Maximally Symmetric Three Higgs Doublet Model

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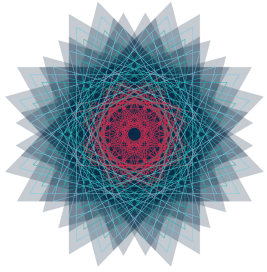


SUSY 2021,
23-28 August 2021

Based on:

- *M. R. Masouminia, A. Pilaftsis and ND, arXiv:2106.03159 [hep-ph].*
- *A. Pilaftsis and ND, PRD99 (2019) 115014; PRD101 (2020) 095008.*

Outline

- The Three Higgs Doublet Model (3HDM); the Type-V 3HDM
- Exact SM Alignment limit in 3HDM
- *Natural* SM Alignment limit by symmetries of 3HDM
- Maximally Symmetric 3HDM (MS-3HDM)
- Quartic couplings unification; predictions of MS-3HDM
- Summary and conclusions

3HDM

- 3HDM consists of three $\mathrm{SU}(2)_L$-preserving scalar doublets Φ_1, Φ_2 and Φ_3.
- Potential for 3HDM [Botella, Silva, '95]

$$V = \underbrace{-\sum_{i,j=1}^{3} m_{ij}^2 \left(\Phi_i^\dagger \Phi_j\right)}_{9 \text{ terms}} + \frac{1}{2} \underbrace{\sum_{i,j\,k,l=1}^{3} \lambda_{ijkl} \left(\Phi_i^\dagger \Phi_j\right)\left(\Phi_k^\dagger \Phi_l\right)}_{45 \text{ terms}}$$

  ◇ Symmetries → reducing the number of parameters

- Based on different vacuum structures [G. Cree, H. E. Logan, '11]

3HDM Yukawa Types	up-type quarks	down-type quarks	charged leptons
Type-I	Φ_1	Φ_1	Φ_1
Type-II	Φ_2	Φ_1	Φ_1
Type-III	Φ_2	Φ_1	Φ_2
Type-IV	Φ_1	Φ_1	Φ_2
Type-V	Φ_2	Φ_1	Φ_3

3HDM Type-V

- The Yukawa Lagrangian of the Type-V 3HDM

$$-\mathcal{L}_Y \;=\; y_d^{ij}\overline{Q}_L^i\Phi_1 d_R^j \;+\; y_u^{ij}\overline{Q}_L^i\widetilde{\Phi}_2 u_R^j \;+\; y_e^{ij}\overline{L}_L^i\Phi_3\, e_R^j \;+\; \text{H.c.}$$

- CP-conserving VEVs

$$\langle\Phi_1^0\rangle = v\cos\beta_1\cos\beta_2, \quad \langle\Phi_2^0\rangle = v\sin\beta_1\cos\beta_2, \quad \langle\Phi_3^0\rangle = v\sin\beta_2$$

- Two ratios of the doublet VEVs:

$$\tan\beta_1 = \frac{v_2}{v_1}, \quad \tan\beta_2 = \frac{v_3}{\sqrt{v_1^2+v_2^2}}, \quad v = \sqrt{v_1^2+v_2^2+v_3^2}$$

Physical States

- There are nine physical states:
  - Four charged scalars ($h^{\pm}_{1,2}$): $\begin{cases} \text{Masses} & \rightarrow & M_{h_1^{\pm}} & M_{h_2^{\pm}} \\ \text{Mixing angle} & \rightarrow & \sigma \end{cases}$
  - Two CP-odd scalars ($a_{1,2}$): $\begin{cases} \text{Masses} & \rightarrow & M_{a_1} & M_{a_2} \\ \text{Mixing angle} & \rightarrow & \rho \end{cases}$
  - Three CP-even scalars (H, $h_{1,2}$): $\begin{cases} \text{Masses} & \rightarrow & M_H & M_{h_1} & M_{h_2} \\ \text{Mixing angles} & \rightarrow & \alpha_1 & \alpha_2 & \alpha \end{cases}$
- The mass ordering: $M_H \leq M_{h_1} \leq M_{h_2}$

