Fermion Pair Production at e^-e^+ Linear Collider Experiments in GUT Inspired Gauge-Higgs Unification

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Short summary of this talk

We discuss fermion pair production at e^-e^+ linear collider experiments with polarized e^- and e^+ beams in the GUT inspired gauge-Higgs unification (GHU).

(1) Due to the Z' bosons coupling to quarks and leptons with large parity violation, several observables in $e^-e^+ \to f\bar{f}(= \mu^-\mu^+, ...)$ processes have large polarization dependence.

(2) Deviations from the SM are testable for the Kaluza-Klein (KK) mass scale up to $\sim$15 TeV in the early stage of ILC.

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Gauge-Higgs unification (GHU) and Z' bosons

Gauge Higgs unification (GHU)

- Higgs boson as the zero mode of the 5th dimensional component of the 5D gauge field [2–7, Y.Hosotani'83;A.T.Davies,A.McLachlan'88;H.Hatanaka et al'98;...]

$SU(3) \times SO(5) \times U(1)$ GHU in the 5D Randall-Sundrum warped space (GHU A-model) [8–20, K.Agashe et al.'05;A.D.Medina et al.'07;Y.Hosotani et al.'08;...]

- Phenomenologies at EW scale are almost the same as the SM [12, 13, 15, 16, S.Funatsu et al.'13-16;...] for $\theta_H \lesssim 0.11$, $m_{\rm KK} \gtrsim 9$TeV;

  E.g., Higgs couplings of quarks, leptons, W and Z bosons are approximately the SM values times $\cos\theta_H$; the deviation is about 1% for $\theta_H \simeq 0.1$.

- KK excited neutral vector bosons $\gamma^{(n)}$, $Z^{(n)}$, $Z_R^{(n)}$ $(n \geq 1)$ can be identified with Z' bosons. ($\gamma^{(1)}$, $Z^{(1)}$, $Z_R^{(1)}$ have KK scale masses $O(m_{\rm KK})$.)

GHU A-model at e^-e^+ colliders

[17, 21–25, S.Funatsu et al.'17;J.Yoon,M.E.Peskin'18;S.Funatsu'19;F.Richard'18;A.Irles et al'20]

- Z' bosons couple to quarks and leptons with large parity violation. Right-handed quarks and leptons have rather large couplings to Z' boson.

$\Rightarrow$ Amplitudes of $e^-e^+ \to f\bar{f}(=\mu^-\mu^+,...)$ have large polarization dependence.

$\Rightarrow$ Signal strength of cross section and forward-backward (FB) asymmetry becomes larger by using initial polarized e^- and e^+ states.

$\Rightarrow$ The masses up to about 10TeV can be explored at 250 GeV ILC with 250 fb^{-1} data. [17, 22, 23, S.Funatsu et al.'17;J.Yoon,M.E.Peskin'18;S.Funatsu'19]

In the talk, we discuss fermion pair production at e^-e^+ linear collider experiments with polarized e^- and e^+ beams in the GUT inspired GHU (B-model).

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GUT inspired GHU (GHU B-model)

GUT inspired GHU (B-model) [18–20, S.Funatsu et al.'19;...].

- Matter content inspired by $SO(11)$ GHGUT ($SO(11) \supset SU(3) \times SO(5) \times U(1)$) [26, 27, Y.Hosotani,N.Yamatsu'15';...].
- Quarks and leptons belong to spinor $\mathbf{4}$ or singlet $\mathbf{1}$ of $SO(5)$ in the B-model. (They belong to vector $\mathbf{5}$ of $SO(5)$ in the GHU A-model.)
- Electro-weak symmetry breaking pattern is the same as that in the A-model

$$SO(5) \times U(1)_X \xrightarrow{BCs} \underbrace{SO(4)}_{SU(2)_L \times \cancel{SU(2)_R}} \times U(1)_X \xrightarrow{\theta_H \neq 0} U(1)_{\rm EM}.$$

- First KK photon $\gamma^{(1)}$, $Z^{(1)}$, $Z_R^{(1)}$ have about KK scale masses.

$SU(3) \times SO(5) \times U(1)$ GHU: A- and B-models

	B-model (=GUT inspired)	A-model
Quark Lepton	$(\mathbf{3},\mathbf{4})_{\frac{1}{6}}$ $(\mathbf{3},\mathbf{1})^{+}_{-\frac{1}{3}}$ $(\mathbf{3},\mathbf{1})^{-}_{-\frac{1}{3}}$ $(\mathbf{1},\mathbf{4})_{-\frac{1}{2}}$	$(\mathbf{3},\mathbf{5})_{\frac{2}{3}}$ $(\mathbf{3},\mathbf{5})_{-\frac{1}{3}}$ $(\mathbf{1},\mathbf{5})_{0}$ $(\mathbf{1},\mathbf{5})_{-1}$
Dark fermion	$(\mathbf{3},\mathbf{4})_{\frac{1}{6}}$ $(\mathbf{1},\mathbf{5})^{+}_{0}$ $(\mathbf{1},\mathbf{5})^{-}_{0}$	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$
Brane fermion	$(\mathbf{1},\mathbf{1})_{0}$	$(\mathbf{3},[\mathbf{2},\mathbf{1}])_{\frac{7}{6},\frac{1}{6},-\frac{5}{6}}$ $(\mathbf{1},[\mathbf{2},\mathbf{1}])_{\frac{1}{2},-\frac{1}{2},-\frac{3}{2}}$
Brane scalar	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$	$(\mathbf{1},[\mathbf{1},\mathbf{2}])_{\frac{1}{2}}$

B-model = the $SO(11)$ GUT inspired model [18–20, S.Funatsu et al.'19-20];
A-model = the (non-GUT inspired) previous model [12–17, S.Funatsu et al.'13-17].

