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Supersymmetric Alignment Models for $(g\text{-}2)_\mu$

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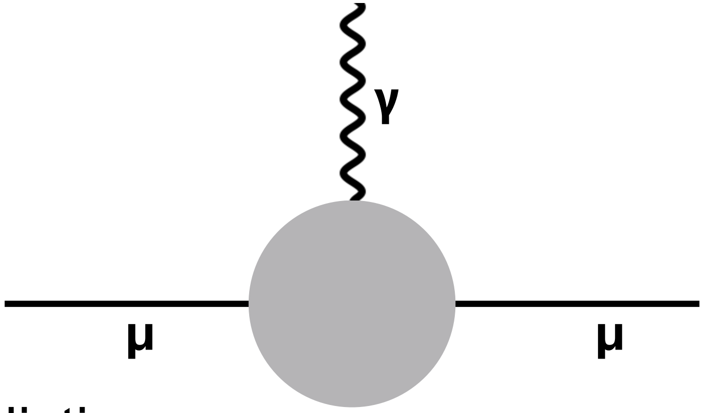
T. D. Lee Institute & Shanghai Jiao Tong U.

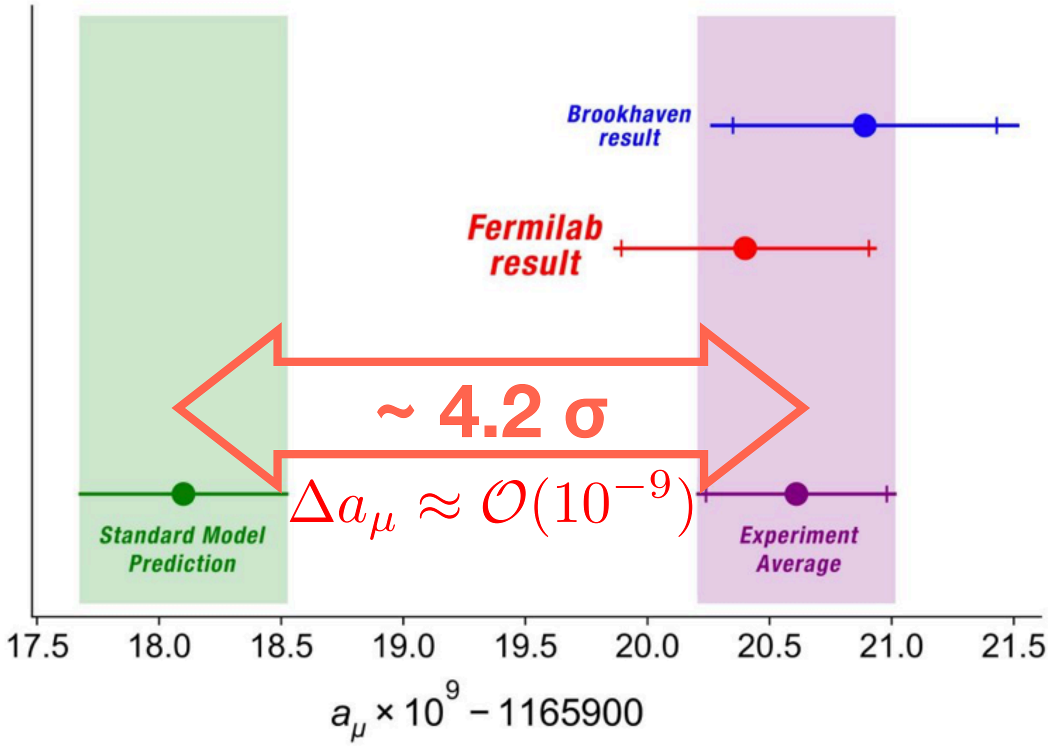
Based on Y. Nakai (TDLI&SJTU), M. Reece (Harvard U.),
arXiv: 2107.10268 .

Muon g-2 anomaly

- g-factor : spin-magnetic field interaction
  - g=2 : tree level
  - a=(g-2)/2 : radiative correction