Couplings to Gauge Bosons

- Three different SM-like Higgs H, h_1, h_2 scenarios can be realised.
- The couplings to the EW gauge bosons ($V = Z, W^{\pm}$):

$$
\begin{aligned}
g_{HVV} &= \cos\alpha_2 \cos\beta_2 \cos(\beta_1 - \alpha_1) + \sin\alpha_2 \sin\beta_2 \\
g_{h_1VV} &= \cos\alpha \cos\beta_2 \sin(\beta_1 - \alpha_1) + \sin\alpha \Big[\cos\alpha_2 \sin\beta_2 \\
&\quad - \cos\beta_2 \sin\alpha_2 \cos(\beta_1 - \alpha_1) \Big] \\
g_{h_2VV} &= \cos\alpha \Big[\cos\alpha_2 \sin\beta_2 - \cos\beta_2 \sin\alpha_2 \cos(\beta_1 - \alpha_1) \Big] \\
&\quad - \sin\alpha \cos\beta_2 \sin(\beta_1 - \alpha_1)
\end{aligned}
$$

- The sum rule: $g^2_{HVV} + g^2_{h_1VV} + g^2_{h_2VV} = 1$.

SM Alignment limit in 3HDM

- The SM alignment limit can be realized in three different scenarios:
  - The *canonical* SM-like H scenario ($g_{HVV} = 1$):

    (i) $\beta_1 = \alpha_1, \quad \beta_2 = \alpha_2$

    (ii) $\beta_2 = \alpha_2 = \pm\pi/2$

  - The SM-like h_1 scenario ($g_{h_1VV} = 1$):

    (i) $\beta_1 = \alpha_1, \quad \beta_2 = \alpha_2 \pm \pi/2, \quad \alpha = \pm\pi/2$

    (ii) $\beta_1 = \alpha_1 + \pi/2, \quad \beta_2 = \alpha, \quad \alpha_2 = 0$

    (iii) $\beta_1 = \alpha_1 + \pi/2, \quad \beta_2 = \alpha = 0$

    (iv) $\beta_2 = \alpha = \pm\pi/2, \quad \alpha_2 = 0$

  - The SM-like h_2 scenario ($g_{h_2VV} = 1$):

    (i) $\beta_1 = \alpha_1, \quad \beta_2 = \alpha_2 + \pi/2, \quad \alpha = 0$

    (ii) $\beta_1 = \alpha_1 + \pi/2, \quad \beta_2 = \alpha + \pi/2, \quad \alpha_2 = 0$

    (iii) $\beta_1 = \alpha_1 \pm \pi/2, \quad \beta_2 = 0, \quad \alpha = \mp\pi/2$

    (iv) $\beta_2 = \pi/2, \quad \alpha_2 = \alpha = 0$

SM Alignment limit in 3HDM

- The SM alignment limit can be realized in three different scenarios:

  - The *canonical* SM-like H scenario ($g_{HVV} = 1$):

    ✓ **(i) $\boldsymbol{\beta_1 = \alpha_1, \quad \beta_2 = \alpha_2}$**

    (ii) $\beta_2 = \alpha_2 = \pm\pi/2$

  - The SM-like h_1 scenario ($g_{h_1VV} = 1$):

    (i) $\beta_1 = \alpha_1, \quad \beta_2 = \alpha_2 \pm \pi/2, \quad \alpha = \pm\pi/2$

    (ii) $\beta_1 = \alpha_1 + \pi/2, \quad \beta_2 = \alpha, \quad \alpha_2 = 0$

    (iii) $\beta_1 = \alpha_1 + \pi/2, \quad \beta_2 = \alpha = 0$

    (iv) $\beta_2 = \alpha = \pm\pi/2, \quad \alpha_2 = 0$

  - The SM-like h_2 scenario ($g_{h_2VV} = 1$):

    (i) $\beta_1 = \alpha_1, \quad \beta_2 = \alpha_2 + \pi/2, \quad \alpha = 0$

    (ii) $\beta_1 = \alpha_1 + \pi/2, \quad \beta_2 = \alpha + \pi/2, \quad \alpha_2 = 0$

    (iii) $\beta_1 = \alpha_1 \pm \pi/2, \quad \beta_2 = 0, \quad \alpha = \mp\pi/2$

    (iv) $\beta_2 = \pi/2, \quad \alpha_2 = \alpha = 0$

SM Alignment limit in 3HDM

The CP-even mass matrix in the Higgs basis $\rightarrow \widehat{\mathcal{M}}_S^2 = \begin{pmatrix} \widehat{A} & \widehat{C}_1 & \widehat{C}_2 \\ \widehat{C}_1 & \widehat{B}_1 & \widehat{C}_3 \\ \widehat{C}_2 & \widehat{C}_3 & \widehat{B}_2 \end{pmatrix}$

