Axion Dark Matter

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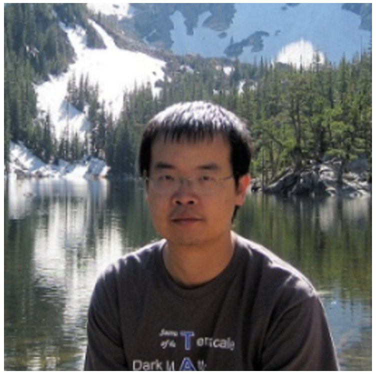
based on work done in collaboration with

- Qiaoli Yang 2009
- Ozgur Erken, Heywood Tam and Qiaoli Yang 2012
- Nilanjan Banik 2013
- Elisa Todarello 2017



In the 21st century we ask:

What is dark matter?

In the 17th century we asked:

What is light?

and it took a long time to figure that out

- Newton: light is a stream of particles
- Huygens: light is a wave
- Young: interference patterns show light is a wave

- Newton: light is a stream of particles
- Huygens: light is a wave
- Young: interference patterns show light is a wave
- Planck: no, light is a stream of photons

Particle-Wave duality

Quantum Field Theory gives a satisfactory description:

- a field is a set of oscillators; each describes the oscillations of a particular wave

$$\Psi_{\vec{\alpha}}(\vec{x})e^{-i\omega_{\vec{\alpha}}t}$$

- each oscillator is quantized

$$H = \sum_{\vec{\alpha}} \omega_{\vec{\alpha}}\ a^{\dagger}_{\vec{\alpha}} a_{\vec{\alpha}}$$

Quantum fields

$$\psi(\vec{x},t) = \sum_{\vec{\alpha}} \left(\Psi_{\vec{\alpha}}(\vec{x})a_{\vec{\alpha}}(t) \;+\; \Psi^*_{\vec{\alpha}}(\vec{x})a^\dagger_{\vec{\alpha}}(t)\right)$$

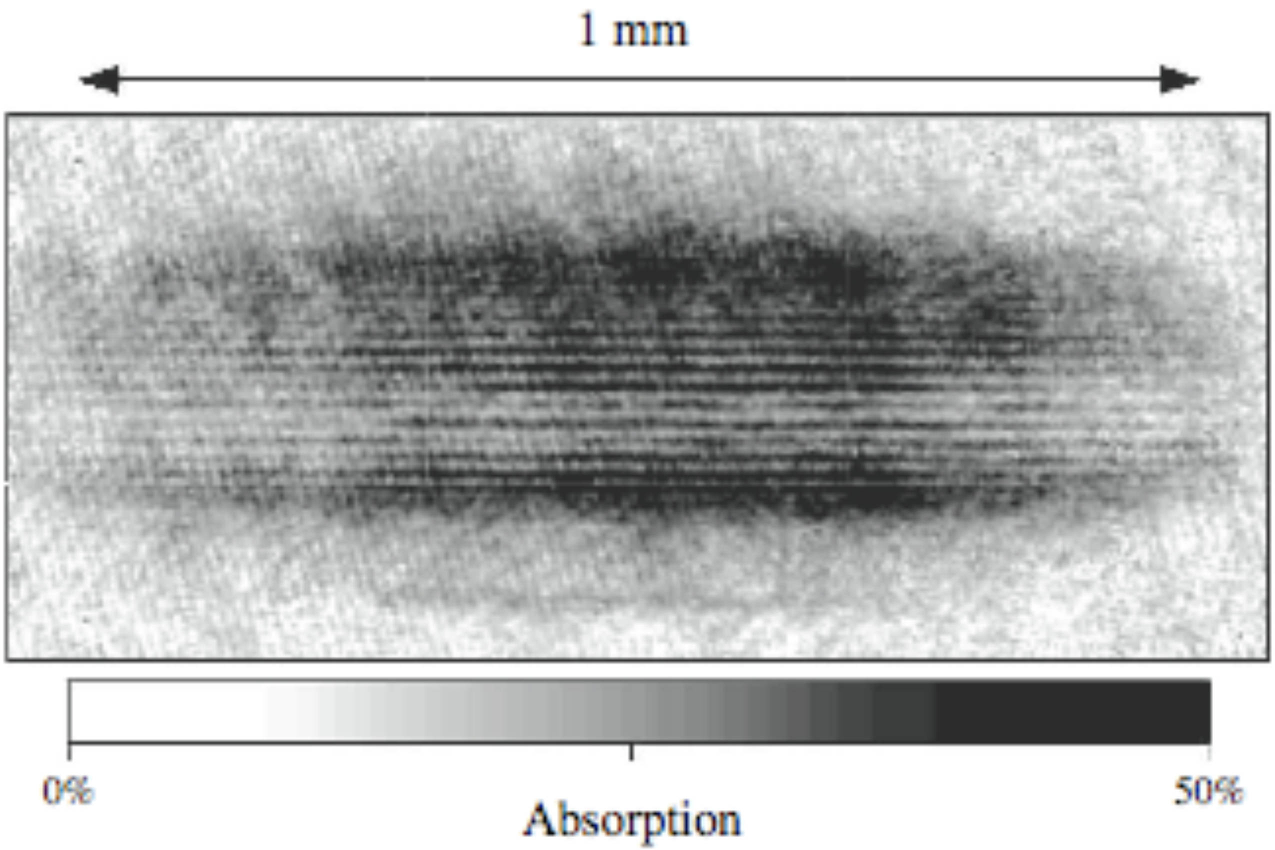
Known quantum fields:

$$e(x) \quad \nu_e(x) \quad \mu(x) \quad \nu_\mu(x) \quad \tau(x) \quad \nu_\tau(x)$$

$$u(x) \quad d(x) \quad s(x) \quad c(x) \quad b(x) \quad t(x)$$

$$A_\mu(x) \quad W^\pm_\mu(x) \quad Z_\mu(x) \quad G^a_\mu(x)$$

$$h(x)$$



from M.R. Andrews, C.G. Townsend, H.-J. Miesner, D.S. Durfee, D.M. Kurn and W. Ketterle, Science 275 (1997) 637.

