

# Gravitational wave propagation beyond GR: waveform distortions and echoes

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# Motivation

- LIGO/VIRGO gravitational wave (GW) detections give a strong constrain on gravity theory beyond GR
- Future: Einstein Telescope, Cosmic Explorer, LISA, TianQin, BBO, DECIGO, PTA
- It's the best era to test GR!

# Motivation

- We study GW propagation if GWs interact with another field
- **Homogenous and isotropic** cosmological background (no interaction with scalar or vector DoF in linear theory, only **tensor** DoF)
- Parametrized equation of motion
- Theoretical Examples: massive bigravity, Yang-Mills theories, Abelian multi-gauge field, Multi-Proca theories (see also Jimenez et al. 2020 for details)

# Equation of Motion

$$\left[ \hat{I} \frac{d^2}{d\eta^2} + \hat{\nu}(\eta) \frac{d}{d\eta} + \hat{C}(\eta) k^2 + \hat{\Pi}(\eta) k + \hat{M}(\eta) \right] \begin{pmatrix} h \\ s \end{pmatrix} = 0$$

velocity
Mass

Friction
Chiral

h: GW field

s: Coupled tensor field

# General Solution: two eigen propagation modes

$$h(\eta, k) = \sum_A h_0(k) f_A(\eta, k) e^{-i\phi_A(\eta, k)}$$

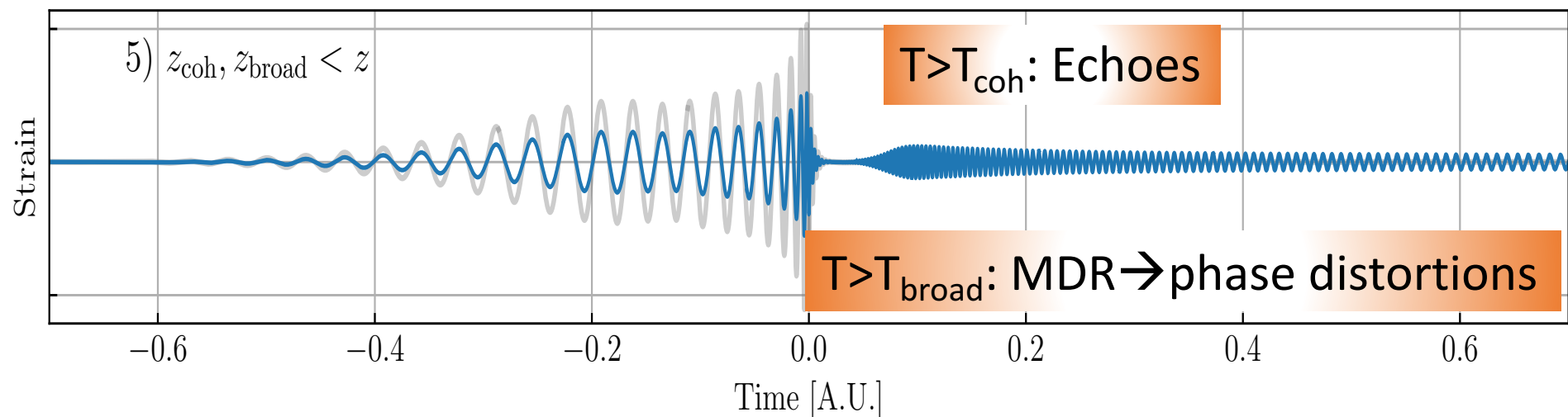
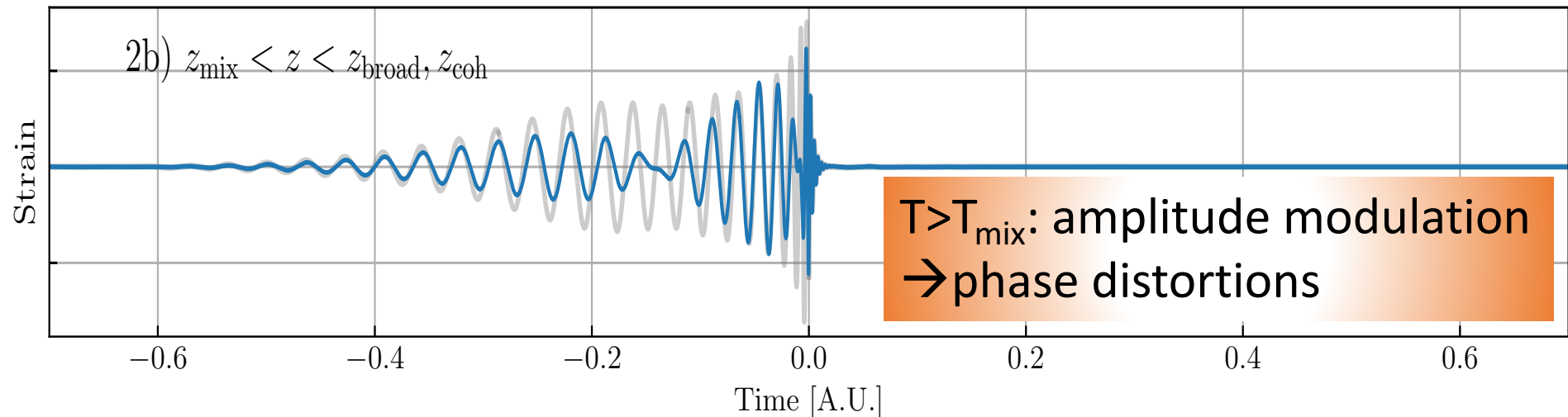
$$\phi_A = \int \omega_A d\eta \quad (\text{GR: } \omega_A = ck)$$

- Constant coefficients: exact analytical solution
- Time-dependent coefficients: WKB approximation
- Two eigenmodes propagate independently in high- $k$  limit.
- The detected signal is the superposition of the two eigenmodes.