- **Exact** *canonical* SM alignment limit ($\beta_1 = \alpha_1,\ \beta_2 = \alpha_2$):
  1. Decoupling of heavy masses $\widehat{B}_{1,2} \gg v$.
  2. $\widehat{C}_{1,2} \rightarrow 0$ that requires the following fine-tuning among the quartic couplings (independent of the values of $\tan\beta_{1,2}$):

$$\begin{aligned} &\lambda_{11} = \lambda_{22} = \lambda_{33}, \\ &\lambda_{1122} = \lambda_{1133} = \lambda_{2233}, \\ &\lambda_{1221} = \lambda_{1331} = \lambda_{2332}, \\ &\lambda_{1212} = \lambda_{1313} = \lambda_{2323} = 2\lambda_{11} - \lambda_{1122} - \lambda_{1221}. \end{aligned}$$

✓ • Natural SM alignment limit by the aid of symmetries of 3HDM.

Symmetries of 3HDM

• The $\mathrm{SU}(2)_L$ scalar doublets of 3HDM $\rightarrow$ the 12-dimentional multiplet-$\mathbf{\Phi}$, realizing $\mathrm{GL}(12, \mathbb{C})$. [Battye, Brawn, Pilaftsis '11; Pilaftsis, ND '19]

$$\mathbf{\Phi}^{\mathsf{T}} = \left(\Phi_1,\, \Phi_2,\, \Phi_3,\, \widetilde{\Phi}_1,\, \widetilde{\Phi}_2,\, \widetilde{\Phi}_3\right)^{\mathsf{T}}$$

• The $\mathrm{SU}(2)_L$ gauge-kinetic terms of $\mathbf{\Phi}$ should remain canonical :

$$\mathrm{GL}(12,\mathbb{C}) \;\rightarrow\; \underbrace{\mathbf{Sp(6)} \otimes \mathbf{SU(2)_L}}_{\textit{Maximal Symmetry}}$$

• There is a *complete* list of 40 accidental symmetries in 3HDM based on the Maximal Symmetry Sp(6). [Pilaftsis, ND '19; Pilaftsis, ND '20]

The list of accidental symmetries for 3HDM potential

No.	Symmetry	Non-zero parameters for 3HDM potentials
1	CP1	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), Re(m_{13}^2), Re(m_{23}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, Re(λ_{1212}), Re(λ_{1313}), Re(λ_{2323}), Re(λ_{1213}), Re(λ_{2113}), Re(λ_{1223}), Re(λ_{2123}), Re(λ_{1323}), Re(λ_{1332}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312}), Re(λ_{1113}), Re(λ_{2213}), Re(λ_{3313}), Re(λ_{1123}), Re(λ_{2223}), Re(λ_{3323})
2	Z_2	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{13}^2$, λ_{1212}, λ_{1313}, λ_{2323}, λ_{1232}, λ_{1113}, λ_{2213}, λ_{3313} and H.c.$\}$
2′	Z_2'	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{23}^2$, λ_{1212}, λ_{1313}, λ_{2323}, λ_{1213}, λ_{1123}, λ_{2223}, λ_{3323} and H.c.$\}$
3	$Z_2 \otimes Z_2'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$, λ_{1313}, λ_{2323} and H.c.$\}$
4	Z_3	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1213}$, λ_{1323}, λ_{2123} and H.c.$\}$
5	Z_4	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$, λ_{1323} and H.c.$\}$
5′	Z_4'	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1313}$, λ_{3212} and H.c.$\}$
6	${}^a\mathrm{U}(1)$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1323}$ and H.c.$\}$
6′	${}^b\mathrm{U}(1)'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{12}^2$, λ_{1212}, λ_{1112}, λ_{2212}, λ_{3312}, λ_{1332} and H.c.$\}$
7	$\mathrm{U}(1) \otimes \mathrm{U}(1)'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}

