Primordial Black Holes in the Excursion Set Theory

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SUSY 2021

27 August 2021

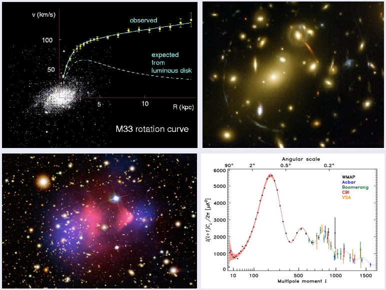
Based on

MNRAS 505 (2021) 1787 [arXiv:2101.07812]

Outline

Dark Matter

Evidences



Properties

- stable
- neutral
- weakly interacting
- right relic density

Candidates

- Axions
- Sterile neutrinos
- WIMPs
- Primordial Black Holes (PBHs)

Primordial Black Holes

Definition

A PBH is a type of black hole that is **not** formed by the gravitational collapse of a star, but by the extreme density of matter present during the Universe's early expansion.

PBHs properties

Mass: $M_{\rm BH} = 10^{15}\left(\frac{t}{10^{-23}\,{\rm s}}\right)\,{\rm g}$ $\qquad M_\odot \simeq 2\times 10^{33}\,{\rm g}$

$$
\begin{aligned}
\text{Planck scale} &\longrightarrow 10^{-5}\,{\rm g}\\
\text{GUT scale} &\longrightarrow 10^{3}\,{\rm g}\\
\text{EW scale} &\longrightarrow 10^{28}\,{\rm g}\\
\text{QCD scale} &\longrightarrow 10^{32}\,{\rm g}
\end{aligned}
$$

Hawking radiation

Temperature: $T_{\rm BH} \approx 10^{-7} \left(\frac{M}{M_\odot}\right)^{-1}$ K

$M > 10^{17}$ g	massless particles
10^{15} g $\lesssim M \lesssim 10^{17}$ g	electrons
10^{14} g $\lesssim M \lesssim 10^{15}$ g	muons
$M < 10^{14}$ g	hadrons

Lifetime: $\tau_{\rm BH} \approx 10^{64} \left(\frac{M}{M_\odot}\right)^{3}$ y

$M_{\rm BH}$	$\tau_{\rm BH}$
A man	10^{-12} s
10^{15} g	10^{10} y
Earth	10^{49} y
Sun	10^{66} y
Milky Way	10^{99} y

Why PBHs are useful?

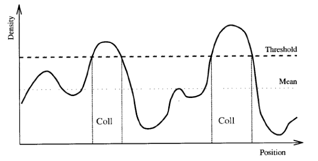
- PBHs as a probe of the early Universe $(M < 10^{15}\,\mathrm{g})$
- PBHs as a probe of gravitational collapse $(M > 10^{15}\,\mathrm{g})$ ✓
  DM candidates $\Omega^0_{\mathrm{PBH}} \lesssim \Omega^0_{\mathrm{CDM}} (= 0.23)$ Source of gravitational waves
- PBHs as a probe of High Energy Physics $(M \sim 10^{15}\,\mathrm{g})$
- PBHs as a probe of quantum gravity $(M \sim 10^{-5}\,\mathrm{g})$
  (DM candidates)

How PBHs form?

- Soft equation of state
- Bubble collisions
- Collapse of cosmic loops
- Fluctuations by inflation

Press-Schechter Formalism

The Press-Schechter formalism is a model for predicting the number density of bound objects of a certain mass.



$$\beta(\geq M) = \gamma \int_{\delta_c}^{\infty} P(\delta;\, M(R))$$

$\delta_c = 0.47$

Gaussian PDF: $P_G(\delta;\, R) = \dfrac{1}{\sqrt{2\pi}\sigma_\delta(R)} \exp\left(-\dfrac{\delta^2(R)}{2\sigma_\delta^2(R)}\right)$