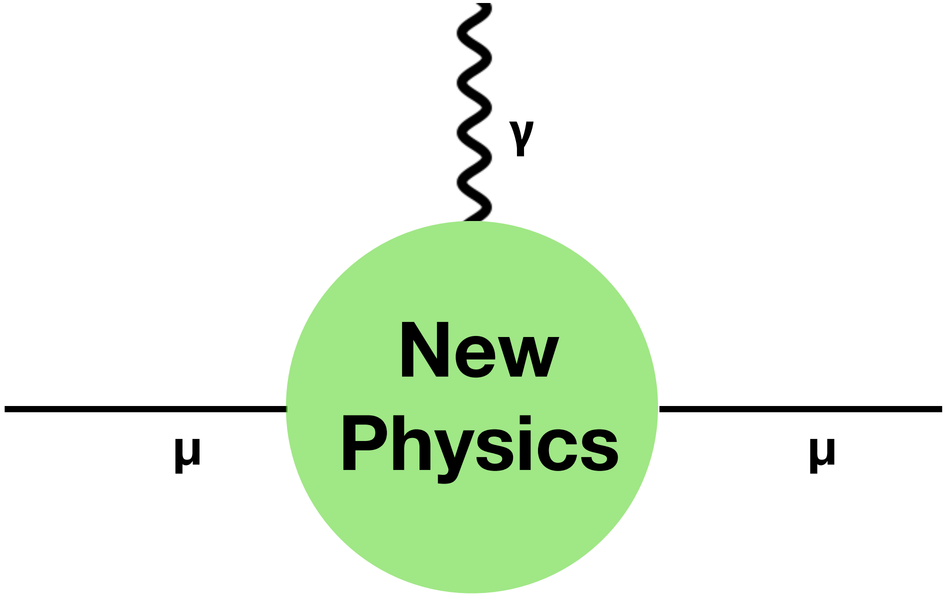
- Muon g-2: experimental results & SM prediction



From Fermilab webpage

New physics contribution

- New physics contribution to muon g-2



$$\Delta a_{\mu,\mathrm{NP}} \approx \frac{1}{16\pi^2}\mathcal{O}(m_\mu^2/m_{\mathrm{NP}}^2)$$

$$\Delta a_{\mu,\mathrm{NP}} \approx \mathcal{O}(10^{-9})$$

$$m_{\mathrm{NP}} \lesssim 1\,\mathrm{TeV}$$

SUSY contributions

- We focus on SUSY models

- electroweakinos-smuon loops

e.g.