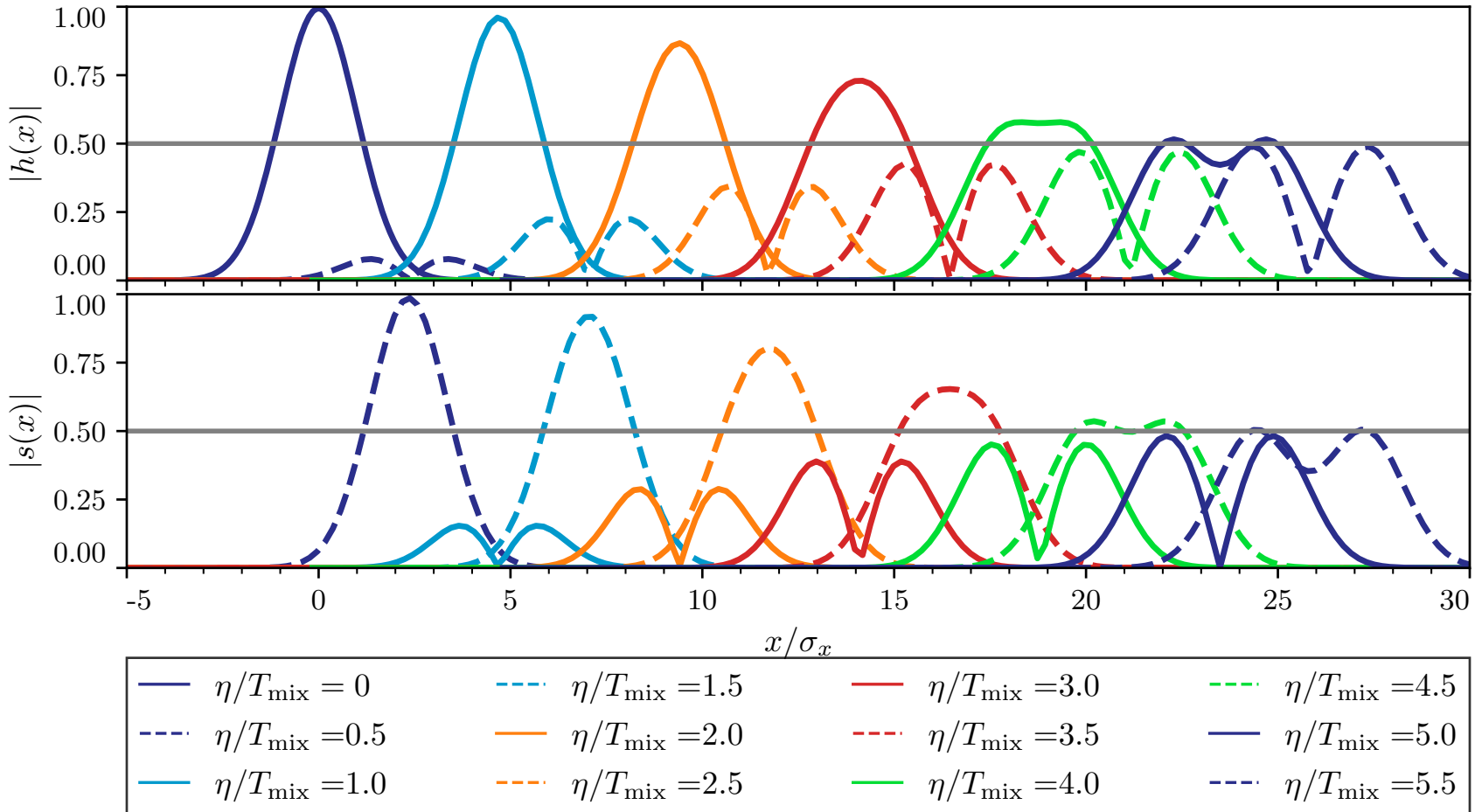
# Phenomenon highlights

- Three timescales: **mixing, coherence, broadening**
- Observational implication:
  - Echoes
  - Phase distortion