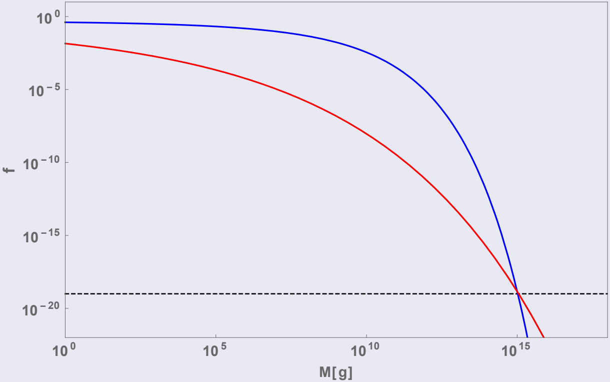
$$\delta^2(k,t) \equiv \mathcal{P}_\delta(k,t) = \frac{4(1+w)^2}{(5+3w)^2}\left(\frac{k}{aH}\right)^4 \mathcal{P}_{\mathcal{R}_c}(k) \qquad w = 1/3$$

$$\mathcal{P}_{\mathcal{R}_c}(k) = \mathcal{P}_{\mathcal{R}_c}(k_0)\left(\frac{k}{k_0}\right)^{n(k)-1}$$

$$\sigma_\delta^2(R) = \int_0^\infty W^2(kR)\mathcal{P}_\delta(k)\frac{\mathrm{d}k}{k} \qquad W(kR) = \exp\left(-k^2R^2/2\right)$$

$$M_{\rm PBH}(k) \simeq 30\, M_\odot \left(\frac{\gamma}{0.2}\right)\left(\frac{g_{*,i}}{10.75}\right)^{-1/6}\left(\frac{k_{\rm PBH}}{2.9\times 10^5\,{\rm Mpc}^{-1}}\right)^{-2}$$

$f(\geq M)$ diagram for the mass range $10^0 - 10^{20}\,\mathrm{g}$



Result

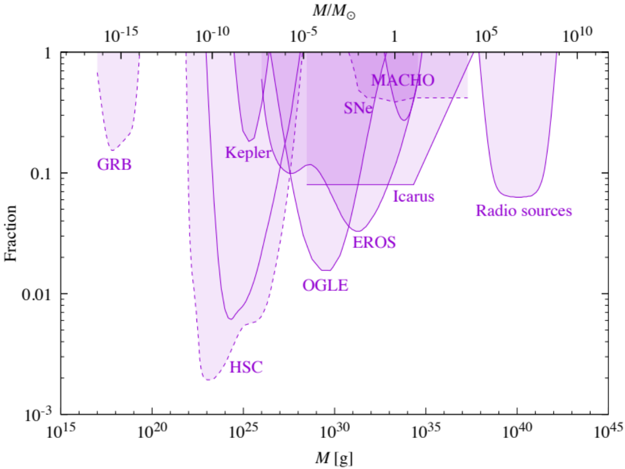
$\beta = \frac{1}{2}\,\mathrm{erfc}\left(\delta_{\mathrm{c}}/\sqrt{2\sigma_\delta^2(R)}\right)$

$n_s(k_{\mathrm{PBH}}) \geq 1.418 \quad \Rightarrow \quad \mathcal{P}_\zeta \simeq 2\times 10^{-2}$ for Gaussian PDF

$$\beta \simeq 3.7\times 10^{-9}\,\gamma^{-1/2}\,\left(\frac{g_{*,i}}{10.75}\right)^{1/4}\left(\frac{M_{\mathrm{PBH}}}{M_\odot}\right)^{1/2} f_{\mathrm{PBH}}, \quad f_{\mathrm{PBH}} \equiv \frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{DM}}}$$

