Renormalization Group analysis of the superradiant growth of the self-interacting axion cloud

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※In this talk, "Axion" is QCD axion or string axion given by following action

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\partial_\mu \phi)^2 - \mu^2 F_a^2 \left(1 - \cos\frac{\phi}{F_a} \right) \right]$$

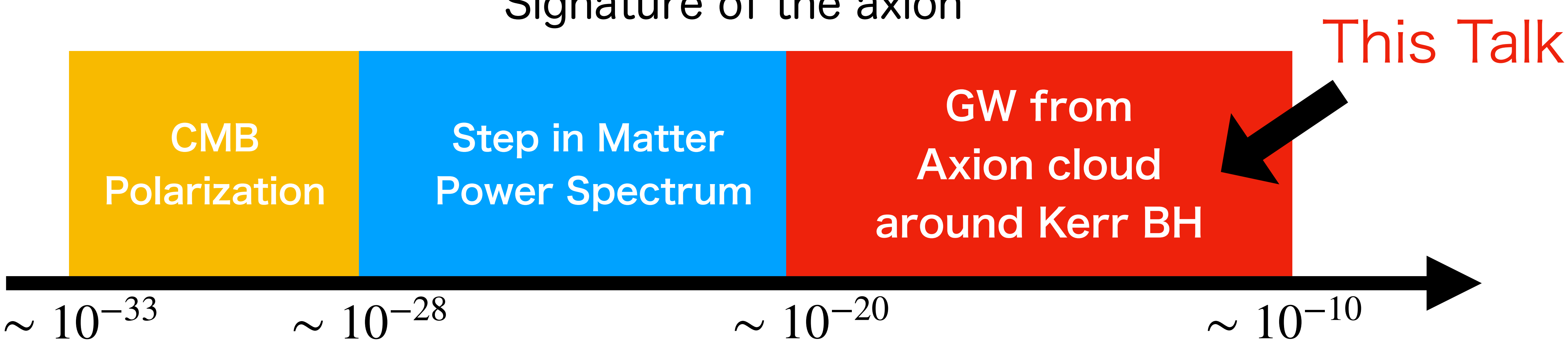
g:Kerr metric μ:Axion mass

ϕ:Axion F_a:Axion decay constant

Introduction

- String Theory→Predicts axion-like particle of mass $10^{-33} \sim 10^{-10}\mathrm{eV}$
- Axion is candidates of Dark Matter
- Observing axion via cosmological/astrophysical phenomena would be happy!

Signature of the axion

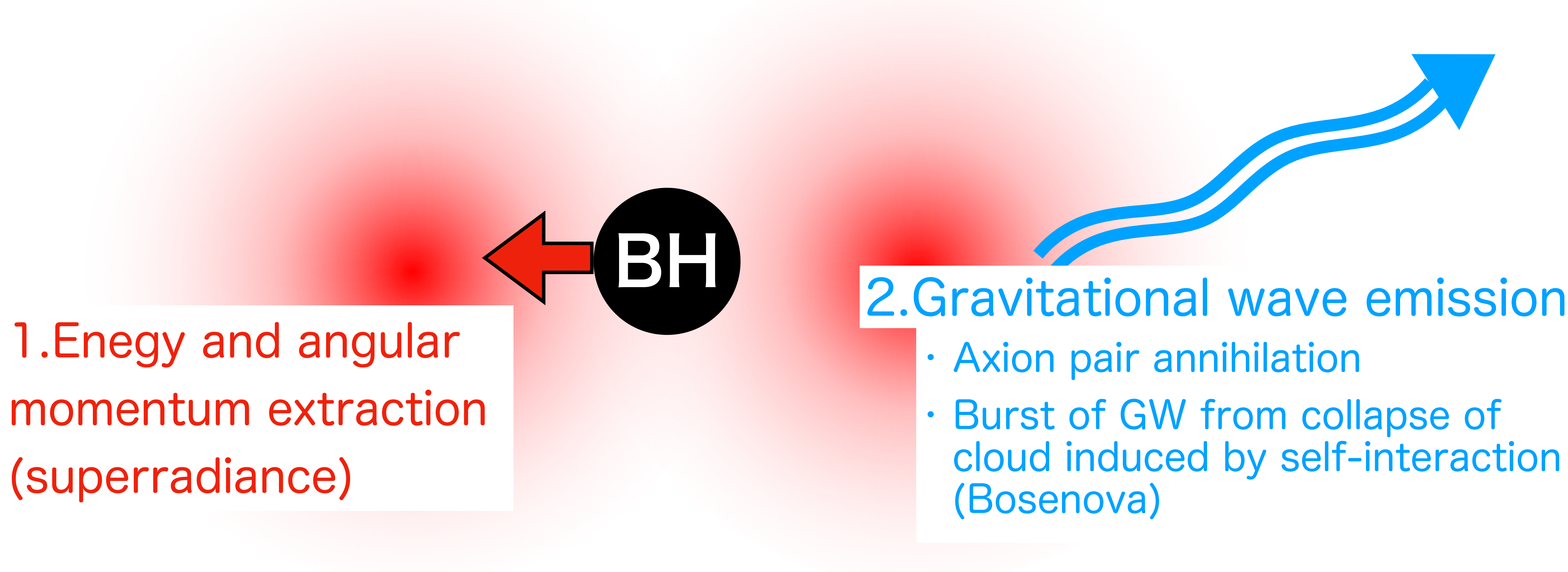


Axion Mass(eV)

(Arvanitaki et. al., 2010)

Axion Cloud

When ultra-light bosonic fields (such as axion) exists, huge condensate of bosonic field is spontaneously made around spinning BH.