QFT Hilbert space

$$|\Psi> \;=\; \sum_{\mathcal{N}_1,\mathcal{N}_2,\ldots} c_{\mathcal{N}_1,\mathcal{N}_2,\ldots} |\mathcal{N}_1, \mathcal{N}_2, \ldots >$$

number of quanta in the 2nd wave

$$|\Psi> \;=\; \sum_{\{\mathcal{N}_{\vec{\alpha}}:\forall\vec{\alpha}\}} c_{\{\mathcal{N}_{\vec{\alpha}}\}} \, |\{\mathcal{N}_{\vec{\alpha}}\} >$$

Single particle quantum mechanics

$$\mathcal{N}_1 = 1$$

$$\mathcal{N}_2 = \mathcal{N}_3 = \mathcal{N}_4 \quad \ldots \quad = 0$$

For one massive non-relativistic particle:

$$i\hbar\, \partial_t \Psi(\vec{x}, t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x}, t)\right)\Psi(\vec{x}, t)$$

For one photon:

$$(\partial_t^2 - \nabla^2)\vec{A}(\vec{x}, t) = 0$$

QFT has two classical limits

- Classical particle limit $\quad \mathrm{QFT} \rightarrow \mathrm{QM} \rightarrow \mathrm{CM}$

$$\hbar \rightarrow 0$$

$$E = \hbar\omega \quad , \quad p = \hbar k = \frac{h}{\lambda} \qquad \text{kept fixed}$$

$$\omega \; , \; k \;\; \rightarrow \;\; \infty$$

- Classical field limit $\quad \mathrm{QFT} \rightarrow \mathrm{CFT}$

$$\hbar \rightarrow 0$$

$$E = \mathcal{N}\hbar\omega \quad , \quad p = \mathcal{N}\hbar k \qquad \text{kept fixed}$$