— GR — Modified propagation

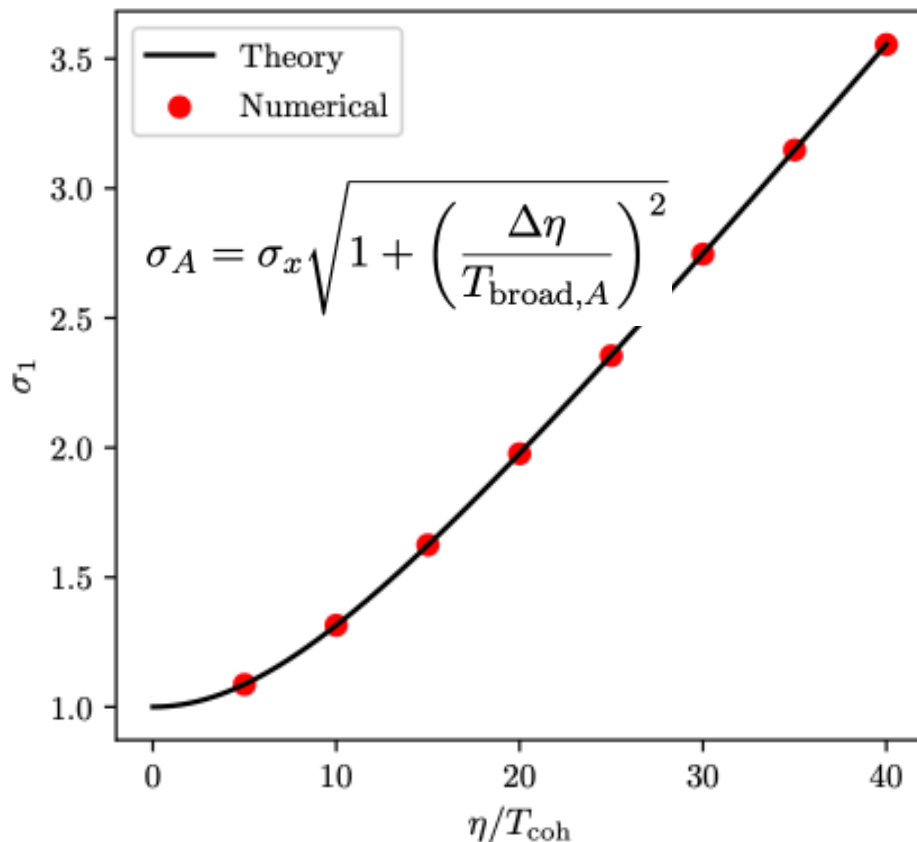
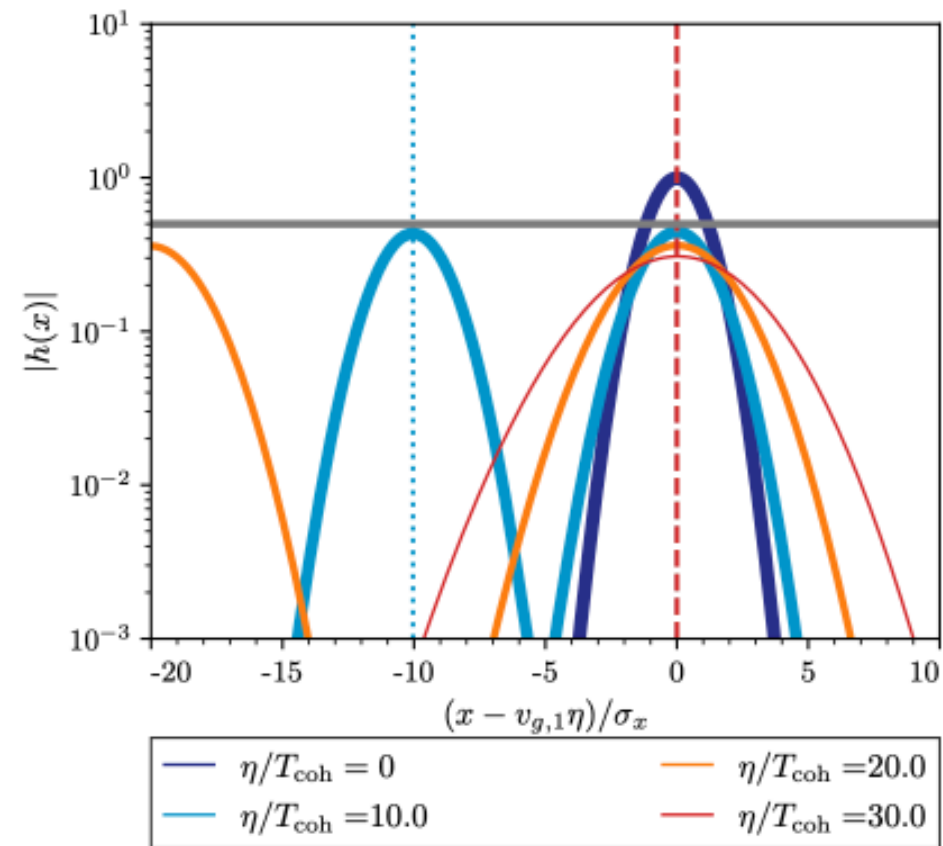


# Echoes: from coherence to decoherence





# Broadening



Real waveforms: k-dependent group velocity  $\rightarrow$  phase distortion

# Summary

- We study GW propagation if GW field interacts with another **tensor** field, obtain general WKB solution
- Three timescales: **mixing, coherence, broadening**
- Observational implication:
  - **Echoes**
  - **Phase distortion**
- Other interesting phenomena in the paper (see [arxiv:2108.10872](https://arxiv.org/abs/2108.10872))
  - Apparent luminosity distance change
  - Polarization oscillations and amplitude/phase birefringence
  - Broadening prevents decoherence



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# BACKUP

# Typical timescales

- Dispersion relation:

$$\omega_A(k) = \omega_A(k_0) + \frac{\partial \omega_A}{\partial k} (k - k_0) + \frac{1}{2} \frac{\partial^2 \omega_A}{\partial k^2} (k - k_0)^2 + \dots$$