- Cloud grows due to superradiant instability.
- Due to angular momentum extraction, BH with large spin is forbidden. We can constrain axion by observing BH spin!
- If we detect characteristic GW from the cloud, this would be the evidence of axion!

Effects of Self-interaction

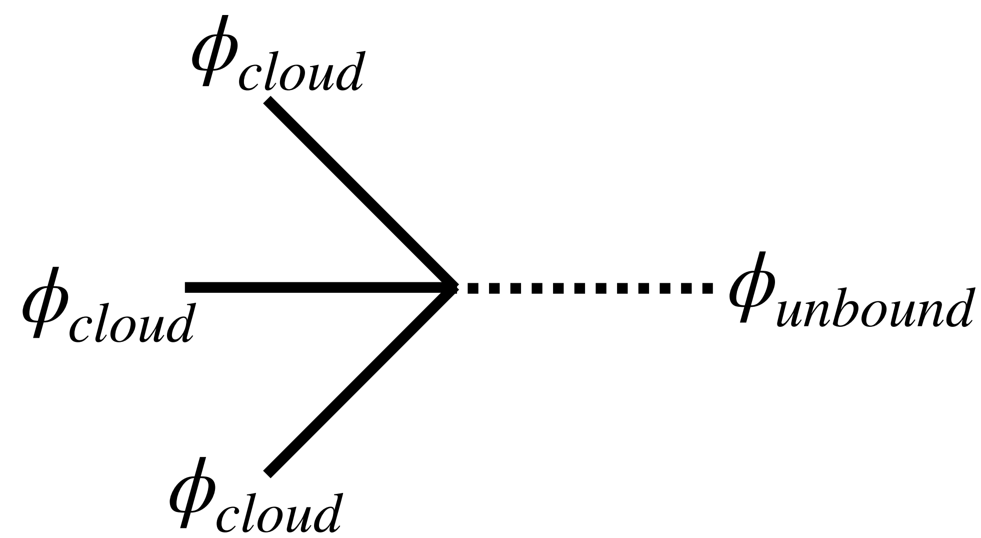
$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\partial_\mu \phi)^2 - \mu^2 F_a^2 \left(1 - \cos \frac{\phi}{F_a} \right) \right]$$

After cloud grows enough, self-interaction works (Arvanitaki et. al.,2010)

1. Scatters axion and dissipates energy to infinity
2. Attractive force modifies shape of cloud. When, attractive force acts strongly, cloud collapse and emits gravitational wave (Bosenova)
3. Induced emission of axion. Introduce another source of dissipation

(Baryakhtar et. al. 2011.11646)

1.