$$\mathcal{N} \rightarrow \infty$$

$$\Delta x \; \Delta p \;\; \geq \;\; \frac{\hbar}{2}$$

- Particle description valid when

$$\Delta x << d$$

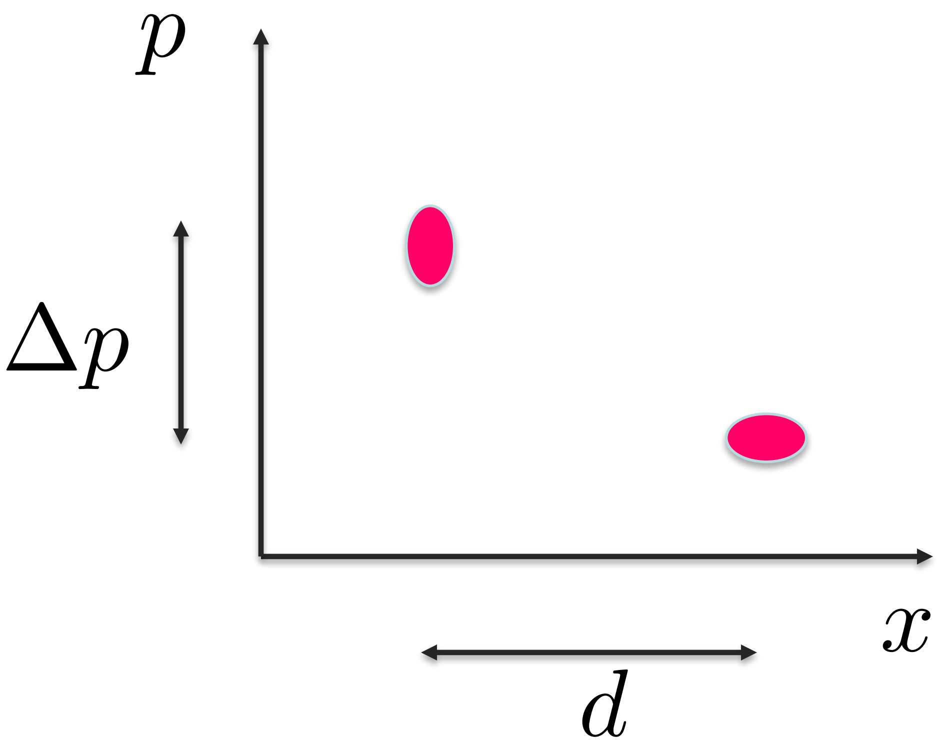
d = interparticle distance

$$d \; \Delta p >> \hbar = \frac{h}{2\pi}$$

$$\mathcal{N} = \frac{h^3}{d^3(\Delta p)^3} << 1$$

$\mathcal{N}$ = phase space density in units of h^{-3}

Phase space



$$n = \frac{1}{d^3}$$ = physical space density

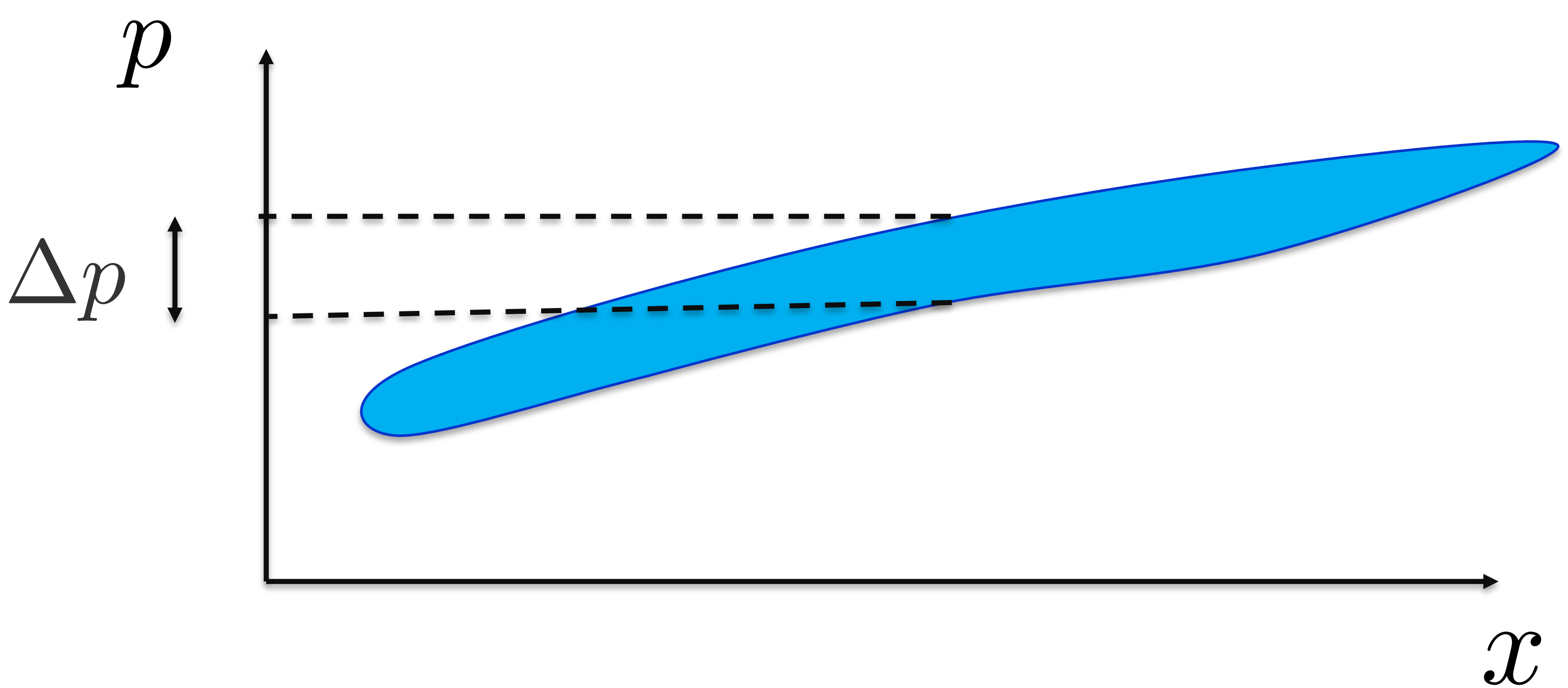
$$\hbar = \frac{h}{2\pi} = 1$$

$$\mathcal{N} = \frac{h^3}{d^3(\Delta p)^3} = \frac{(2\pi)^3 n}{(\Delta p)^3} << 1$$

- Wave description is valid when

$$\mathcal{N} = \frac{(2\pi)^3 n}{(\Delta p)^3} >> 1$$

for Bosons only!



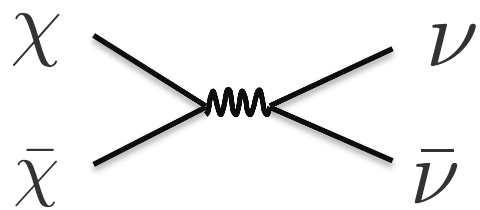
$\mathcal{N}$ = quantum degeneracy

Two dark matter candidates: WIMPs & axions

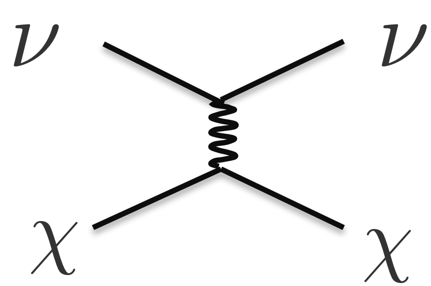
- WIMPs (weakly interacting massive particles)

$$m_W \sim 10\ \mathrm{GeV}$$

produced by freeze-out at $$T_W \sim \frac{m_W}{20}$$



decouple kinetically at $$T_{W\,kin} \sim \mathrm{MeV}$$



WIMPs today

$$\rho_{\rm DM} = \Omega_{\rm DM}\ \rho_{\rm crit}$$

$$\Omega_{\rm DM} \simeq 0.23 \qquad\qquad \rho_{\rm crit} \simeq 10^{-29}\ {\rm gr/cc}$$

$$n_W \simeq 0.13\ \frac{1}{\rm m^3}\left(\frac{10\ {\rm GeV}}{m_W}\right)$$

$$\Delta p_W \equiv m_W \Delta v_W \sim \sqrt{2 m_W T_{W\rm kin}}\ \frac{T_0}{T_{W\rm kin}}$$

$$\Delta v_W \sim 1.3 \cdot 10^{-12} \sqrt{\frac{10 \text{ GeV}}{m_W}} \qquad (c = 1)$$

$$\Delta x_W \sim \frac{1}{\Delta p_W} \sim 15 \mu\text{m} \sqrt{\frac{10 \text{ GeV}}{m_W}} \qquad (\hbar = 1)$$

$$\mathcal{N}_W = \frac{(2\pi)^3 n_W}{(\Delta p_W)^3} \sim 10^{-13} \left(\frac{10 \text{ GeV}}{m_W} \right)^{\frac{5}{2}}$$

Axions

$$m_a \sim 10^{-5} \text{ eV}$$

produced by 'vacuum realignment' during the QCD phase transition, and perhaps also by string and domain wall decay

$$\Delta p_a(t_1) \sim \frac{1}{t_1}$$

$t_1 \sim 2 \cdot 10^{-7} \text{ sec}$ = age of the universe when

$$T_1 \sim 1 \text{ GeV}$$

Axions today

$$n_a(t_0) = \frac{\rho_{\rm DM}(t_0)}{m_a} \sim 1.3 \cdot 10^8 \frac{1}{\rm cm^3}$$

$$\Delta p_a = m_a \Delta v_a \sim \frac{1}{t_1} \frac{10^{-4}\ \rm eV}{\rm GeV}$$