The list of accidental symmetries for 3HDM potential

No.	Symmetry	Non-zero parameter space of 3HDM
8	$Z_2 \otimes \mathrm{U}(1)'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$ and H.c.$\}$
9	CP1$\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, Re(λ_{1212}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312})
10	CP1$\otimes Z_2 \otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, Re(λ_{1212})
11	U(1) $\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}
12	CP2	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$, Re($\lambda_{1313}$)=Re($\lambda_{2323}$), Re($\lambda_{1212}$), $\{\lambda_{1112} = -\lambda_{2212}$ and H.c.$\}$
13	CP2$\otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, Re($\lambda_{1212}$), $\{\lambda_{1112} = -\lambda_{2212}$ and H.c.$\}$
14	SO(2)$_{\phi_1,\phi_2}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$, Re($\lambda_{1313}$) = Re($\lambda_{2323}$), Re($\lambda_{1212}$) = $2\lambda_{11} - (\lambda_{1122} + \lambda_{1221})$,
15	D_3	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, $\lambda_{1331} = \lambda_{2332}$, $\{\lambda_{2131} = -\lambda_{1232}$, λ_{1323} and H.c.$\}$
16	D_4	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, $\{\lambda_{1212}$ and H.c.$\}$, $\lambda_{1331} = \lambda_{2332} = $ Re(λ_{3231})
17	$D_3 \otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}
18	$D_4 \otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, Re(λ_{1212})
19	SO(2)$_{\phi_1,\phi_2} \otimes$ Sp(2)$_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = $ Re(λ_{1212}) $= \lambda_{11} - \frac{1}{2}\lambda_{1122}$
20	SU(2)$_{\phi_1,\phi_2}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, $\lambda_{1122} = 2\lambda_{11} - \lambda_{1221}$, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$

The list of accidental symmetries for 3HDM potential

No.	Symmetry	Non-zero parameter space of 3HDM
21	$\mathrm{SU}(2)_{\phi_1,\phi_2} \otimes \mathrm{Sp}(2)_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, $\lambda_{1122} = 2\lambda_{11} - \lambda_{1221}$, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}
22	$\mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, $\mathrm{Re}(m_{12}^2)$, $\mathrm{Re}(m_{13}^2)$, $\mathrm{Re}(m_{23}^2)$, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}= $\mathrm{Re}(\lambda_{1212})$, λ_{1331}=$\mathrm{Re}(\lambda_{1313})$, λ_{2332}=$\mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1213}) = \mathrm{Re}(\lambda_{2113})$, $\mathrm{Re}(\lambda_{1223}) = \mathrm{Re}(\lambda_{2123})$, $\mathrm{Re}(\lambda_{1323}) = \mathrm{Re}(\lambda_{1332})$, $\mathrm{Re}(\lambda_{1112}), \mathrm{Re}(\lambda_{2212}), \mathrm{Re}(\lambda_{3312})$, $\mathrm{Re}(\lambda_{1113}), \mathrm{Re}(\lambda_{2213}), \mathrm{Re}(\lambda_{3313}), \mathrm{Re}(\lambda_{1123}), \mathrm{Re}(\lambda_{2223}), \mathrm{Re}(\lambda_{3323})$
23	$Z_2 \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, $\mathrm{Re}(m_{13}^2)$, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}= $\mathrm{Re}(\lambda_{1212})$, λ_{1331}=$\mathrm{Re}(\lambda_{1313})$, λ_{2332}=$\mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1113}), \mathrm{Re}(\lambda_{2213}), \mathrm{Re}(\lambda_{3313})$
23′	$Z_2' \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, $\mathrm{Re}(m_{23}^2)$, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}= $\mathrm{Re}(\lambda_{1212})$, λ_{1331}=$\mathrm{Re}(\lambda_{1313})$, λ_{2332}=$\mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1123}), \mathrm{Re}(\lambda_{2223}), \mathrm{Re}(\lambda_{3323})$
24	$Z_2 \otimes Z_2' \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}= $\mathrm{Re}(\lambda_{1212})$, λ_{1331}=$\mathrm{Re}(\lambda_{1313})$, λ_{2332}=$\mathrm{Re}(\lambda_{2323})$
25	$Z_4 \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}= $\mathrm{Re}(\lambda_{1212})$
26	$(\mathrm{CP1} \rtimes S_2) \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\mathrm{Re}(m_{12}^2)$, $\mathrm{Re}(m_{13}^2)$=$\mathrm{Re}(m_{23}^2)$, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}= $\mathrm{Re}(\lambda_{1212})$, λ_{1331}=λ_{2332}=$\mathrm{Re}(\lambda_{1313})$=$\mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1323}) = \mathrm{Re}(\lambda_{1332})$, $\mathrm{Re}(\lambda_{1223}) = \mathrm{Re}(\lambda_{2123})$=$\mathrm{Re}(\lambda_{1213}) = \mathrm{Re}(\lambda_{2113})$, $\mathrm{Re}(\lambda_{3313}) = \mathrm{Re}(\lambda_{3323})$, $\mathrm{Re}(\lambda_{1112}) = \mathrm{Re}(\lambda_{2212})$, $\mathrm{Re}(\lambda_{3312})$, $\mathrm{Re}(\lambda_{1113}) = \mathrm{Re}(\lambda_{1123}) = \mathrm{Re}(\lambda_{2213}) = \mathrm{Re}(\lambda_{2223})$

