QFT bedrock principles and the inverse problem @ future lepton colliders

Benjamin Fuks (LPTHE / Sorbonne Université)

In collaboration with Yiming Liu, Cen Zhang & Shuang-Yong Zhou

Chin.Phys. C45 (2021) 023108 [arXiv:2009.02212]

SUSY 2021 - The 23rd International Conference on Supersymmetry and Unification of Fundamental Interactions

CAS Beijing - 23-28 August 2021

Outline

1. Dim-6 and dim-8 SMEFT operators at future lepton colliders

2. Positivity bounds

3. Applications
   - ★ Testing positivity and core QFT principles
   - ★ Inferring BSM in a model-independent way

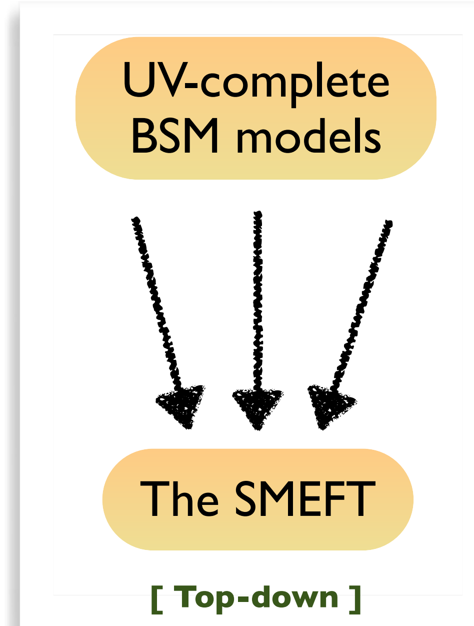
4. Summary

Theoretical context: the SMEFT

✦ The SMEFT: new physics is parametrised as small deviations to the SM

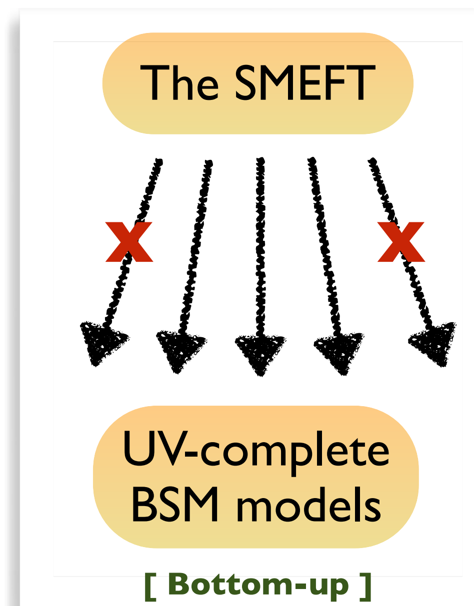
$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{O}_i^{(8)} + \dots$$

- Solely involves the SM fields
- Leading effects usually assumed to be dim-6
- UV-complete setups: also higher-order operators
  - ★ No *a priori* reason to neglect them
  - ★ Other types of powerful constraints [to which dim-6 operators are insensitive]



✦ Choice for the Wilson coefficients

- Coefficients dictated by the UV
  - ★ Taken as free parameters in the SMEFT
  - ★ Not all arbitrary values physical (positivity bounds)



Four-electron operators

✦ Four-electron operators

- ✤ Barely constrained (LEP+SLD): at best 1TeV bounds on dim-6 operators
- ✤ Excellent case for future lepton colliders ($e^+e^- \to e^+e^-$ scattering)

[Han & Skiba (PRD`05)]

✦ Relevant operators at the dim-6 and dim-8 level

$$O_{ee} = (\bar{e}\gamma^\mu e)\ (\bar{e}\gamma_\mu e)$$
$$O_{el} = (\bar{e}\gamma^\mu e)\ (\bar{l}\gamma_\mu l)$$
$$O_{ll} = (\bar{l}\gamma^\mu l)\ (\bar{l}\gamma_\mu l)$$

