Challenges in supersymmetric cosmology

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Problem of scales

- describe high energy (SUSY?) extension of the Standard Model
  unification of all fundamental interactions

- incorporate Dark Energy
  simplest case: infinitesimal (tuneable) +ve cosmological constant [4]

- describe possible accelerated expanding phase of our universe
  models of inflation (approximate de Sitter) [5]

$\Rightarrow$ 3 very different scales besides $M_{Planck}$: [6]

\begin{tabular}{l|c|c|c|c}
  Dark Energy & ElectroWeak & Inflation & Quantum Gravity \\
  meV & TeV & $M_I$ & $M_{Planck}$
\end{tabular}
Supersymmetry

A well motivated proposal

addressing several open problems of the Standard Model

- natural elementary scalars
- realise unification of the three Standard Model forces
- natural dark matter candidate (lightest supersymmetric particle)
- addressing the hierarchy problem
- prediction of light Higgs ($\lesssim 130$ GeV)
- soft UV behavior and important ingredient of string theory

But no experimental indication of any BSM physics at LHC

It is likely to be there at some (more) fundamental level
Relativistic dark energy  70-75% of the observable universe

negative pressure: \( p = -\rho \Rightarrow \) cosmological constant

\[
R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab} \Rightarrow \rho_{\Lambda} = \frac{c^4 \Lambda}{8\pi G} = -p_{\Lambda}
\]

Two length scales:

- \([\Lambda] = L^{-2} \leftarrow \) size of the observable Universe
  \[
  \Lambda_{\text{obs}} \simeq 0.74 \times 3H_0^2/c^2 \simeq 1.4 \times (10^{26} \text{ m})^{-2}
  \]
  Hubble parameter \( \simeq 73 \text{ km s}^{-1} \text{ Mpc}^{-1} \)

- \([\frac{\Lambda \times c^3}{G \hbar}] = L^{-4} \leftarrow \) dark energy length \( \simeq 85 \mu\text{m} \) [2]
Inflation:

Theoretical paradigm consistent with cosmological observations

But phenomenological models with not real underlying theory\(^\text{[2]}\) introduce a new scalar field that drives Universe expansion at early times

Inflaton potential

slow-roll region with $V'$, $V''$ small compared to the de Sitter curvature
Direct connection of inflation and supersymmetry breaking:

identify the inflaton with the partner of the goldstino

Goldstone fermion of spontaneous supersymmetry breaking

while accommodating observed vacuum energy
Inflation in supergravity: main problems

Inflaton: part of a chiral superfield $X$

- slow-roll conditions: the eta problem $\Rightarrow$ fine-tuning of the potential

$$\eta = V''/V, \quad V_F = e^K(|DW|^2 - 3|W|^2), \quad DW = W' + K'W$$

- $K$: Kähler potential, $W$: superpotential
- Planck units: $\kappa = 1$
- canonically normalised field: $K = X\bar{X} \Rightarrow \eta = 1 + \ldots$

- trans-Planckian initial conditions $\Rightarrow$ break validity of EFT
- no-scale type models that avoid the $\eta$-problem

$$K = -3\ln(T + \bar{T}); \quad W = W_0 \Rightarrow V_F = 0$$

- stabilisation of the (pseudo) scalar companion of the inflaton
- chiral multiplets $\Rightarrow$ complex scalars
- moduli stabilisation, de Sitter vacuum, . . .
Inflation from supersymmetry breaking

I.A.-Chatrabhuti-Isono-Knoops ’16, ’17, ’19

Inflaton: goldstino superpartner in the presence of a gauged R-symmetry

- linear superpotential $W = f X \Rightarrow$ no $\eta$-problem

\[
V_F = e^K \left( |DW|^2 - 3|W|^2 \right) = e^K \left( |1 + KX|^2 - 3|X|^2 \right) |f|^2 \quad K = X \bar{X}
\]

\[
= e^{|X|^2} (1 - |X|^2 + \mathcal{O}(|X|^4)) |f|^2 = \mathcal{O}(|X|^4) \quad \Rightarrow \eta = 0 + \ldots
\]

linear $W$ guaranteed by an R-symmetry

- gauge R-symmetry: (pseudo) scalar absorbed by the $U(1)_R$

- inflation around a maximum of scalar potential (hill-top) $\Rightarrow$ small field

- no large field initial conditions

- vacuum energy at the minimum: tuning between $V_F$ and $V_D$
Two classes of models

- Case 1: R-symmetry is restored during inflation (at the maximum)