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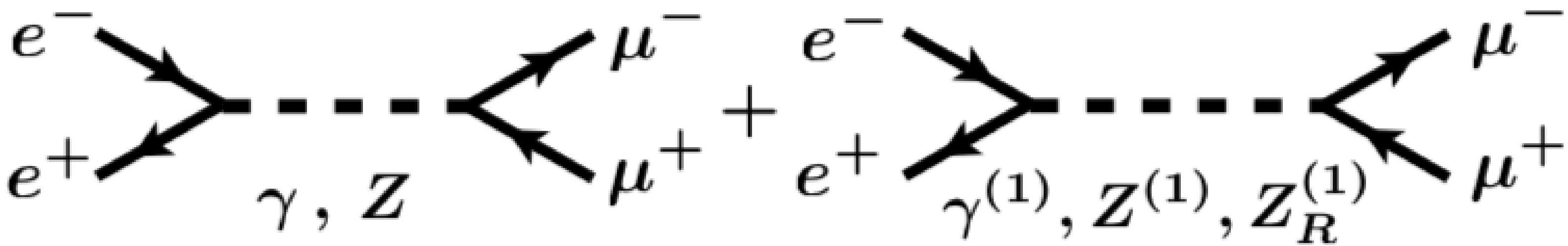
Observables in $e^-e^+ \to \{V_i\} \to f\bar{f}$ ($f\bar{f} = \mu^-\mu^+, \ldots$)



Observables	(Symbol)	Unpolarized	Polarized
Cross sections	$(\sigma^{f\bar{f}})$	Yes	Yes
FB asymmetries	$(A_{FB}^{f\bar{f}})$	Yes	Yes
LR asymmetries	$(A_{LR}^{f\bar{f}})$	No	Yes
LR FB asymmetries	$(A_{LR,FB}^{f\bar{f}})$	No	Yes

Forward-backward (FB) asymmetries, left-right (LR) and LR FB asymmetries are studied in e.g., Refs. [28–32, B.Schrempp et al'88;D.C.Kennedy et al'89;SLD Collaboration'94-95].

Polarized e^-e^+ to $f\bar{f}$ production

s-channel process of $e^-e^+ \to f\bar{f}$ expressed by 4 quantities $Q_{e_X f_Y}$ $(X, Y = L, R)$.

$$Q_{e_X f_Y} = \sum_i \frac{g^X_{V_i e} g^Y_{V_i f}}{(s - m^2_{V_i}) + i m_{V_i} \Gamma_{V_i}}, \quad P_{\text{eff}} = \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-} P_{e^+}},$$

Obs.	Linear combination of observed amplitudes (Overall factor is omitted, $\sqrt{s} \gg m_f$)
σ	$(1 - P_{\text{eff}})(\lvert Q_{e_L f_L}\rvert^2 + \lvert Q_{e_L f_R}\rvert^2) + (1 + P_{\text{eff}})(\lvert Q_{e_R f_R}\rvert^2 + \lvert Q_{e_R f_L}\rvert^2)$
A_{FB}	$(1 - P_{\text{eff}})(\lvert Q_{e_L f_L}\rvert^2 - \lvert Q_{e_L f_R}\rvert^2) + (1 + P_{\text{eff}})(\lvert Q_{e_R f_R}\rvert^2 - \lvert Q_{e_R f_L}\rvert^2)$
A_{LR}	$\lvert Q_{e_L f_L}\rvert^2 + \lvert Q_{e_L f_R}\rvert^2 - \lvert Q_{e_R f_R}\rvert^2 - \lvert Q_{e_R f_L}\rvert^2$
$A_{LR,FB}$	$\lvert Q_{e_L f_L}\rvert^2 - \lvert Q_{e_L f_R}\rvert^2 - \lvert Q_{e_R f_R}\rvert^2 + \lvert Q_{e_R f_L}\rvert^2$

In the following, we will show the results of $e^-e^+ \to \mu^-\mu^+$ process with polarized e^-e^+, $(P_{e^-}, P_{e^+}) = (\mp 0.8, \pm 0.3)$ at $\sqrt{s} = 250$ GeV with 250 fb^{-1} data in the GHU B-model.