Dim-6

$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)$$
$$O_3 = D^\alpha(\bar{e}l)\ D_\alpha(\bar{l}e)$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\ \partial_\alpha(\bar{l}\gamma_\mu l)$$
$$O_5 = D^\alpha(\bar{l}\gamma^\mu\tau^I l)\ D_\alpha(\bar{l}\gamma_\mu\tau^I l)$$

Dim-8

- ✤ Other dim-8 operators omitted (dim-6 structure, flavour, *etc.*)

Constraining the SMEFT at future ee colliders

✦ Differential $e^+e^- \to e^+e^-$ cross section at the dim-8 level

- ✤ Interferences between the SM and the SMEFT operators
- ✤ Squared dim-6 contributions (no interferences for $m_e \to 0$)

$$d\sigma = d\sigma_{\rm SM} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} d\sigma_i^{(6)} + \sum_i \frac{{C_i^{(6)}}^2}{\Lambda^4} d\sigma_{ii}^{(6)} + \sum_i \frac{C_i^{(8)}}{\Lambda^4} d\sigma_i^{(8)}$$

✦ Setup for computations of the cos θ spectrum

- ✤ LEP2: measurements at 2%
  - ↝ Assumption: 1% at future colliders for each cos θ bin
- ✤ 25 bins, we ignore the most forward bin
- ✤ All future colliders considered (except $\sqrt{s} = m_Z$), full luminosity
- ✤ Statistical uncertainties included

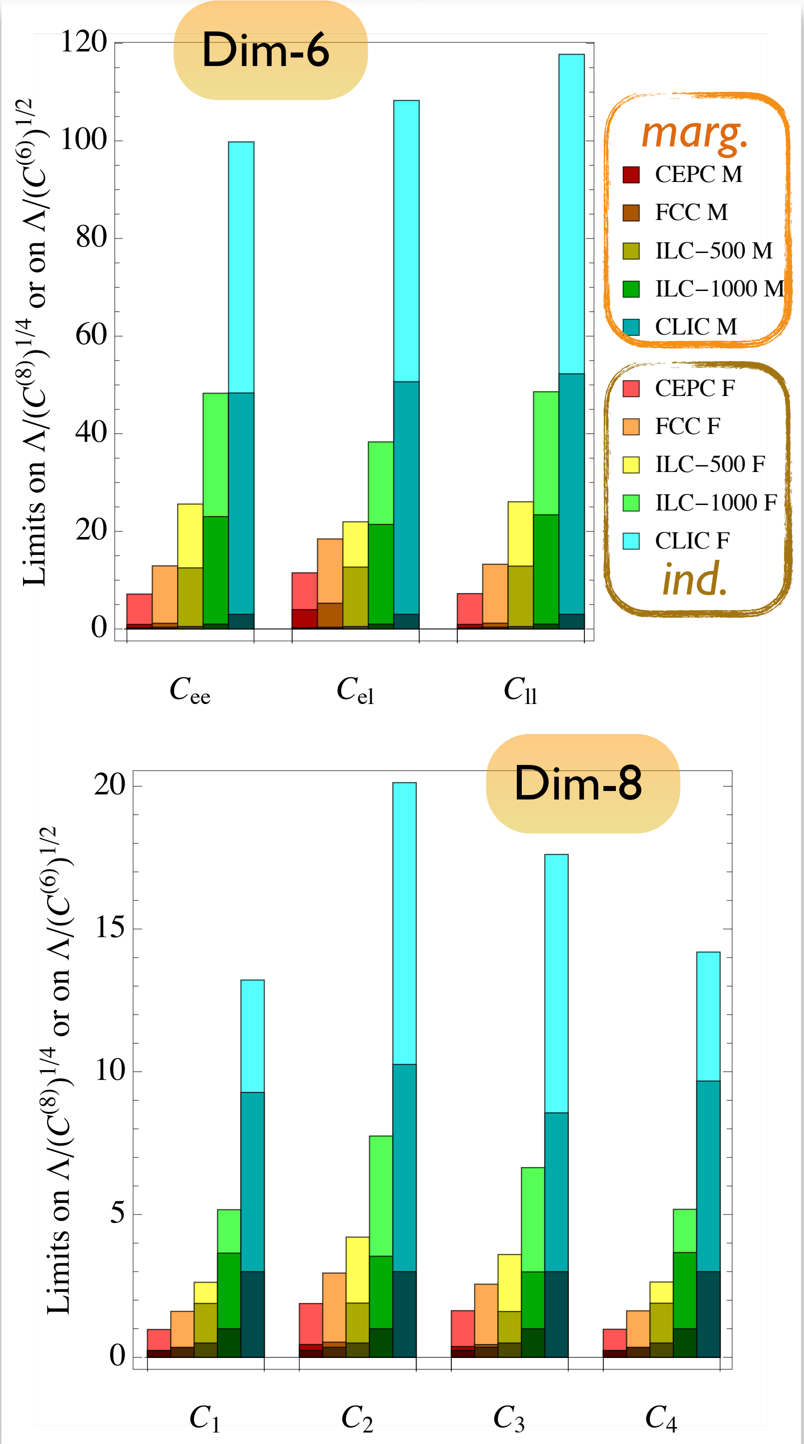
[ILC white paper (2013)]
[De Blas, Durieux, Grojean, Gu & Paul (JHEP`19)]

