}{}^{\beta\ldots} \quad , \quad \alpha,\beta,\ldots = 1,\ldots,N$
  2. Invariant tensors: $\epsilon_{\alpha_1\ldots\alpha_N}$ or $\epsilon^{\beta_1\ldots\beta_N}$ (completely anti-symmetric)
  3. SU(N) invariants are products of tensors with indices up and down summed, for example: $A_{\alpha\beta_1\beta_2}\, B^{\alpha}{}_{\beta_3\ldots\beta_N}\, \epsilon^{\beta_1\ldots\beta_N}$
- Very similar in E_6:
  1. Tensors with upper and/or lower indices:
     $T_{\alpha\ldots}{}^{\beta\ldots} \quad , \quad \alpha,\beta,\ldots = 1,\ldots,27$
  2. Invariant tensors: $d_{\alpha\beta\gamma}$ or $d^{\alpha\beta\gamma}$ (completely symmetric)
  3. E_6 invariants are products of tensors with indices up and down summed, for example: $A_{\alpha\beta\gamma}\, B^{\alpha}{}_{\delta}\, d^{\beta\gamma\delta}$

The lowest dimensional representations of E_6:

$$
\begin{array}{rcl}
27^\mu & \ldots & \text{fundamental} \\
78^\mu{}_\nu & \ldots & \text{adjoint } (= (t^A)^\mu{}_\nu 78^A) \\
351^{\mu\nu} = -351^{\nu\mu} & \ldots & \text{two indices antisymmetric} \\
351'^{\mu\nu} = +351'^{\nu\mu} & \ldots & \text{two indices symmetric } (d_{\lambda\mu\nu} 351'^{\mu\nu} = 0) \\
650^\mu{}_\nu & \ldots & (650^\mu{}_\mu = (t^A)^\nu{}_\mu 650^\mu{}_\nu = 0)
\end{array}
$$

In red the main heroes of this talk

- Where do SM fermions live in E_6?

  Decompose 27 of E_6 under SO(10)

$$27_F = 16 + 10 + 1$$

$$\begin{aligned} 16 &= SM\ fermions + RH\ neutrino \\ 10 &= exotic\ vectorlike\ pairs\ of\ quarks\ and\ leptons \\ 1 &= RH\ neutrino \end{aligned}$$

- Why we concentrate on 27, $351'$, 650 Higgses?

Generic Yukawa sector in E_6

In all generality three types of Yukawas

$$\mathcal{L}_{Yukawa} = 27_i \left(Y_{27}^{ij}\, 27 + Y_{351'}^{ij}\, 351' + Y_{351}^{ij}\, 351 \right) 27_j$$

$$Y_{27,351'} = Y_{27,351'}^T \qquad \text{symmetric}$$

$$Y_{351} = -Y_{351}^T \qquad \text{antisymmetric}$$

Completely analogous to SO(10):

$$\mathcal{L}_{Yukawa} = 16_i \left(Y_{10}^{ij}\, 10 + Y_{126}^{ij}\, 126 + Y_{120}^{ij}\, 120 \right) 16_j$$

$$Y_{10,126} = Y_{10,126}^T \qquad \text{symmetric}$$

$$Y_{120} = -Y_{120}^T \qquad \text{antisymmetric}$$

In fact

$$\begin{aligned} \mathbf{27} &= 1 + \mathbf{10} + \mathbf{16} \\ \mathbf{351'} &= 1 + \mathbf{10} + \overline{\mathbf{16}} + 54 + \mathbf{126} + \mathbf{144} \\ \mathbf{351} &= \mathbf{10} + \overline{\mathbf{16}} + \mathbf{16} + 45 + \mathbf{120} + \mathbf{144} \end{aligned}$$

The antisymmetric 351 contribution (similar as 120 in SO(10)) seems less promising so we will concentrate on the symmetric 27 and $351'$ from now on.