[Setup] Parameter set GHU(B)

Name	θ_H [rad.]	$m_{\rm KK}$ [TeV]	z_L	k [GeV]	$m_{\gamma^{(1)}}$ [TeV]	$\Gamma_{\gamma^{(1)}}$ [TeV]	$m_{Z^{(1)}}$ [TeV]	$\Gamma_{Z^{(1)}}$ [TeV]	$m_{Z_R^{(1)}}$ [TeV]	$\Gamma_{Z_R^{(1)}}$ [TeV]
B$^{\rm L}$	0.10	11.0	2.0×10^{8}	6.9×10^{11}	8.7	2.1	8.7	4.8	8.4	0.60
B	0.10	13.0	3.9×10^{11}	1.6×10^{15}	10.2	3.3	10.2	7.8	10.0	0.82
B$^{\rm H}$	0.10	15.0	2.7×10^{15}	1.3×10^{19}	11.7	4.9	11.7	11.8	11.5	1.3

f	g^L_{Zf}	g^R_{Zf}	$g^L_{Z^{(1)}f}$	$g^R_{Z^{(1)}f}$	$g^L_{Z_R^{(1)}f}$	$g^R_{Z_R^{(1)}f}$	$g^L_{\gamma^{(1)}f}$	$g^R_{\gamma^{(1)}f}$
e	-0.3058	0.2629	-1.7621	-0.0584	-1.0444	0	-2.7587	0.1071
μ	-0.3058	0.2629	-1.6778	-0.0584	-0.9969	0	-2.6268	0.1071
τ	-0.3058	0.2629	-1.6218	-0.0584	-0.9652	0.0001	-2.5391	0.1070

- The parameter set GHU(B) $(\theta_H, m_{\rm KK}) = (0.10, 13{\rm TeV})$ is the benchmark in our calculation. (Coupling constants in units of $g_w = e/\sin\theta_W^0$.)

  We first fix the values of θ_H and $m_{\rm KK}$, and then determine the other parameters. (For more information, see Ref. [1, S.Funatsu et al.'20].)

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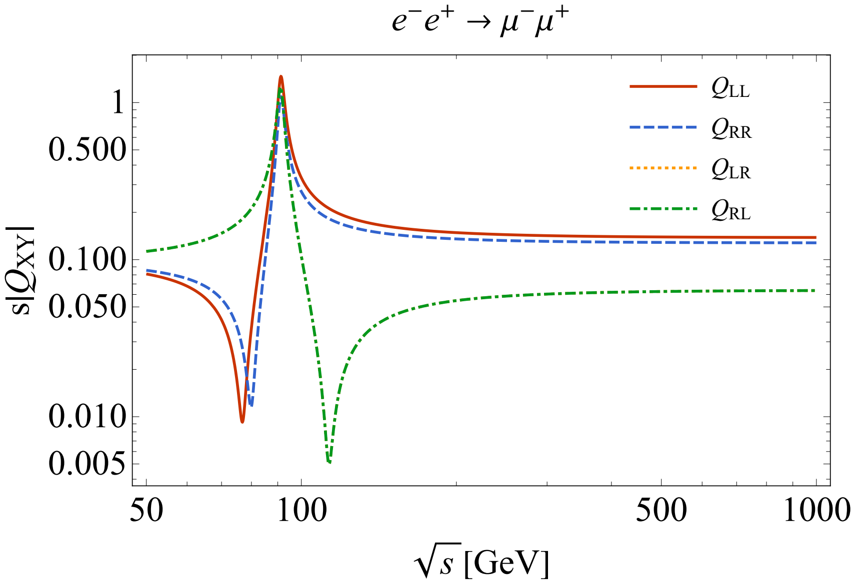
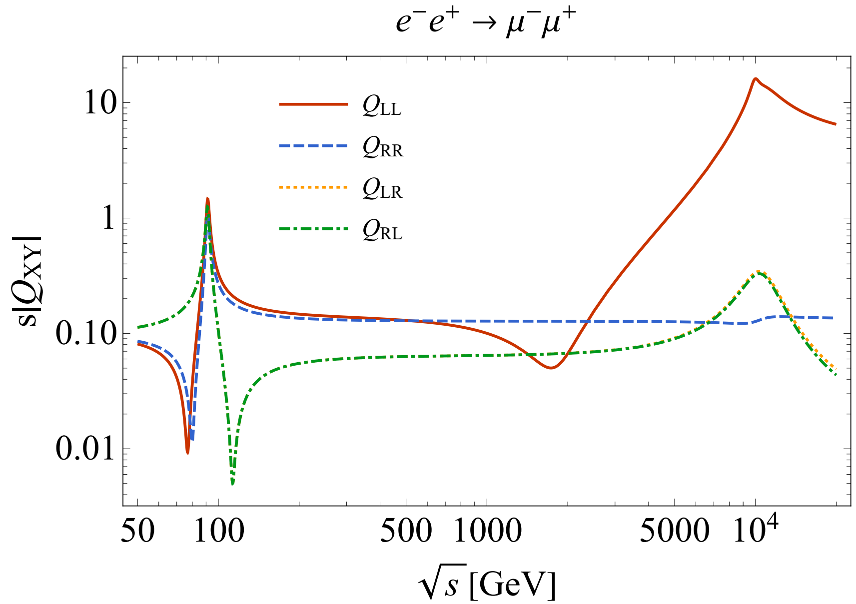
[Calculation:1/5] Amplitude

$$Q_{e_X f_Y} = \sum_{V_i} \frac{g_{Ve}^X g_{Vf}^Y g_w^2}{(s - m_V^2) + i m_V \Gamma_V}, \quad V_i = \begin{cases} \gamma, Z, & \text{for SM}, \\ \gamma, Z, \gamma^{(1)}, Z^{(1)}, Z_R^{(1)} & \text{for GHU} \end{cases}.$$



