Quantum Gravity and the Swampland

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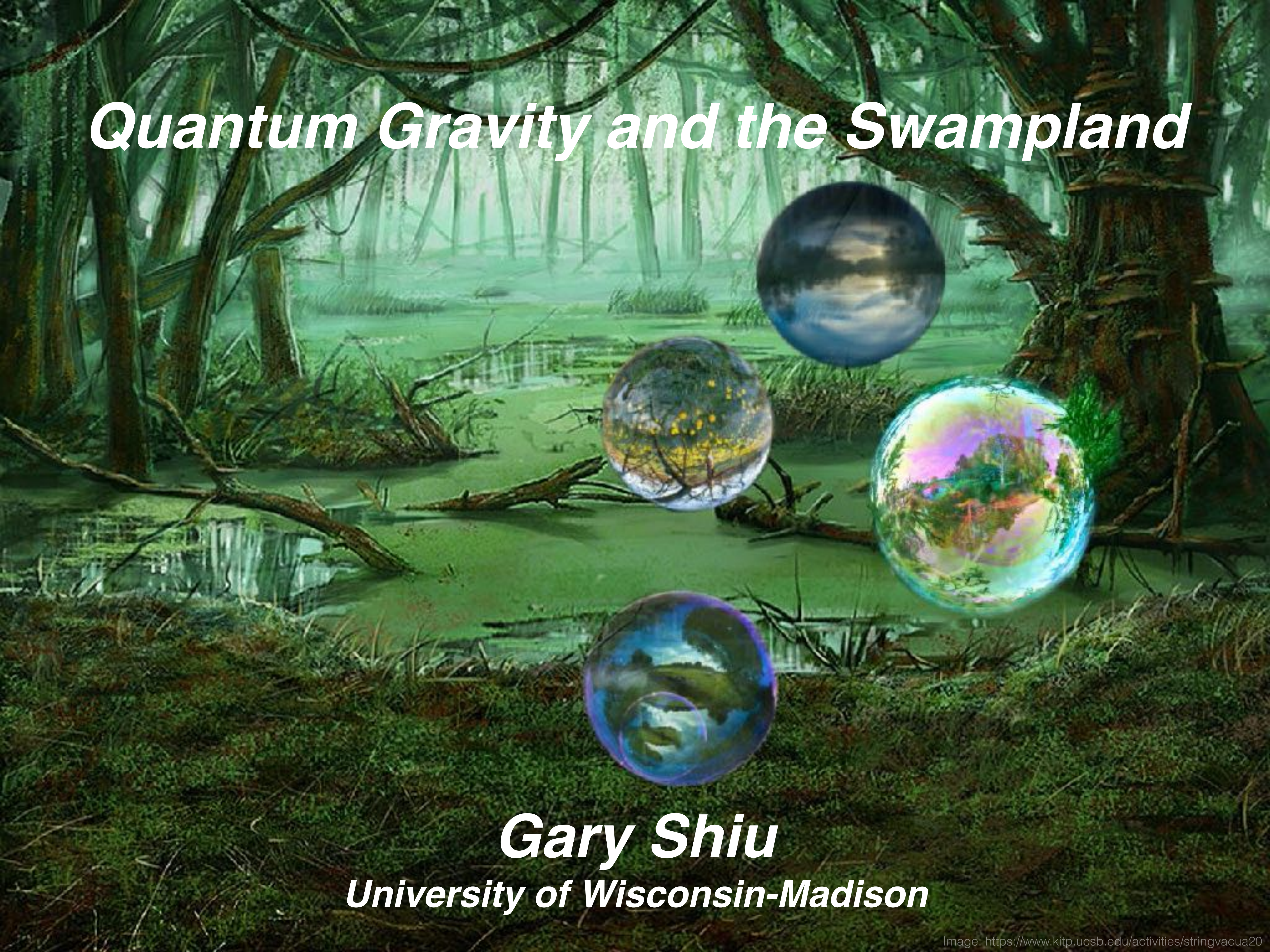