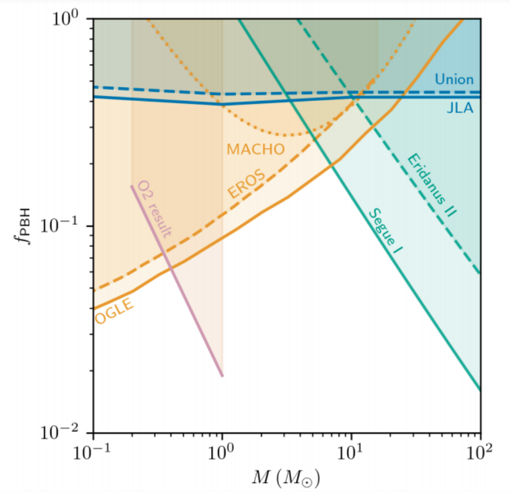
Gravitational Lensing

PRD 100 (2019) 063521



- $10^{-11}\,M_\odot \lesssim M_{\rm PBH} \lesssim 10^{-6}\,M_\odot$ with $f_{\rm PBH} \sim 10^{-3}$,
- $10^{-6}\,M_\odot \lesssim M_{\rm PBH} \lesssim 10^{-3}\,M_\odot$ with $f_{\rm PBH} \sim 10^{-2}$,
- $10^{-3}\,M_\odot \lesssim M_{\rm PBH} \lesssim 10^{-1}\,M_\odot$ with $f_{\rm PBH} < 0.04$.

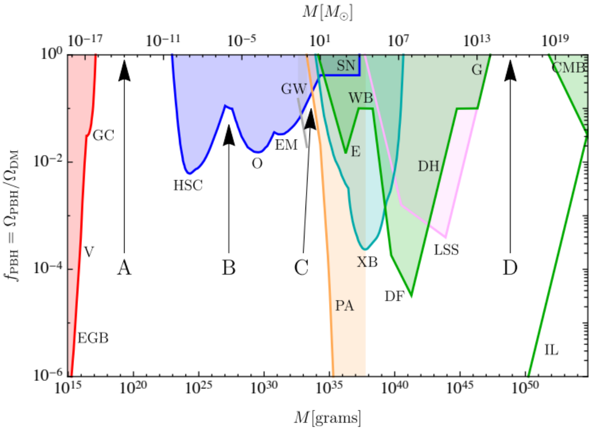
Gravitational Waves



- $0.2\,M_\odot \lesssim M_{\rm PBH} \lesssim 1.0\,M_\odot$, corresponding to at most 0.16 or 0.02 of the DM, respectively.

No constraints

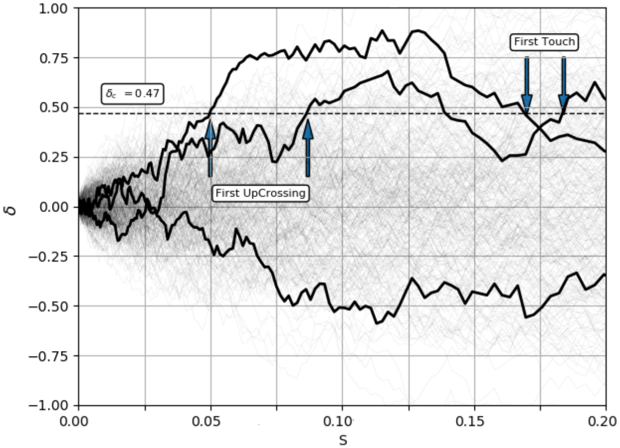
arXiv:2002.12778



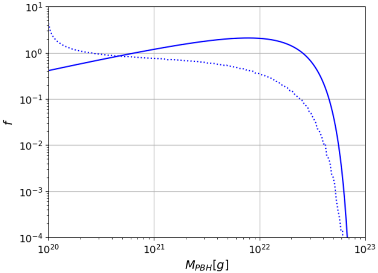
- asteroid mass range $10^{16}\,\mathrm{g} \lesssim M_{\rm PBH} \lesssim 10^{17}\,\mathrm{g}$,
- sublunar mass range $10^{20}\,\mathrm{g} \lesssim M_{\rm PBH} \lesssim 10^{24}\,\mathrm{g}$,
- intermediate mass range $10\,M_\odot \lesssim M_{\rm PBH} \lesssim 10^{3}\,M_\odot$,
- stupendously large black holes (SLABs) $M_{\rm PBH} \geq 10^{11}\,M_\odot$.