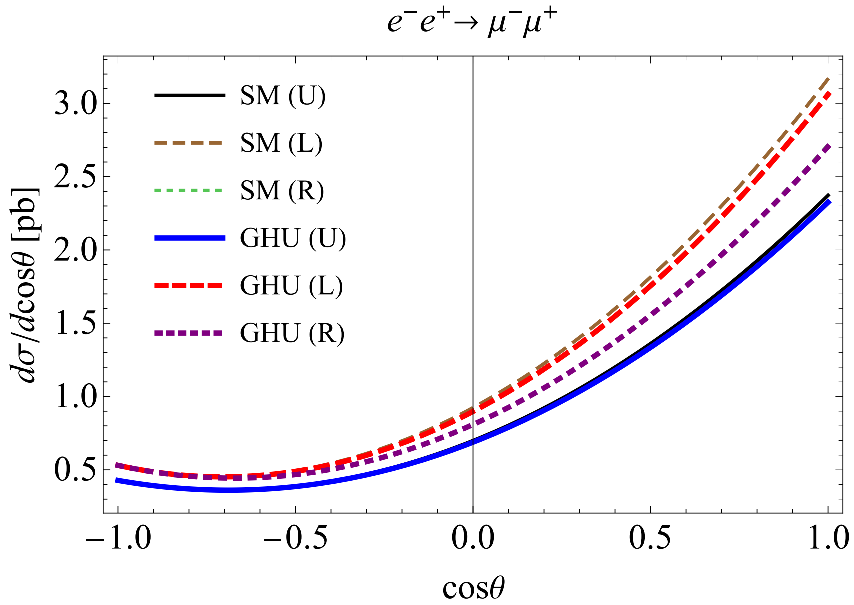


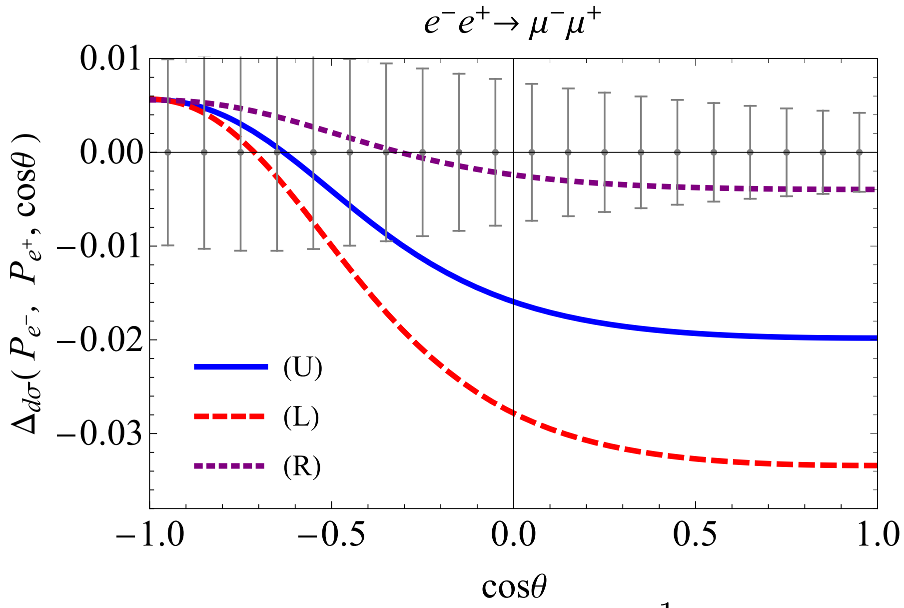
- The amplitude $s|Q_{XY}|$ $(X, Y = L, R)$ vs $\sqrt{s}$ [GeV] for the SM (left figure) and the GHU (B) (right figure) are shown, Q_{XY} stands for $Q_{e_X f_Y}(e^- e^+ \to \mu^- \mu^+)$.

[Calculation:2/5] Differential cross section

$$\frac{d\sigma^{f\bar{f}}_{LR}}{d\cos\theta}(\cos\theta) \overset{m_f=0}{=} \frac{s}{32\pi}\left\{(1+\cos\theta)^2 \; |Q_{e_L f_L}|^2 + (1-\cos\theta)^2 \; |Q_{e_L f_R}|^2 \right\},$$

$$\frac{d\sigma^{f\bar{f}}_{RL}}{d\cos\theta}(\cos\theta) \overset{m_f=0}{=} \frac{s}{32\pi}\left\{(1+\cos\theta)^2 \; |Q_{e_R f_R}|^2 + (1-\cos\theta)^2 \; |Q_{e_R f_L}|^2 \right\}.$$

