# Anapole Moment of Majorana Fermions and Implications for Direct Detection of Neutralino Dark Matter


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with A. Ibarra and R. Nagai
(work in progress)

Technical University of Munich (TUM)

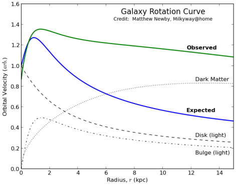

26.08.2021

SFB 1258 Neutrinos Dark Matter Messengers

# Outline

# Dark Matter: Observational Evidence

- Rotation curves of galaxies



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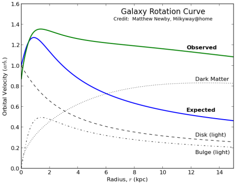


Credit: *NASA*

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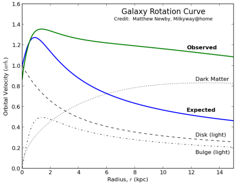


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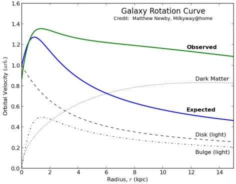


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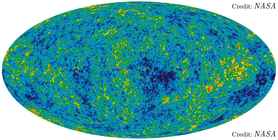

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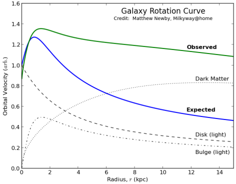


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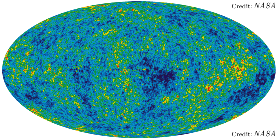

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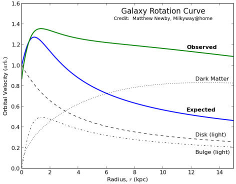
- What is it: Particle (WIMP, FIMP, axion,...?), MOND,...?

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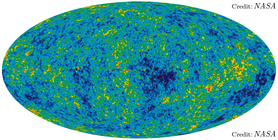

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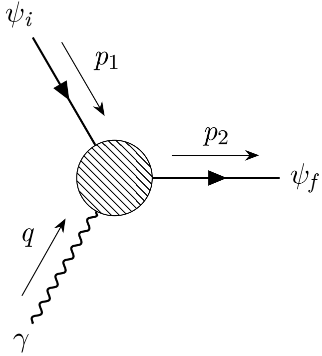
- Cross section depends on model: magnetic moment, anapole moment,...?

# Anapole Moment: Basics I

- Effective interaction vertex



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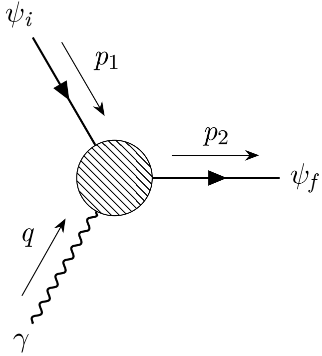


- $M_\mu^{fi}(q) =$
$(\gamma_\mu - q_\mu \not{q}/q^2)\left[f_Q^{fi}(q^2) + f_A^{fi}(q^2) q^2 \gamma_5\right] - i\sigma_{\mu\nu} q^\nu \left[f_M^{fi}(q^2) + i f_E^{fi}(q^2)\gamma_5\right]$

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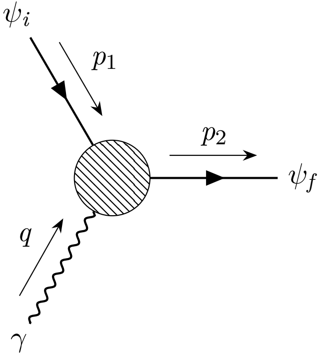


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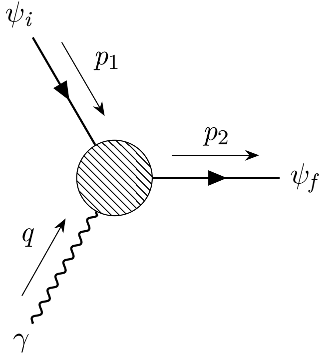


