Linking the supersymmetric standard model to the cosmological constant

Yu-Cheng QIU

The Hong Kong University of Science and Technology

August 23, 2021

In collaboration with **Shing Yan Li** (MIT) & **S. -H. Henry Tye** (HKUST, Cornell U).
Based on: (Published) 2006.16620 & 2010.10089

Outline

Puzzles

- Smallness of positive Cosmological Constant

$$\Lambda_{\rm obs} \simeq +10^{-120} M_{\rm Pl}^4 \sim \left(10^{-30}\, M_{\rm Pl}\right)^4 \ .$$

- Higgs Mass Hierarchy

$$m_h = 125\,\text{GeV} \sim 10^{-16} M_{\rm Pl} \ .$$

General Idea

- **String theory**: M_S and no other parameter.
- Wrapped Geometry $\rightarrow$ **New Scale**
- **KKLT**: arbitrary $\Lambda > 0$.
- **Racetrack Kähler Uplift (RKU)**: $P(\Lambda \rightarrow 0^+) \sim \Lambda^{-1+k}$ with $0 < k < 1$.
- **Electroweak SSB**: $V_h \sim -m_{\text{EW}}^4$
- **Supersymmetric Standard Model**: $V_{\cancel{\text{susy}}} \sim +m_{\cancel{\text{susy}}}^4$.

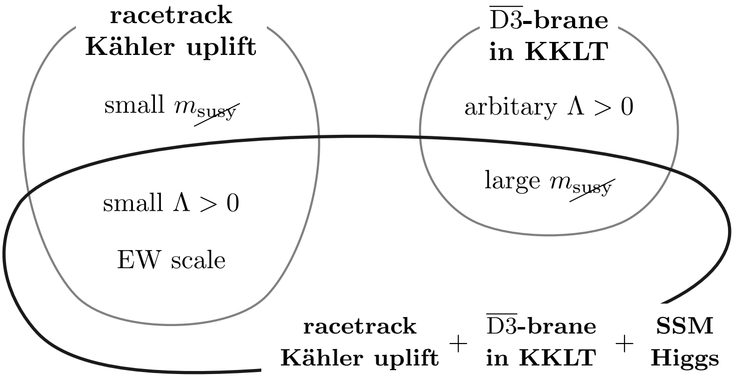

Figure: Relations among the 3 pillars of the model.

Model

In units where $M_{\mathrm{Pl}} = 1$.

$$V_F = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3 W \bar{W} \right)$$

$$K = -2 \ln \left(\mathcal{V} + \frac{\xi}{2} \right) + H_i^\dagger H_i + \cdots \,, \qquad W = \mathscr{W} + W_{\mathrm{np}}(T)$$

where

$$\mathcal{V} = \left(\frac{M_{\mathrm{Pl}}}{M_S} \right)^2 = \left(T + \overline{T} \right)^{3/2} \qquad \xi = -\frac{\zeta(3)}{4\sqrt{2}} \chi(\mathcal{M}) \left(S + \overline{S} \right)^{3/2} > 0$$

$$\mathscr{W} = W_0(U_i, S) + \mu H_u H_d \qquad W_{\mathrm{np}}(T) = A e^{-aT} + B e^{-bT}$$

Approximated potential

- Large Volume Scenario, $\frac{\xi}{\mathcal{V}} \ll 1$.

$$V(T) = V_F + \Delta V \ , \quad \Delta V = V_3 + D_h + S_h$$

After neglecting higher-order and doubly-suppressed terms. The potential of $T = t + i\tau$ could be written as

$$V(T) \simeq \left(-\frac{a^3 A \mathcal{N} \mathscr{W}}{2} \right) \lambda(x, y) \ ,$$

$$\lambda(x, y) = -\frac{e^{-x}}{x^2} \cos y - \frac{\beta}{z} \frac{e^{-\beta x}}{x^2} \cos(\beta y) + \frac{C}{x^{9/2}} + \frac{D}{x^2} \ ,$$

$$C = -\frac{3\xi a^{3/2} \mathscr{W}}{32\sqrt{2} A} \ , \quad D = -\frac{\mathscr{D}}{2aA\mathcal{N}\mathscr{W}}$$

where $x = at \sim \mathcal{O}(100)$, $y = a\tau$, $\beta = b/a$ and $z = A/B < 0$.
$\mathcal{N}$ is the overall constant from stabilization of U_i and S.