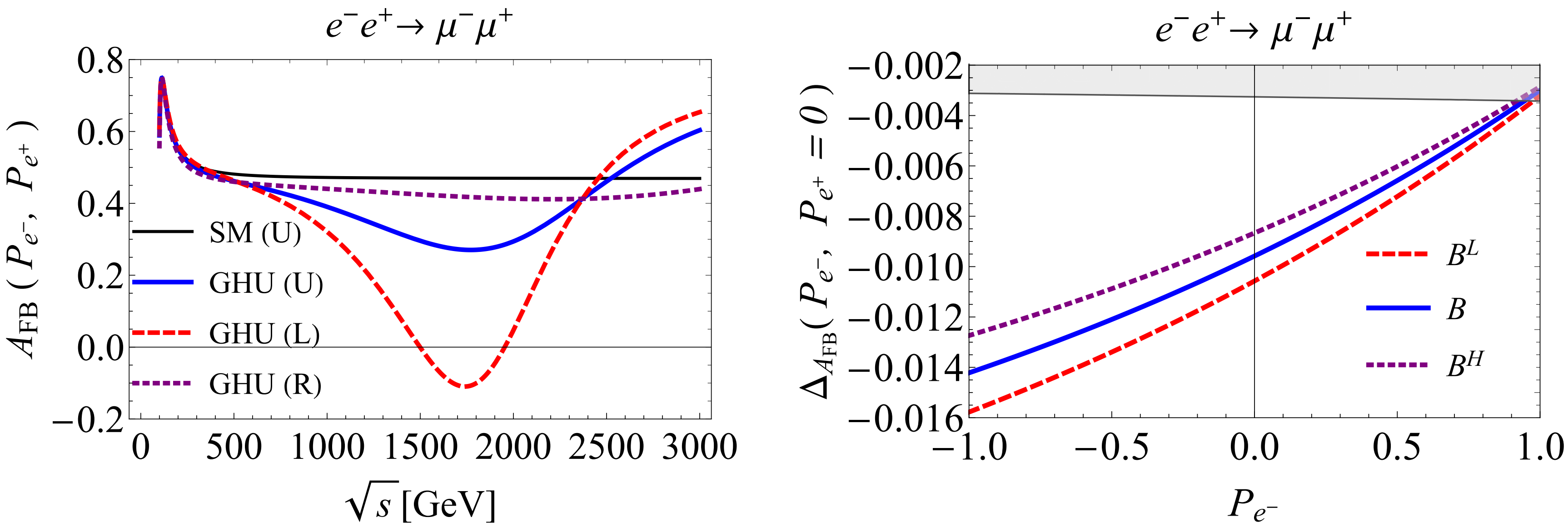




- $(P_{e^-}, P_{e^+}) = (0,0)(U), (-0.8,+0.3)(L), (+0.8,-0.3)(R)$; $\sqrt{s} = 250$ GeV, 250 fb^{-1}

[Calculation:3/5] Forward-backward asymmetry

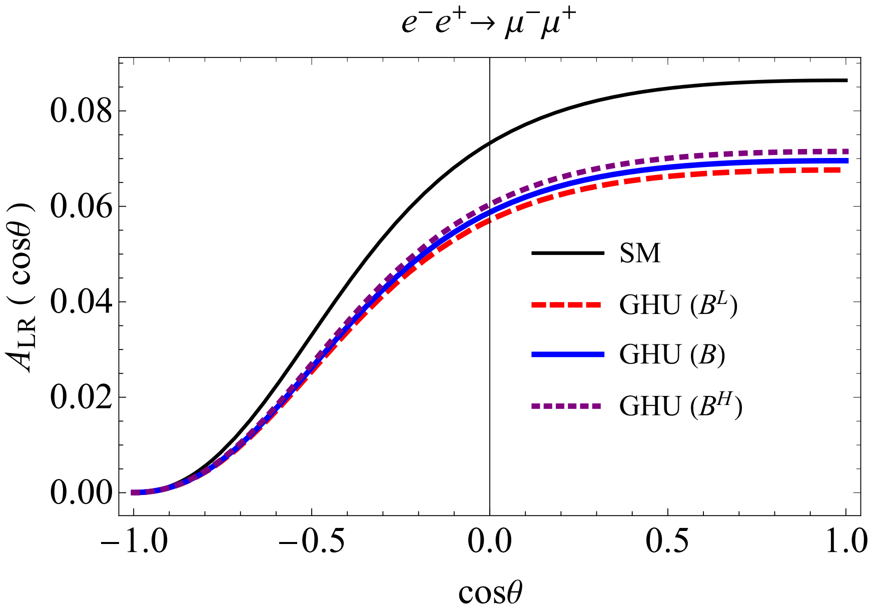
$$A_{FB}^{f\bar{f}}(P_{e^-}, P_{e^+}) = \frac{\sigma_F^{f\bar{f}}(P_{e^-}, P_{e^+}) - \sigma_B^{f\bar{f}}(P_{e^-}, P_{e^+})}{\sigma_F^{f\bar{f}}(P_{e^-}, P_{e^+}) + \sigma_B^{f\bar{f}}(P_{e^-}, P_{e^+})}.$$