✦ Constraints on the vector of Wilson coefficients *C*

- ✤ Hypothesis $\boldsymbol{C}_0$
- ✤ Would-be observation $\boldsymbol{C}$

↝ $\chi^2(\boldsymbol{C}, \boldsymbol{C}_0)$ ↝ $[\boldsymbol{C}_{\rm min}, \boldsymbol{C}_{\rm max}]$ range at 2σ ↝ BSM scale Λ_c

Results for the SM hypothesis $C_0 = 0$



✦ Dimension-6 coefficients

- ✤ Sensitivity to very large scales
  - ★ 1–2 orders of magnitude larger than √s
- ✤ Similar findings as for LEP2
- ✤ Marginalised limits a factor of a few weaker
  - ★ OK wrt to the EFT validity

✦ Dimension-8 coefficients

- ✤ Sensitivity to scales of 1–10 TeV
  - ★ 5 times larger than √s
- ✤ Marginalised limits a factor of a few weaker
  - ★ CEPC/FCC-ee: smaller than √s for C_1/C_4
    - ↝ almost identical LLLL and RRRR cross sections
    - ↝ almost flat direction
  - ★ The EFT is valid (check of the next terms in $d\sigma$)

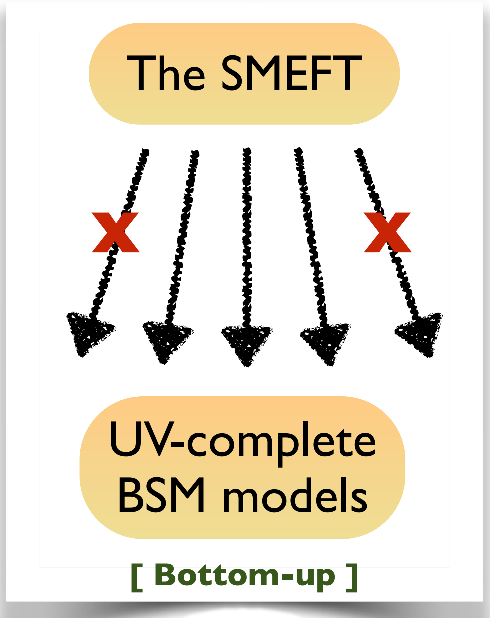
$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e) \quad O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\,\partial_\alpha(\bar{l}\gamma_\mu l)$$

- ✤ Implications for UV physics: positivity bounds

Positivity bounds: generalities

- ✦ Not all SMEFT scenarios $\equiv$ UV completions
  - ✤ Wilson coefficients not arbitrary
  - ✤ Some non-physical regions in the SMEFT parameter space