- Case 2: R-symmetry is (spontaneously) broken everywhere and restored at infinity
  
  example: \( S = \ln X \)
Case 1: R-symmetry restored during inflation

maximum at the origin with small $\eta$ by a correction to the Kähler potential

$$\mathcal{K}(X, \bar{X}) = \kappa^{-2} X \bar{X} + \kappa^{-4} A (X \bar{X})^2 \quad A > 0$$

$$\mathcal{W}(X) = \kappa^{-3} f X$$

$$f(X) = 1 \quad (+\beta \ln X \text{ to cancel anomalies but } \beta \text{ very small})$$

$$\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D$$

$$\mathcal{V}_F = \kappa^{-4} f^2 e^{X \bar{X}(1+AX \bar{X})} \left[ -3X \bar{X} + \frac{(1 + X \bar{X}(1 + 2AX \bar{X}))^2}{1 + 4AX \bar{X}} \right]$$

$$\mathcal{V}_D = \kappa^{-4} \frac{q^2}{2} \left[ 1 + X \bar{X}(1 + 2AX \bar{X}) \right]^2$$

Assume inflation happens around the maximum $|X| \equiv \rho \simeq 0$
Predictions

slow-roll parameters \( q \approx 0 \)

\[
\eta = \frac{1}{\kappa^2} \left( \frac{V''}{V} \right) = -4A + \mathcal{O}(\rho^2) \quad [14]
\]

\[
\epsilon = \frac{1}{2\kappa^2} \left( \frac{V'}{V} \right)^2 = 16A^2 \rho^2 + \mathcal{O}(\rho^4) \approx \eta^2 \rho^2
\]

\( \eta \) naturally small since \( A \) is a correction

inflation starts with an initial condition for \( \phi = \phi_* \) near the maximum and ends when \( |\eta| = 1 \)

\[
\Rightarrow \text{number of e-folds } N = \int_{\text{end}}^{\text{start}} \frac{V}{V'} = \kappa \int \frac{1}{\sqrt{2\epsilon}} \approx \frac{1}{|\eta_*|} \ln \left( \frac{\rho_{\text{end}}}{\rho_*} \right) \quad [19]
\]

Planck '15 data: \( \eta \approx -0.02 \Rightarrow N \gtrsim 50 \) naturally
Predictions

amplitude of density perturbations \[ A_s = \frac{\kappa^4 V_*}{24\pi^2 \epsilon_*} = \frac{\kappa^2 H_*^2}{8\pi^2 \epsilon_*} \]

spectral index \[ n_s = 1 + 2\eta_* - 6\epsilon_* \approx 1 + 2\eta_* \]

tensor - to - scalar ratio \[ r = 16\epsilon_* \]

Planck ’15 data : \[ \eta \approx -0.02, \ A_s \approx 2.2 \times 10^{-9}, \ \mathcal{N} \gtrsim 50 \]

\[ \Rightarrow r \lesssim 10^{-4}, \ H_* \lesssim 10^{12} \text{ GeV} \quad \text{assuming } \rho_{\text{end}} \lesssim 1/2 \]

Question: can a ‘nearby’ minimum exist with a tiny +ve vacuum energy?

Answer: Yes in a ‘weaker’ sense: perturbative expansion \(^{[10]}\)

valid for the Kähler potential but not for the slow-roll parameters

need D-term contribution and next (cubic) correction in \( \mathcal{K} \)
Microscopic Model

Fayet-Iliopoulos model based on a $U(1)$ R-symmetry in supergravity

two chiral multiplets $\Phi_{\pm}$ of charges $q_{\pm}$ and mass $m$ and FI parameter $\xi$

$$W = m \Phi_+ \Phi_-$$

R-symmetry $\Rightarrow q_+ + q_- \neq 0$

Higgs phase: $\langle \Phi_- \rangle = v \neq 0$

Limit of small SUSY breaking compared to the $U(1)$ mass: $m^2 \ll q_- v^2$

integrate out gauge superfield $\rightarrow$ EFT for the goldstino superfield $\Phi_+$

$$W = m v \Phi_+ ; \quad K = \Phi_+ \Phi_+ + A (\Phi_+ \Phi_+)^2 + B (\Phi_+ \Phi_+)^3 + \cdots$$

parameter space allows realistic inflation

and a nearby minimum with tuneable energy
Fayet-Iliopoulos (FI) D-terms in supergravity

D-term contribution: positive contribution to $\eta \Rightarrow$ should stay small

its role: not important for inflation

- $U(1)$ absorbs the pseudoscalar partner of inflaton
- allows tuning the EW vacuum energy