$$\mathcal{L}_{Yukawa} = \begin{pmatrix} 16 & 10 & 1 \end{pmatrix} Y_{27} \begin{pmatrix} 10 & 16 & 0 \\ 16 & 1 & 10 \\ 0 & 10 & 0 \end{pmatrix} \begin{pmatrix} 16 \\ 10 \\ 1 \end{pmatrix}$$

$$+ \begin{pmatrix} 16 & 10 & 1 \end{pmatrix} Y_{351'} \begin{pmatrix} 126 + 10 & 144 & \overline{16} \\ 144 & 54 & 10 \\ \overline{16} & 10 & 1 \end{pmatrix} \begin{pmatrix} 16 \\ 10 \\ 1 \end{pmatrix}$$

- if spinorial $(16, \overline{16}, 144)$ vevs large $\mathcal{O}(M_{GUT}) \rightarrow$
  - mixing between $\bar{5} \in 16$ and $\bar{5} \in 10$ $(d^c,\ L)$
  - mixing between $1 \in 1$ and $1 \in 16$ (ν^c)
  - several new Higgs doublets (not only in 10 and 126)
- $M^U_{3\times 3}$, $M^D_{6\times 6}$, $M^E_{6\times 6}$, $M^N_{15\times 15} \rightarrow$ light $(M_{U,D,E,N})_{3\times 3}$

This case seems really minimal: 27 and $351'$ that contribute to Yukawa terms could participate in symmetry breaking !

In fact the SM singlets:

$$\begin{aligned} 27 &: \quad V, S \\ 351' &: \quad V'_1, V'_2, V'_3, S'_1, S'_2 \end{aligned}$$

are at least in principle enough to go right to the SM

V, V'_i : vectorial vevs (from 10, 45, 54, 120, 126, ... of SO(10))

S, S'_i : spinorial vevs (from 16, 144,... of SO(10))

If spinorial vevs $= 0$ $\rightarrow$ spinorial number conserved (like R-parity in SO(10))

$\rightarrow$ the lightest fermion or boson with odd spinorial parity a dark matter candidate (for ex. in 27 one among 16_H, 10_F, 1_F)

Symmetry breaking

Look for the pattern $E_6 \to \text{SM}$.

The simplest renormalizable potential made of $351' + 27$ contains 11 real and 6 complex parameters:

$$
\begin{aligned}
V \quad = \quad & \underbrace{{M_{27}}^2\, 27^i\, 27^*_i + \ldots}_{2\ real\ parameters} \\
+ \quad & \underbrace{m_1\, d_{ijk}\, 27^i\, 27^j\, 27^k + h.c. + \ldots}_{3\ complex\ parameters} \\
+ \quad & \underbrace{\kappa_1\, \left(27^i\, 27^*_i\right)^2 + \ldots}_{9\ real\ parameters} \\
+ \quad & \underbrace{\kappa_{10}\, d_{ijk}\, 27^i\, 27^j\, 27^*_l\, 351'^{kl} + h.c. + \ldots}_{3\ complex\ parameters}
\end{aligned}
$$

SM singlets:

$$\begin{aligned} 27 &: \quad V, S \\ 351' &: \quad V_1', V_2', V_3', S_1', S_2' \end{aligned}$$

Assume vanishing spinorial vevs (DM candidate)

Single intermediate state Pati Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$

Then only 3 vevs:

$V_1' \in 54_{SO(10)} \in 351'_{E_6}$

$V \in 1_{SO(10)} \in 27_{E_6}$

$V_2' \in 1_{SO(10)} \in 351'_{E_6}$

- Looking for numerical solution, varying potential parameters:
  No minima found
- Problem: always some tachyonic state ($m^2 < 0$)
- Numerical evidence that there is no such PS minimum

Want to do a more general RG analysis of possible symmetry pattern

$$E_6 \to intermediate\ scale \to SM$$

Only a single intermediate scale assumed

Consider the following E_6 subgroups as intermediate scales:

1. $SU(3)_C \times SU(3)_L \times SU(3)_R \times P_{LR}$
2. $SU(3)_C \times SU(3)_L \times SU(3)_R \times P_{CR}$
3. $SU(3)_C \times SU(3)_L \times SU(3)_R \times P_{CL}$
4. SU(6)×SU(2) (ordinary solution)
5. SU(6)×SU(2) (flipped solution)
6. SU(6)×SU(2) (flipped left-right solution)
7. SO(10)×U(1) (ordinary solution)
8. SO(10)×U(1) (flipped solution)
9. $SU(4)_C \times SU(2)_L \times SU(2)_R \times P_{LR}$
10. $SU(4)_C \times SU(2)_L \times U(1)_R$ (ordinary solution)
11. $SU(4)_C \times SU(2)_L \times U(1)_R$ (flipped solution)

RG evolution of gauge couplings $g_{1,2,3}$ from M_Z(SM) to see if

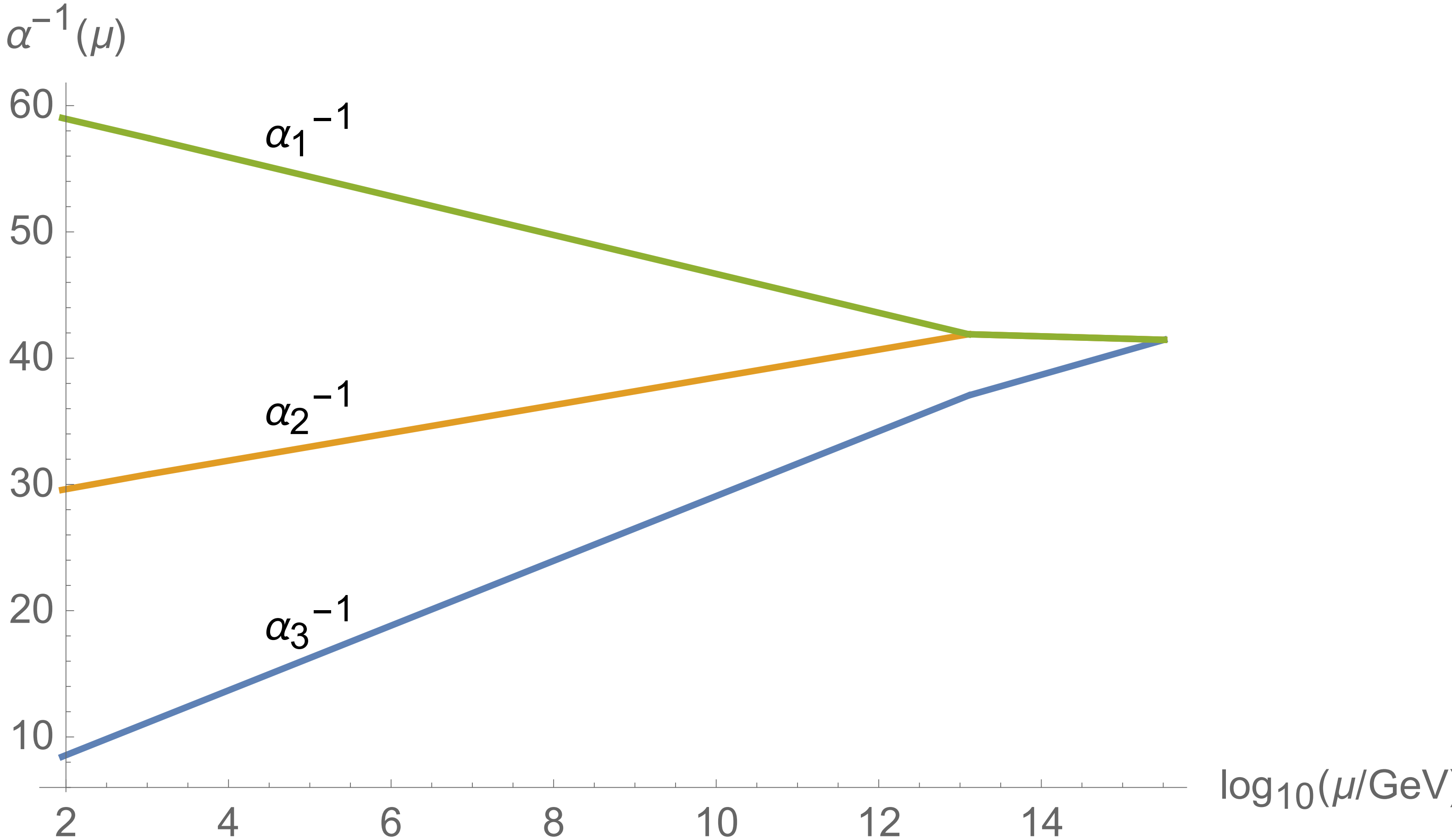
- unification possible at M_{GUT} (E_6)
- $M_{GUT} \gtrsim 10^{15.5}$ GeV