The list of accidental symmetries for 3HDM potential

No.	Symmetry	Non-zero parameter space of 3HDM
27	$D_4 \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22} = \frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221} =Re(λ_{1212})
28	$\mathrm{Sp}(2)_{\phi_1+\phi_2} \otimes \mathrm{Sp}(2)_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}= Re(λ_{1212}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312})
29	$\mathrm{Sp}(2)_{\phi_1\phi_2}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22} = \frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, $\lambda_{1331} = \lambda_{2332}$
30	$\mathrm{Sp}(2)_{\phi_1\phi_2} \otimes \mathrm{Sp}(2)_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22} = \frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}
31	A_4	$\mu_1^2 = \mu_2^2 = \mu_3^2$, $\lambda_{11} = \lambda_{22} = \lambda_{33}$, $\lambda_{1122} = \lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = \lambda_{1331} = \lambda_{2332}$, $\{\lambda_{1212} = \lambda_{1313} = \lambda_{2323}$, and H.c.$\}$
32	S_4	$\mu_1^2 = \mu_2^2 = \mu_3^2$, $\lambda_{11} = \lambda_{22} = \lambda_{33}$, $\lambda_{1122} = \lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = \lambda_{1331} = \lambda_{2332}$, Re($\lambda_{1212}$) = Re($\lambda_{1313}$) = Re($\lambda_{2323}$)
✓ 33	SO(3)	$\mu_1^2 = \mu_2^2 = \mu_3^2$, $\lambda_{11} = \lambda_{22} = \lambda_{33}$, $\lambda_{1122} = \lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = \lambda_{1331} = \lambda_{2332}$, Re($\lambda_{1212}$) = Re($\lambda_{1313}$) = Re($\lambda_{2323}$) = $2\lambda_{11} - (\lambda_{1122} + \lambda_{1221})$
34	$S_4 \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2 = \mu_2^2 = \mu_3^2$, $\lambda_{11} = \lambda_{22} = \lambda_{33} = \frac{1}{2}\lambda_{1122} = \lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = \lambda_{1331} = \lambda_{2332}$=Re($\lambda_{1212}$) = Re($\lambda_{1313}$) = Re($\lambda_{2323}$)
35	$\Delta(54)$	$\mu_1^2 = \mu_2^2 = \mu_3^2$, $\lambda_{11} = \lambda_{22} = \lambda_{33}$, $\lambda_{1122} = \lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = \lambda_{1331} = \lambda_{2332} = \lambda_{11} - 2\lambda_{1122}$, $\{\lambda_{1213} = \lambda_{2123} = \lambda_{3231}$ and H.c.$\}$
36	$\Sigma(36)$	$\mu_1^2 = \mu_2^2 = \mu_3^2$, $\lambda_{11} = \lambda_{22} = \lambda_{33}$, $\lambda_{1122} = \lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = \lambda_{1331} = \lambda_{2332}$, Re($\lambda_{1213}$) = Re($\lambda_{1323}$) = Re($\lambda_{1232}$)= $\frac{3}{4}(2\lambda_{11} - \lambda_{1122} - \lambda_{1221})$

The list of accidental symmetries for 3HDM potential

No.	Symmetry	Non-zero parameter space of 3HDM
37	$\mathrm{Sp}(2)_{\phi_1}\otimes\mathrm{Sp}(2)_{\phi_2}\otimes\mathrm{Sp}(2)_{\phi_3}$	$\mu_1^2,\ \mu_2^2,\ \mu_3^2,\ \lambda_{11},\ \lambda_{22},\ \lambda_{33},\ \lambda_{1122},\ \lambda_{1133},\ \lambda_{2233}$
38	$\mathrm{Sp}(4)\otimes\ \mathrm{Sp}(2)$	$\mu_1^2=\mu_2^2,\ \mu_3^2,\ \lambda_{11}=\lambda_{22}=\frac{1}{2}\lambda_{1122},\ \lambda_{33},\ \lambda_{1133}=\lambda_{2233}$
✓ 39	$\mathrm{SU}(3)$	$\mu_1^2=\mu_2^2=\mu_3^2,\ \lambda_{11}=\lambda_{22}=\lambda_{33},\ \lambda_{1122}=\lambda_{1133}=\lambda_{2233},$ $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}=2\lambda_{11}-\lambda_{1122}$
✓ 40	$\mathrm{Sp}(6)$	$\mu_1^2=\mu_2^2=\mu_3^2,\ \lambda_{11}=\lambda_{22}=\lambda_{33}=\frac{1}{2}\lambda_{1122}=\frac{1}{2}\lambda_{1133}=\frac{1}{2}\lambda_{2233}$