**Question:** is it possible to have inflation by SUSY breaking via D-term?

the inflaton should belong to a massive vector multiplet as before

FI-term in supergravity very restrictive:

it gives a large positive mass to the inflaton

A new FI term was written recently: Cribiori-Farakos-Tournoy-Van Proeyen ’18

gauge invariant at the Lagrangian level but non-local

becomes local and very simple in the unitary gauge
A new FI term

Global supersymmetry:

\[ \mathcal{L}_{\text{FI}}^{\text{new}} = \xi_1 \int d^4 \theta \frac{W^2 \bar{W}^2}{D^2 W^2 \bar{D}^2 \bar{W}^2} \mathcal{D} \bar{W} = -\xi_1 D + \text{fermions} \]

It makes sense only when \( <D> \neq 0 \Rightarrow \text{SUSY broken by a D-term} \)

Supergravity generalisation: straightforward

unitarity gauge: goldstino = \( U(1) \) gaugino = 0 \( \Rightarrow \) standard sugra \( -\xi_1 D \)

Pure sugra + one vector multiplet \( \Rightarrow \)

\[ \mathcal{L} = R + \bar{\psi}_\mu \sigma^{\mu\nu\rho} D_\rho \psi_\nu + m_{3/2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu - \frac{1}{4} F_{\mu\nu}^2 - \left( -3m_{3/2}^2 + \frac{1}{2} \xi_1^2 \right) \]

- \( \xi_1 = 0 \Rightarrow \text{AdS supergravity} \)
- \( \xi_1 \neq 0 \) uplifts the vacuum energy and breaks SUSY
  
  e.g. \( \xi_1 = \sqrt{6} m_{3/2} \Rightarrow \text{massive gravitino in flat space} \)
Net result: $\xi_1 \to \xi_1 e^{K/3}$

The new and standard FI terms can co-exist in a particular Kähler basis

$$K = X\bar{X} + b \ln X\bar{X} + A (X\bar{X})^2$$

previous model: $b = 1$ in a different basis $\Rightarrow (A = 0)$ [10]

$$V_D = \frac{q^2}{2} \left( \rho^2 + b + \xi \rho^2 e^{\frac{2b}{3} e^{\frac{1}{3} \rho^2}} \right)^2$$

new FI term

$\xi = \xi_1 / q$

$b$ : standard FI constant

Case $f = 0$ (pure D-term potential) $\Rightarrow$ model of inflation on D-term

maximum at $\rho = 0 \Rightarrow b = 3$ and $\xi \leq -1$
\[ \nu_D = \frac{q^2}{2} \left[ 3 + \rho^2 \left( 1 + \xi e^{\frac{1}{3}\rho^2} \right) \right]^2 \]

- \( \xi = -1 \): effective charge of \( X \) vanishes
  
  \((1 + \xi)\) plays the role of the correction \( A \) to Kähler potential

- supersymmetric minimum at \( D=0 \)
  
  new FI-term singular \( \Rightarrow \) need (small) \( f \neq 0 \)
Case $f \neq 0$:
- minimum is lifted up and SUSY is broken by both D and F
Predictions for inflation

slow-roll parameters

\[ \eta = \frac{4(1 + \xi)}{3} + O(\rho^2) \]

\[ \epsilon = \frac{16}{9}(1 + \xi)^2 \rho^2 + O(\rho^4) \sim \eta^2 \rho^2 \]

\[ N \sim \frac{1}{|\eta_*|} \ln \left( \frac{\rho_{\text{end}}}{\rho_*} \right) \]

⇒ same main results as before (F-term dominated inflation) !! [11]

However allowing higher order correction to the Kähler potential
one can obtain \( r \) as large as 0.015 (near the experimental bound)
New FI-term introduces a cosmological constant in the absence of matter

Presence of matter $\Rightarrow$ non trivial scalar potential  
net result: $\xi_1 \to \xi_1 e^{K/3}$ 
but breaks Kähler invariance

However new FI-term in the presence of matter is not unique

Question: can one modify it to respect Kähler invariance?

Answer: yes, constant FI-term + fermions as in the absence of matter

$\Rightarrow$ constant uplift of the potential  
I.A.-Chatrabhuti-Isono-Knoops '18

Jang-Porrati '21

In general $\xi_1 \to \xi_1 f(m_{3/2}[\phi, \bar{\phi}])$  
I.A.-Rondeau '99

It can also be written in $N = 2$ supergravity  
I.A.-Derendinger-Farakos-Tartaglino Mazzucchelli '19
Conclusions

General class of models with inflation from SUSY breaking:

identify inflaton with goldstino superpartner

- (gauged) R-symmetry restored (case 1)
  small field, avoids the $\eta$-problem, no (pseudo) scalar companion
  a nearby minimum can have tuneable positive vacuum energy

- D-term inflation is also possible using a new FI term
  it allows for a positive uplifting of the scalar potential
  it can lead to large $r$ of primordial gravitational waves

Open question: string theory realisation

gauged R-symmetry, ordinary vs new FI-terms, explicit models