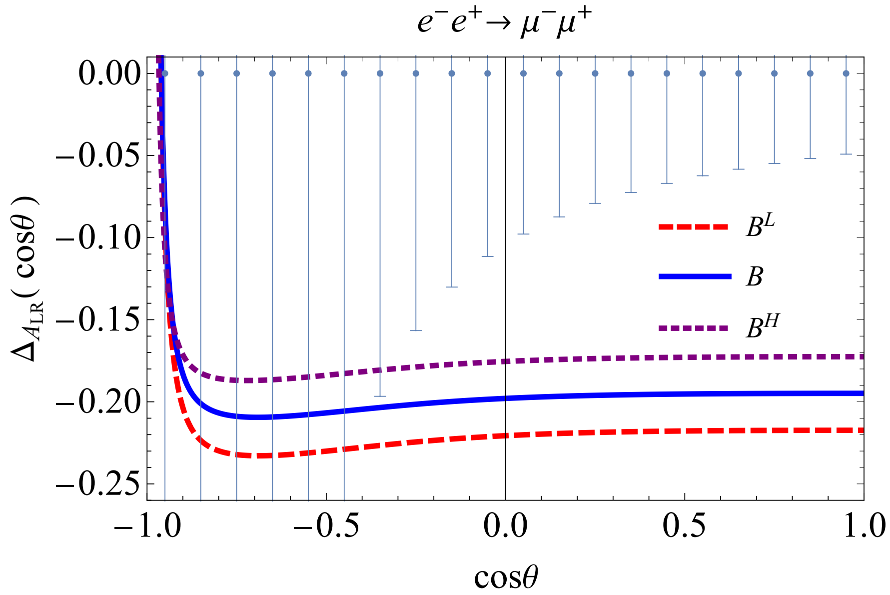


- (left figure) $(P_{e^-}, P_{e^+}) = (0,0)(U), (-0.8,+0.3)(L), (+0.8,-0.3)(R)$;
  (right figure) GHU $\{(B^L),(B),(B^H)\} \leftrightarrow m_{\rm KK} = \{11,13,15\}$TeV; $\sqrt{s} = 250$ GeV, 250 fb^{-1}.

[Calculation:4/5] Left-right asymmetry

$$A_{LR}^{f\bar{f}}(\cos\theta) = \frac{\sigma_{LR}^{f\bar{f}}(\cos\theta) - \sigma_{RL}^{f\bar{f}}(\cos\theta)}{\sigma_{LR}^{f\bar{f}}(\cos\theta) + \sigma_{RL}^{f\bar{f}}(\cos\theta)}.$$

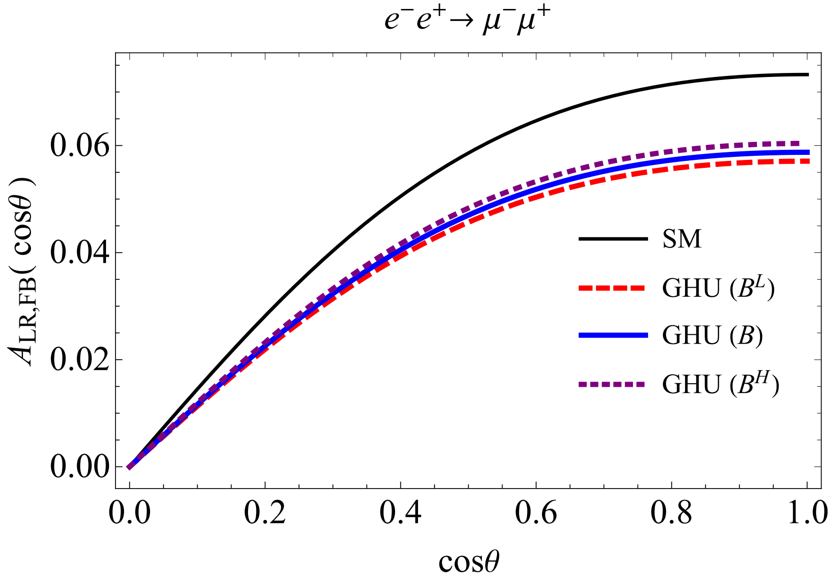


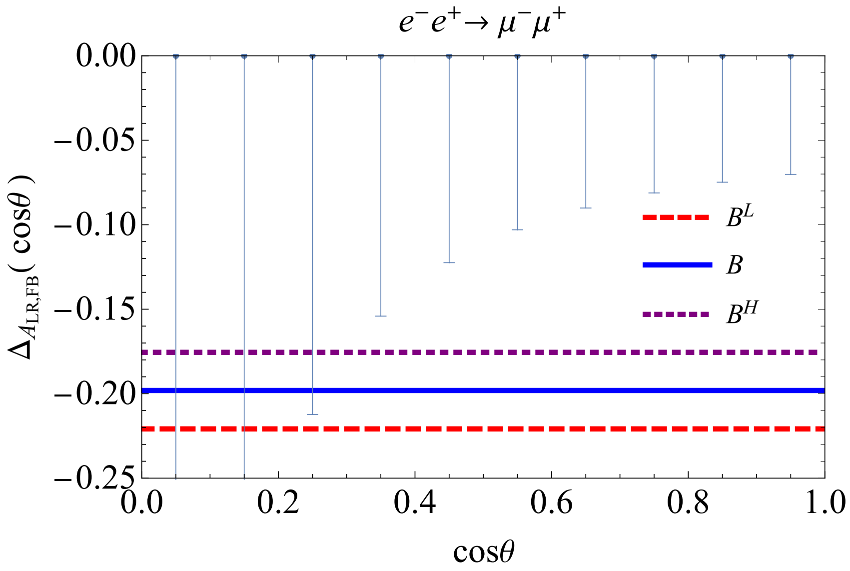


- GHU $\{(B^L), (B), (B^H)\} \leftrightarrow m_{\rm KK} = \{11, 13, 15\}$TeV; $\sqrt{s} = 250$ GeV, 250 fb^{-1}; $(P_{e^-}, P_{e^+}) = (\mp 0.8, \pm 0.3)$