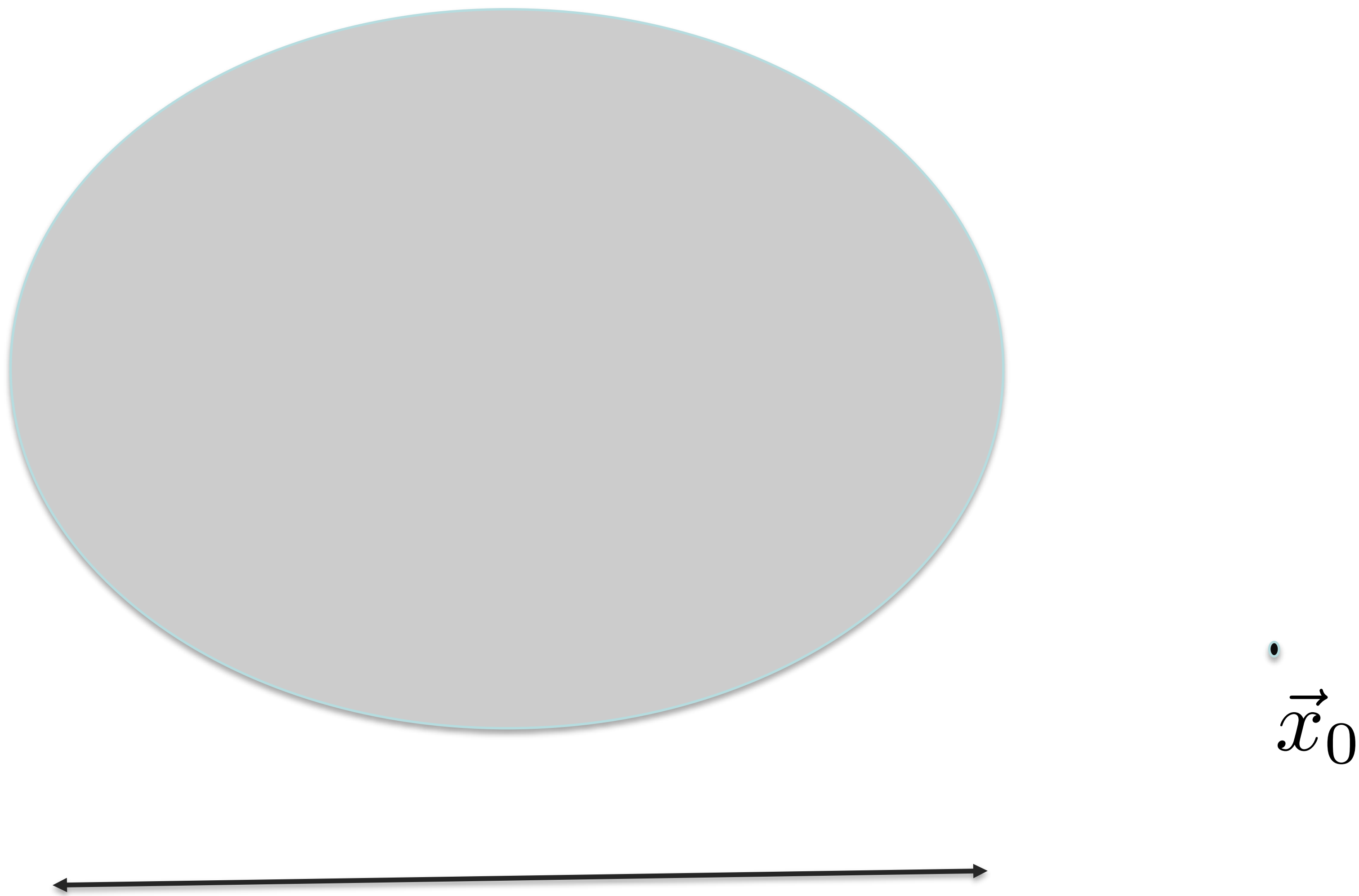
$$\Delta v_a \sim 3 \cdot 10^{-17} \sim 10^{-6}\ \frac{\rm cm}{\rm sec} \sim \frac{30\ \rm cm}{\rm year}$$

$$\Delta x_a \sim \frac{1}{\Delta p_a} \sim 0.7 \cdot 10^{17} \text{ cm} \simeq 0.02 \text{ pc}$$

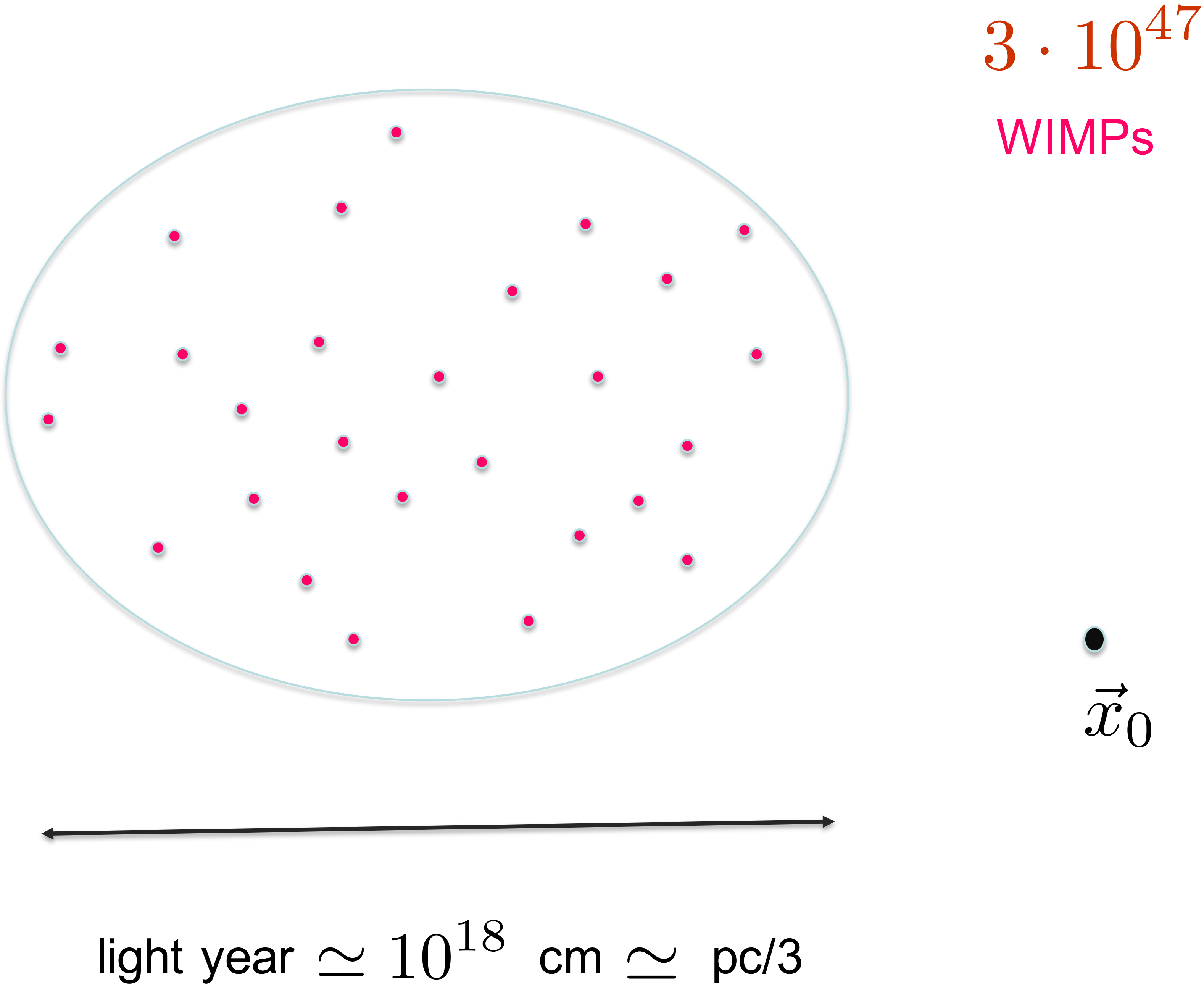
$$\mathcal{N}_a = \frac{(2\pi)^3 n_a}{(\Delta p_a)^3} \sim 10^{61}$$ (!)