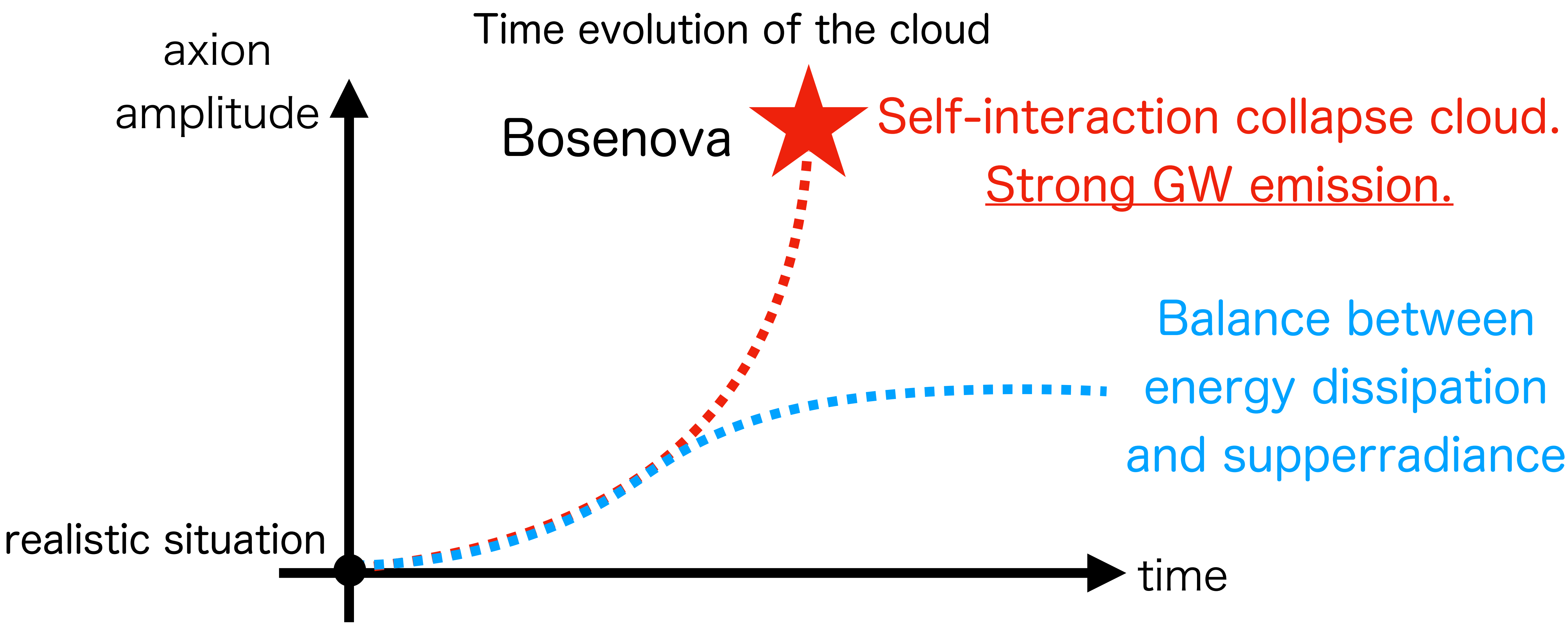


2. $V_{\text{axion}} \sim \frac{1}{2}\mu^2\phi^2 - \frac{\mu^2}{4!F_a^2}\phi^4 + \ldots$

Attractive

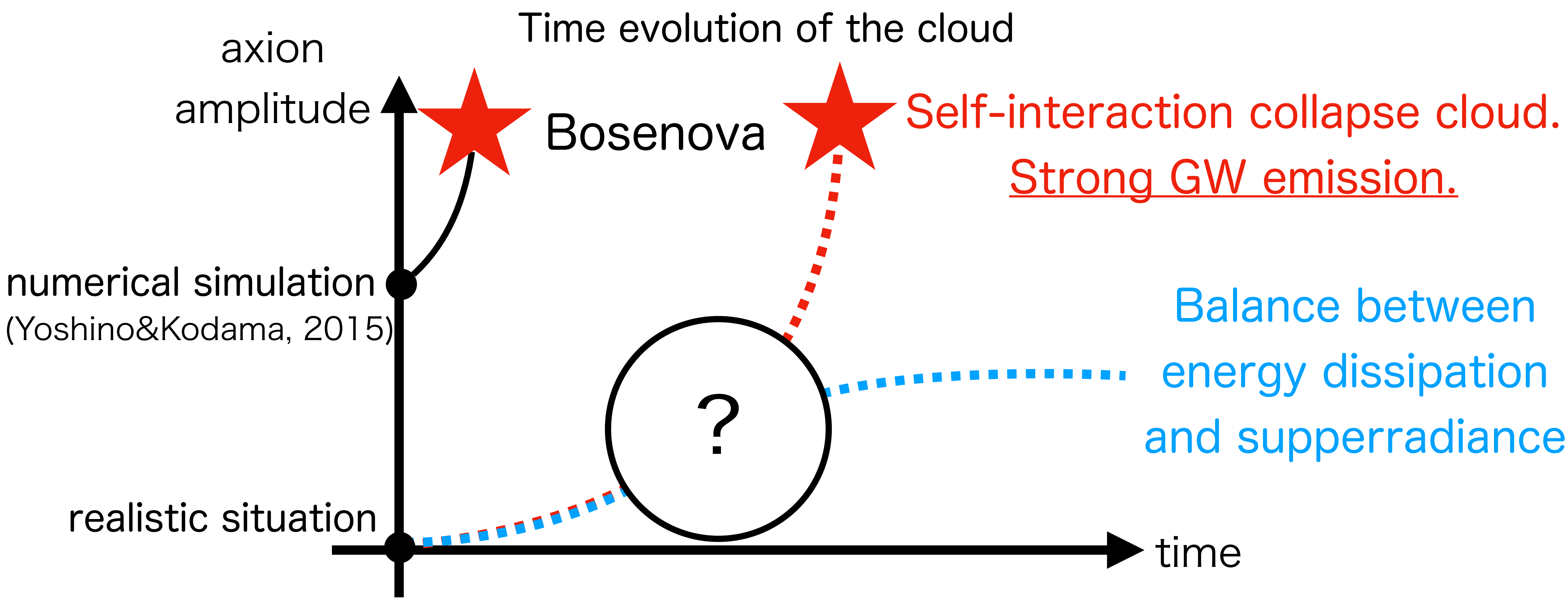
These effects make evolution of the cloud complicated and non-trivial!

Evolution of the cloud



Motivation

Q:What kind of state is realized in realistic situation?