[Calculation:5/5] LR FB asymmetries

$$A_{LR,FB}^{f\bar{f}}(\cos\theta) = \frac{\left[\sigma_{LR}^{f\bar{f}} - \sigma_{RL}^{f\bar{f}}\right](\cos\theta) - \left[\sigma_{LR}^{f\bar{f}} - \sigma_{RL}^{f\bar{f}}\right](-\cos\theta)}{\left[\sigma_{LR}^{f\bar{f}} + \sigma_{RL}^{f\bar{f}}\right](\cos\theta) + \left[\sigma_{LR}^{f\bar{f}} + \sigma_{RL}^{f\bar{f}}\right](-\cos\theta)} \overset{m_f \to 0}{\propto} \frac{2\cos\theta}{1+\cos^2\theta}$$





- GHU $\{(B^L), (B), (B^H)\} \leftrightarrow m_{\rm KK} = \{11, 13, 15\}$TeV; $\sqrt{s} = 250$ GeV, 250 fb^{-1}

Summary

We discussed fermion pair production at e^-e^+ linear collider experiments with polarized e^- and e^+ beams in the GUT inspired gauge-Higgs unification (GHU B-model).

(1) Due to the Z' bosons coupling to quarks and leptons with large parity violation, several observables in $e^-e^+ \to f\bar{f}(=\mu^-\mu^+, ...)$ processes have large polarization dependence.

(2) Deviations from the SM are testable for the Kaluza-Klein (KK) mass scale up to ∼15 TeV in the early stage of ILC.

[Observable:1/6] Differential cross section

Differential cross section for $e^-e^+ \to (V_i) \to f\bar{f}$:

$$\frac{d\sigma^{f\bar{f}}}{d\cos\theta}(P_{e^-}, P_{e^+}, \cos\theta) \propto (1-P_{\rm eff})\underbrace{\frac{d\sigma^{f\bar{f}}_{LR}}{d\cos\theta}(\cos\theta)}_{e^-_L e^+_R \to (V_i) \to f\bar{f}} + (1+P_{\rm eff})\underbrace{\frac{d\sigma^{f\bar{f}}_{RL}}{d\cos\theta}(\cos\theta)}_{e^-_R e^+_L \to (V_i) \to f\bar{f}},$$

For $e^-_L e^+_R \to f\bar{f}$ and $e^-_R e^+_L \to f\bar{f}$ ($\sqrt{s} \gg m_f$),

$$\frac{d\sigma^{f\bar{f}}_{LR}}{d\cos\theta}(\cos\theta) \underset{\sim}{\propto} (1+\cos\theta)^2 \, |Q_{e_L f_L}|^2 + (1-\cos\theta)^2 \, |Q_{e_L f_R}|^2 ,$$

$$\frac{d\sigma^{f\bar{f}}_{RL}}{d\cos\theta}(\cos\theta) \underset{\sim}{\propto} (1+\cos\theta)^2 \, |Q_{e_R f_R}|^2 + (1-\cos\theta)^2 \, |Q_{e_R f_L}|^2 ,$$

$$P_{\rm eff} := \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-}P_{e^+}}, \quad Q_{e_X f_Y} := \sum_i \frac{g^X_{V_i e} g^Y_{V_i f}}{(s - m^2_{V_i}) + i m_{V_i}\Gamma_{V_i}}.$$

[Observable:2/6] Total cross section

Total cross section for $e^- e^+ \to (V_i) \to f\bar{f}$:

$$\sigma_{\mathrm{tot}}^{f\bar{f}}(P_{e^-}, P_{e^+}) = \int_{-\cos\theta_{\max}}^{+\cos\theta_{\max}} \frac{d\sigma^{f\bar{f}}}{d\cos\theta}(P_{e^-}, P_{e^+}, \cos\theta)\ d\cos\theta$$

$$\propto\!\!\!\!\sim (1 - P_{\mathrm{eff}})\sigma_{LR}^{f\bar{f}} + (1 + P_{\mathrm{eff}})\sigma_{RL}^{f\bar{f}}.$$

Total cross section for $e_L^- e_R^+ \to f\bar{f}$ and $e_R^- e_L^+ \to f\bar{f}$ ($\sqrt{s} \gg m_f$):

$$\sigma_{LR}^{f\bar{f}} \propto\!\!\!\!\sim |Q_{e_L f_L}|^2 + |Q_{e_L f_R}|^2, \quad \sigma_{RL}^{f\bar{f}} \propto\!\!\!\!\sim |Q_{e_R f_R}|^2 + |Q_{e_R f_L}|^2.$$