- Mixing:  $T_{mix} |\omega_1 - \omega_2| \sim 2\pi$

- $T > T_{mix}$ : oscillations due to mixing

- Coherence:  $T_{coh} |v_{g1} - v_{g2}| \sim \sigma_A$ ,  $v_{g,A} = \frac{\partial \omega_A}{\partial k}$

- $T > T_{coh}$ : echoes

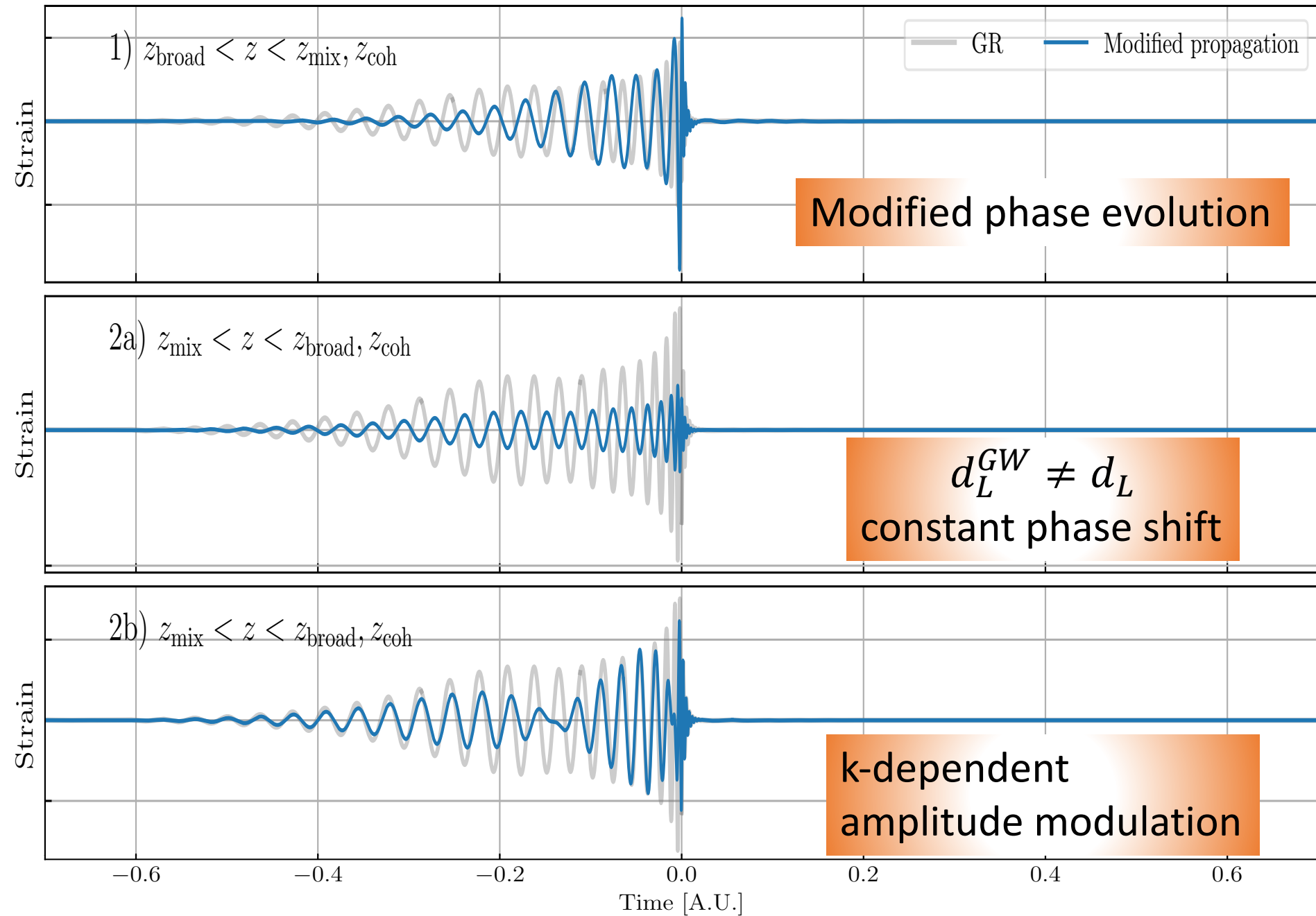
- Broadening:  $T_{broad,A} \left| \frac{\partial^2 \omega_A}{\partial k^2} \right| \sim \sigma_x^2$