- ✦ Core QFT principles (analyticity, unitarity, locality, Lorentz)
  - ✤ The optical theorem + dispersion relation: positivity bounds
  - ✤ Constraints on the allowed SMEFT scenarios



- ✦ Positivity bounds and future measurements: applications
  - ✤ Measurement of a positivity bound violation
    - ↝ breakdown of the fundamental principles of QFT
    - ↝ the SMEFT invalid (TeV scale new physics)
  - ✤ Observations in agreement with the positivity bounds
    - ↝ Inferring / excluding the existence of UV new physics model-independently
    - ↝ Towards solving the inverse problem

[Zhang & Zhou (2020)]

Derivation of the positivity bounds

[Zhang & Zhou (PRD`19)]

✦ Context: forward scattering amplitudes $M_{ij\to kl}(s)$

- Dispersion relation from Cauchy's integral formula, analyticity, unitarity of $M_{ij\to kl}$
- Subtraction of the poles and the low-energy part ($Q \ll \epsilon\Lambda$)

$$M^{ijkl} = \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}s^2} M_{ij\to kl}(s=Q^2/2) - \text{poles} - \text{low energy}$$
$$= \boxed{\int_{(\epsilon\Lambda)^2}^{\infty} \frac{\mathrm{d}\mu}{2i\pi} \frac{\mathrm{Disc} M_{ij\to kl}(\mu)}{\left(\mu - \frac{1}{2}Q^2\right)^3} + [\mathrm{j} \leftrightarrow \mathrm{l}]} \quad \textbf{Positive}$$

✦ Re-organisation of the dispersion relation

- Hermiticity
- Optical theorem

$$M_{ij\to kl}(s-i\epsilon) = M^*_{kl\to ij}(s+i\epsilon)$$
$$M_{ij\to kl} - M_{kl\to ij} = i{\sum_X}' M_{ij\to X} M^*_{kl\to X}$$

✦ Final formula

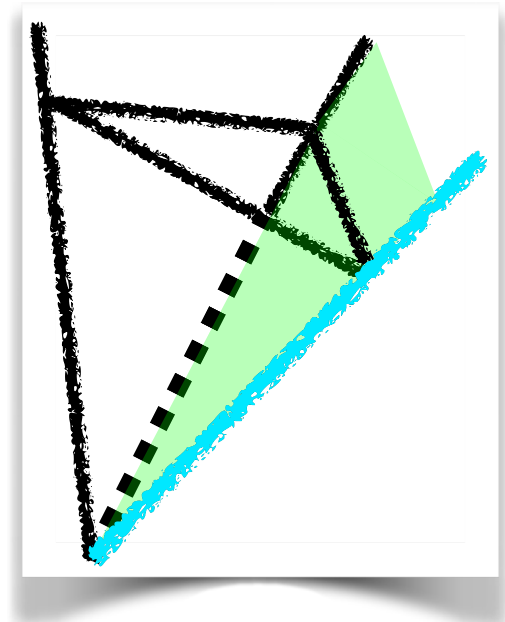
$$M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} \mathrm{d}\mu \sum_{Z \text{ in } \mathbf{r}}{}' \frac{|\langle Z|\mathcal{M}|\mathbf{r}\rangle|^2}{\pi\left(\mu - \frac{1}{2}Q^2\right)^3} P_{\mathbf{r}}^{i(j|k|l)}$$

- Sum over intermediate Z-states
- Decomposition over SO(2) rotation reps. around the scattering axis

Convex geometry an its link to UV completions

✦ More about convex geometry

✤ Positivity defines a convex cone in the SMEFT space
[set closed under additions and positive scalar multiplications]



✤ Two representations

★ Faces: the positivity bounds themselves
↝ Hahn-Banach separation theorem: a convex cone can be specified by a set of linear inequalities

★ Convex hull of extremal rays (Krein-Milman theorem)
[An extremal ray cannot be trivially split into other cone elements]
↝ The extremal rays are the cone generators

[Bi, Zhang & Zhou (JHEP`19)]