Image: https://www.kitp.ucsb.edu/activities/stringvacua20

Quantum Field Theory Meets Gravity

- Initial excitement in string theory was not only that it offers a framework for QFT and gravity to meet but also due to its sense of **uniqueness**.
- Anomaly cancellation **[Green, Schwarz, '84]**: $E_8 \times E_8$, SO(32), $U(1)^{496}$ and $E_8 \times U(1)^{248}$ gauge groups, the former 2 are realized by the **heterotic string** **[Gross, Harvey, Martinec, Rohm,'85].**
- **Calabi-Yau compactification** **[Candelas, Horowitz, Strominger, Witten, '85]:**

TABLE 1
Known examples of six (real)-dimensional manifolds with SU(3) holonomy together with some of their properties

Manifold	χ	$b_{1,1}$	$b_{2,1}$	Known holomorphic discrete symmetries that act freely	Number of zero modes
$Y_{(4;5)}$	-200	1	101	$Z_5 \times Z_5$	203
$Y_{(5;4,2)}$	-176	1	89		179
$Y_{(5;3,3)}$	-144	1	73	$Z_3 \times Z_3$	147
$Y_{(6;3,2,2)}$	-144	1	73		147
$Y_{(7;2,2,2,2)}$	-128	1	65	$Z_2 \times Z_2 \times Z_2; Z_8$	131
Y	-8	1	5	none	11
Z	$+72$	36	0		36

All these manifolds have $b_0 = 1$, $b_1 = 0$, $b_{0,2} = b_{2,0} = 0$, $b_{2,1} = b_{1,2}$ and $b_{0,3} = b_{3,0} = 1$.

