Warming up Cold Inflation

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based on arXiv:2107.07517

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Warming Up Cold Inflation

- Punchline: Generic inflaton couplings can turn cold axion inflation models warm!

- Dissipative effects will source a thermal bath (at some $T_{\rm eq}$)

- This equilibrium will be reached regardless of initial conditions

- The bath can alter the predictions and dynamics of inflation for large parts of parameter space

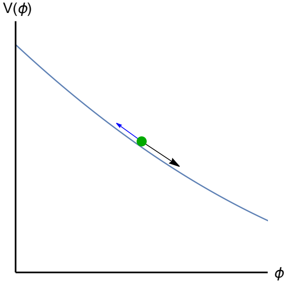
Warm Inflation Dynamics

- Scalar equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

- Friedmann equation:

$$H^2 - \frac{1}{3M_{\rm pl}}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right) = 0$$



Warm Inflation Dynamics

- Scalar equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon(T)\dot{\phi} + V'(\phi) = 0$$

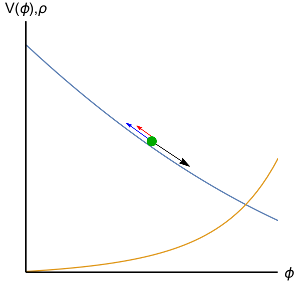
- Friedmann equation:

$$H^2 - \frac{1}{3M_{\rm pl}}\left(\rho_R + \frac{\dot{\phi}^2}{2} + V(\phi)\right) = 0$$

- Dissipation $\Upsilon(T)$ sources bath:

$$\dot{\rho}_R + 4H\rho_R - \Upsilon(T)\dot{\phi}^2 = 0,$$

where $\rho_R = \frac{\pi^2 g_*}{30} T^4$



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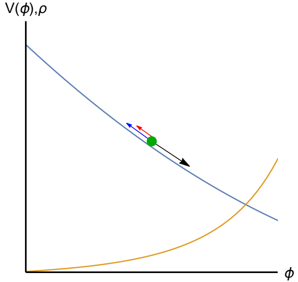

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- Can solve for slow-roll equilibrium

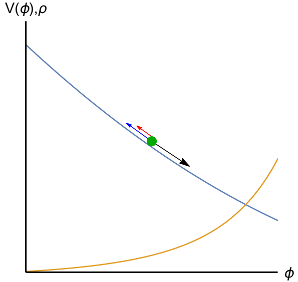
Slow Roll Approximation

- Slow roll parameters:

$$\epsilon_W = \frac{1}{1+Q} \cdot \frac{1}{2} \left(\frac{V'}{V} \right)^2 M_{\rm pl}^2 \ll 1$$

$$\eta_W = \frac{1}{1+Q} \cdot \left(\frac{V''}{V} \right) M_{\rm pl} \ll 1,$$

where $Q = \Upsilon / 3H$



- Inflation ends at $\epsilon_W = 1$

Axion in a Thermal Bath

- Axion coupled to $SU(N_c)$ gauge sector:

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2g^2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} - \frac{\phi}{16\pi^2 f}\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu} - V(\phi)$$

- No thermal correction to V because of ϕ shift symmetry

- Thermal friction

$$\Upsilon(T) \sim \frac{T^3}{f^2}$$

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$$-\bar{\psi}(\not{D} - m_f)\psi$$

- No thermal correction to V because of ϕ shift symmetry

- Thermal friction

$$\Upsilon(T) \sim \frac{m_f^2 T}{f^2}$$

- Light fermions ($m_f \ll T$) lead to suppression of friction

Warm Inflation is an Attractor Solution

- If inflation begins in a thermal bath, it will quickly reach T_{eq}
  - If $T_0 > T_{\text{eq}}$, it will redshift in $\ln(T_0/T_{\text{eq}})$ e-folds
  - If $H < T_0 < T_{\text{eq}}$, thermal friction will heat bath within Hubble time [1910.07525 Berghaus, Graham, Kaplan]

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  - If $H < T_0 < T_{\text{eq}}$, thermal friction will heat bath within Hubble time [1910.07525 Berghaus, Graham, Kaplan]
- If inflation begins in vacuum, can still source thermal bath!
  - Rolling inflaton causes exponential production of gauge bosons [0908.4089 Anber & Sorbo]
  - Can generate $\rho_R \gg H^4$ before self-interactions take over
  - If self-interactions thermalize, then $T_0 > H$ so above case applies!

Beginning from Vacuum

- Three conditions for potential thermalization:
  - Nonlinearities take over within a Hubble time — upper bound on f
  - Nonlinearities take over before inflaton slows — lower bound on f
  - Enough energy density for a bath with $T > H$ — upper bound on f

Inflationary Regimes

- Quantities of interest:
  - $Q = \Upsilon/3H$: dynamics dominated by Hubble or bath
  - T vs. H: predictions dominated by Hubble or bath
  - If H or $T > f$, then EFT breaks down!