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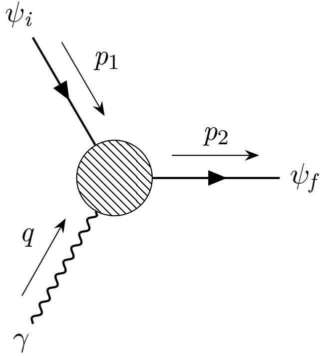


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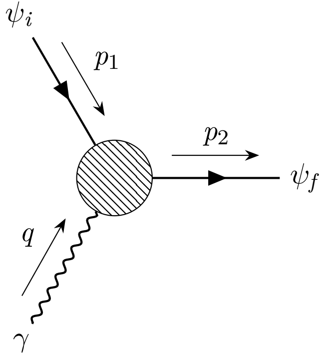


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- $\psi_i \neq \psi_f$: transition formfactors
- $\psi_i = \psi_f$ (diagonal) and $\psi$ Majorana: only the anapole is non-vanishing!

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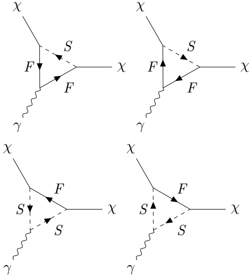

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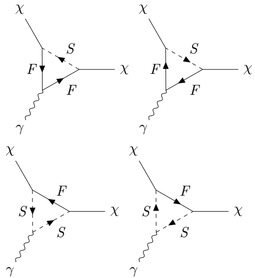

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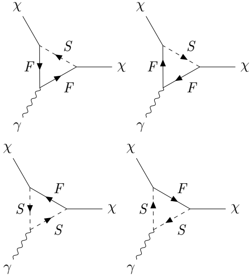

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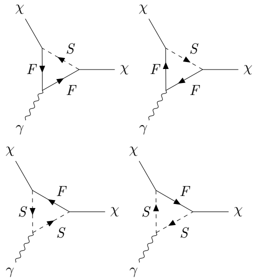

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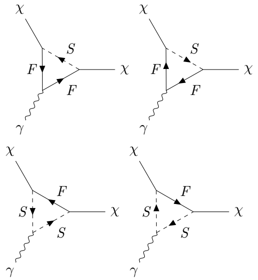

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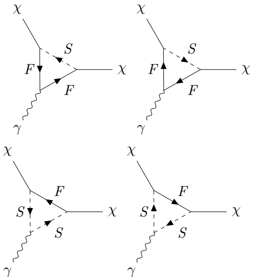

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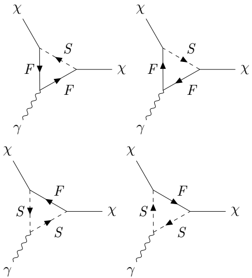
  - Majorana: currently unconstrained, future DD and collider
- DD is most sensitive, what is the reach?



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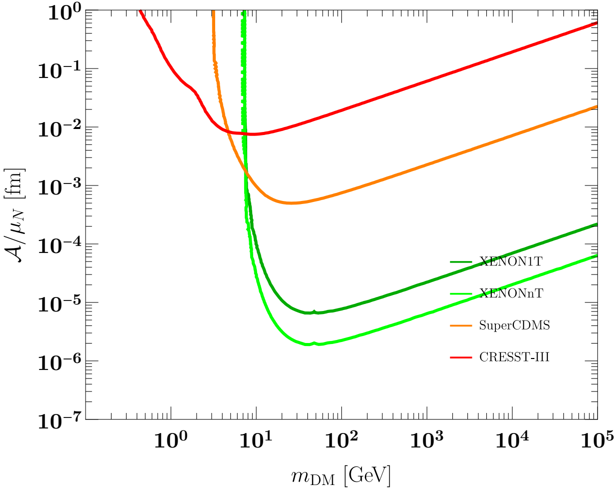