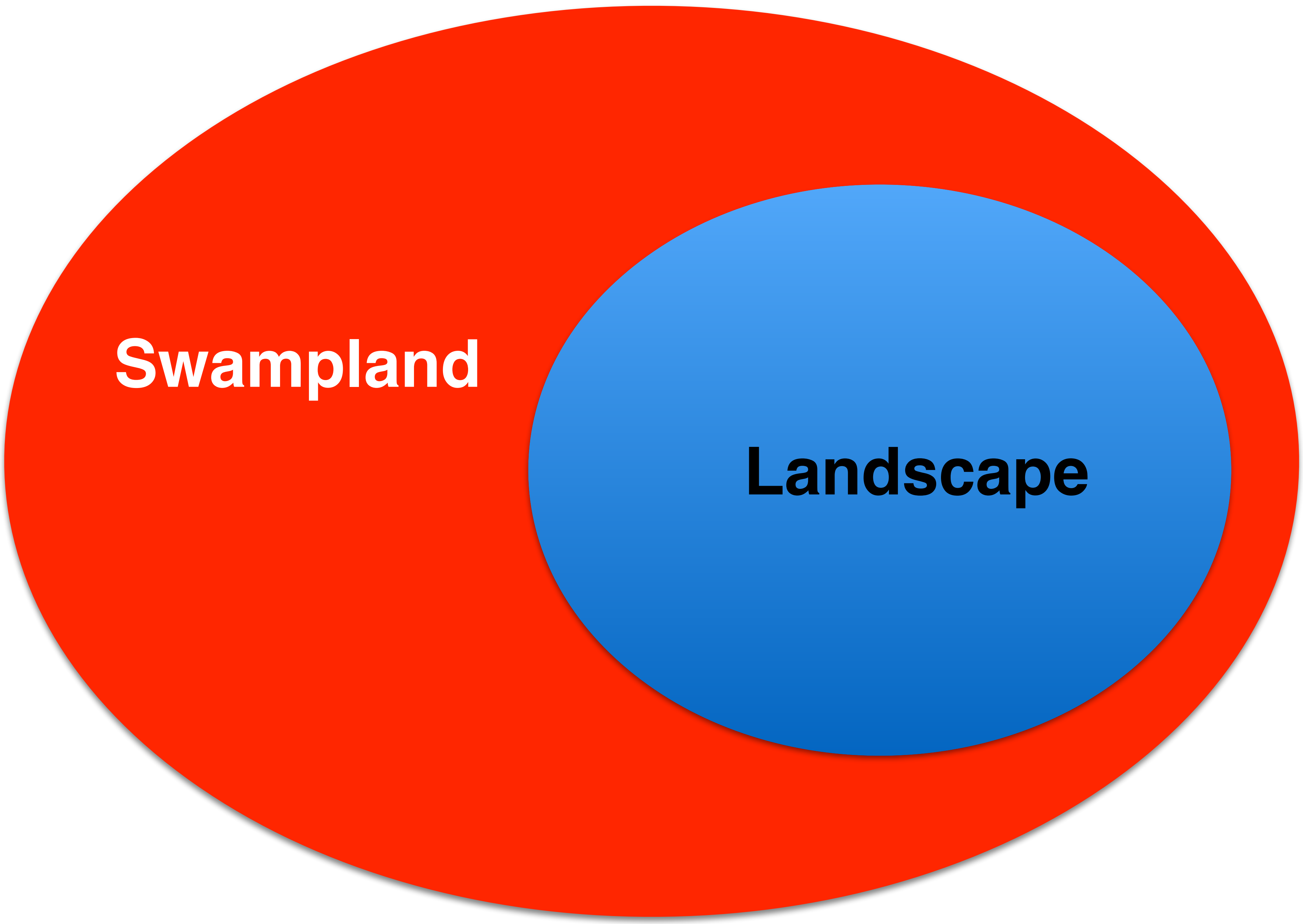
String Theory Landscape

String Theory Landscape

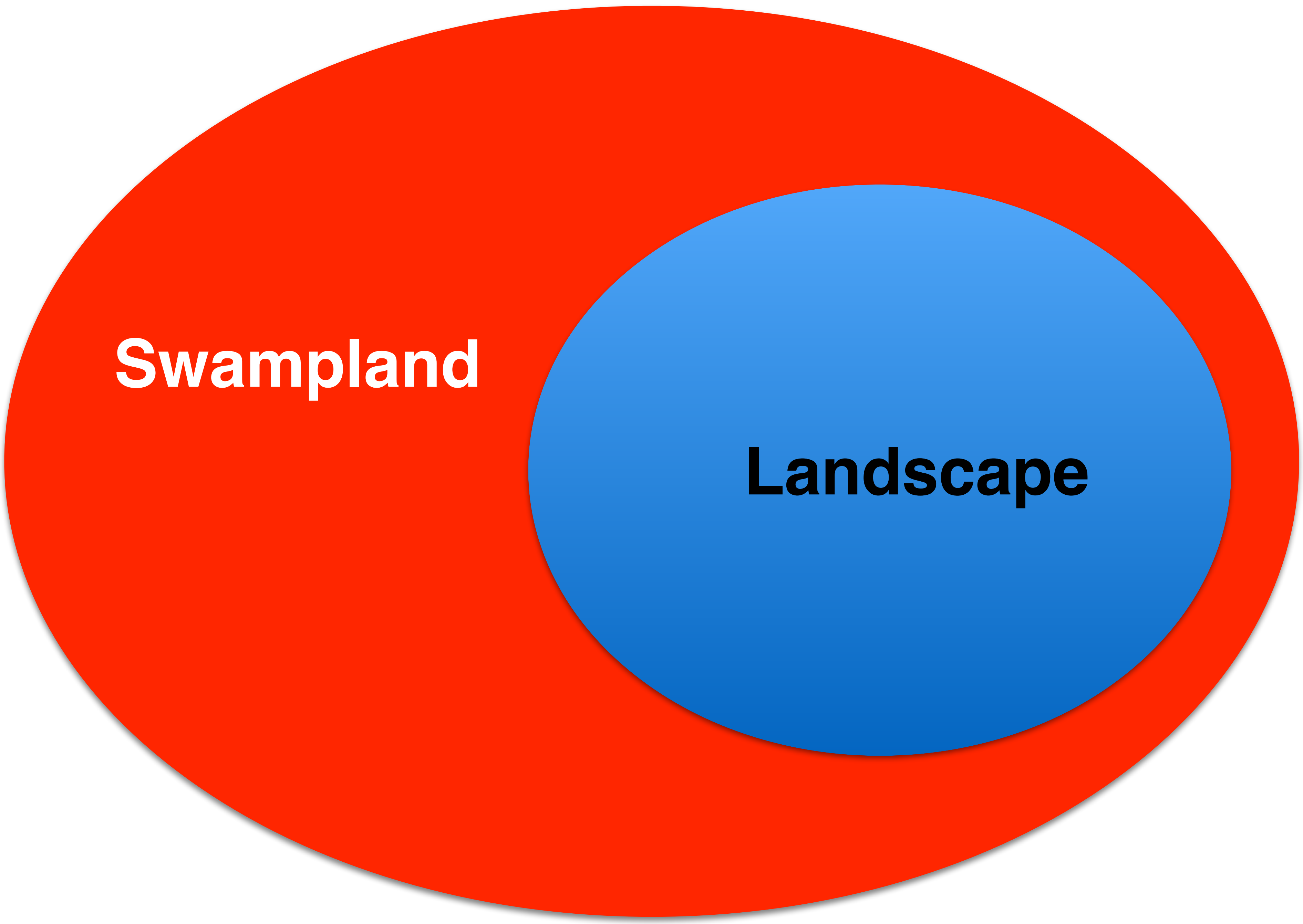
String Theory Landscape

Landscape vs Swampland

Landscape vs Swampland



Landscape vs Swampland



What properties delineate the landscape from the swampland?

What are the phenomenological implications?

Swampland Criteria

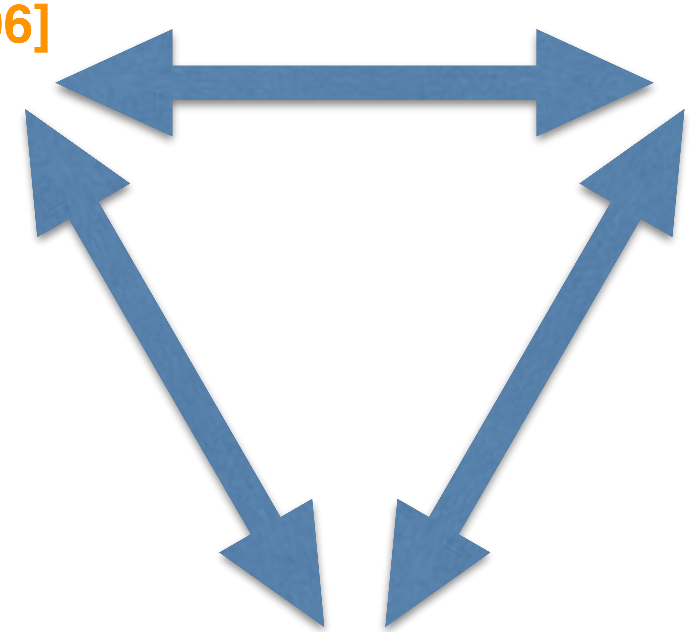
These criteria do not follow from purely low-energy EFT considerations.
Why are they necessary for consistency of quantum gravitational theories?

Weak Gravity Conjecture

[Arkani-Hamed, Motl, Nicolis, Vafa. ’06]

Distance Conjecture

[Ooguri, Vafa. ’06]



dS Conjecture

[Obied, Ooguri, Spodyneiko, Vafa,’18]; [Ooguri, Palti, GS, Vafa, ’18]

Inconsistencies can sometimes be probed by a string/brane, even though they are not apparent from a particle viewpoint **[Kim, GS, Vafa, ’19]**

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa ‘06

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

- The conjecture:

"Gravity is the Weakest Force"

- This is a scale-dependent statement, but as we'll see, the WGC comes with a UV cutoff Λ (magnetic WGC).