- Numerical simulation tells both case happens. They are determined by initial condition and axion parameters (mass and decay constant)
- Hard to run a long term simulation due to difference in time scale $\omega_R \gg \omega_I$

Perturbative analysis of cloud evolution

(Omiya,Takahashi,Tanaka:2012.03473)

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\partial_\mu \phi)^2 - \mu^2 F_a^2 \left(1 - \cos\frac{\phi}{F_a} \right) \right]$$

g:Kerr metric

ϕ:Axion

μ:Axion mass

F_a:decay constant

EoM: $\Box_g \phi - \mu^2 F_a^2 \sin\frac{\phi}{F_a} = 0$

$$\xrightarrow{\phi \ll F_a} \left(\Box_g - \mu^2 \right) \phi \sim -\lambda \phi^3,\ \lambda = \frac{\mu^2}{6F_a^2}$$

- We will solve this equation perturbatively in λ
- As a first step, we treat axion cloud with single superradiant mode

Naive perturbation

Expand axion as $\phi = \phi_{(0)} + \lambda \phi_{(1)} + \cdots$

$\mathcal{O}(\lambda^0)$:$(\Box_g - \mu^2)\phi_{(0)} = 0$

We take bound state as 0th order solution

$$\phi_{(0)} = \underline{A(t_0)}\underline{e^{-i\omega_0 t}\psi_{\text{cloud}}(r,\theta,\phi)} + \text{c.c.} \qquad (\omega_{0,R} \gg \omega_{0,I})$$

Amplitude

Same bound state appears in 0th and 1st order

$\mathcal{O}(\lambda)$:$(\Box_g - \mu^2)\phi_{(1)} = -\phi_{(0)}^3$

$$\phi_{(1)} = -3\underline{C^{(1)}}A|A|^2 e^{2\omega_{0,I}t}\underline{e^{-i\omega_0 t}\psi_{\text{cloud}}(r,\theta,\phi)} + \text{c.c.} + \ldots$$

$C^{(1)}$:Diverge in $\omega_I \to 0$ limit

$\ldots$:other inhomogeneous solution and initial condition

Applying RG method

Solution up to $\mathcal{O}(\lambda)$

$$\phi = \left(A(t_0) - \underline{3\lambda C^{(1)} A |A|^2 e^{2\omega_{0,I} t}} \right) e^{-i\omega_0 t} \psi_{\rm cloud}(r, \theta, \phi) + {\rm c.c.} + \ldots$$

$C^{(1)}$ is huge ($\mathcal{O}(\omega_{0,I}^{-1})$) and breaks perturbative solution.

Eliminate this divergence at $t = t_0$ by adding homogeneous solution (RG method (Chen et. al.,1994, Kunihiro,1995)).

$$\phi = \left(A(t_0) - 3\lambda C^{(1)} A |A|^2 \left(e^{2\omega_I t} - e^{2\omega_{0,I} t_0} \right) - \underline{3\lambda \delta C^{(1)} A |A|^2 e^{2\omega_{0,I} t_0}} \right) \times e^{-i\omega_0 t} \psi_{\rm cloud}(r, \theta, \phi) + \ldots$$

Freedom of adding non-divergent solution (Scheme dependence of RG)

Applying RG method

RG equation

$$\frac{\partial \phi}{\partial t_0} = 0 \quad \Rightarrow \quad \frac{\partial A(t_0)}{\partial t_0} = -6\lambda\omega_I \tilde{C}^{(1)} e^{2\omega_I t_0} A|A|^2$$

$$\left(\tilde{C}^{(1)} = C^{(1)} + \delta C^{(1)}\right)$$

To consider the energy dissipation, we do same procedure up to $\mathcal{O}(\lambda^2)$

Evolution equation of axion cloud with self-interaction

$$\frac{\partial A(t_0)}{\partial t_0} = -6\lambda\omega_I \tilde{C}^{(1)} e^{2\omega_I t_0} A|A|^2 + 12\lambda^2\omega_I \left(C^{(2)} + \delta C^{(2)} - \frac{3}{2}(C^{(1)} - \delta C^{(1)})\left(2\tilde{C}^{(1)} + \tilde{C}^{(1)*}\right) \right) e^{4\omega_I t_0} A|A|^4 \, .$$