Axion dark matter is an extremely degenerate Bose gas.

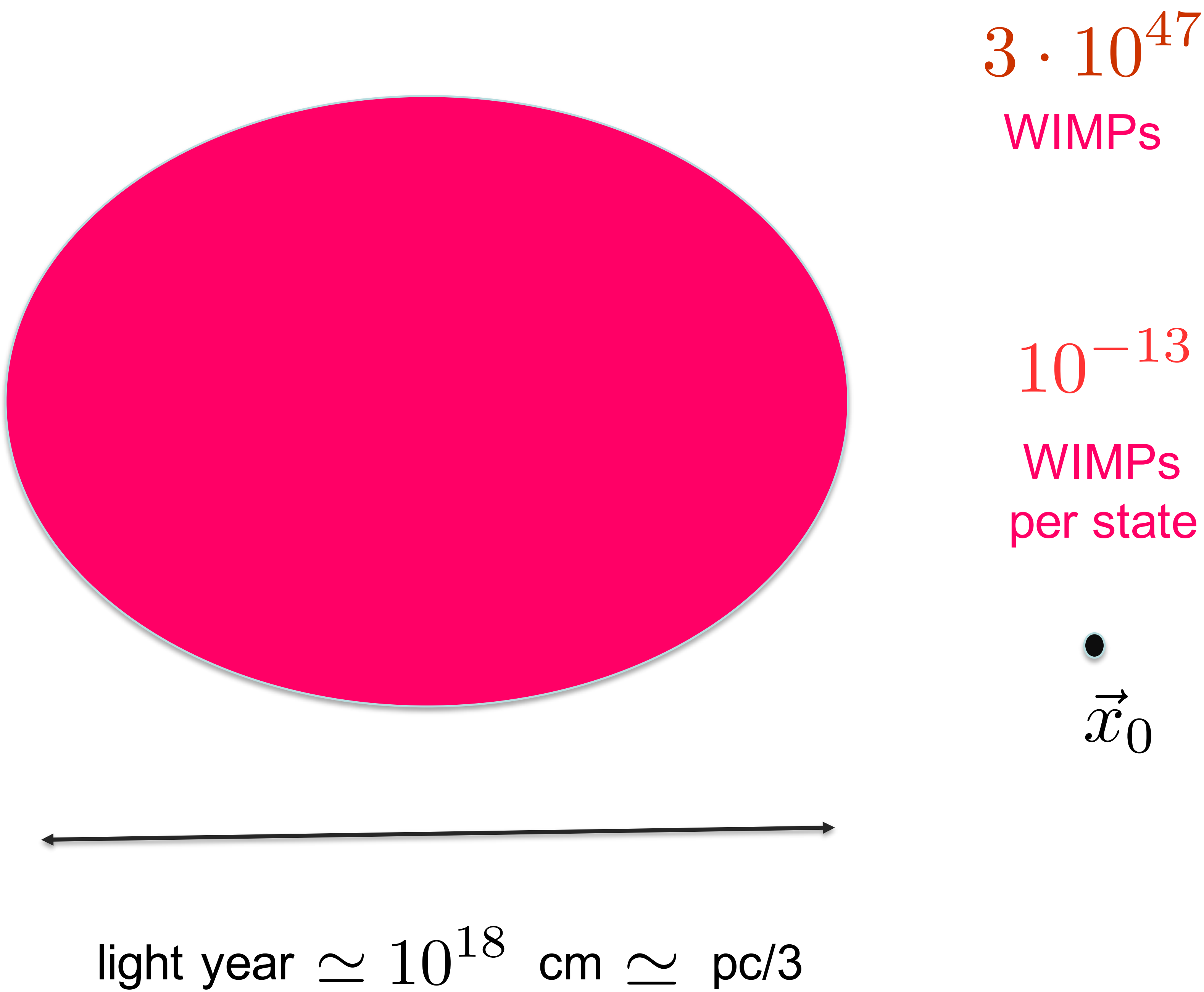
Does it behave the same way as WIMP dark matter in astrophysical contexts?