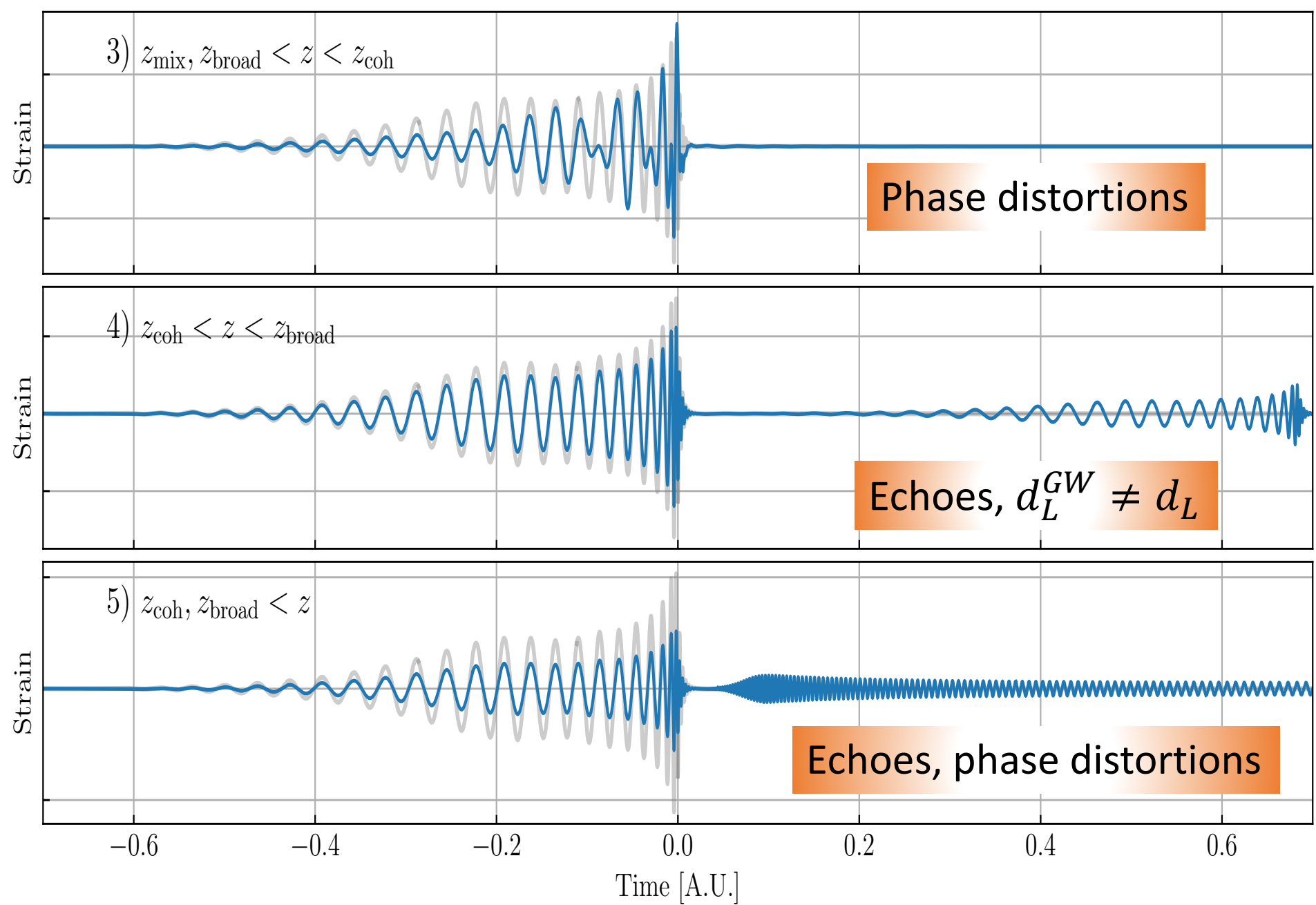
- $T > T_{broad}$ : phase distortions

# Observation: for real binary coalescence signal

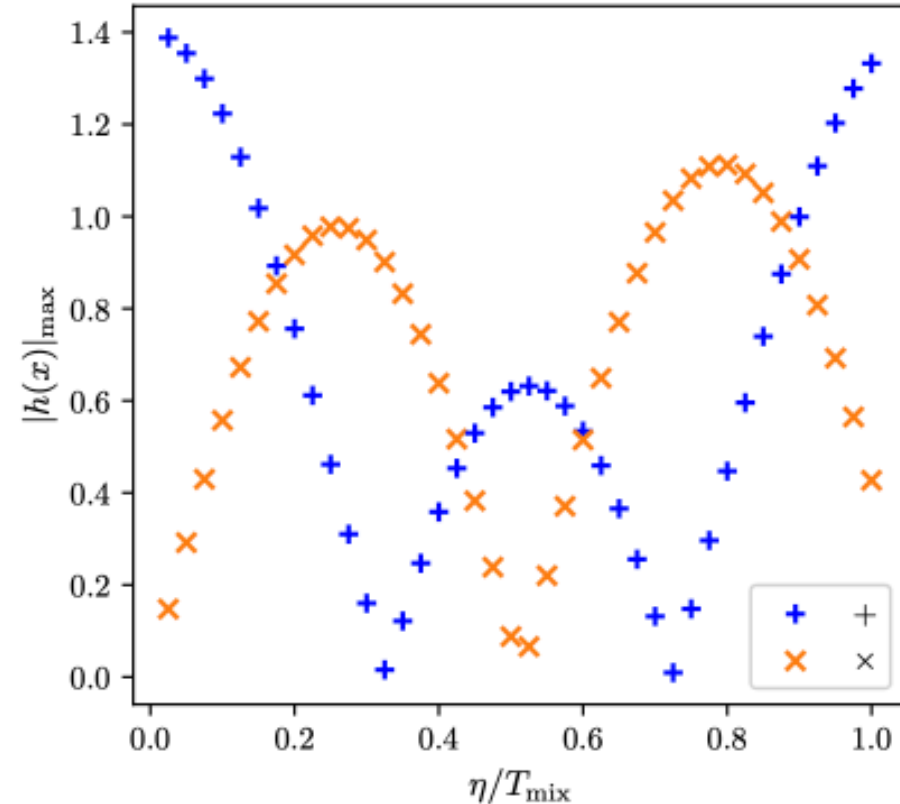
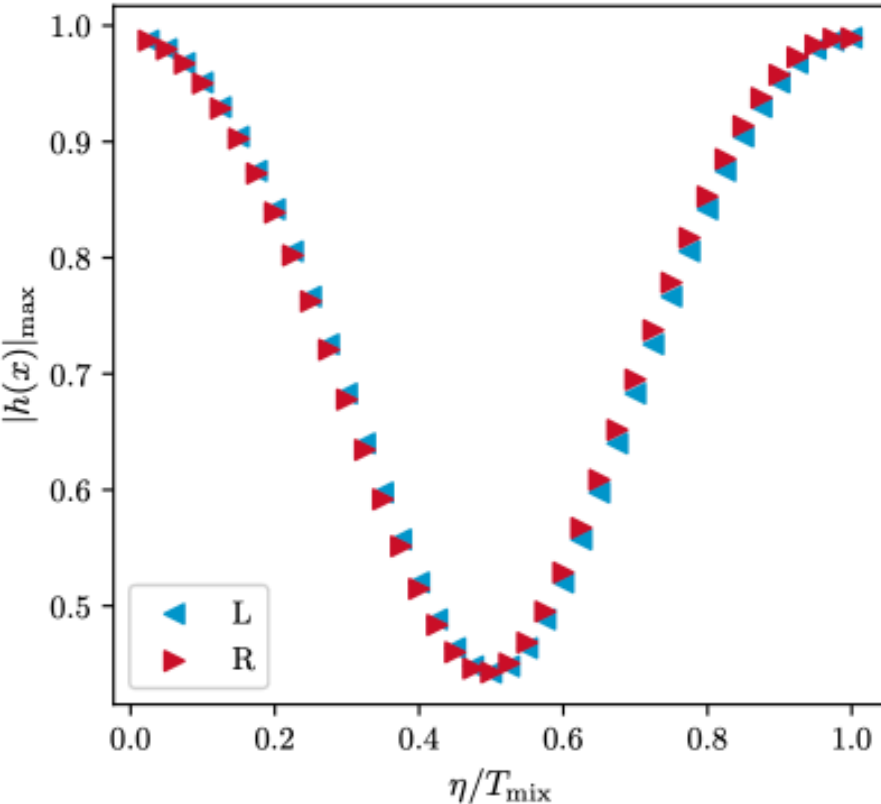
	Regime	Observables
0)	$z \ll z_{\text{mix}}, z_{\text{broad}}, z_{\text{coh}}$	Unmodified waveform
1)	$z_{\text{broad}} < z \ll z_{\text{mix}}, z_{\text{coh}}$	Single event with modified phase evolution
2a)	$z_{\text{mix}} < z < z_{\text{broad}}, z_{\text{coh}}$	Single event with $d_L^{\text{GW}} \neq d_L$ and constant phase shift, or
2b)		frequency-dependent amplitude modulation with phase distortions
3)	$z_{\text{mix}}, z_{\text{broad}} < z < z_{\text{coh}}$	Single event with modified phase evolution
4)	$z_{\text{coh}} < z < z_{\text{broad}}$	Echoes with different arrival times and $d_L^{\text{GW}}$
5)	$z_{\text{coh}}, z_{\text{broad}} < z$	Echoes with different arrival times and phase distortions