Applying RG method

Introduce $A = \mathscr{A} e^{-\omega_I t} e^{-i\int \delta\omega dt}$

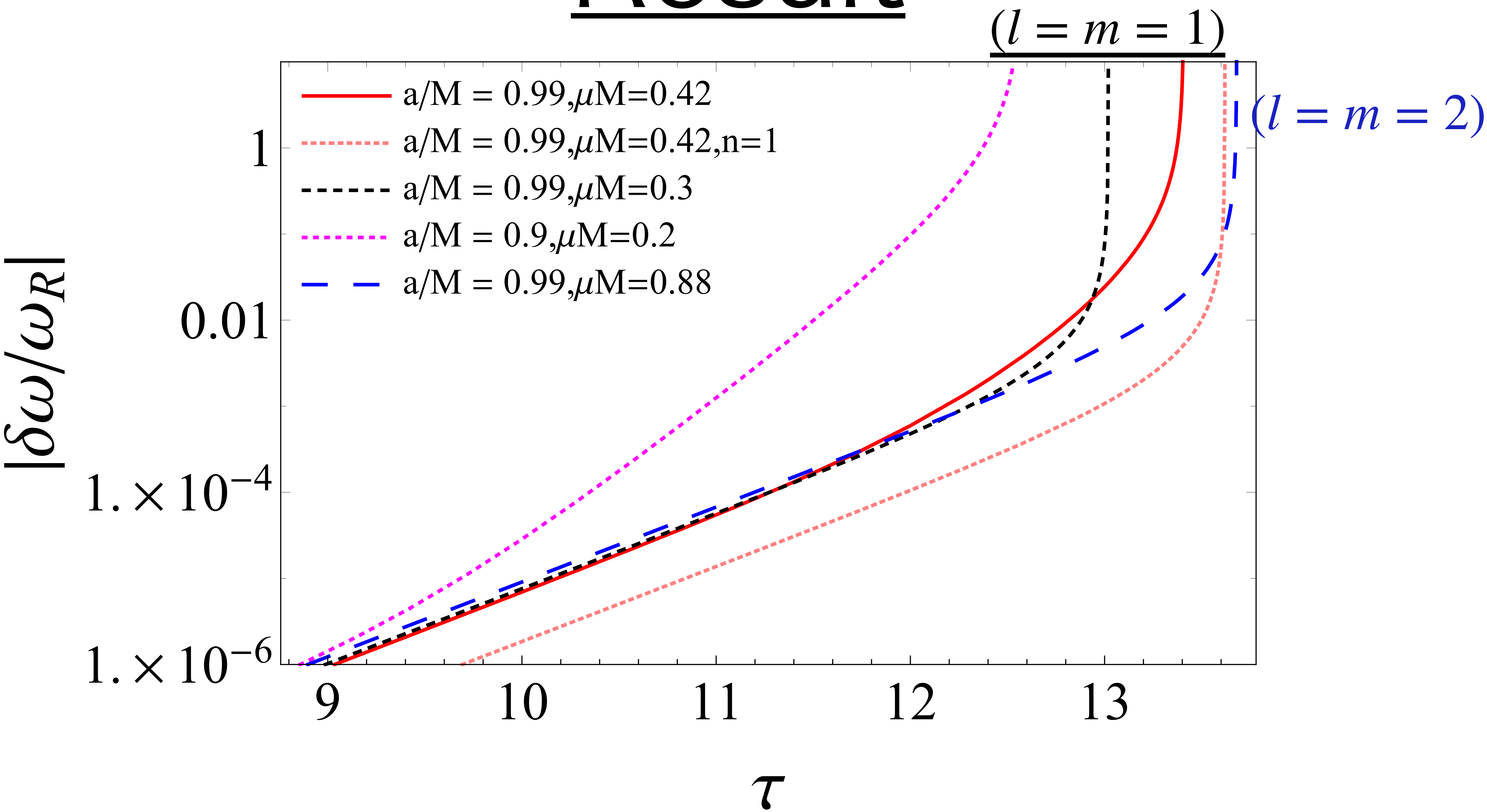
$$\frac{1}{\omega_I}\frac{\partial \mathscr{A}}{\partial t} = \mathscr{A} - 6\lambda \mathrm{Re}[C^{(1)}]\mathscr{A}^3 + 12\lambda^2 \mathrm{Re}\left[C^{(2)} - \frac{3}{2}\left(2C^{(1)2} + |C^{(1)}|^2\right)\right]\mathscr{A}^5 \ .$$

$$\frac{\delta\omega}{\omega_I} = 6\lambda \mathrm{Im}[C^{(1)}]\mathscr{A}^2 - 12\lambda^2 \mathrm{Im}\left[C^{(2)} - \frac{3}{2}\left(2C^{(1)2} + |C^{(1)}|^2\right)\right]\mathscr{A}^4 \ .$$

Scheme: $\delta C = 0$

We can tell break down of perturbation theory by size of $\delta\omega$

Result



- Instability is accelerated and $\delta\omega/\omega_R$ grows indefinitely
- At least in perturbation theory, energy dissipation by self-interaction won't stop the superradiant growth→sign of bosenova？

Reason of explosion

$C^{(2)}$ = term from $(\omega = 3\omega_0)$+term from $(\omega = \omega_0 + 2i\omega_{0,I})$

Satisfies $\mathrm{Re}[3\omega_0] > \mu$.
Dissipates energy

Satisfies $\mathrm{Re}[\omega_0] < \mu$.
Trapped by gravitational potential

- Energy dissipation to infinity do not work efficiently.
  → Dissipation to infinity is too weak to stop the instability!

- Attractive interaction between cloud and excited trapped modes lowers the energy of the cloud. → Instability is accelarated.

- Within the adiabatic approximation, excited modes never falls back to BH (satisfies $\omega < m\Omega_H$). → Dissipation to BH do not occurs.