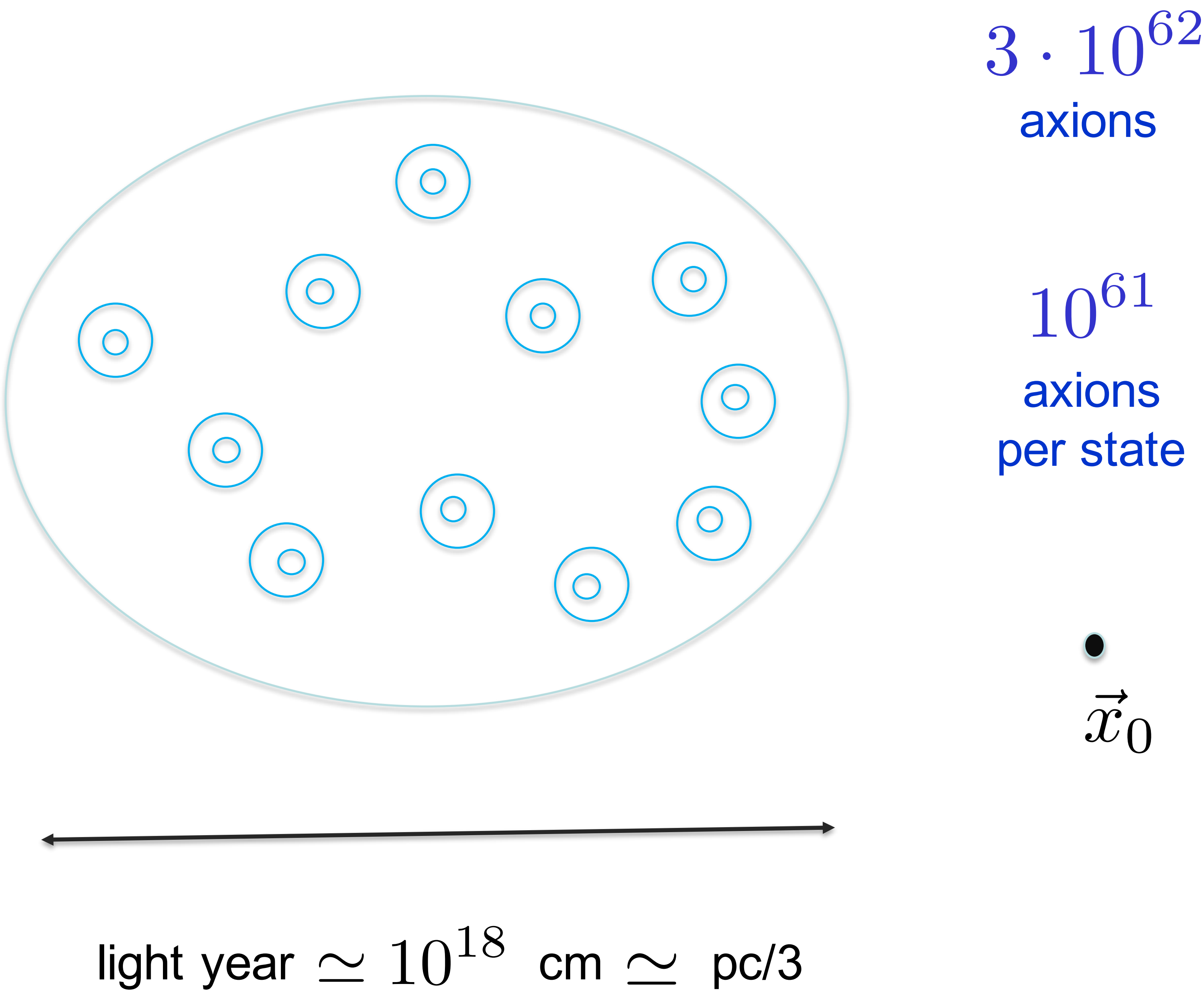


light year $\simeq 10^{18}$ cm $\simeq$ pc/3



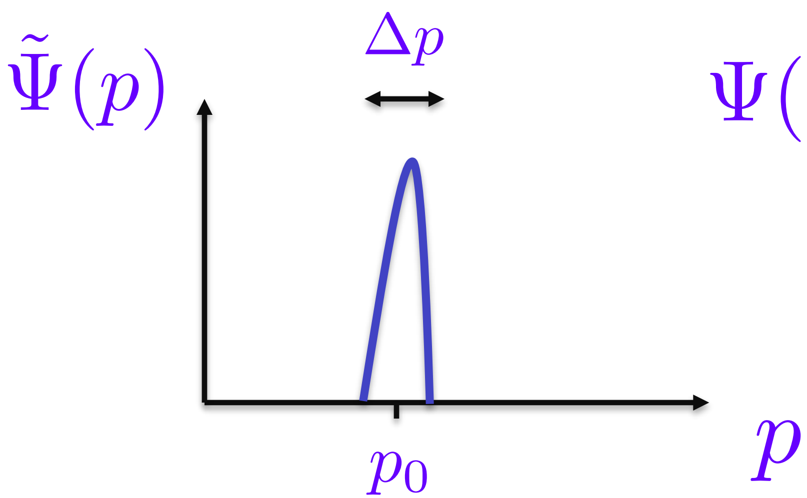
$3 \cdot 10^{47}$
WIMPs
$\vec{x}_0$
light year $\simeq 10^{18}$ cm $\simeq$ pc/3


$3 \cdot 10^{47}$
WIMPs
10^{-13}
WIMPs
per state
$\vec{x}_0$
light year $\simeq 10^{18}$ cm $\simeq$ pc/3

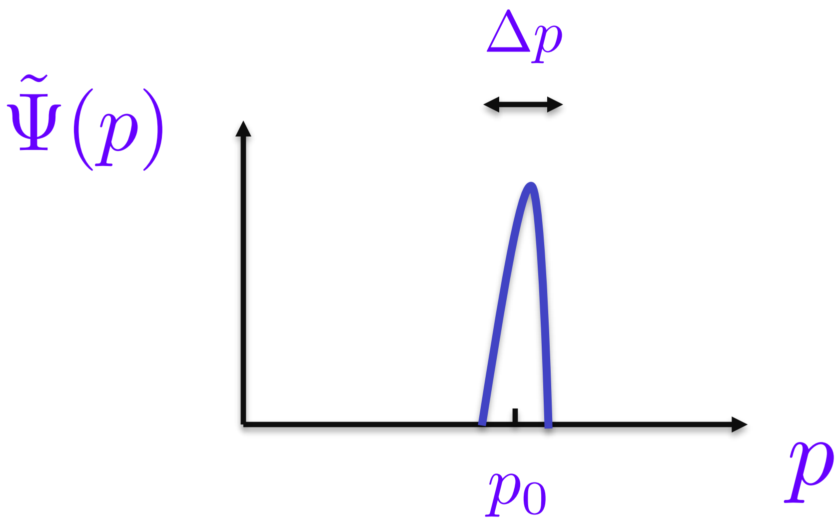

$3 \cdot 10^{62}$
axions
10^{61}
axions
per state
$\vec{x}_0$
light year $\simeq 10^{18}$ cm $\simeq$ pc/3

In the axion case, when the object size becomes of order $\frac{1}{\Delta p_a}$, only a limited number of configurations are possible

$$\ell \equiv \frac{1}{\Delta p_a}$$ = correlation length



$$\Psi(\vec{x}) = \int d^3p \; \tilde{\Psi}(\vec{p}) \; e^{i\vec{p}\cdot\vec{x}}$$



$$\Psi(\vec{x}) = \int d^3p \; \tilde{\Psi}(\vec{p}) \; e^{i\vec{p}\cdot\vec{x}}$$

$$\rho(\vec{x}) = m_a \; |\Psi(\vec{x})|^2$$

$$= \int d^3p \int d^3p' \; \Psi(\vec{p})\Psi(\vec{p}\,')^* \; e^{i(\vec{p}-\vec{p}\,')\cdot\vec{x}}$$