Stabilization Condition

The minimum solution of $T \propto x + iy$ is given by equation

$$\partial_x \lambda = \partial_y \lambda = 0$$

which gives $x_0 = x_0(z, \beta, C, D)$ and $y_0 = 0$. Accordingly, $\lambda_{\text{ext}} = \lambda_{\text{ext}}(z, \beta, C, D)$. One could also express everything in $(x_0, \lambda_{\text{ext}}, \beta, D)$.
Stable condition

$$\partial_x^2 \lambda \geq 0\ , \quad \partial_y^2 \lambda \geq 0\ , \quad \partial_{xy}^2 \lambda \geq 0\ ,$$

which turn out to be very informative.

Stabilization Condition

$$\partial_x^2 \lambda\big|_{\rm ext} \propto e^{-x}\frac{2(\beta-1)}{9\beta x}\left(1-\frac{5(\beta+1)}{2\beta x}+\frac{3}{2\beta x}+\cdots\right)+\frac{5D}{9x^2}\left(1-\frac{2}{\beta x}+\cdots\right)-\lambda_{\rm ext}\geq 0$$

$$\partial_y^2 \lambda\big|_{\rm ext} \propto -e^{-x}\frac{2(\beta-1)}{9\beta x}\left(1-\frac{5(\beta+1)}{2\beta x}\right)-\frac{5D}{9x^2}+\lambda_{\rm ext}\geq 0$$

$$\partial_x\partial_y\lambda\big|_{\rm ext}=\partial_y\partial_x\lambda\big|_{\rm ext}=0\ ,$$

which reduce to

$$\lambda_{\rm min}(x,\beta,D)\leq\lambda_{\rm ext}\leq\lambda_{\rm max}(x,\beta,D)$$

stabilization Condition

$$\lambda_{\rm max} \simeq e^{-x}\frac{2(\beta-1)}{9\beta x}\left(1-\frac{5(\beta+1)}{2\beta x}+\frac{3}{2\beta x}+\cdots\right)+\frac{5D}{9x^2}\left(1-\frac{2}{\beta x}+\cdots\right) ,$$
$$\lambda_{\rm min} = e^{-x}\frac{2(\beta-1)}{9\beta x}\left(1-\frac{5(\beta+1)}{2\beta x}\right)+\frac{5D}{9x^2}$$

There is a Hidden constraint.

The existence of solution ($\lambda_{\rm max} > \lambda_{\rm min}$) indicates that

$$D \leq D_{\rm max} \simeq \frac{3}{10}xe^{-x}\frac{\beta-1}{\beta} ,$$

- $D < -D_{\rm max}$ ensures the existence of solution but a AdS one.
- $|D| < D_{\rm max}$ indicates a cancellation between Higgs SSB and SUSY-breaking.
- $D > D_{\rm max}$ Decompactified. (No solution for x, or T.)

Physical picture

We only interested in dS solution.
Cosmological constant of this model consists of two parts.

$$\Lambda = \Lambda_\xi + \Delta V \ ,$$

• Λ_ξ is the Kähler uplift contribution providing the statistical preferred exponentially small dS solution.

• $\Delta V < \alpha \Lambda_\xi \ , \quad \alpha \sim \mathcal{O}(1)$ implies any other contributions should essentially cancel within themselves. Otherwise, fine-tuning is introduced.

• Assume $\Delta V \to 0$, then one would conclude that

$$m_{\not{\text{susy}}} \simeq m_{\text{EW}} \simeq 100\,\text{GeV}$$

and $M_{\not{\text{susy}}} \simeq 100 m_{\not{\text{susy}}}$ is the unwrapped $\overline{\text{D3}}$-brane tension, which could be responsible for soft terms.

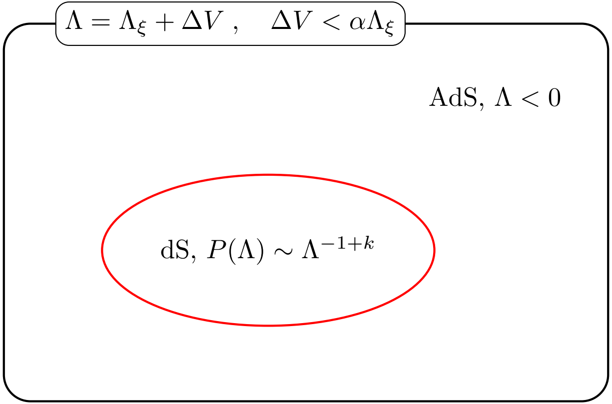

Figure: Hidden condition in dS solution.