Assumptions:

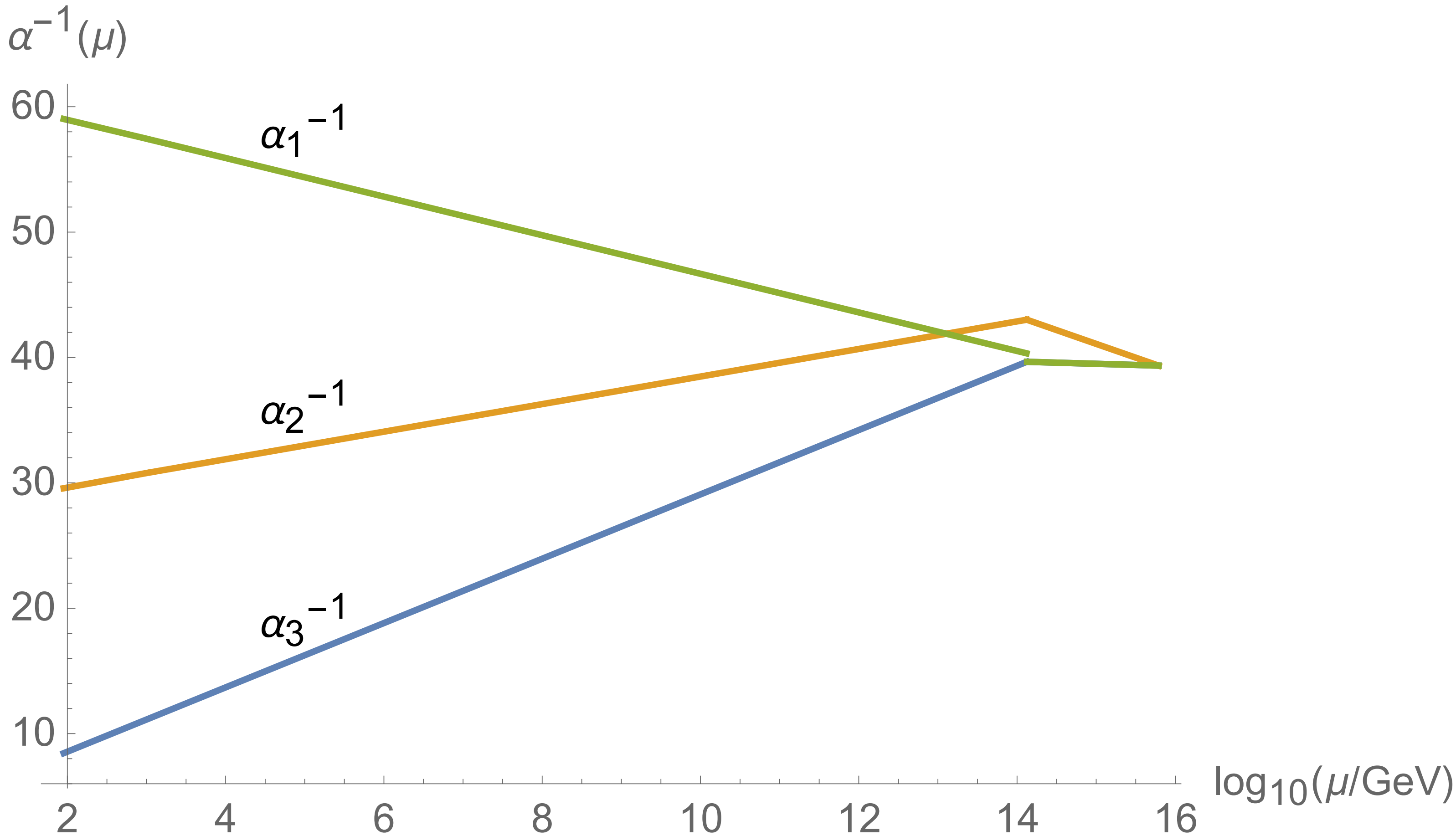
- the minimal survival hypothesis (only particles needed for symmetry breaking are at a given scale - no extra fine-tuning if not necessary)
- 1-loop and no thresholds is a good enough approximation