✦ Extremal rays and physics

✤ Convex hull of {$\mathbf{x}_i$} ($\mathbf{x}_i$ denoting the extremal rays)

$$\text{all } \mathbf{x} \quad \text{such that} \quad \mathbf{x} = \sum_i \omega_i \mathbf{x}_i$$
$$\text{with } \omega_i \geq 0 \text{ and } \sum_i \omega_i = 1$$

★ Extremal rays ≡ generators of all UV-completable SMEFT setups

★ Extremal ray ≡ SM + one particle species

Seeking UV physics with convex geometry

✦ 'SM + *n* particles' setup

✤ Integrating out ↝ **C** = (C_1, C_2, …, C_5)

★ n > 1: not an extremal ray (extremal rays cannot be split)

✤ UV models generated from 'SM + 1 particle' extensions

✤ Localisation within the cone

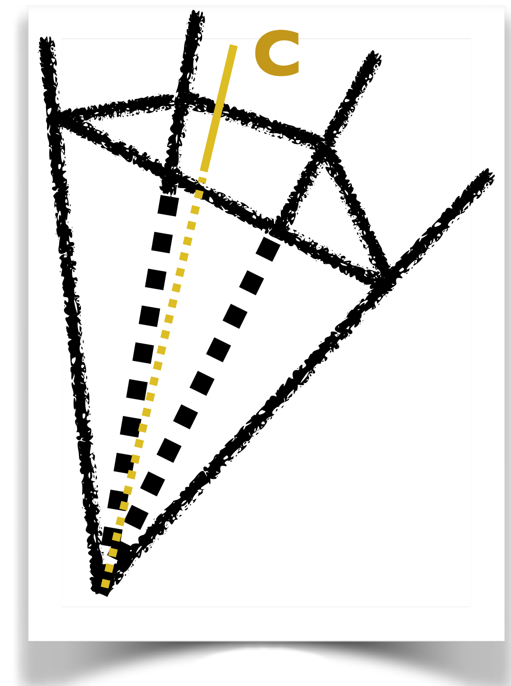
★ Extremal rays: one particle SM extensions

↝ One ray for different EW/spin reps.

★ Faces of the cone: two-particle SM extensions

★ Inside the cone: more complex SM extensions

★ Outside the cone: violation of positivity, QFT core principles



Model-independent conclusions after adding future lepton collider expectations

Positivity bounds - interpretations

✦ Conditions on the Wilson coefficients

$$\boxed{M^{ijkl}} = \boxed{\int_{(\epsilon\Lambda)^2}^{\infty} \mathrm{d}\mu \sum_{Z \text{ in } \mathbf{r}}{}' \frac{|\langle Z|\mathcal{M}|\mathbf{r}\rangle|^2}{\pi\left(\mu - \frac{1}{2}M^2\right)^3} P_{\mathbf{r}}^{i(j|k|l)}}$$

Quadratic in $C^{(8)}$ | Positive

Direct calculations: derivation of the viable SMEFT space regions

Geometry: additional model-independent constraints on new physics

✤ We can compute M^{ijkl} in the SMEFT

- ★ From any elastic fwd scattering amp.
- ★ The second-order derivative > 0
- ★ Arbitrary superpositions
- ★ Constraints on the coefficients

✤ We can use convex geometry

- ★ The rhs written with $C^{(8)}$ (for given $\mathbf{r}$) ⤳ $\{\mathbf{c}_\mathbf{r}\}$ forms a convex cone
- ★ The lhs written with $C^{(8)}$: $\mathbf{C}$ ⤳ $\mathbf{C} \in \text{cone}(\mathbf{c}_\mathbf{r})$

Positivity bounds for $e^+e^- \to e^+e^-$ collisions

✦ Direct computations (reproduced with convex geometry)