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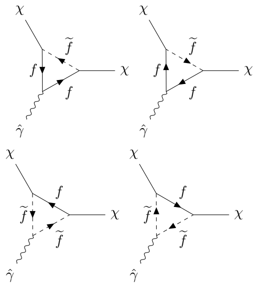

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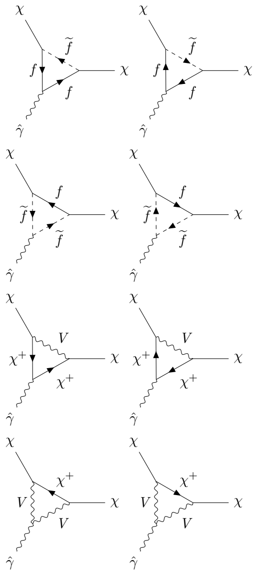

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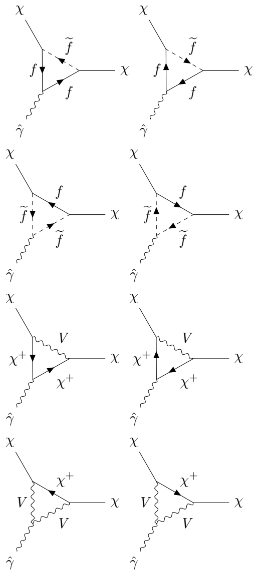

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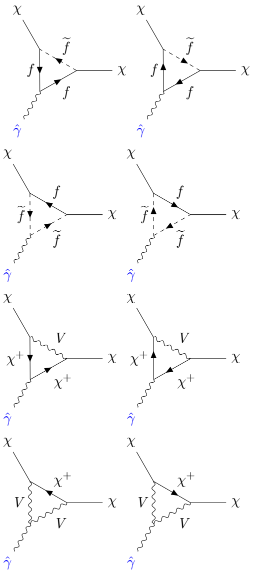
- Background Field Method: $\gamma \to \hat{\gamma}$

[Works by Cornwall, Papavassiliou, Bernabeu, Rosado, Vidal, Binosi... See review 0909.2536]



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- Contributions to anapole moment:

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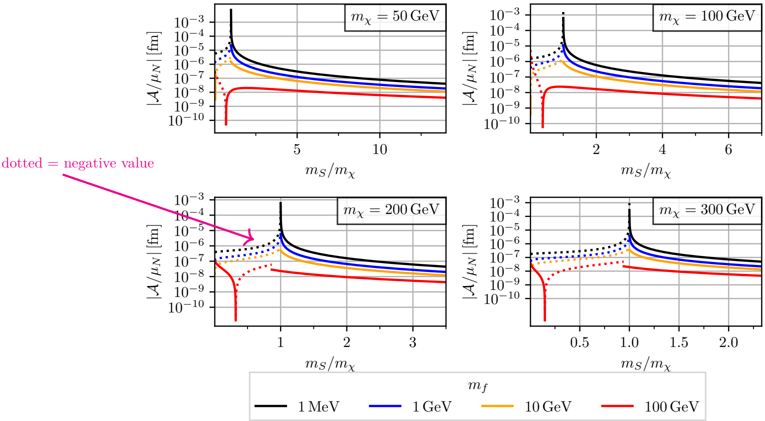
- with $\mu = m_f / m_\chi$, $\eta_{(V)} = m_{S(V)} / m_\chi$
- $\mathcal{F}_{S,V}$ boosted for $\mu \approx 1$ and $\eta \ll 1$ (or vice versa)