Main effects: echoes, phase distortions,  
oscillations and birefringence





# Polarization: chiral mixing



Initial: pure + polarization

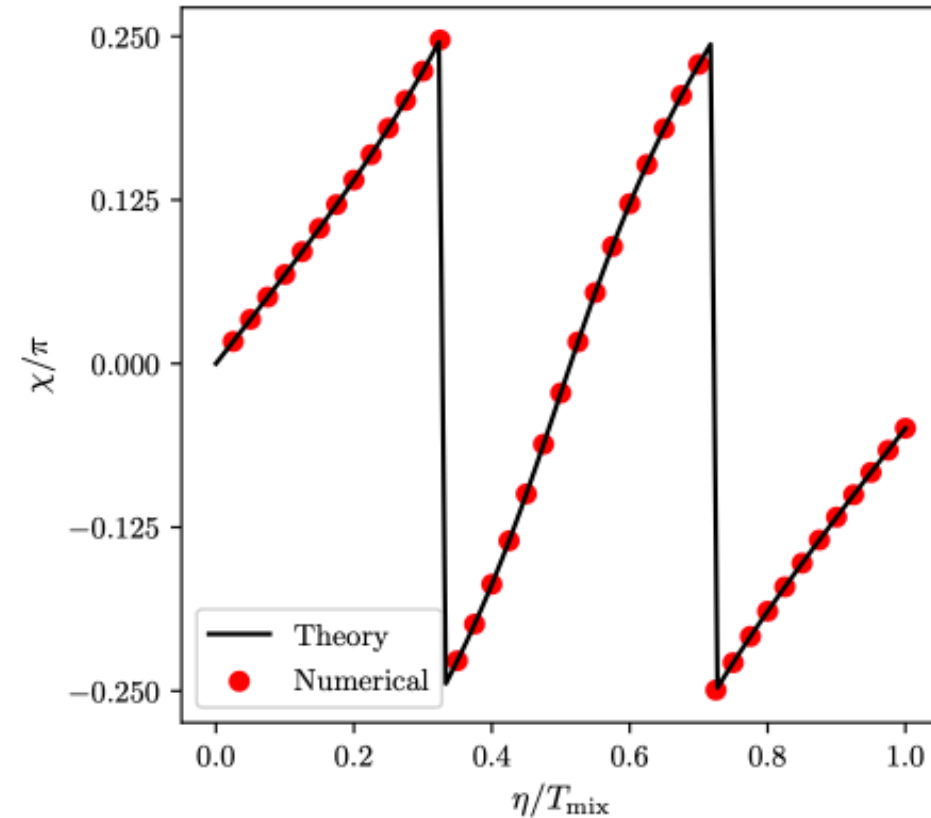
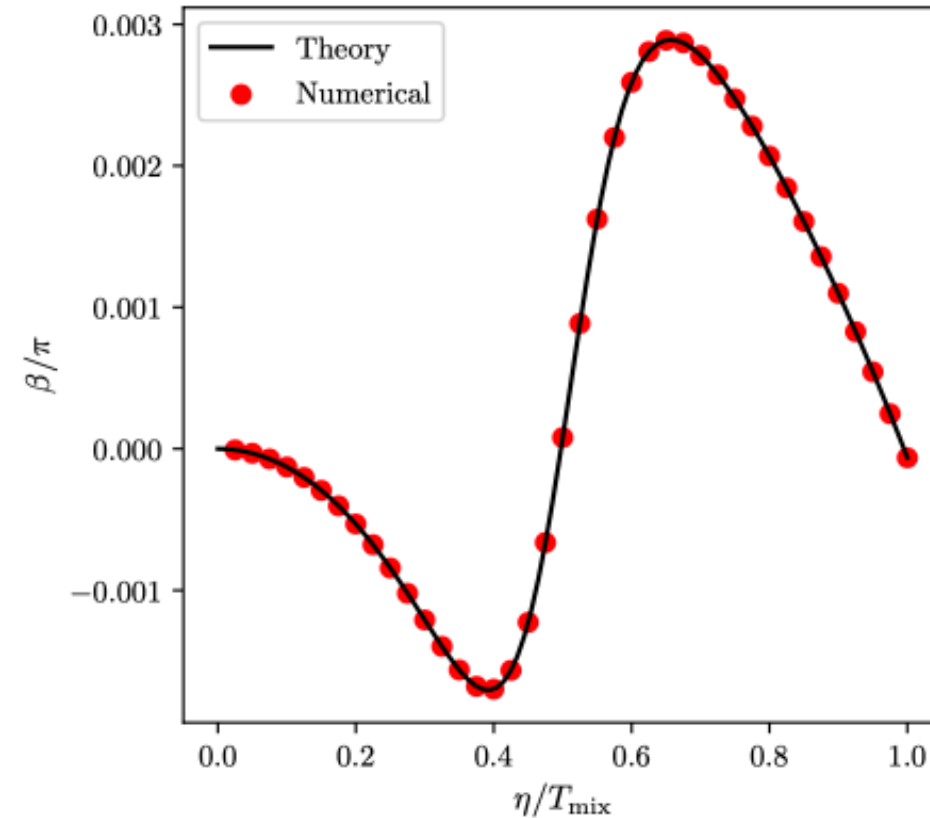
Propagation: oscillate between + and X