- For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m} M_P \geq \text{“}1\text{”} \equiv \frac{Q_{Ext}}{M_{Ext}} M_P$$

- This implies extremal BHs can decay, even though the remnant problem (which applies to global symmetry) does not arise here.

- Applying the WGC to magnetically charged states imply:

$$q_{mag} \sim 1/g, \quad m_{mag} \sim \Lambda/g^2 \quad \Rightarrow \quad \Lambda \lesssim g(\Lambda) M_P$$

WGC for p-form Symmetry

- One can generalize the WGC for 1-form gauge fields to the WGC for (p+1)-form gauge fields which couple to p-branes:

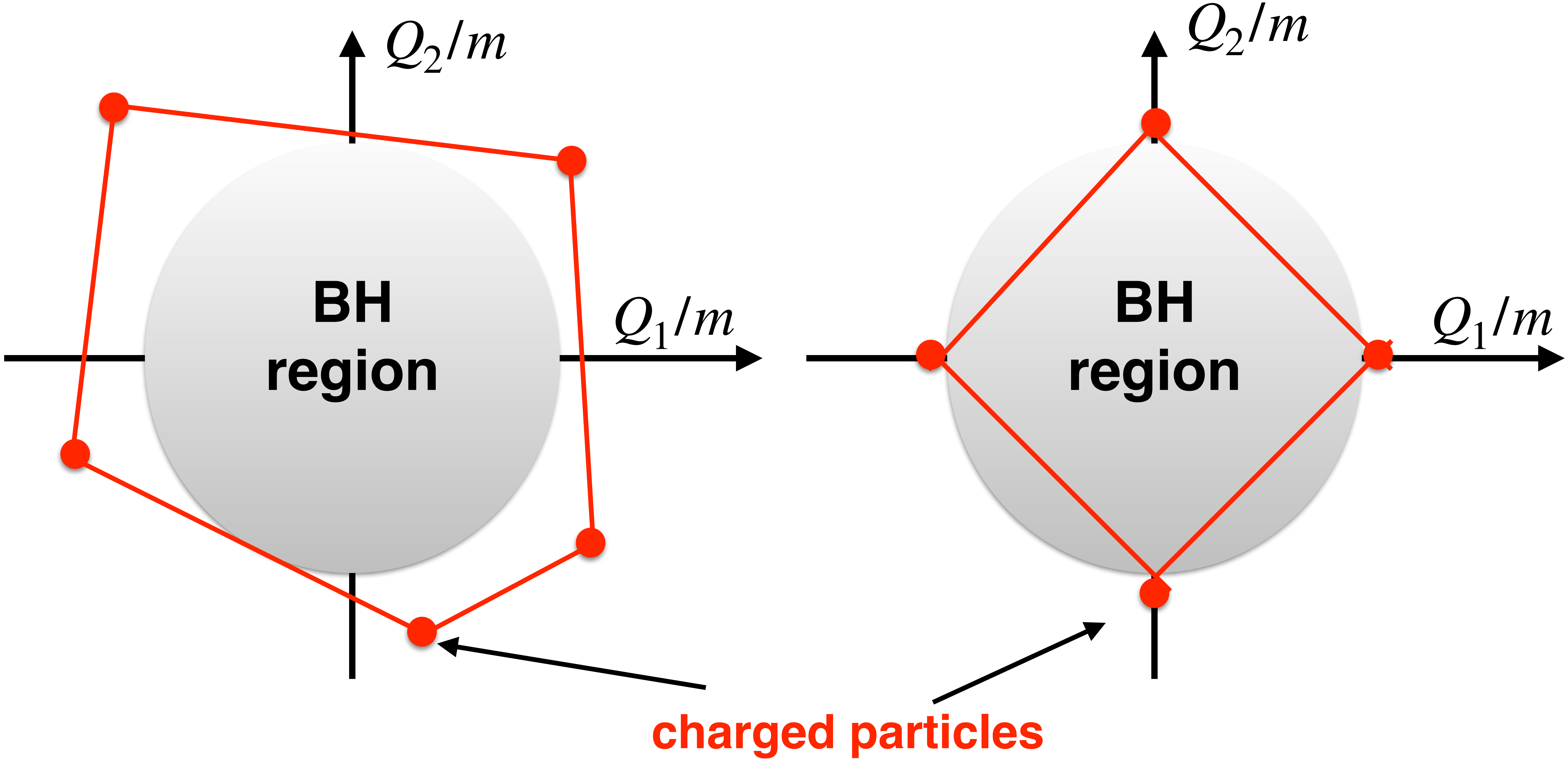
$$\frac{Q_p}{T_p} \geq \left(\frac{Q_p}{T_p}\right)_{\rm Ext}$$