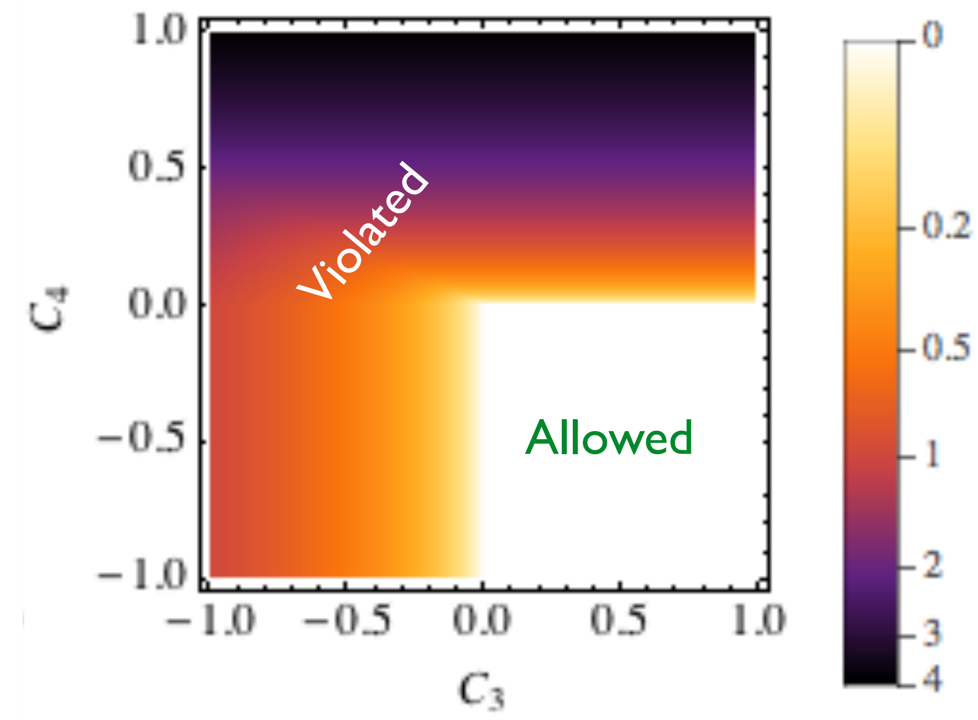
From specific helicities

$$M(e_R e_R \to e_R e_R) \to C_1 \leq 0$$
$$M(e_L e_L \to e_L e_L) \to C_4 + C_5 \leq 0$$
$$M(e_R \bar{e}_L \to e_R \bar{e}_L) \to C_3 \geq 0$$
$$M(e_L \nu_L \to e_L \nu_L) \to C_5 \leq 0$$

From given superpositions

$$|f_\pm\rangle \propto (C_4 + C_5)^{1/4}|e_R\rangle + C_1^{1/4}|\bar{e}_L\rangle$$

$$2\sqrt{C_1(C_4+C_5)} > C_2$$
$$2\sqrt{C_1(C_4+C_5)} > -(C_2+C_3)$$



The constraints define a cone in the SMEFT space
Distance of a point outside ≡ amount of positivity violation

Positivity bounds at the FCC-ee

✦ Assumptions

✤ Assumption 1: nature picks a given $\boldsymbol{C_0} = (C_1, C_2, \ldots, C_5)$ [within the bounds or not]

✤ Assumption 2: $\boldsymbol{C_{exp}}$ is measured at the FCC-ee and is consistent with $\boldsymbol{C_0}$