cannot change much over a distance $\ell = \dfrac{1}{\Delta p}$

In the axion case, the gravitational fields are necessarily large

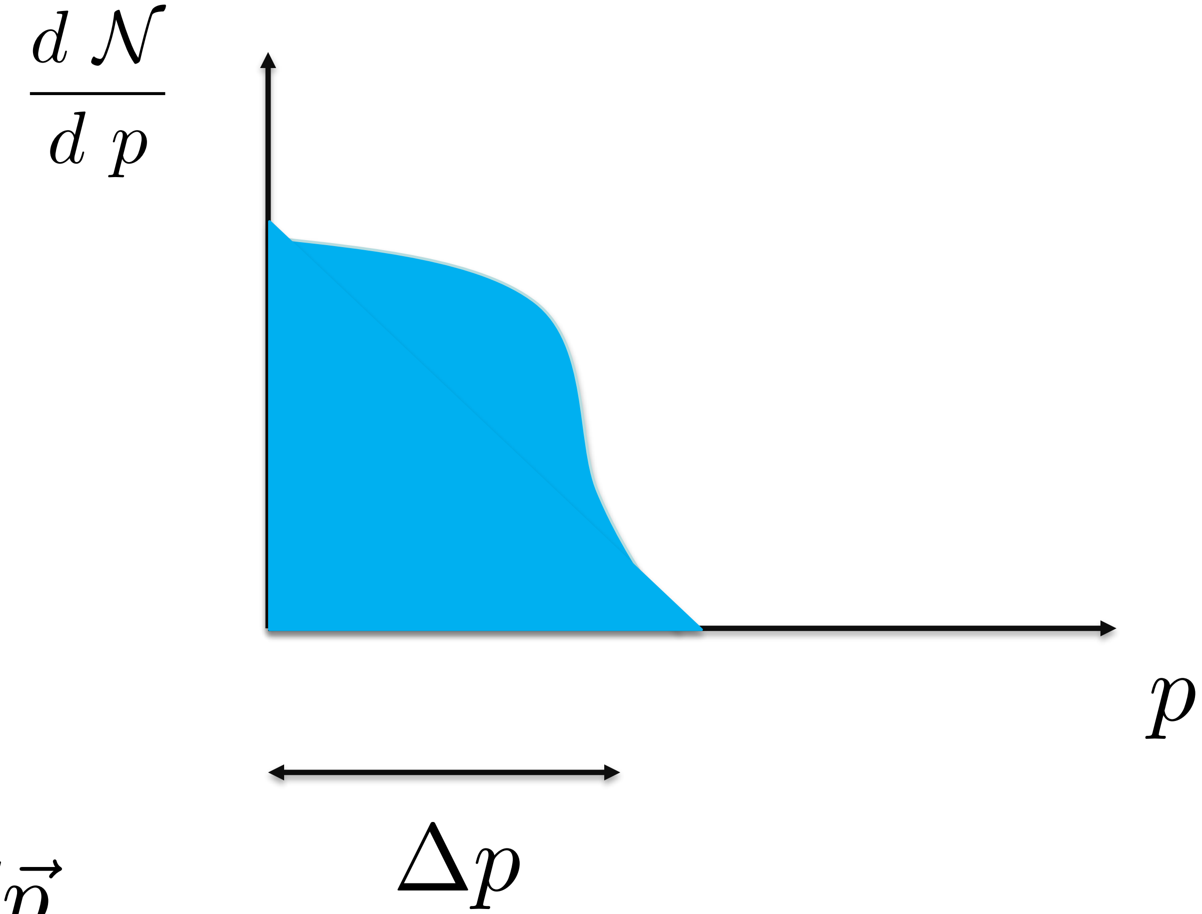
$$\delta g \sim 4\pi G \rho \, \ell$$

regardless of their average value.

For example in a homogeneous universe

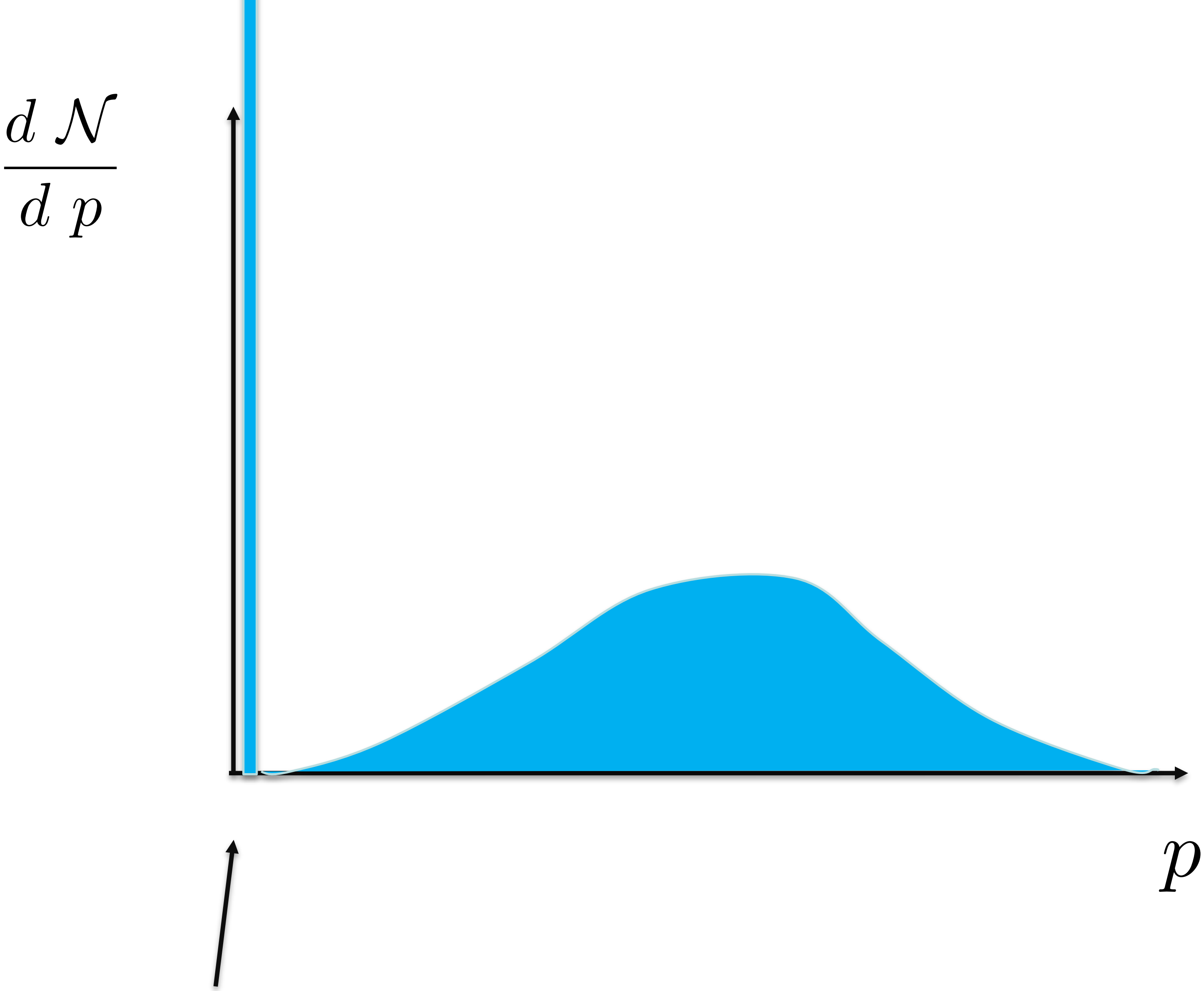
$\vec{g} = 0$ in the WIMP case

$\bar{\vec{g}} = 0$ in the axion case, but the typical gravitational field is δg



$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\tau \sim \frac{\Delta p}{m\delta g} \sim 24 \text{ sec}$$



axion Bose-Einstein condensate (contains almost all the axions)