Excursion Set Theory

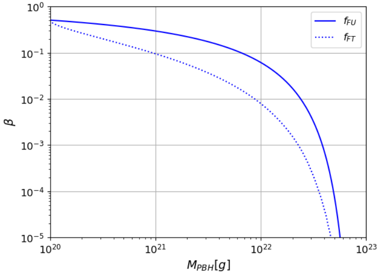
arXiv:2101.07812



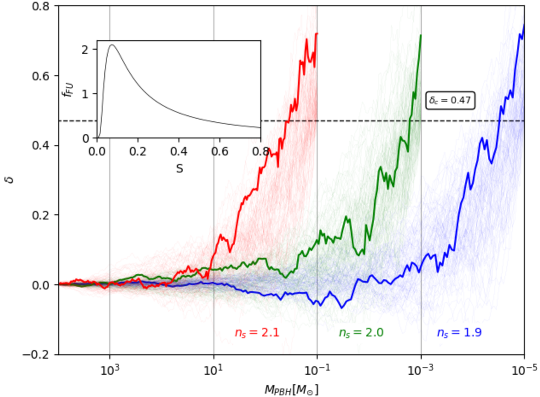
$$f_{\mathrm{FU}}(S,\,\delta_c) = \frac{1}{\sqrt{2\pi}}\,\frac{\delta_c}{S^{3/2}}\exp\left(-\frac{\delta_c^2}{2S}\right)$$



f
$M_{PBH}[g]$



β
$M_{PBH}[g]$
f_{FU}
f_{FT}



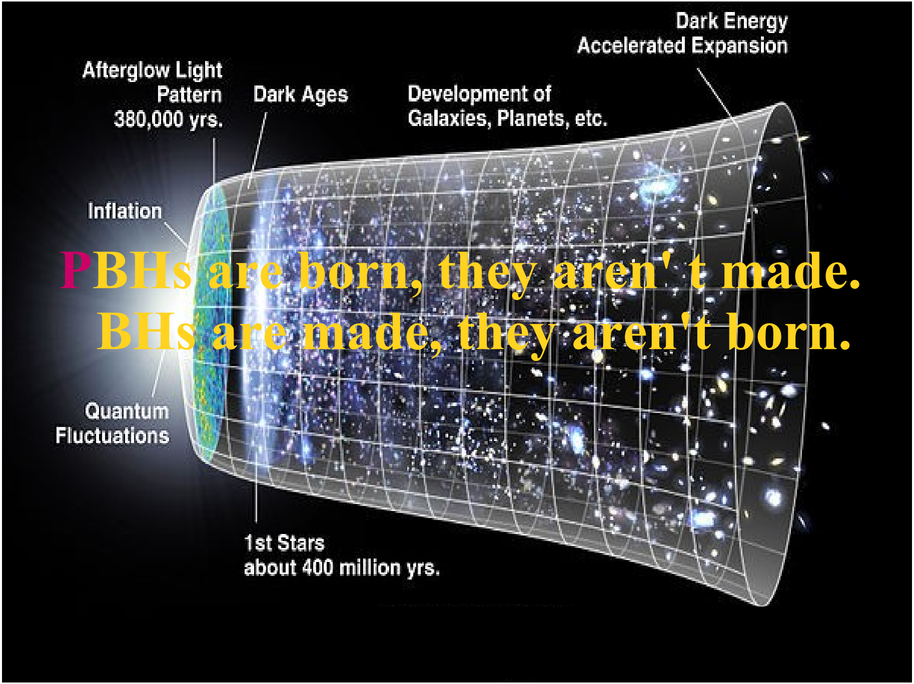
δ
$M_{PBH}[M_\odot]$
f_{FU}
S
$\delta_c = 0.47$
$n_s = 2.1$
$n_s = 2.0$
$n_s = 1.9$