Spontaneous SUSY-breaking

- In a Calabi-Yau orientifold in Type IIB string theory, D3-brane introduces a nilpotent superfield X, i.e., $X^2 = 0$. Kallosh and Wrase (2014) and others.

- By introducing a term into superpotential, one would have an uplift term like in KKLT setup

$$W = X m_s^2 \quad \rightarrow \quad V_X = \frac{m_s^4}{(T + \bar{T})^2}$$

Perfect Square Potential

• Introducing $W = X\left(m_s^2 + \gamma H_u H_d\right) + \mu H_u H_d + \cdots$ and nilpotent condition $X^2 = 0$, one would have

$$V = \left|m_s^2 + \gamma H_u H_d\right|^2 + \cdots \ .$$

• μ^2-term is projected out by $X\overline{H} =$ chiral.

• D-term potential for Higgs is projected out by imposing $X[(H_u)_i - \epsilon_{ij}(\overline{H}_d)^j] = 0$. 2010.10089

• In this model, the cosmological constant is obtained from stabilization of geometrical sector.

• EW SSB happened in V_h naturally gives $(\Delta V)_{\text{min}} = 0$ thanks to the $|\cdots|^2$.

Axi-Higgs model

in collaboration with **Leo WH Fung, Lingfeng Li, Tao Liu, Hoang Nhan Luu** and **S.-H.Henry Tye**

Consider superpotential $W \supset X\left(m_s^2 + \gamma H_u H_d\right)$, where parameter m_s and γ is in principle determined by geometrical sector (U_i, S_i), which intrinsically include axion-like fields. Thus

$$V_X \to \left| m_s^2 G(a) - \kappa K(a) h^\dagger h \right|^2 = \left| K(a) \left[m_s^2 F(a) - \kappa h^\dagger h \right] \right|^2 ,$$

where to leading order, with proper normalization $G(a=0) = K(a=0) = 1$,

$$G(a) = 1 + \frac{ga^2}{M_{\mathrm{Pl}}^2} , \quad K(a) = 1 + \frac{ka^2}{M_{\mathrm{Pl}}^2} , \quad F(a) = \frac{G(a)}{K(a)} \simeq 1 + \frac{Ca^2}{M_{\mathrm{Pl}}^2} ,$$

and $C = g - k$ is a constant whose positivity is undetermined.

Evolving Higgs VEV

Mass of a is small, $m_a \sim 10^{-29}\,\text{eV}$, which means that the field profile evolves with the expansion of the universe. This means that we could consider Higgs profile as evolving VEV, which is a function of $a(t)$.

$$\delta v(t) = \frac{v(t) - v_0}{v_0} = [F(a(t))]^{1/2} - 1 \simeq \frac{Ca(t)^2}{2M_{\text{Pl}}^2} \ ,$$

where $v_0 = \sqrt{2}m_s/\sqrt{\kappa} = 246\,\text{GeV}$ and $a(t)$ is determined by differential equation

$$\ddot{a} + 3H(t)\dot{a} + \frac{\partial V_a}{\partial a} = 0 \ .$$

Detailed explanation in 2102.11257 and 2105.01631.

Summary

- Smallness of $\Lambda_{\rm obs}$ is statistically preferred in presence of Kähler uplift.
- The positive contribution to Λ from SUSY-breaking and negative contribution from Higgs SSB should cancel each other to get a stable dS solution.
- With the help of Nilpotent superfield X, one could make above contribution exactly cancel while preserving small Λ.
- This naturally gives us a perfect square form of Higgs potential and ultra-light axion could be easily incorporated into it, which leads to the phenomena of shifting Higgs VEV and resolving several puzzles in modern cosmology.

Thank you for your attention.