[Ivanov, Vdeovin, '12; Keus et al, '13; Ivanov, Varzielas, '19;$\cdots$]$\rightarrow$ Discrete symmetries in 3HDM

[Pilaftsis, '16]$\rightarrow$ Continuous symmetries in nHDM

[ND, Pilaftsis, '19]$\rightarrow$ The complete list of accidental symmetries in 3HDM

Symmetries for Natural SM Alignment Limit in 3HDM

- Symmetries for the **Natural** SM alignment in 3HDM (independent of the values of $\tan\beta_{1,2}$): [Pilaftsis '16; ND,Pilaftsis '20]

  (i) Sp(6) (ii) SU(3) (iii) SO(3)

- **The most economical version is the Maximally Symmetric 3HDM**
  - The 3HDM $\mathrm{Sp}(6)\otimes\mathrm{SU}(2)_L$-invariant potential:

$$V_{\text{MS-3HDM}} = -\mu_1^2\left(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2\right) + \lambda_{11}\left(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2\right)^2$$

  - The mixing angles that diagonalise the heavy sectors of the CP-even, CP-odd and charged- scalar mass matrices are all equal, i.e.

$$\boldsymbol{\alpha = \rho = \sigma}$$

Symmetry Breaking

- The Maximal Symmetry breaks by RG effects and soft Sp(6)-breaking bilinear mass terms m_{ij}^2:

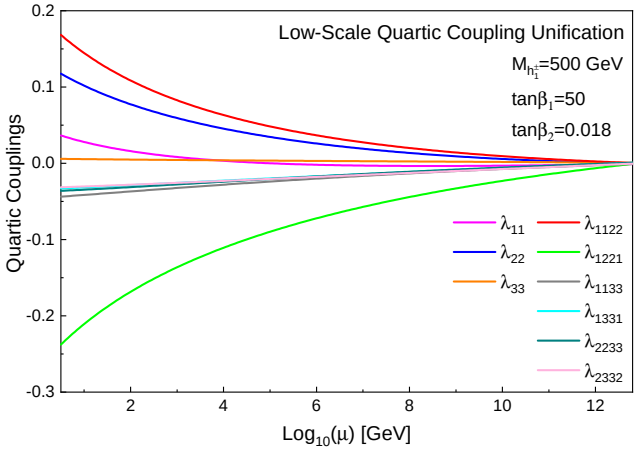
$$\begin{aligned}
\mathrm{Sp}(6) \otimes \mathrm{SU}(2)_L &\xrightarrow{y_u, y_d, y_e \neq 0} \mathrm{Sp}(2) \otimes \mathrm{Sp}(2) \otimes \mathrm{Sp}(2) \otimes \mathrm{SU}(2)_L \\
&\xrightarrow{g' \neq 0} \quad \mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)'_{\mathrm{PQ}} \otimes \mathrm{U}(1)_{\mathrm{Y}} \otimes \mathrm{SU}(2)_L \\
&\xrightarrow{\langle \Phi_{1,2,3} \rangle \neq 0} \mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)'_{\mathrm{PQ}} \otimes \mathrm{U}(1)_{\mathrm{em}} \\
&\xrightarrow{m_{ij}^2 \neq 0} \quad \mathrm{U}(1)_{\mathrm{em}}
\end{aligned}$$

- The non-zero soft terms m_{12}^2, m_{13}^2 and m_{23}^2 lift up the masses of heavy states:

$$M_H^2 \simeq 2\lambda_{22} v^2 \qquad M_{h_1}^2 \simeq M_{a_1}^2 \simeq M_{h_1^\pm}^2 \uparrow \qquad M_{h_2}^2 \simeq M_{a_2}^2 \simeq M_{h_2^\pm}^2 \uparrow$$