# Anapole Moment: Model-Independent Results (Scalar)




- $c_L = 1,\ c_R = 0,\ Q_f = -1$, colorless

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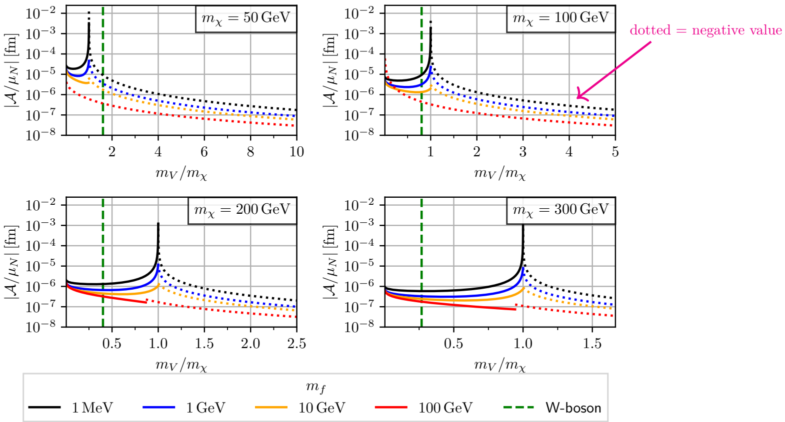


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- Two Higgs doublets with fermionic higgsinos as partners
- Mass eigenstates of neutral gauginos+higgsinos are neutralinos $\rightarrow$ lightest is DM candidate $\chi$ ($\tilde{\chi}_1^0$)

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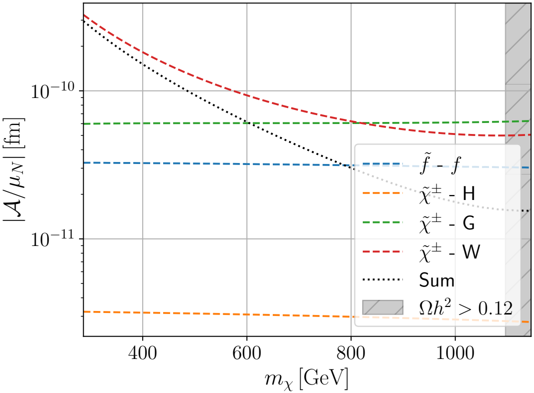
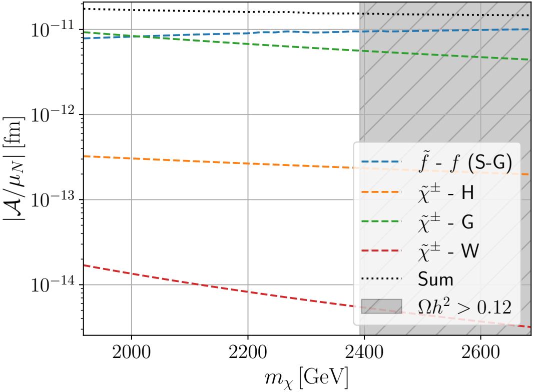
# MSSM: Anapole Moment of $\tilde{\chi}_1^0$ in SUGRA



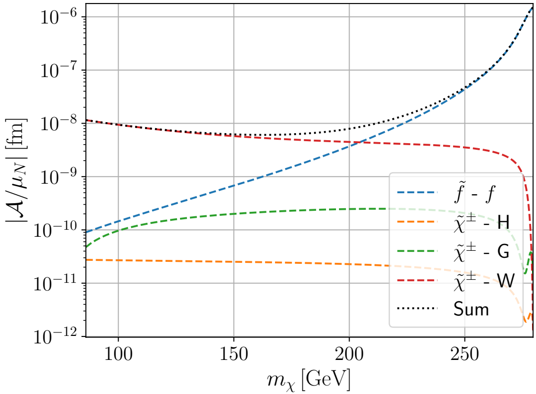

- $m_0 = 9\,\text{TeV}$ , $m_{1/2} \in [2550, 3000]\,\text{GeV}$, $A_0 = 3\,\text{TeV}$, $\tan\beta = 10$, $\text{sgn}\mu = +1$
- The higgsino-nature of $\tilde{\chi}_1^0$ enhances the chargino-W contribution