✦ Strategy

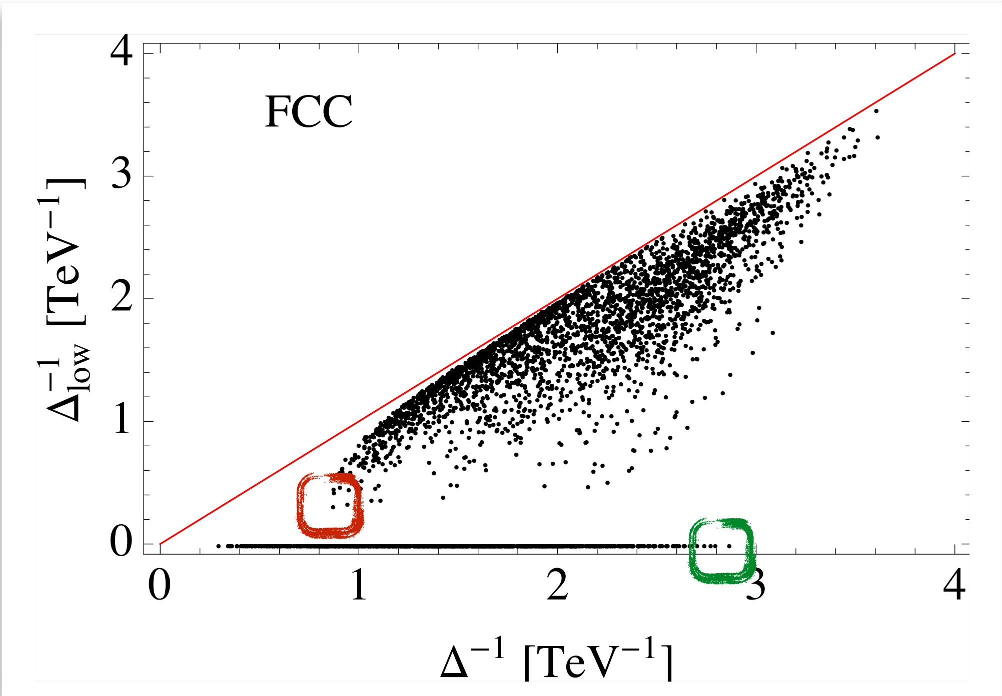
✤ The measurements yield bounds for positivity violation $\Delta^{-1} \in [\Delta^{-1}_{\rm low}, \Delta^{-1}_{\rm high}]$

✤ The lower bound is a conservative estimate

★ = 0: Compatible with the positivity bounds

★ > 0: Observation of positivity violation

✦ FCC-ee sensitivity to a positivity violation scale Δ *[scan over $\boldsymbol{C_0}$]*



✤ $\boldsymbol{\Delta}$ connected with the scale at which QFT bedrock principles are violated

✤ Smallest $\boldsymbol{\Delta}^{-1}$ for a non-zero $(\boldsymbol{\Delta}_{\rm low})^{-1}$

★ Positivity violation occurring below 1.2 TeV has a chance to be detected

✤ Largest $\boldsymbol{\Delta}^{-1}$ for a zero $(\boldsymbol{\Delta}_{\rm low})^{-1}$

★ Positivity violation occurring below 360 GeV is guaranteed to be detected

UV physics from precision at future ee colliders

✦ Assumptions

- ✤ Assumption 1: nature picks a given $\boldsymbol{C_0} = (C_1, C_2, \ldots, C_5)$ [blue dot]
- ✤ Assumption 2: $\boldsymbol{C_{exp}}$ is measured at some lepton collider [green area]
- ✤ The positivity bounds define a convex cone in the SMEFT space [yellow cone]

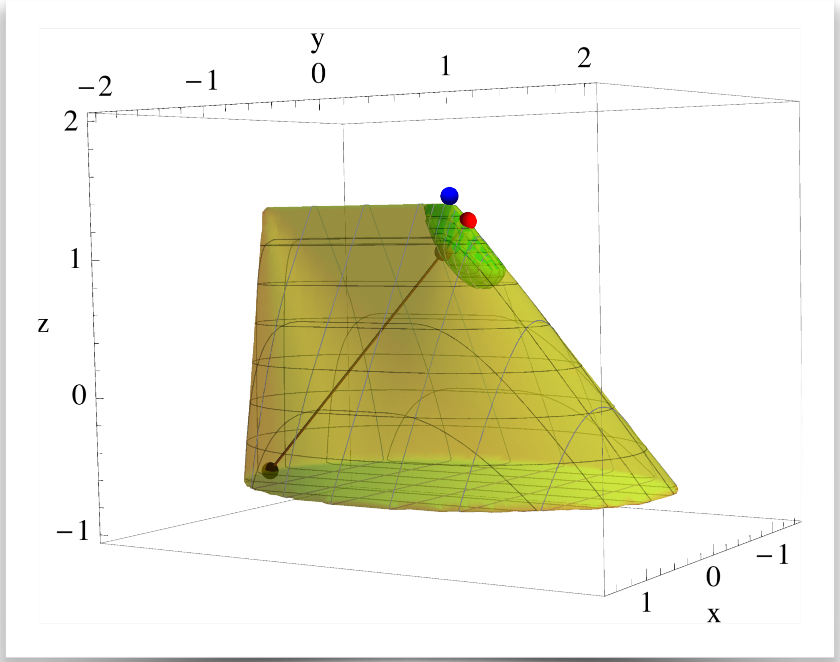
✦ Towards excluding a class of particle X'