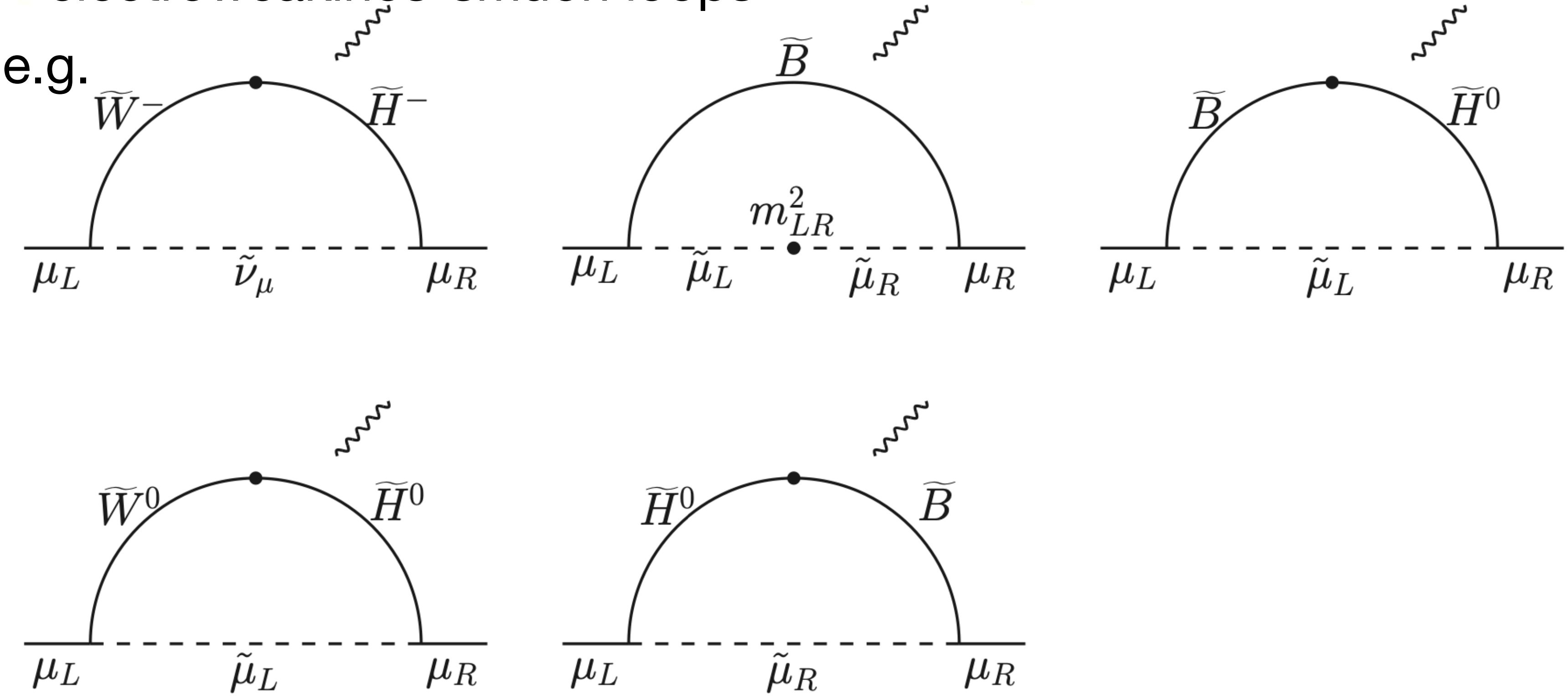


Figure from Gi. Cho, K. Hagiwara, Y. Matsumoto, D. Nomura (2011)

FCNC

e.g. Gravity mediation

$$\delta \sim 0.3$$

$$m^2_{\text{slepton}} \sim \tilde{m}^2 \begin{pmatrix} 1 & \delta & \delta \\ \delta & 1 & \delta \\ \delta & \delta & 1 \end{pmatrix}$$

$\mu \to e + \gamma$ ➜ $\tilde{m} \gg 1\,\text{TeV}$

Alignment

Alignment : a mechanism to suppress FCNCs

- In the basis that Yukawa diagonal

$$Y_e \sim \begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \end{pmatrix}$$

$$m^2_{\mathrm{slepton}} \sim \tilde{m}^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } A_e \propto Y_e$$

FCNC constraints can be relaxed by appoximate alignment

Alignment

Horizontal (flavor) symmetries

— Flavor structure of SM fermions + alignment

- Approximate alignment of quark-squark by horizontal symmetries

  Nir and Seiberg (1993)

- Approximate alignment of lepton-slepton by horizontal symmetries

  Ben-Hamo and Nir (1994)

  Grossman and Nir (1995)

SM flavor structure

- CKM matrix is presented in the Wolfenstein's parameterization — the order of magnitude of mixing angle is given by

$$|V_{12}^{\rm CKM}| \sim \lambda \ , \quad |V_{23}^{\rm CKM}| \sim \lambda^2 \ , \quad |V_{13}^{\rm CKM}| \sim \lambda^3$$

$$\lambda \sim 0.2$$

- Quark mass ratio can be also expressed in powers of λ

$$m_c/m_t \sim \lambda^3 \ , \quad m_u/m_t \sim \lambda^6 - \lambda^7,$$

$$m_b/m_t \sim \lambda^2 \ , \quad m_s/m_b \sim \lambda^2 \ , \quad m_d/m_b \sim \lambda^4$$

- Lepton mass ratio and mixing angles can be also expressed by λ

Horizontal symmetry

- Hierarchy and smallness of quark and lepton sector parameters

’tHooft Naturalness

Small numbers are natural only if an exact symmetry is acquired when they are set to zero

- Let us consider **U(1)H horizontal symmetry** that acts on quarks and leptons

$H(X)$: U(1)H charge of X

- e.g. up-type Yukawa terms

$$W \sim H_u Q_i \bar{U}_j$$

$$H(H_u) + H(Q_i) + H(\bar{U}_j) \neq 0$$

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- e.g. up-type Yukawa terms

$$W \sim \cancel{H_u Q_i \bar{U}_j}$$

$$H(H_u) + H(Q_i) + H(\bar{U}_j) \neq 0$$

Horizontal symmetry

- Introducing flavon "S" which has a non-zero U(1)H charge and VEV

$$H(S) \neq 0$$

$$\langle S \rangle = \lambda \, \Lambda_{\rm UV} \;\; (\lambda \sim 0.2)$$

- Fermion mass ratio and mixing can be explained by couplings with flavon

$$c.f. \;\; W \sim S^{a_{ij}} H_u Q_i \bar{U}_j \quad (\Lambda_{\rm UV} = 1 \text{ unit})$$

$$Y_u = c^u_{ij} \langle S \rangle^{a_{ij}}$$

A simple model

- Introducing a flavon S1

$$H(S_1) = -1$$

- Assuming non-negative U(1)H charges for matter,
  the mass ratios and mixing matrices are estimated by simple formula

$$|V_{ij}^{\mathrm{PMNS}}| \sim \lambda^{|H(L_i)-H(L_j)|}$$

- We also obtain soft mass squared matrices

$$M^2_{\tilde{L}\,ij} = \tilde{m}^2_\ell\,\lambda^{|H(L_i)-H(L_j)|}$$

— relation between mixing matrices and soft mass squared

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— relation between mixing matrices and soft mass squared