We get the following results:

- no unification: cases 3, 4, 5, 7, 8
- too low M_{GUT}: cases 6, 9, 10, 11
- acceptable solution: cases 1, 2



$$\mathrm{SU}(3)_C\times\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R\times P_{LR} \quad : \quad M_{GUT} = 3.2\times 10^{15}\ \mathrm{GeV}$$



$$\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times P_{CR} \quad : \quad M_{GUT} = 6.1 \times 10^{15}\ \mathrm{GeV}$$

So the only realistic intermediate states from RG flow are trinifications

- $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times P_{LR}$
- $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times P_{CR}$

These states can be obtained from E_6 only by the vevs of the real representation 650 (it has trinification singlets)

Need for extra Higgs: 650

It has singlets under all relevant maximal subgroups

$$\begin{aligned} E_6 \to SU(3) \times SU(3) \times SU(3) \quad &: \quad 650 = (1,1,1) + \dots \\ E_6 \to SU(6) \times SU(2) \quad &: \quad 650 = (1,1) + \dots \\ E_6 \to SO(10) \times U(1) \quad &: \quad 650 = (1,0) + \dots \end{aligned}$$

(an extra 351 or 78 could not help)

The most general renormalisable potential of 650 has 8 real parameters:

$$
\begin{aligned}
V \quad = \quad & \underbrace{M_{650}^2 \, Tr\left(650^2\right)}_{1 \; real \; parameter} \\
+ \quad & \underbrace{m_1 \, Tr\left(650^3\right) + \ldots}_{2 \; real \; parameters} \\
+ \quad & \underbrace{\lambda_1 \, \left(Tr\left(650^2\right)\right)^2 + \ldots}_{5 \; real \; parameters}
\end{aligned}
$$

650 has 11 SM singlets, but only 2 trinification singlets:

$s \ldots$ symmetric under P_{LR}

$a \ldots$ anti-symmetric under P_{LR}

$$V(s,a) = \frac{1}{2}M^2\left(s^2+a^2\right) - \frac{1}{3}m\,s\left(s^2-3a^2\right) + \frac{1}{4}\lambda\left(s^2+s^2\right)^2$$

with $M^2 < 0$, $m > 0$, $\lambda > 0$ linear combinations of M^2_{650}, m_i and λ_k

Apart from the trivial solution minimisation gives

$$\begin{aligned} s &= \frac{m \pm \sqrt{m^2 - 4\lambda M^2}}{2\lambda} \\ a &= 0 \end{aligned}$$

$E_6 \to$ trinification easily possible

Important: $a = 0 \to$ parity preserved

Open questions

- Can we really prove that the model with only $27 + 351'$ Higgses does not work? Or, otherwise, can we find a counter-example?
- Can we prove that realistic solutions need spinorial vevs to vanish? This would mean an automatic possibility for dark matter (fine-tuning is needed to make the right abundance, i.e. $M_{DM} \approx$ TeV)
- We showed a successful intermediate trinification scale in the theory with 650, but did not perform yet the next step of symmetry breaking to SM. Can we live without tachyons?
- A thorough study of proton decay and Yukawa sector

Conclusions

- This is the first attempt of a detailed study of the symmetry breaking pattern in non-supersymmetric E_6
- The minimal Yukawa needs at least **27** and **351'** Higgs reps
- The minimal Higgs potential with only 27 and 351' seems not to work
- An additional 650 Higgs is needed, which leads to the trinification intermediate scale
- absence of spinorial vevs provides a dark matter candidate