- ✤ The extremal ray corresponding to X': $\boldsymbol{c_{X'}}$ [black dot]
- ✤ We determine $\boldsymbol{C_{max}}$ so that λ is maximised

$$\max_{\lambda}\left[\vec{C}^{(8)}_{\rm exp} - \lambda \vec{c}^{\,(8)}_{X'} \in \mathcal{C}\right]$$

[brown dot; red dot]

- ✤ $\lambda_{\max} \equiv$ the max X' contribution to $\boldsymbol{C_0}$



✦ Examples: the no new physics case $\boldsymbol{C_0} \equiv (0,0,0,0,0)$ @ FCC-ee

X'	$\lambda_{\max}$	$M_{X'}/\sqrt{g_{X'}}$	X'	$\lambda_{\max}$	$M_{X'}/\sqrt{g_{X'}}$
$\mathbf{2}_{1/2}$ scalar	0.4267	≥ 1.23 TeV	$\mathbf{1}_2$ scalar	0.7257	≥ 1.08 TeV
$\mathbf{1}_1$ scalar	0.6897	≥ 1.09 TeV	$\mathbf{1}_0$ vector	0.3627	≥ 1.29 TeV
$\mathbf{2}_{-3/2}$ vector	0.2427	≥ 1.42 TeV	$\mathbf{1}_0$ axial-vector	0.3551	≥ 1.30 TeV

Model-independent bounds!

Summary

✦ The SMEFT parameter space not entirely physical

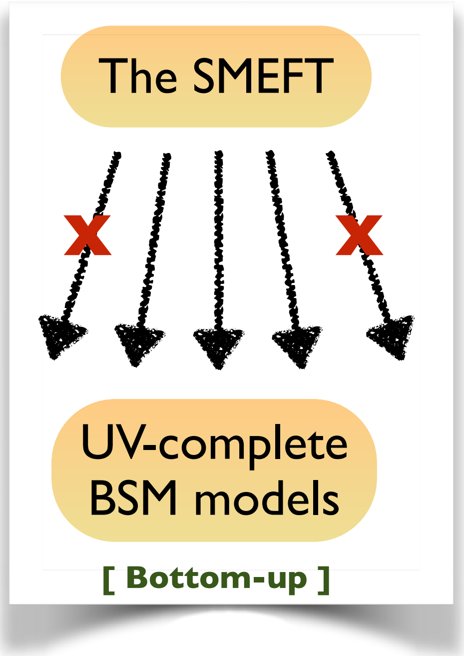
- ✤ Wilson coefficients not arbitrary
- ✤ Some configurations have no UV completion

✦ Positivity bounds built from:

- ✤ Core QFT principles (analyticity, unitarity, locality, Lorentz inv.)
- ✤ Optical theorem and dispersion relation

✦ Applications

- ✤ Testing fundamentals of QFT
- ✤ Model-independent inference of the existence of new physics
- ✤ Solution to the inverse problem



✦ Example: ee→ee scattering at the FCC-ee

- ✤ Probing positivity violation at 360 GeV (guaranteed) and 1.2 TeV (possible)
- ✤ Exclusion of UV physics model-independently (for scales up to about 1 TeV)