FCNC in lepton sector

- Mixing matrices - soft mass matrices relation

$$|V_{12}^{\mathrm{PMNS}}| \sim \lambda \longrightarrow M_{\tilde{L}\,12}^{2} \sim \tilde{m}_{\ell}^{2}\lambda$$

$$\boxed{\lambda \sim 0.2 \longrightarrow \tilde{m}_{\ell} \gg 1\,\mathrm{TeV}}$$

$$\mu \to e + \gamma$$

Our work

Y. Nakai, M. Reece, M.S. (2021)

We study horizontal symmetry models to explain muon g-2

- one horizontal symmetry and several flavons
- two horizontal symmetries and several flavons

One horizontal symmetry

- **Avoiding a relation between mixing matrix and soft mass**

$$|V_{ij}^{\rm PMNS}| \sim \lambda^{|H(L_i)-H(L_j)|} \qquad M^2_{\tilde{L}\,ij} = \tilde{m}_\ell^2\,\lambda^{|H(L_i)-H(L_j)|}$$

- To avoid the relation, **we consider multi-flavons without S1**

$$S_2,\ \bar{S}_2,\ S_3$$

$$H(S_2) = -2,\ H(\bar{S}_2) = +2\ ,\ H(S_3) = -3$$

$$\langle S_2\rangle \sim \lambda^2\ ,\ \langle \bar{S}_2\rangle \sim \lambda^2\ ,\ \langle S_3\rangle \sim \lambda^3$$

One horizontal symmetry

- The relation is avoided?

$$M^2_{\tilde{L}\,12}\tilde{L}_1\tilde{L}_2^\dagger$$

$$\mathcal{L} \sim \qquad\qquad\qquad \tilde{L}_1\tilde{L}_2^\dagger$$

+1 = **H(L1)-H(L2)=1**

One horizontal symmetry

- The relation is avoided?

$$M^2_{\tilde{L}\,12}\tilde{L}_1\tilde{L}_2^\dagger$$

$$\mathcal{L} \sim -\tilde{m}_\ell^2 S_3 \bar{S}_2\, \tilde{L}_1 \tilde{L}_2^\dagger$$

-1 +1 = **H(L1)-H(L2)=1**

One horizontal symmetry

- The relation is avoided?

$$M^2_{\tilde{L}\,12}\tilde{L}_1\tilde{L}_2^\dagger$$

$$\mathcal{L} \sim -\tilde{m}_\ell^2 S_3 \bar{S}_2\, \tilde{L}_1 \tilde{L}_2^\dagger$$

-1 +1 = **H(L1)-H(L2)=1**

↓

$$M^2_{\tilde{L}\,12} = \tilde{m}_\ell^2\,\lambda^5 \quad \cancel{= \tilde{m}_\ell^2\,\lambda}$$

One horiozontal symmetry

- A charge assignment

Q_1	Q_2	Q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$	$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
(3)	(2)	(0)	(4)	(1)	(0)	(−7)	(−4)	(0)

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$
(2)	(1)	(1)	(3)	(−3)	(−1)

- Reasonable fermion mass ratio and mixing angles are obtained
- Soft mass squared matrices are relatively suppressed
- $\mu \to e + \gamma$ constraint is still stringent

$$\tilde{m}_\ell \gtrsim 5\,\mathrm{TeV}$$

Two horizontal symmetries

$$U(1)_{H_1} \times U(1)_{H_2}$$

- Two flavons

$$\begin{array}{cc} S_1 & S_2 \\ (-1,0) & (0,-1) \end{array}$$

- Flavon VEVs

$$\langle S_1 \rangle \sim \lambda, \; \langle S_2 \rangle \sim \lambda^2$$

- An example charge assignment

$$\begin{array}{cccccc} Q_1 & Q_2 & Q_3 & \bar{u}_1 & \bar{u}_2 & \bar{u}_3 \\ (3,0) & (0,1) & (0,0) & (-2,3) & (1,0) & (0,0) \end{array}$$

$$\begin{array}{ccc} \bar{d}_1 & \bar{d}_2 & \bar{d}_3 \\ (-3,2) & (2,-1) & (0,0) \end{array}$$

$$\begin{array}{cccccc} L_1 & L_2 & L_3 & \bar{e}_1 & \bar{e}_2 & \bar{e}_3 \\ (5,0) & (0,2) & (0,2) & (-4,2) & (2,-2) & (0,-2) \end{array}$$