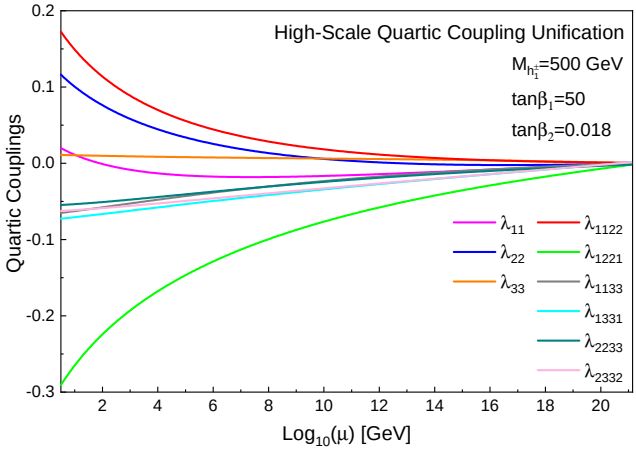
The Unification of Quartic Couplings

- The Sp(6) symmetry is realized at high scale 10^{13} GeV.
- At the threshold scale SM is realized.

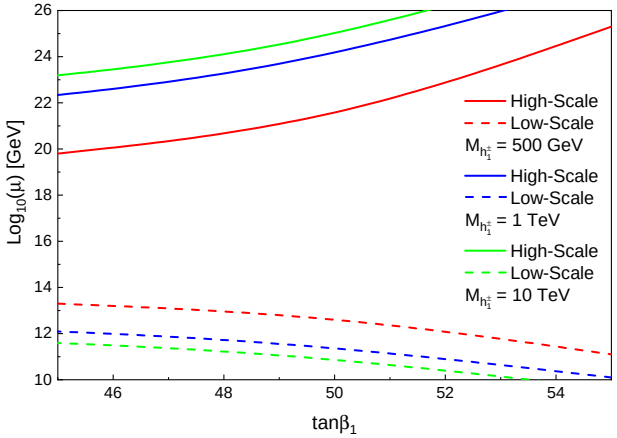


The Unification of Quartic Couplings

- At the scale $\mu_X \sim 10^{13}$ GeV, λ_{22} turns to negative but leaves the EW vacuum metastable. [ND,Pilaftsis '19]
- At the scale $\mu_X \sim 10^{21}$ GeV, the Sp(6) symmetry can be realized too.



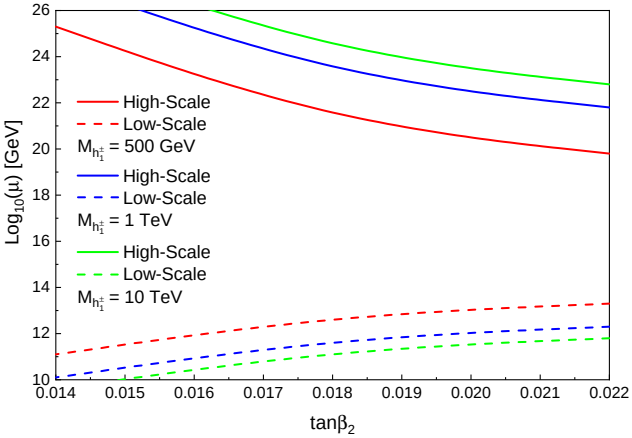
The Unification of Quartic Couplings



This unified framework (based on Type-V 3HDM) for different $\tan\beta_1$ and different $M_{h_1^\pm}$.

- The scales of LS become lower for higher $\tan\beta_1$ and vice versa for HS unifications.
- Similar trend happens for higher $M_{h_1^\pm}$.

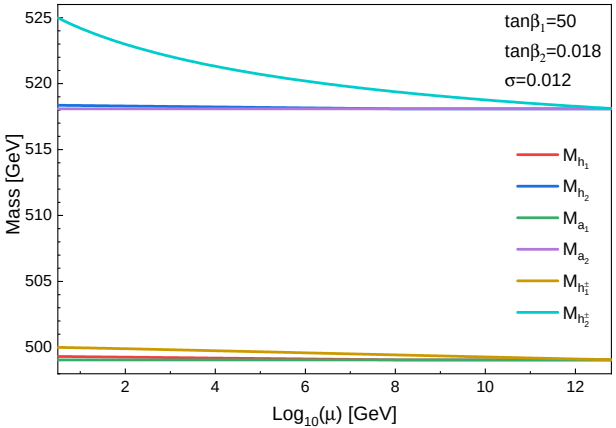
The Unification of Quartic Couplings



This unified framework (based on Type-V 3HDM) for lower values of $\tan\beta_1$ requires higher values of $\tan\beta_2 \rightarrow$ restricts $\tan\beta_{1,2}$.