Scheme dependence

Instead of Minimal Subtraction($\delta C = 0$),

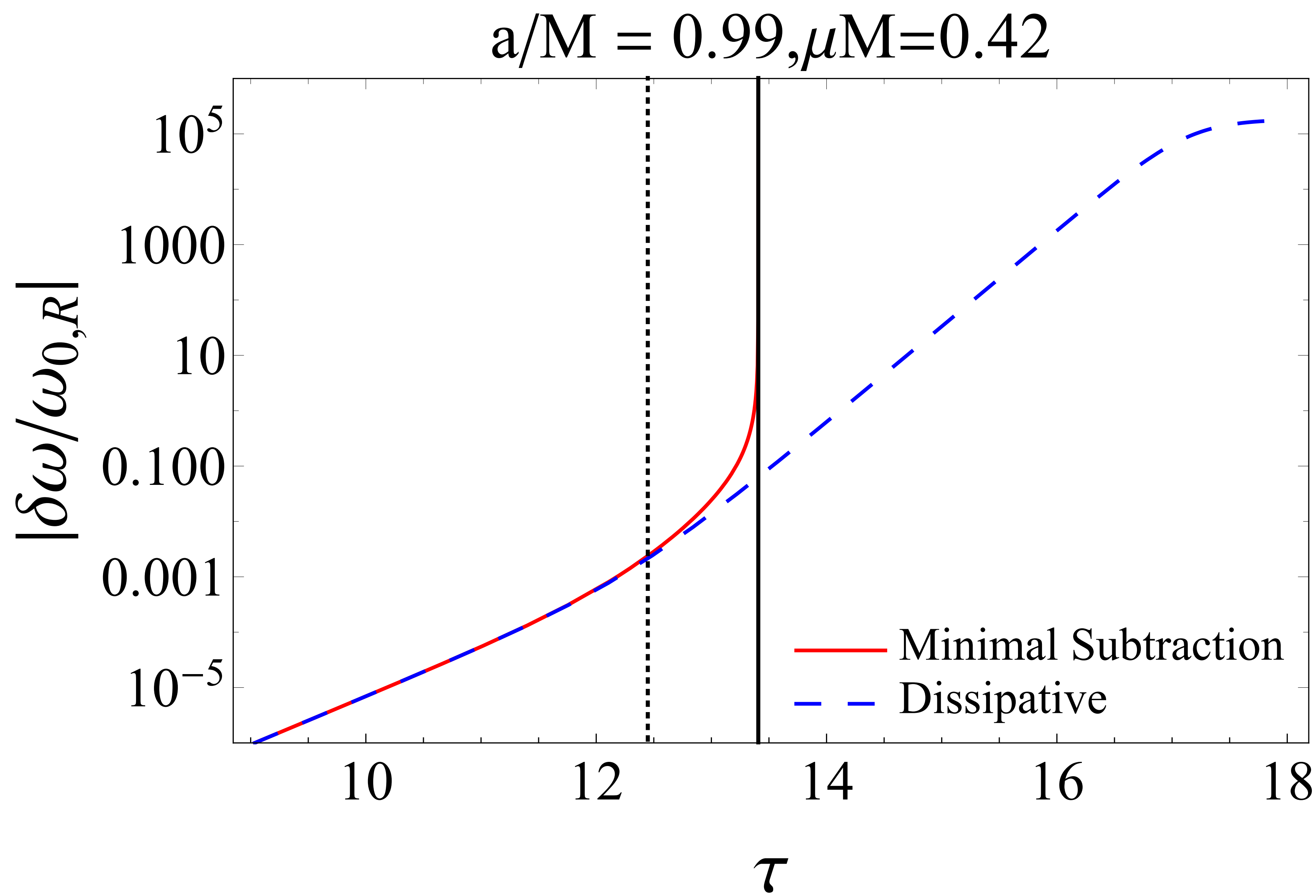
we can take the scheme in which the time evolution of the amplitude is totally governed by the dissipation to infinity.

$$\frac{1}{\omega_{0,I}}\frac{\mathrm{d}|\mathcal{A}|}{\mathrm{d}t} = |\mathcal{A}| + 12\lambda^2 C^{(2)\mathrm{diss}}|\mathcal{A}|^5 \ ,$$

$$\frac{\delta\omega}{\omega_{0,I}} = 6\lambda\mathrm{Im}\left[C^{(1)}\right]|\mathcal{A}|^2 - 12\lambda^2\left(\mathrm{Im}\left[\hat{C}^{(2)}\right] + 3\mathrm{Re}\left[C^{(1)}\right]\mathrm{Im}\left[C^{(1)}\right]\right)|\mathcal{A}|^4 \ .$$

$C^{(2)diss}$:dissipative part in $C^{(2)}$

Scheme dependence



- $\delta\omega/\omega_R$ goes to some constant value, but very large
  →Break down of perturbation theory
- Perturbation theory breaks down when two scheme gives different answer→After $\tau \sim 13$, perturbation theory breaks down.