Two horizontal symmetries

- Soft mass matrices are suppressed well

$$M^2_{\tilde{L}} \sim \tilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^9 \\ \lambda^9 & 1 & 1 \\ \lambda^9 & 1 & 1 \end{pmatrix}, \quad M^2_{\tilde{\bar{e}}} \sim \tilde{m}^2 \begin{pmatrix} 1 & \lambda^{14} & \lambda^{12} \\ \lambda^{14} & 1 & \lambda^2 \\ \lambda^{12} & \lambda^2 & 1 \end{pmatrix}$$

- $\mu \to e + \gamma$

$$\tilde{m}_\ell < 1\,\mathrm{TeV}$$

- However, eEDM constraint is also stringent…

$$\tilde{m}_\ell \gg 1\,\mathrm{TeV}$$

Spontaneous CP violation

- Suppression of eEDM by SCPV

- Consider an additional flavon

$$\begin{array}{c} S_N \\ (-N, 0) \end{array}$$

- Superpotential is

$$W_S = Z(aS_N^2 + bS_N S_1^N + cS_1^{2N}) \,,$$

- SN can obtain a complex phase

$$\frac{\langle S_N \rangle}{\langle S_1 \rangle^N} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad b^2 - 4ac < 0$$

$$|\langle S_N \rangle| \sim \lambda^N$$

eEDM

- Flavon with complex phase

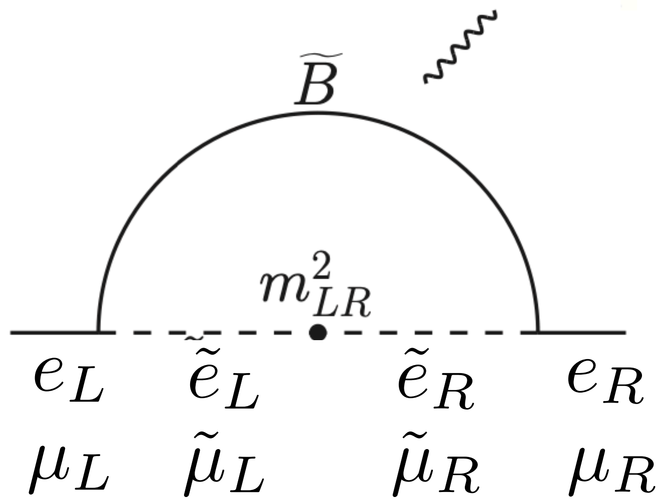
$$S_4$$
$$(-4, 0)$$

- Dominant flavor diagonal contribution
- Electron electric dipole moment

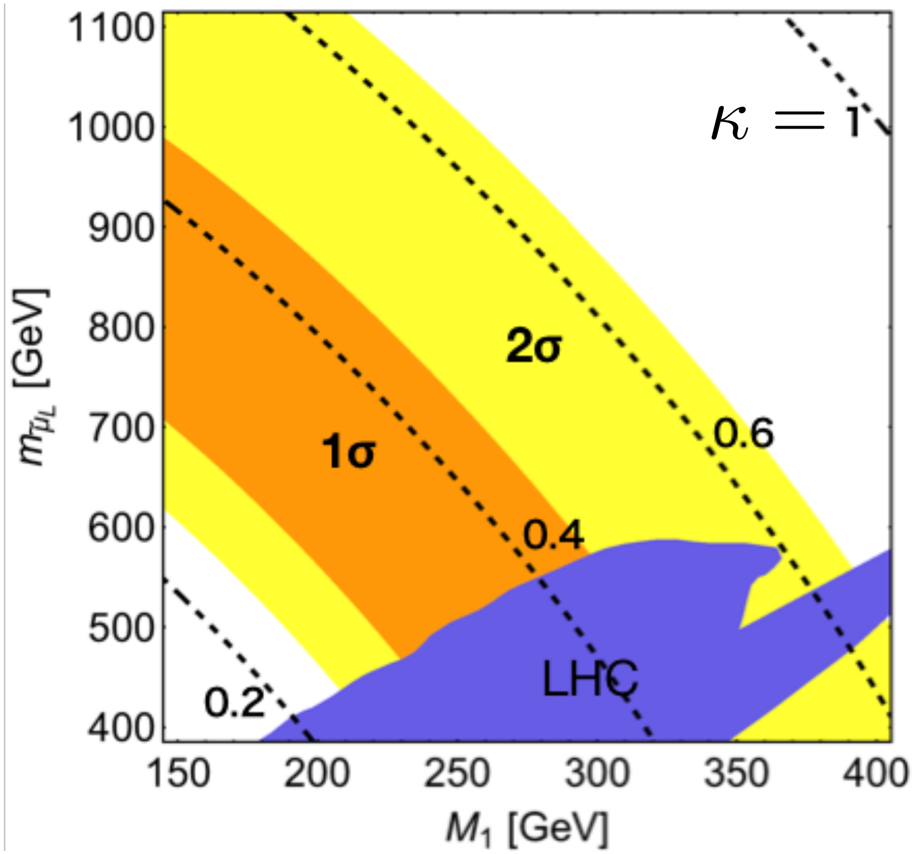
$$|d_e^{\rm SUSY}| \simeq 10^{-24} \left(\frac{a_\mu}{2\times 10^{-9}}\right) |\arg(\mu)|\, e\,{\rm cm}$$