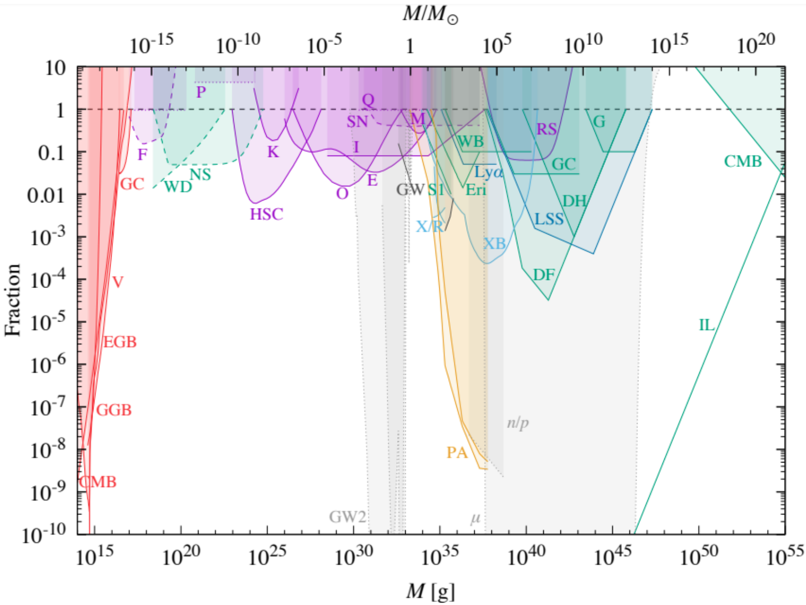
Results

	mass range	$f_{\rm PBH}$	lower bound of mass range	spectral index EST	spectral index PS
asteroid mass range	$(10^{16} - 10^{17})$ g	1	10^{16} g	1.490	1.616
sublunar mass range	$(10^{20} - 10^{24})$ g	1	10^{20} g	1.540	1.703
			10^{22} g	1.600	1.756
Subaru HSC	$(10^{-11} - 10^{-6})\ M_\odot$	10^{-3}	$10^{-10}\ M_\odot$	1.604	1.560
			$10^{-8}\ M_\odot$	1.666	1.605
OGLE	$(10^{-6} - 10^{-3})\ M_\odot$	10^{-2}	$10^{-6}\ M_\odot$	1.729	1.757
			$10^{-4}\ M_\odot$	1.845	1.835
EROS/MACHO	$(10^{-3} - 10^{-1})\ M_\odot$	0.04	$10^{-3}\ M_\odot$	1.862	1.947
			$10^{-2}\ M_\odot$	1.942	1.970
sub-Solar mass range	$(0.2 - 1.0)\ M_\odot$	0.02	$0.2\ M_\odot$	2.018	2.046
			$0.6\ M_\odot$	2.115	2.078
intermediate mass range	$(10^{1} - 10^{3})\ M_\odot$	1	$10\ M_\odot$	2.130	2.408
			$10^{2}\ M_\odot$	2.235	2.514
SLABs	$\geq 10^{11}\ M_\odot$	1	$10^{11}\ M_\odot$	5.220	5.598
			$10^{12}\ M_\odot$	6.660	6.891

Conclusions

- The fluctuation which arise at inflation are the most likely source of PBHs.
- The power spectrum at scale of PBHs formation should be much higher than the one in the observed scales.
- We can form PBHs in the EST with any mass.
- PBH cannot be the whole fraction of dark matter.


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
PBHs are born, they aren' t made.
BHs are made, they aren't born.
Quantum
Fluctuations
1st Stars
about 400 million yrs.


M/M⊙
10-15
10-10
10-5
1
105
1010
1015
1020
Fraction
10
1
0.1
0.01
10-3
10-4
10-5
10-6
10-7
10-8
10-9
10-10
1015
1020
1025
1030
1035
1040
1045
1050
1055
M [g]
P
Q
SN
M
F
GC
NS
WD
K
HSC
I
O
E
GW
S1
WB
Lyα
Eri
RS
G
GC
CMB
DH
LSS
X/R
XB
DF
V
EGB
GGB
CMB
IL
n/p
PA
GW2
μ