# MSSM: Anapole Moment of $\tilde{\chi}_1^0$ in AMSB




- $m_0 = 25\,\text{TeV},\ m_{3/2} \in [600, 850]\,\text{TeV}\,\tan\beta = 5,\ \text{sgn}\mu = +1$
- $\tilde{\chi}_1^0$ is wino like with degenerate chargino mixing angles $\Rightarrow v_L \approx v_R$

# MSSM: Anapole Moment of $\tilde{\chi}_1^0$ in pMSSM




- $\mu \in [100, 2500]$ GeV, $\mathcal{M}_L = 280$ GeV, $M_1 = 285$ GeV, $M_2 = 300$ GeV, $M_3 = \mathcal{M}_{\tilde{\tau}_{L/R}} = \mathcal{M}_{\tilde{Q}} = 3$ TeV, $A_t = 4$ TeV, $A_b = A_\tau = 0$, $m_A = 5$ TeV, $\tan\beta = 50$ defined @ 3 TeV
- For $m_\chi \lesssim 200$ GeV: $m_\chi \approx m_{\chi_1^+}$ and $v_L \neq v_R$, for $m_\chi \gtrsim 200$ GeV: $\chi$ is bino-like

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- In pMSSM scenarios it can be sizeable, via scalar and/or vector contribution

# Thank you for your attention

Questions?

SFB 1258 Neutrinos Dark Matter Messengers

# Extra: Analytical Anapole Expressions I

- Anapole functions:

$$\mathcal{A}_S = \frac{e}{96\pi^2 m_\chi^2} Q_f \left[ |c_L|^2 - |c_R|^2 \right] \mathcal{F}_S(\mu, \eta)$$

$$\mathcal{A}_V = -\frac{e}{48\pi^2 m_\chi^2} \left[ |v_L|^2 - |v_R|^2 \right] \mathcal{F}_V(\mu, \eta_V)$$

- with $\mu = m_f/m_\chi$, $\eta_{(V)} = m_{S(V)}/m_\chi$ and

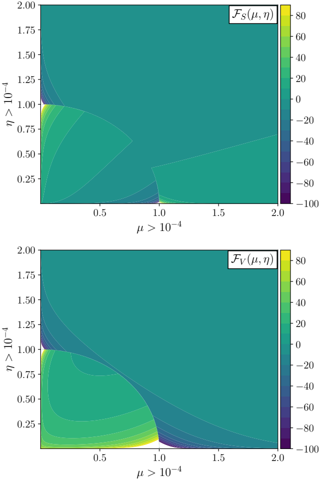
$$\mathcal{F}_S(\mu, \eta) = \frac{3}{2} \log\left(\frac{\eta^2}{\mu^2}\right) - (3\eta^2 - 3\mu^2 + 1) f(\mu, \eta),$$

$$\mathcal{F}_V(\mu, \eta_V) = \frac{3}{2} \log\left(\frac{\mu^2}{\eta_V^2}\right) + (3\eta_V^2 - 3\mu^2 - 7) f(\mu, \eta_V)$$

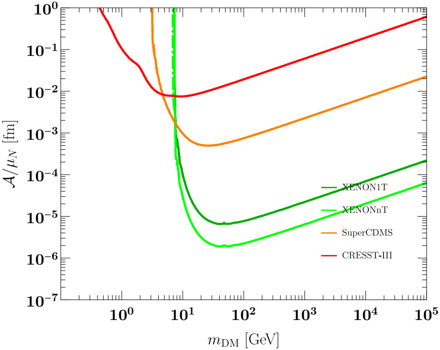
# Extra: Analytical Anapole Expressions II

- Scalar- and vector contribution governed by

$$f(\mu,\eta) = \frac{1}{2}\int_0^1 \frac{\mathrm{d}x}{x\eta^2 + (1-x)\mu^2 - x(1-x)}$$
$$= \begin{cases} \frac{1}{\sqrt{\Delta}}\operatorname{arctanh}\left(\frac{\sqrt{\Delta}}{\mu^2+\eta^2-1}\right) & \Delta > 0 \\ \frac{1}{\sqrt{|\Delta|}}\arctan\left(\frac{\sqrt{|\Delta|}}{\mu^2+\eta^2-1}\right) & \Delta < 0 \\ \frac{2}{(\mu^2-\eta^2)^2-1} & \Delta = 0 \end{cases}$$