- CP phase in mu-term

$$\mu = |\mu|(1 + i\kappa\,\lambda^8)$$



Result



future expected reach

$$|d_e| = 10^{-30}\, e\,\text{cm}$$

$$\arg(\mu) = \kappa\,\lambda^8$$

- decoupling of $\tilde{e}_i$

- parameters of

$$\tan\beta = 50,\ \ \mu = M_2 = 2M_1$$

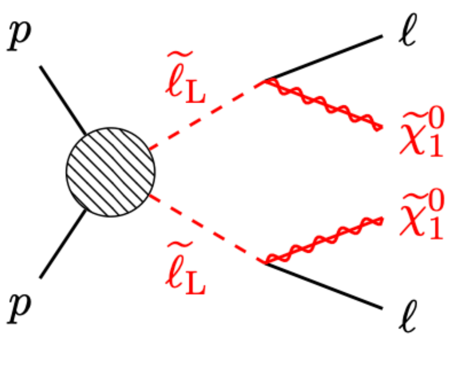
Future eEDM experiment will search for favored region

Conclusion

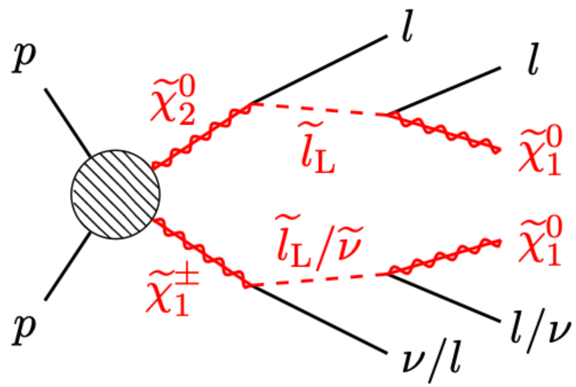
- New physics to explain muon g-2
- SUSY model is a prime candidate
- Horizontal symmetry provides fermion flavor structure and suppressed FCNCs = alignment
- A simple model predicts large FCNCs
- We considered SUSY horizontal symmetry models by one or two horizontal symmetries with multi-flavons
- We have developed models where muon g-2 can be explained by avoiding FCNC and CP observable constraints
- Future eEDM experiment will search for the favored parameter space

Back up

Collider constraint



(a) SLSL



(e) NC/3L

From M. Endo, K. Hamaguchi, S. Iwamoto, T. Kitahara (2021)

Flavor & CP observables

Observable	Experimental bound	Model with S_3	Model with S_4
$\lvert\epsilon_K\rvert$	$2.228(11) \times 10^{-3}$ [56]	$\sim 10^{-3}$	$\sim 10^{-7}$
$\lvert\Delta M_D\rvert$	$0.63^{+0.27}_{-0.29} \times 10^{-14}$ GeV [56]	$\sim 5 \times 10^{-17}$ GeV	$\sim 5 \times 10^{-17}$ GeV
nEDM	$\leqslant 10^{-26}\ e\,\mathrm{cm}$ [62]	$\sim 10^{-28}\ e\,\mathrm{cm}$	$\sim 10^{-28}\ e\,\mathrm{cm}$
$\mathrm{Br}(\mu \to e + \gamma)$	$\leqslant 4.2 \times 10^{-13}$ [64]	$\sim 10^{-16}$	$\sim 10^{-16}$
eEDM	$\leqslant 1.1 \times 10^{-29}\ e\,\mathrm{cm}$ [23]	$\sim 5 \times 10^{-29}\ e\,\mathrm{cm}$	$\sim 10^{-30}\ e\,\mathrm{cm}$

TABLE I. CP and flavor observables and their current experimental bounds. The estimation of the SUSY contribution to each observable in the models with $U(1)_{H_1} \times U(1)_{H_2}$ is also shown. We take $\widetilde{m}_q = M_3 = 5\,\mathrm{TeV}$, $\widetilde{m}_\ell = M_{1,2} = \mu = 500\,\mathrm{GeV}$ and $\tan\beta = 50$. The typical mass scales of the trilinear soft SUSY breaking terms are taken as $\widetilde{m}_q$, $\widetilde{m}_\ell$ for squarks and sleptons, respectively.