[Observable:3/6] Forward-backward asymmetry

Forward-backward asymmetry for $e^- e^+ \to (V_i) \to f\bar{f}$:

$$A_{FB}^{f\bar{f}}(P_{e^-}, P_{e^+}) = \frac{\sigma_F^{f\bar{f}}(P_{e^-}, P_{e^+}) - \sigma_B^{f\bar{f}}(P_{e^-}, P_{e^+})}{\sigma_F^{f\bar{f}}(P_{e^-}, P_{e^+}) + \sigma_B^{f\bar{f}}(P_{e^-}, P_{e^+})},$$

Forward/backward cross section for $\sigma_{F/B}^{f\bar{f}}(P_{e^-}, P_{e^+})$ $(\sqrt{s} \gg m_f)$

$$A_{FB}^{f\bar{f}}(P_{e^-}, P_{e^+}) \simeq \frac{3}{4}\frac{B_1 - B_2}{B_1 + B_2},$$

$$B_1 = (1 - P_{\text{eff}}) \, |Q_{e_L f_L}|^2 + (1 + P_{\text{eff}}) \, |Q_{e_R f_R}|^2 ,$$

$$B_2 = (1 - P_{\text{eff}}) \, |Q_{e_L f_R}|^2 + (1 + P_{\text{eff}}) \, |Q_{e_R f_L}|^2 .$$

[Observable:4/6] Left-right (LR) asymmetry

LR asymmetry for $e^-e^+ \to (V_i) \to f\bar{f}$ [29, 33, B.Schrempp et al.'88;...]:

$$A_{LR}^{f\bar{f}}(\cos\theta) = \frac{\sigma_{LR}^{f\bar{f}}(\cos\theta) - \sigma_{RL}^{f\bar{f}}(\cos\theta)}{\sigma_{LR}^{f\bar{f}}(\cos\theta) + \sigma_{RL}^{f\bar{f}}(\cos\theta)}.$$

Observable LR asymmetry:

$$A_{LR}^{f\bar{f}}(P_{e^-}, P_{e^+}, \cos\theta) = \frac{\sigma^{f\bar{f}}(P_{e^-}, P_{e^+}, \cos\theta) - \sigma^{f\bar{f}}(-P_{e^-}, -P_{e^+}, \cos\theta)}{\sigma^{f\bar{f}}(P_{e^-}, P_{e^+}, \cos\theta) + \sigma^{f\bar{f}}(-P_{e^-}, -P_{e^+}, \cos\theta)}.$$

The relations between the above two asymmetries :

$$A_{LR}^{f\bar{f}}(\cos\theta) = \frac{1}{P_{\mathrm{eff}}} A_{LR}^{f\bar{f}}(P_{e^-}, P_{e^+}, \cos\theta).$$

[Observable:5/6] Left-right (LR) asymmetry

LR asymmetry for $e^-e^+ \to (V_i) \to f\bar{f}$ [29, 33, B.Schrempp et al.'88;...]:

$$A_{LR}^{f\bar{f}} = \frac{\sigma_{LR}^{f\bar{f}} - \sigma_{RL}^{f\bar{f}}}{\sigma_{LR}^{f\bar{f}} + \sigma_{RL}^{f\bar{f}}}.$$

For $\sqrt{s} \gg m_f$,

$$A_{LR}^{f\bar{f}} \simeq \frac{\left[\,|Q_{e_L f_L}|^2 + |Q_{e_L f_R}|^2\,\right] - \left[\,|Q_{e_R f_R}|^2 + |Q_{e_R f_L}|^2\,\right]}{\left[\,|Q_{e_L f_L}|^2 + |Q_{e_L f_R}|^2\,\right] + \left[\,|Q_{e_R f_R}|^2 + |Q_{e_R f_L}|^2\,\right]}.$$

[Observable:6/6] LR FB asymmetry

LR FB asymmetry [28–32, D.C.Kennedy et al.'89;SLD Collaboration'94'95]:

$$A^{f\bar{f}}_{LR,FB}(\cos\theta) = \frac{\left[\sigma^{f\bar{f}}_{LR} - \sigma^{f\bar{f}}_{RL}\right](\cos\theta) - \left[\sigma^{f\bar{f}}_{LR} - \sigma^{f\bar{f}}_{RL}\right](-\cos\theta)}{\left[\sigma^{f\bar{f}}_{LR} + \sigma^{f\bar{f}}_{RL}\right](\cos\theta) + \left[\sigma^{f\bar{f}}_{LR} + \sigma^{f\bar{f}}_{RL}\right](-\cos\theta)}.$$

For $\sqrt{s} \gg m_f$,

$$A^{f\bar{f}}_{LR,FB}(\cos\theta) \simeq \frac{2\cos\theta}{1+\cos^2\theta} \frac{\left[|Q_{e_Lf_L}|^2 - |Q_{e_Lf_R}|^2\right] - \left[|Q_{e_Rf_R}|^2 - |Q_{e_Rf_L}|^2\right]}{|Q_{e_Lf_L}|^2 + |Q_{e_Lf_R}|^2 + |Q_{e_Rf_R}|^2 + |Q_{e_Rf_L}|^2}.$$

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