Summary

- Studied evolution of the self-interacting axion cloud, which is important for future axion search with gravitational wave
- Used dynamical RG method to solve the problem
- Instability of axion cloud with single mode is accelerated by the self-interaction in weakly non-linear regime
  - Need further investigation of cloud with multiple modes
  - Need non-perturbative analysis for the final fate of the cloud
- Constructed solution valid for longer time period than naive perturbation
  - More realistic initial condition for the numerical simulation

Back up

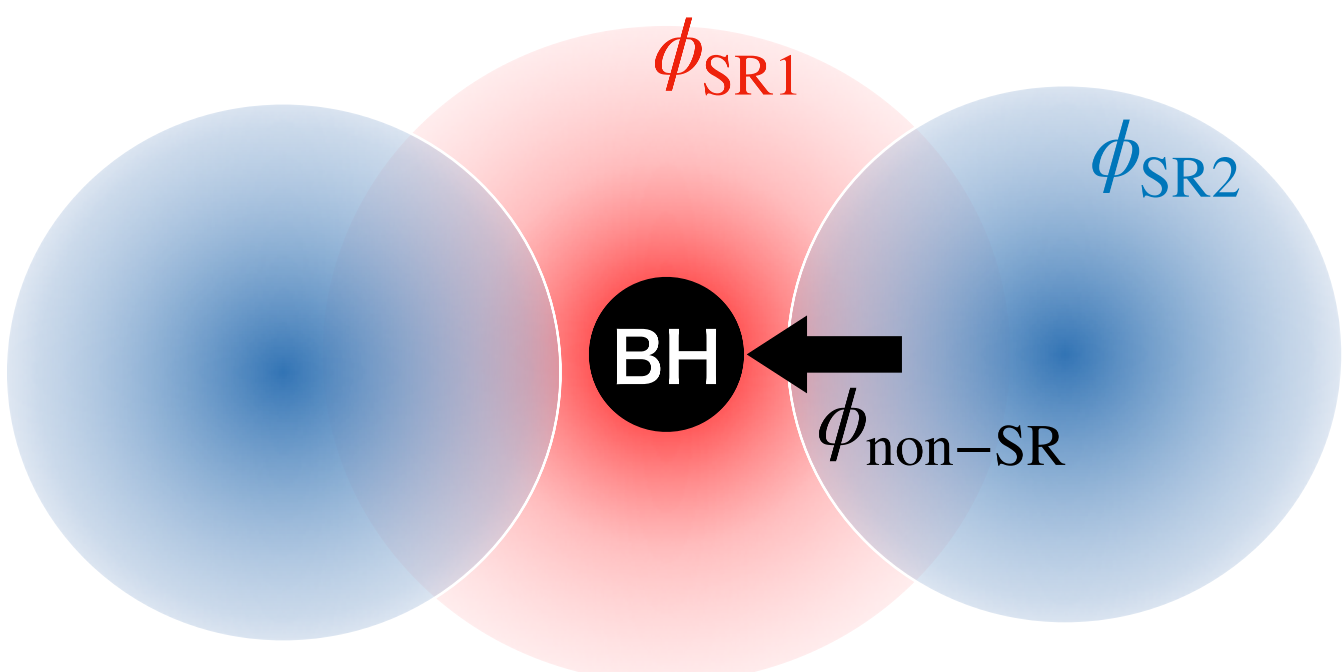
Other possible effects

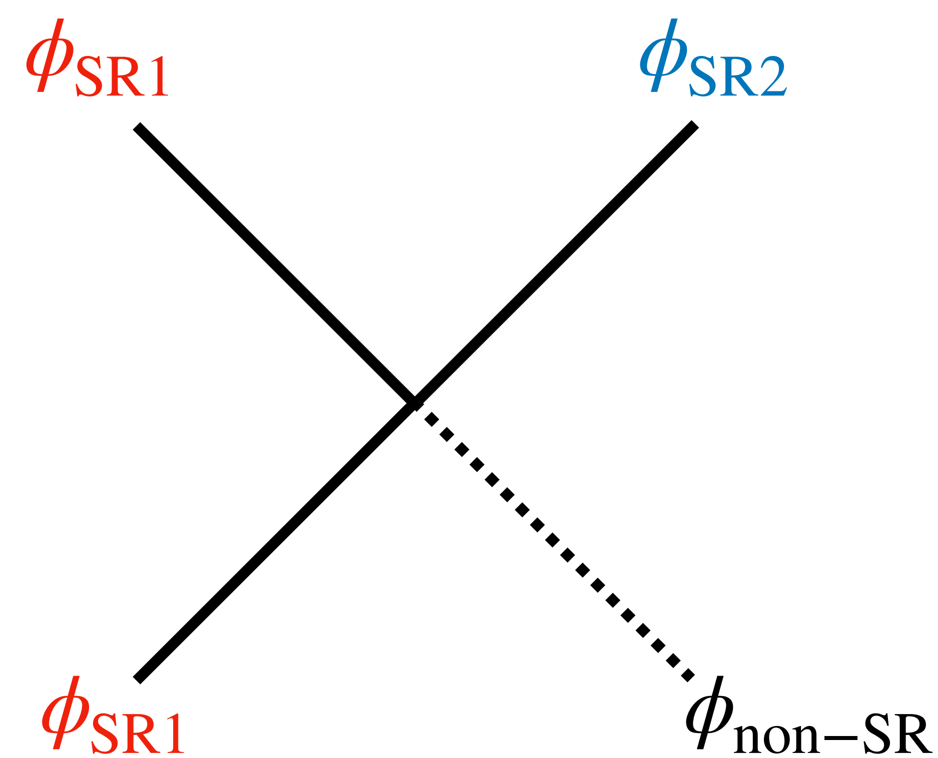
Q:Does our analysis completely capture the effect of self-interaction in perturbative regime?