Q_1	Q_2	Q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$
$(3,0)$	$(0,1)$	$(0,0)$	$(-2,3)$	$(1,0)$	$(0,0)$

$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
$(-3,2)$	$(2,-1)$	$(0,0)$

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$
$(5,0)$	$(0,2)$	$(0,2)$	$(-4,2)$	$(2,-2)$	$(0,-2)$

Flavor structure

$$Y_u \sim \begin{pmatrix} \lambda^7 & \lambda^4 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & \lambda & 1 \end{pmatrix}, \qquad Y_d \sim \begin{pmatrix} \lambda^4 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix},$$

$$Y_e \sim \begin{pmatrix} \lambda^5 & 0 & 0 \\ 0 & \lambda^2 & 1 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \qquad Y_\nu \sim \begin{pmatrix} \lambda^{10} & \lambda^9 & \lambda^9 \\ \lambda^9 & \lambda^8 & \lambda^8 \\ \lambda^9 & \lambda^8 & \lambda^8 \end{pmatrix}.$$

$$M^2_{\widetilde{Q}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^5 & \lambda^3 \\ \lambda^5 & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \; M^2_{\widetilde{\bar{u}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^8 \\ \lambda^9 & 1 & \lambda \\ \lambda^8 & \lambda & 1 \end{pmatrix},$$

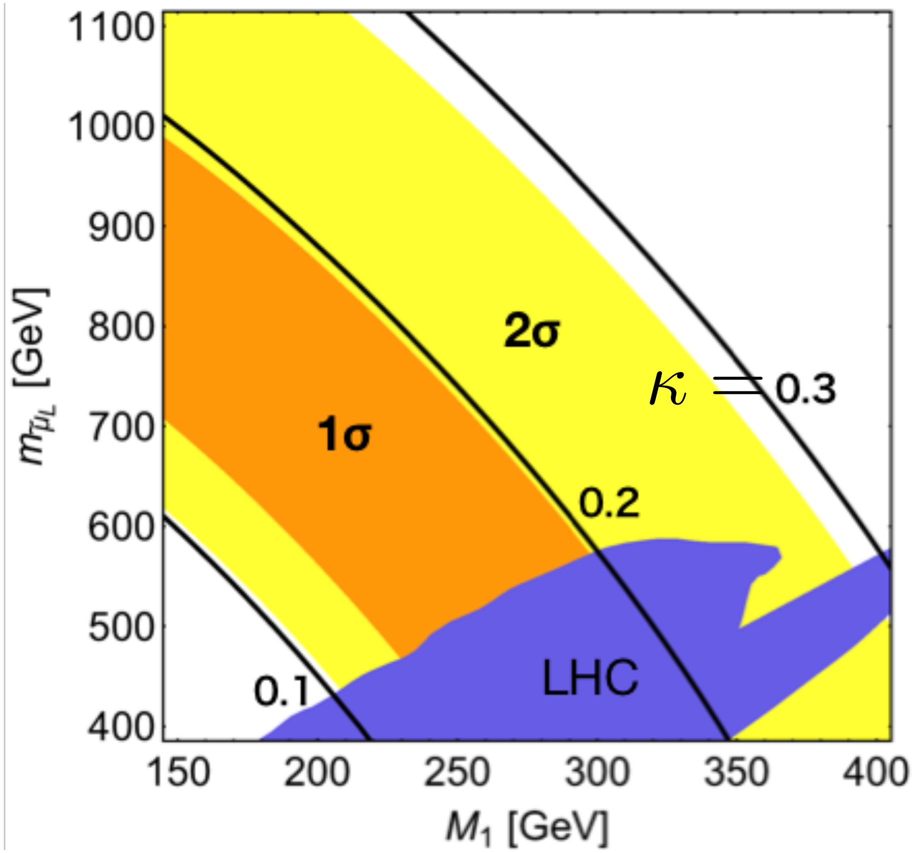
$$M^2_{\widetilde{\bar{d}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^{11} & \lambda^7 \\ \lambda^{11} & 1 & \lambda^4 \\ \lambda^7 & \lambda^4 & 1 \end{pmatrix},$$

$$M^2_{\widetilde{L}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^9 \\ \lambda^9 & 1 & 1 \\ \lambda^9 & 1 & 1 \end{pmatrix}, \; M^2_{\widetilde{\bar{e}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^{14} & \lambda^{12} \\ \lambda^{14} & 1 & \lambda^2 \\ \lambda^{12} & \lambda^2 & 1 \end{pmatrix}.$$