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- Inflationary regimes:
  - Strong warm inflation: $Q > 1$ and $T > H$
  - Weak warm inflation: $Q < 1$ and $T > H$
  - Cold inflation: $T < H$
  - Broken: $H > f$
  - Thermally Broken: $T > f > H$

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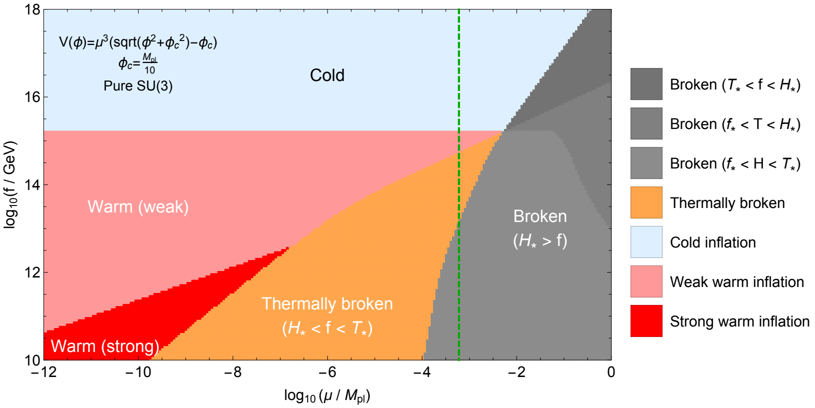
  - Broken: $H > f$
  - Thermally Broken: $T > f > H$
- All quantities evaluated at $N_* = 60$ e-folds before inflation ends

Parameter Space – Pure $SU(3)$ Case

$$V(\phi) = \mu^3(\sqrt{\phi^2 + \phi_c^2} - \phi_c), \quad \phi_c = M_{\rm pl}/10$$

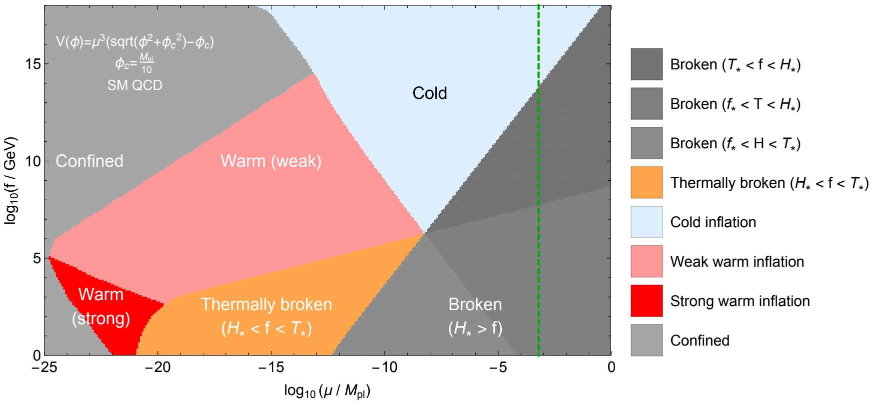


$$N_c = 3, \alpha = 0.1, g_* = 17, N_* = 60$$

Green line indicates normalization which matches observed CMB amplitude in cold inflation

Parameter Space – Standard Model Case

$$V(\phi) = \mu^3\left(\sqrt{\phi^2 + \phi_c^2} - \phi_c\right), \quad \phi_c = M_{\rm pl}/10$$



$N_c = 3$, T-dependent α and g_*, $m_f = 1$ MeV, $N_* = 60$

Green line indicates normalization which matches observed CMB amplitude in cold inflation
Light gray region indicates that QCD has confined ($T \lesssim 0.1$ GeV)

Summary

- Dissipative effects generate thermal bath during inflation

- Bath is an attractor solution, which is insensitive to initial conditions

- Thermal effects can ruin cold inflation even for small couplings!

- Public Service Anouncement: Thermal effects must be accounted for when building inflation models

Thank You!

Slow Roll Equilibrium

$$\dot{\phi} \approx -\frac{V'(\phi)}{3H(1+Q)}$$

$$H^2 \approx \frac{V(\phi)}{3M_{\rm pl}^2}$$

$$\rho_R \approx \frac{3Q\dot{\phi}^2}{4}$$

Beginning from Low Temperature

- Can show that bath is generated within a Hubble time, using only:

$$\dot{\rho}_R + 4H\rho_R - \Upsilon\dot{\phi}^2 = 0$$

$$\rho_R \sim T^4, \quad \Upsilon \sim T^3$$

- If T begins small, we can neglect $4H\rho_R$

$$\dot{\rho}_R \sim \Upsilon\dot{\phi}^2 \implies \dot{T} \sim \dot{\phi}^2 > \dot{\phi}^2_{\mathrm{eq}}$$

- Therefore the time to equilibrium t_{eq} is at most

$$T_{\mathrm{eq}} \gtrsim \dot{\phi}^2_{\mathrm{eq}} t_{\mathrm{eq}}$$

- At equilibrium, we can neglect $\dot{\rho}_R$

$$HT^4_{\mathrm{eq}} \sim T^3_{\mathrm{eq}}\dot{\phi}^2_{\mathrm{eq}} \implies t_{\mathrm{eq}} \lesssim H^{-1}$$

- If T begins large, bath will dilute to equilibrium in $\ln(T_0/T_{\mathrm{eq}})$ e-folds
- Caveat: assumes thermalization rate $\Delta \sim T$ greater than H

Beginning from Vacuum (detailed)

- Three conditions for potential thermalization:
  - Do nonlinearities become relevant within a Hubble time?

$$\frac{f}{M_{\rm pl}} \ll \mathcal{O}(10^{-1})\alpha\sqrt{\epsilon_V}$$

  - Do nonlinearities become relevant before inflaton slows down?

$$\frac{f}{M_{\rm pl}} \gg \mathcal{O}(10^{-2})\sqrt[4]{\alpha^3\epsilon_V}\sqrt{\frac{H}{M_{\rm pl}}}$$

  - Is there enough energy density to source a bath with $T > H$?

$$\frac{f}{M_{\rm pl}} \ll \mathcal{O}(1)N_c^2\alpha^{11/4}g_*^{-1/4}\epsilon_V^{1/2}$$