$$h_+ = \frac{1}{\sqrt{2}}(h_L + h_R)$$

$$h_\times = \frac{i}{\sqrt{2}}(h_L - h_R)$$

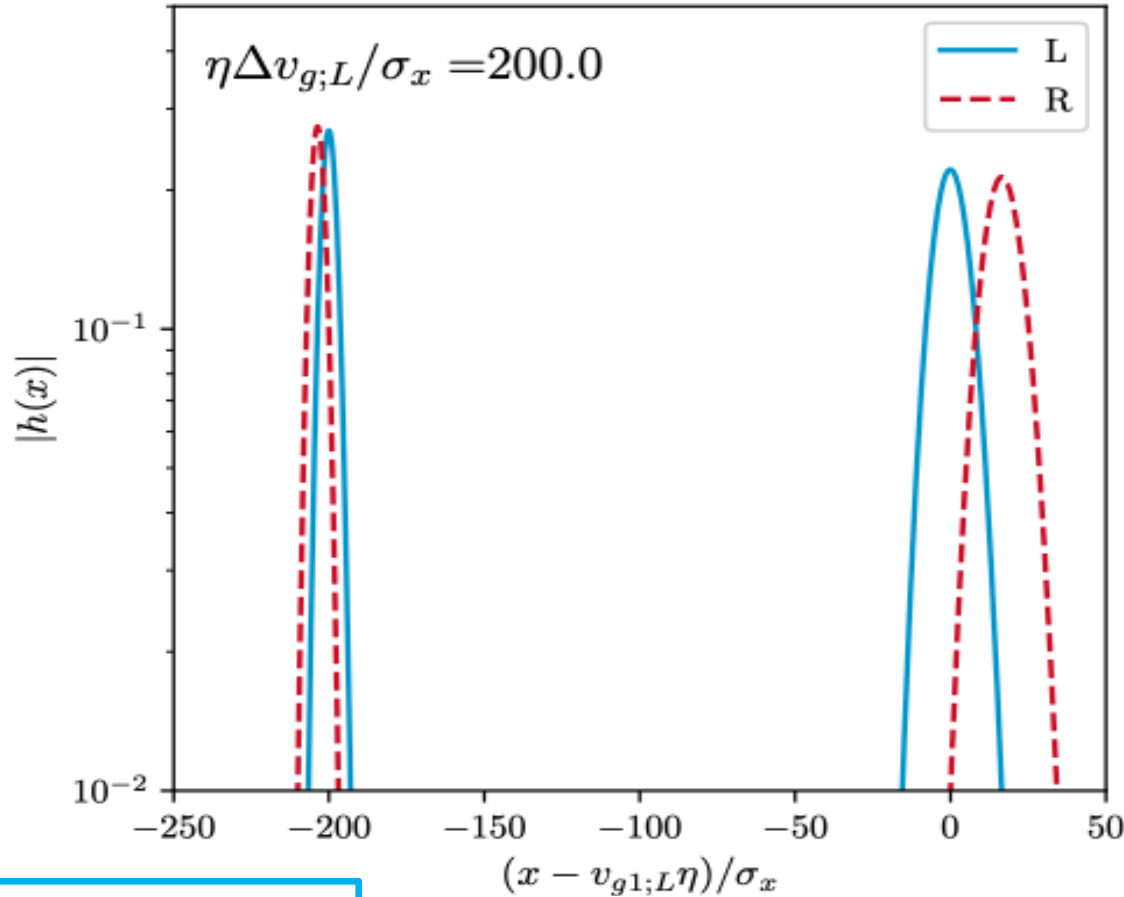


# Polarization: chiral mixing



$\beta$ : degree of circular polarization (amplitude birefringence)  
 $\chi$ : orientation of elliptical polarization (phase birefringence)

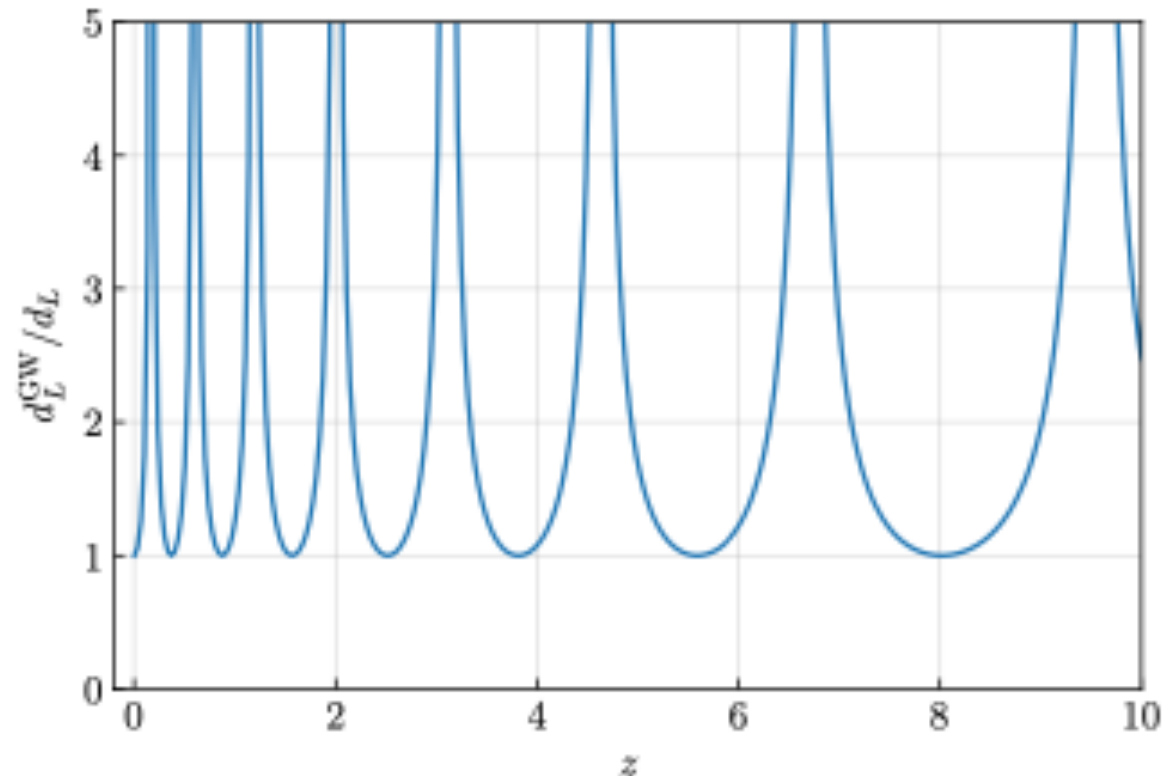
# Broadening prevents decoherence



Criteria for **ever**  
**decoherence**:  $RHS > 1$

$$\frac{\int_{\eta_0}^{\eta_0 + \Delta\eta} \Delta v_g(k_0) d\eta}{\sigma_A} \rightarrow \frac{\Delta v_g(k_0) T_{\text{broad,A}}(k_0)}{\sigma_x}$$

# Observation: apparent luminosity distance (friction mixing)



# Observation: birefringence (chiral mixing)