Result

- Decoupling of $\tilde{e}_i$, $\tan\beta = 50, \ \mu = M_2 = 2M_1$

S_3



- mu-phase

$$\mu = |\mu|(1 + i\kappa\,\lambda^6)$$

- current experimental constraint

$$|d_e| = 1.1 \times 10^{-29}\, e\,\mathrm{cm}$$

Most of parameter space is excluded without small κ

Another charge assignment

- Charge assignment

$$\begin{array}{cccccc} L_1 & L_2 & L_3 & \bar{e}_1 & \bar{e}_2 & \bar{e}_3 \\ (7,0) & (2,2) & (0,3) & (-6,2) & (-2,-1) & (0,-3) \end{array}$$

- Yukawa matrices

$$Y_e \sim \begin{pmatrix} \lambda^5 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad Y_\nu \sim \begin{pmatrix} \lambda^{14} & \lambda^{13} & \lambda^{13} \\ \lambda^{13} & \lambda^{12} & \lambda^{12} \\ \lambda^{13} & \lambda^{12} & \lambda^{12} \end{pmatrix}$$

- Soft mass squared mass matrices

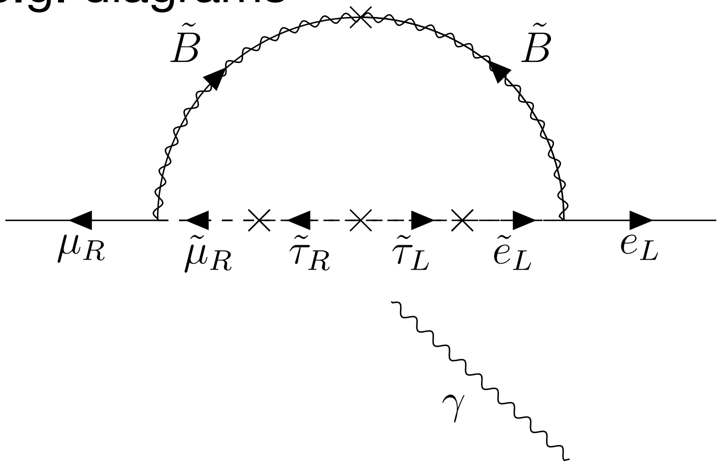
$$M^2_{\widetilde{L}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^{13} \\ \lambda^9 & 1 & \lambda^4 \\ \lambda^{13} & \lambda^4 & 1 \end{pmatrix}, \; M^2_{\widetilde{\bar{e}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^{10} & \lambda^{16} \\ \lambda^{10} & 1 & \lambda^6 \\ \lambda^{16} & \lambda^6 & 1 \end{pmatrix}$$

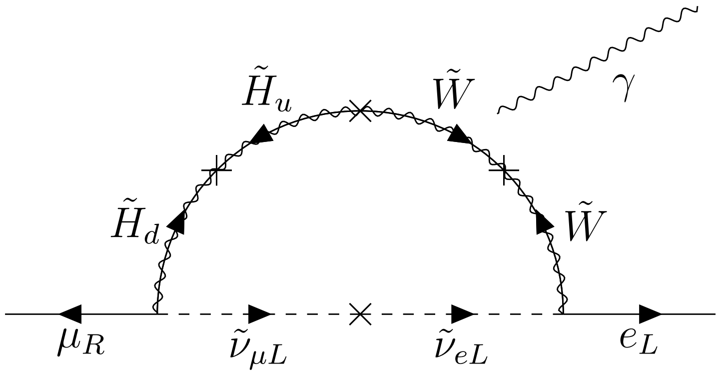
FCNC

FCNC constraints are stringent

- Flavor changing process
  - $\mu \to e + \gamma$

  e.g. diagrams





- Constraints on slepton masses

$$m^2_{\text{slepton}} \sim \tilde{m}^2 \begin{pmatrix} 1 & \delta & \delta \\ \delta & 1 & \delta \\ \delta & \delta & 1 \end{pmatrix} \quad \delta \sim 0.3 \quad \rightarrow \quad \tilde{m} \gg 1\,\text{TeV}$$

Two horizontal symmetries

$$U(1)_{H_1} \times U(1)_{H_2}$$

- Two flavons

$$S_1 \quad (-1, 0) \qquad S_2 \quad (0, -1)$$

- Flavon VEVs

$$\langle S_1 \rangle \sim \lambda, \langle S_2 \rangle \sim \lambda^2$$

- Fermion mass ratio

$$H(L_3) + H(\bar{e}_3) = 0, \quad H(L_2) + H(\bar{e}_2) = 2, H(L_1) + H(\bar{e}_1) = 5$$

$$H(L_1) - H(L_2) = H(L_1) - H(L_3) = 1, \quad H(L_2) - H(L_3) = 0$$

— Definition of H

$$H = H_1 + 2H_2$$