A:Seems not. Our analysis interactions between different clouds were investigated.

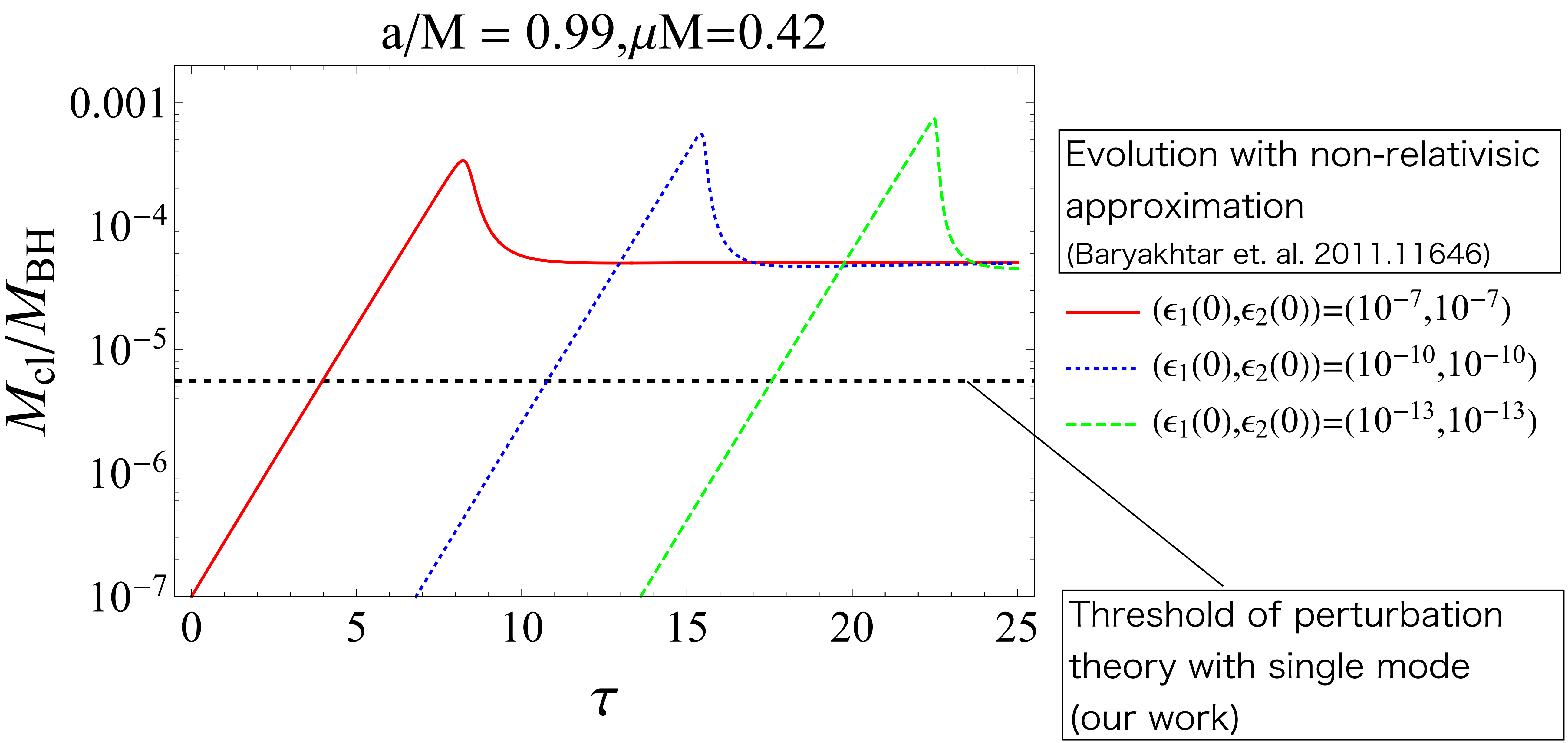
- This interaction introduces another source of dissipation
- But analysis has been done only in non-relativistic regime

(Baryakhtar et. al. 2011.11646)





Other possible effects



Even with new effects, non-linearity becomes strong.

Need further investigation!

Self-interaction vs backreaction

When cloud becomes dense, is self-interaction are dominant than GW emission becomes dominant?

Compare them by energy emission

$$\frac{\dot{E}_{\rm self}}{\dot{E}_{\rm grav}} \sim \frac{\omega^2(\frac{\mu^2}{F_a^2}\Phi^3)^2}{G\omega^4\Phi^4} \sim 10^6(\mu M)^4 \frac{M_{\rm cl}}{M_{BH}} \left(\frac{F_a/M_{\rm pl}}{10^{-2}}\right)^{-4}$$

When M_{cl} is not so small than$M$, then self-interaction dominates

Terminating parameter region