- $40 \lesssim \tan\beta_1 \lesssim 55$.
- $0.013 \lesssim \tan\beta_2 \lesssim 0.022$.
- Type-V 3HDM is the most restrictive version.
- The bound on $\tan\beta_1$ for other types is broadened to just be higher than > 1.

Mass Spectrum



Only **five** input parameters are required to define the full mass spectrum of MS-3HDM:

- The charged Higgs masses $M_{h^{\pm}_{1,2}}$, the mixing angle between $M_{h^{\pm}_1}$ and $M_{h^{\pm}_2}$, i.e. σ and $\tan\beta_{1,2}$.

Misalignment

The deviation from the SM alignment limit:

$$\Theta_1 = \frac{\widehat{C}_1}{\widehat{A} - \widehat{B}_1} \qquad \Theta_2 = \frac{\widehat{C}_2}{\widehat{A} - \widehat{B}_2} \qquad \Theta_{12} = \frac{\widehat{C}_1}{\widehat{A} - \widehat{B}_2}$$

Gauge bosons couplings

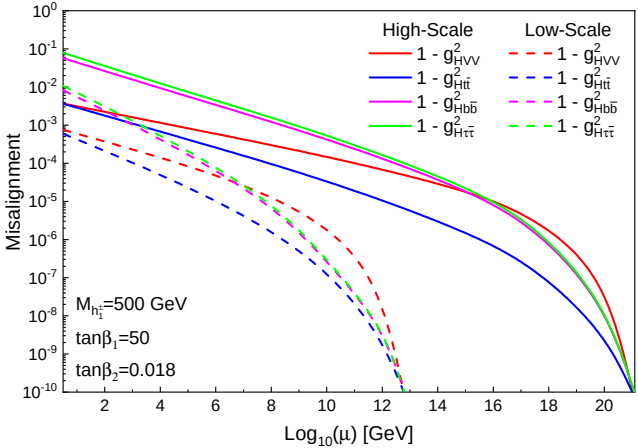
$$\begin{aligned} g_{HVV} &\simeq 1 - \frac{\Theta_1^2}{4}, \\ g_{h_1VV} &\simeq c_\alpha c_{\beta_2}\,\Theta_1 + \frac{s_\alpha}{4}\,\Theta_2\,(1+\Theta_1), \\ g_{h_2VV} &\simeq -s_\alpha c_{\beta_2}\,\Theta_1 + \frac{c_\alpha}{4}\Theta_2\,(1+\Theta_1). \end{aligned}$$

Fermions couplings

$$\begin{aligned} g_{Hd\bar{d}} &\simeq 1 - (t_{\beta_1} + t_{\beta_2})\,\Theta_1 - (1 - t_{\beta_1} t_{\beta_2})\,\Theta_2^2, \\ g_{Hu\bar{u}} &\simeq 1 - t_{\beta_1}^{-1}\,\Theta_1 + \frac{t_{\beta_2}}{2}(1 - t_{\beta_1}^{-1}\,\Theta_1)\,\Theta_2 \\ &\quad - \frac{1}{8}(4\Theta_1^2 + \Theta_2^2), \\ g_{Hl\bar{l}} &\simeq 1 - t_{\beta_2}^{-1}\frac{2\,\Theta_2}{4 + \Theta_1\,\Theta_{12}} - \frac{2\,\Theta_2^2}{(4 + \Theta_1\,\Theta_{12})^2}. \end{aligned}$$

Misalignment

- The higher-scale unification has a larger deviation from the SM.
- Both high and low scale unification scenarios are within experimental limits.
- g_{HVV} and g_{Htt} approach their SM values ~ 1.
- Misalignment for g_{Hbb} and $g_{H\tau\tau}$ are only within 2σ CL.



Summary and Conclusions

- I have discussed 3HDM and identified all limits for exact SM alignment.
- In the 3HDM the SM alignment limit can be achieved naturally by the aid of symmetries.
- 3HDM has 40 distinct accidental symmetries.
- The most economic version is the MS-3HDM.
- The MS-3HDM is a minimal extension of the SM governed by only **five** additional parameters: the charged Higgs masses $M_{h^{\pm}_{1,2}}$, their mixing angle and $\tan\beta_{1,2}$.
- The quartic couplings can be unified at some high scale near Planck scale.
- The misalignment predictions are consistent with experimental constraints.

$\star\star\star$

◇ *A. Pilaftsis and ND, PRD99 (2019) 115014; PRD101 (2020) 095008.*

◇ *M. R. Masouminia, A. Pilaftsis and ND, arXiv:2106.03159 [hep-ph].*

Thank You!