Constrained superfield

For a superfield $X = x + \sqrt{2}\theta G + \theta\theta F^X$ satisfying nilpotent constraint $X^2 = 0$, components are constrained by equations

$$x^2 = 0 \, , \quad x G_\alpha = 0 \, , \quad 2x F^X - GG = 0 \, .$$

- Trivial solution states that $x = G_\alpha = F^X = 0$.
- For $F^X \neq 0$, one could conclude that

$$x = \frac{GG}{2F^X} \, .$$

This means that scalar component x of X is projected out. When considering scalar potential in the system, one could simply let $x \to 0$.

Complete model

In units where $M_{\rm Pl} = 1$.

$$K = -2\ln\left[\left(T + \overline{T} - X\overline{X} - n_u H_u^\dagger H_u - n_d H_d^\dagger H_d + K_{\rm matter}\right)^{3/2} + \frac{\xi}{2}\right]$$
$$W = W_0\left(U_i, S\right) + W_{\rm np}(T) + \tilde{\mu} H_u H_d - X\left(\tilde{m}_s^2 + \tilde{\gamma} H_u H_d\right) + W_{\rm matter}$$
$$W_{\rm np}(T) = Ae^{-aT} + Be^{-bT}\ , \quad \xi = -\frac{\zeta(3)}{4\sqrt{2}}\chi(\mathcal{M})\left(S + \overline{S}\right)^{3/2} > 0\ .$$

with superfield constraints

$$X^2 = 0\ , \quad X\overline{H} = \text{chiral}\ , \quad X\left[(H_u)_i - \epsilon_{ij}(\overline{H}_d)^j\right] = 0\ , \quad XQ_i = XL_i = XW_\alpha = 0\ .$$

Scalar potential is given by

$$V = V_T + V_X + V_{H,F} + V_D = V_T + \Delta V \ ,$$

where

$$\begin{aligned} V_T &= e^K K^{T\overline{T}} |D_T W|^2 - 3e^K |W|^2 \\ V_X &= K_{X\overline{X}} F^X \overline{F}^{\overline{X}} + \left(K_{T\overline{X}} F^T \overline{F}^{\overline{X}} + \text{c.c}\right) \\ V_{H,F} &= K_{H\overline{H}} F^H \overline{F}^{\overline{H}} + \left(K_{H\overline{I}} F^H \overline{F}^{\overline{I}} + \text{c.c.}\right) \\ V_{H,D} &= \sum_a \frac{1}{2} g_a^2 D^{a2} \end{aligned}$$

Superfield constraint gives us

$$\langle X|_{\theta=\bar{\theta}=0}\rangle = 0\,, \quad \langle F^H\rangle = 0\,, \quad h_u^+ = \sqrt{\frac{n_u}{n_d}}\bar{h}_d^-\,, \quad h_u^0 = \sqrt{\frac{n_u}{n_d}}\bar{h}_d^0\,.$$

Therefore

$$\Delta V = V_{F,H} + V_{H,D} + V_X = V_X = \left| m_s^2 - \kappa h^\dagger h \right|^2\,,$$

where

$$m_s = \tilde{m}_s \left[3\left(T + \overline{T}\right)^2 \right]^{-1/2}\,, \quad \kappa = \tilde{\gamma}\,(27 n_u n_d)^{-1/2} \sqrt{\frac{n_d}{n_u}}\,,$$

and

$$h = h_u = \left(\frac{3n_u}{T + \overline{T}} \right)^{1/2} H_u|_{\theta=\bar{\theta}=0}$$

Axi-Higgs Potential

Scalar potential in the model is

$$V = V_a + V_h \ ,$$

where

$$V_a = m_a^2 f_a^2 \left(1 - \cos\frac{a}{f_a}\right) \simeq \frac{1}{2} m_a^2 a^2 - \frac{1}{24}\frac{m_a^2}{f_a^2} a^4 + \cdots \ ,$$

$$V_h = \left| m_s^2 F(a) - \kappa h^\dagger h \right|^2 \ , \quad F(a) = 1 + \frac{Ca^2}{M_{\rm Pl}^2} \ .$$

Neglect three Goldstone directions and let $h^\dagger h \to \frac{1}{2}\phi^2$, then

$$V \simeq \frac{1}{2} m_a^2 a^2 + |B(a,\phi)|^2 \ , \quad B = m_s^2 \left(1 + \frac{Ca^2}{M_{\rm Pl}^2}\right) - \frac{1}{2}\kappa\phi^2 \ .$$