Thermalization occurs due to gravitational interactions

PS + Q. Yang, PRL 103 (2009) 111301

$$\Gamma_g \equiv \frac{1}{\tau} \sim 4\pi G\rho\ell \; m \; \frac{1}{\Delta p} = 4\pi G\rho m\ell^2$$

compare with $\quad H(t) = \frac{1}{2t}$

During the QCD phase transition

$$\frac{\Gamma_g(t_1)}{H(t_1)} \sim 4 \cdot 10^{-7} \left(\frac{10^{-5}\ \mathrm{eV}}{m_a} \right)^{\frac{2}{3}}$$

but

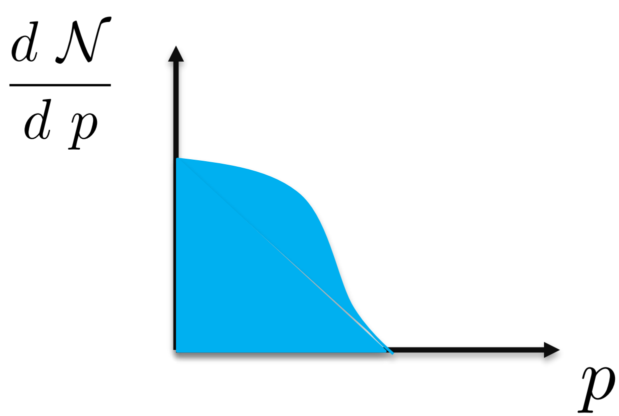
$$\Gamma_g \sim 4\pi G n m^2 \ell^2 \propto n\ \ell^2 \propto a(t)^{-1}$$

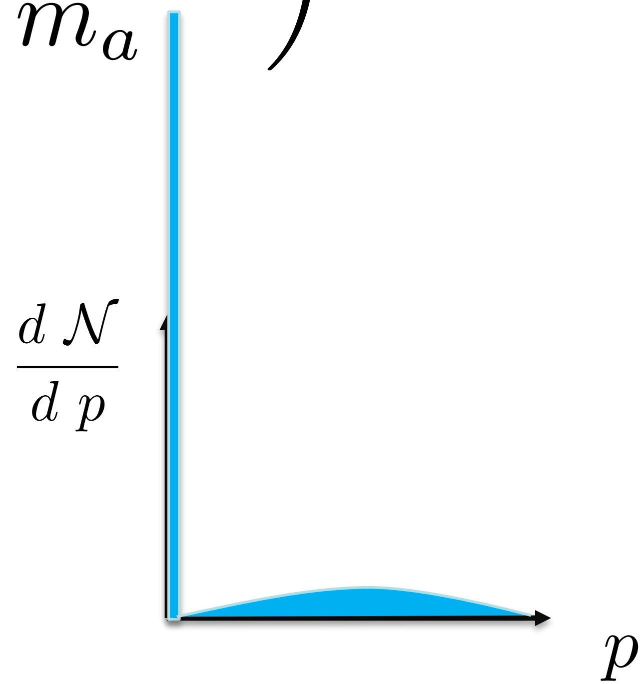
whereas

$$H = \frac{1}{2t} \sim a(t)^{-2}$$

Gravitational interactions thermalize the axions and cause them to form a BEC when the photon temperature

$$T_\gamma \sim 400 \text{ eV} \left(\frac{10^{-5} \text{ eV}}{m_a} \right)^{\frac{1}{2}}$$





How does axion dark matter differ from WIMP dark matter?

In the linear regime of evolution of density perturbations, before multi-streaming and caustic formation

WIMP dark matter

$$\partial_t \, n(\vec{x},t) + \vec{\nabla} \cdot (n(\vec{x},t)\vec{v}(\vec{x},t)) = 0$$

$$\partial_t \vec{v} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\vec{\nabla} \Phi(\vec{x},t)$$

Newtonian gravitational potential

If all axions are in the same state and remain in that state

$$\Psi(\vec{x}, t)$$

$$\partial_t \; n(\vec{x}, t) + \vec{\nabla} \cdot (n(\vec{x}, t)\vec{v}(\vec{x}, t)) = 0$$

$$\Psi(\vec{x}, t) = A(\vec{x}, t) \; e^{i\beta(\vec{x}, t)}$$

$$n(\vec{x}, t) = A(\vec{x}, t)^2$$

$$\vec{v}(\vec{x}, t) = \frac{1}{m}\vec{\nabla}\beta(\vec{x}, t)$$

$$\partial_t \vec{v} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\vec{\nabla} \Phi(\vec{x}, t) - \vec{\nabla} q(\vec{x}, t)$$

$$q(\vec{x}, t) = -\frac{1}{m^2} \frac{\nabla^2 \sqrt{n(\vec{x}, t)}}{\sqrt{n(\vec{x}, t)}}$$

In the absence of gravity,
a packet of axions initially at rest will tend to spread,

whereas the analogous packet of WIMPs just sits there

the axion dark matter fluid has a Jeans length

$$\lambda_{\mathrm{J}} = (16\pi G\rho m^2)^{-\frac{1}{4}}$$

$$\simeq \; 10^{14}\ \mathrm{cm} \left(\frac{10^{-5}\ \mathrm{eV}}{m}\right)^{\frac{1}{2}} \left(\frac{\rho}{10^{-29}\ \mathrm{gr/cc}}\right)^{\frac{1}{4}}$$

M.Y. Khlopov, B.A. Malomed and
Y.B. Zeldovich, MNRAS 215 (1985) 575

On time scales larger than

$$\tau \sim \frac{1}{4\pi G\rho \; m \; \ell^2}$$

the axions thermalize, i.e. they move from one state to another.

Vorticity can be generated and is expected to be generated because the lowest energy state for given total angular momentum is a state of rigid rotation in the angular variables.

谢谢

Thank you