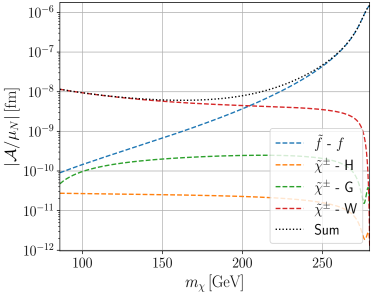


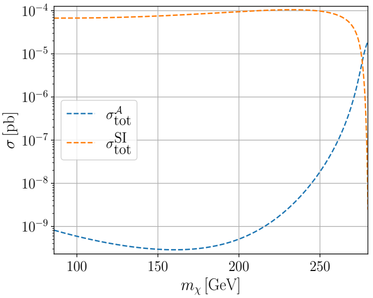
# Extra: Scattering Rate for the Anapole Interaction

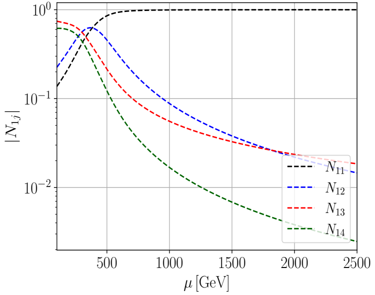


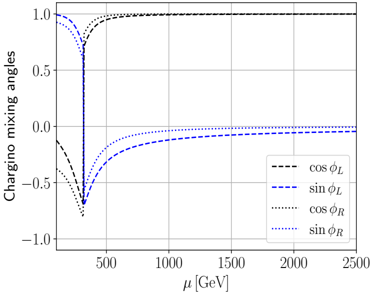

$$\frac{d\sigma}{dE_R} = \alpha_{\rm EM}\,\mathcal{A}^2\left[Z^2\left(2m_T - \left(1+\frac{m_T}{m_\chi}\right)^2\frac{E_R}{v^2}\right)F_Z^2(q^2) + \frac{1}{3}\frac{m_T}{m_\chi^2}\left(\frac{\bar{\mu}_T}{\mu_N}\right)^2\frac{E_R}{v^2}F_D^2(q^2)\right]$$

# Extra: Anapole Moment of $\tilde{\chi}_1^0$ in pMSSM: Details

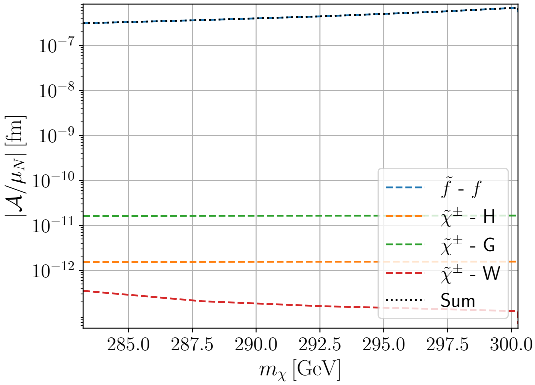


# Extra: Anapole Moment of $\tilde{\chi}_1^0$ in pMSSM-b




- $M_1 \in [290, 1000]$ GeV, $\mathcal{M}_L = 300$ GeV, $M_2 = 285$ GeV, $\mu = 5$ TeV, $M_3 = \mathcal{M}_{\tilde{\tau}_{L/R}} = \mathcal{M}_{\tilde{Q}} = 2$ TeV, $A_t = 5$ TeV, $A_b = A_\tau = 0$, $m_A = 5$ TeV, $\tan\beta = 50$ defined @ 3 TeV
- $\tilde{\chi}_1^0$ is wino-like, $\sin\phi_{L/R} \approx 0$

# Extra: Anapole Moment of $\tilde{\chi}_1^0$ in pMSSM-b: Details