- The 0-form gauge field (axion) case **(-1 form symmetry)** is most interesting (axion inflation) but subtle as the “branes” that couple to it are **instantons**.
- Obtaining an axion by duality **[Brown, Cottrell, GS, Soler, ‘15]** or dimensional reduction **[Heidenreich, Reece, Rudelius, ’16]** suggests that the above inequality can indeed be extrapolated to:

$$f \cdot S_{\rm inst} \leq \mathcal{O}(1) M_P$$

- Attempt to give a more direct argument for the -1 form WGC **[Andriolo, Huang, Noumi, Ooguri, GS, ’20]** : the “extremal bound” is set by the action-to-charge ratio of the macroscopic semi-wormhole.

Convex Hull Condition

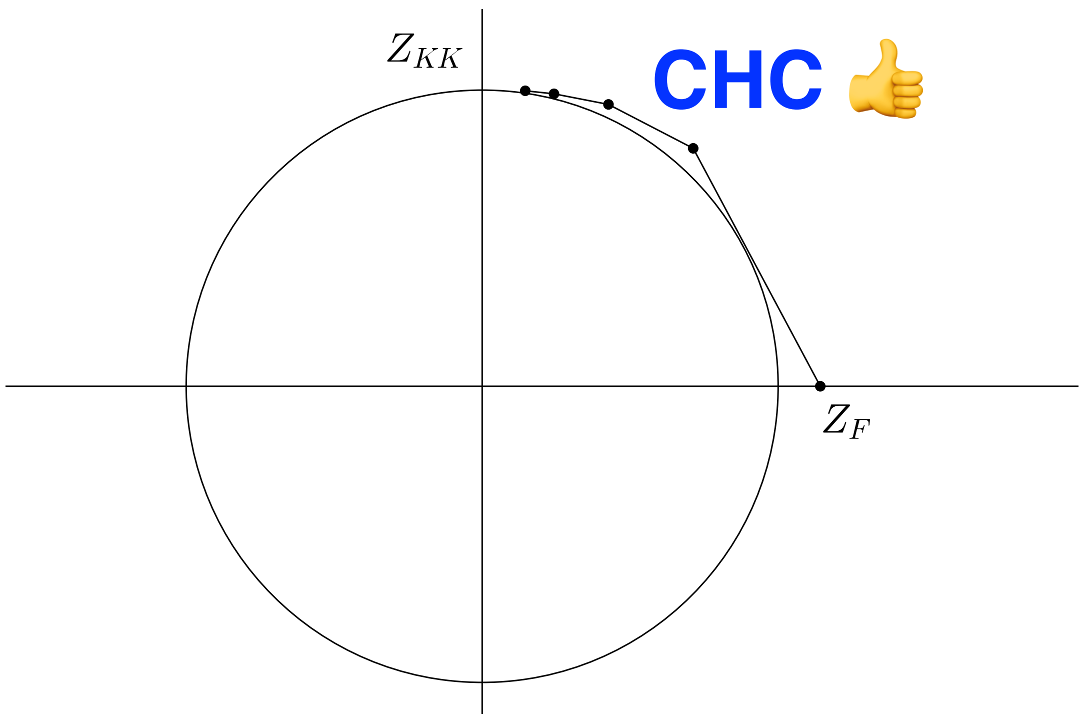


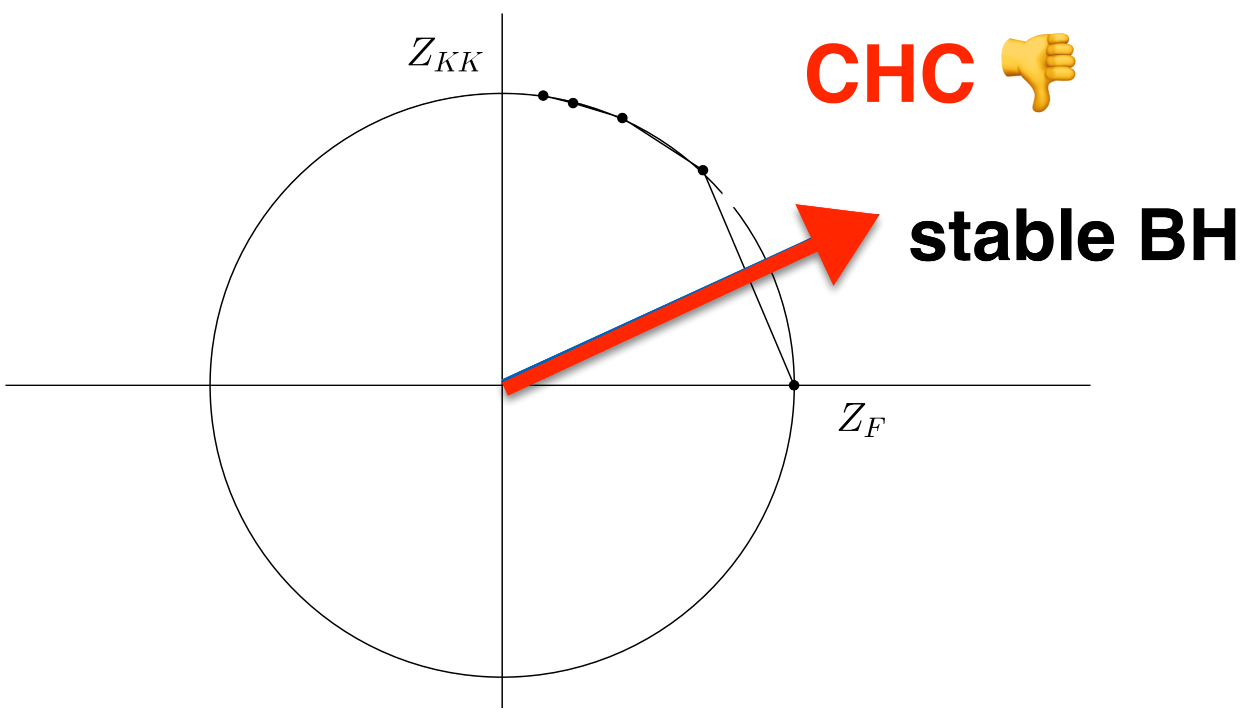
[Cheung, Remmen, '14]

Constraints on multi-axion inflation models [Brown, Cottrell, GS, Soler, '15]

Tower/Sub-Lattice WGC

- Compactifying a theory on a circle gives rise to an additional $U(1)_{KK}$. Apply the WGC to black holes with general charges.
- Infinite tower of (super)extremal KK states. Charge-to-mass ratio depends on the radius:

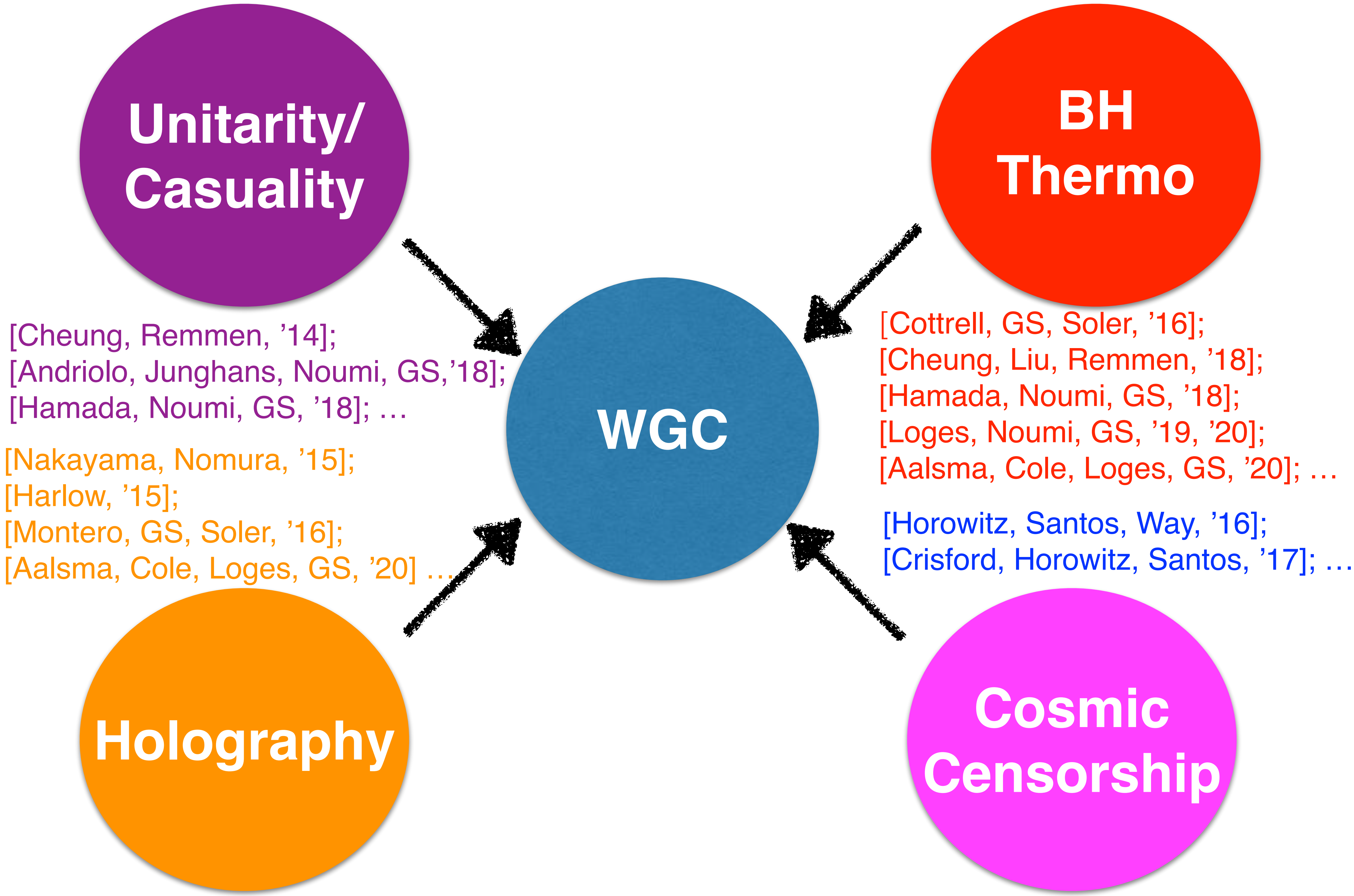




- Convex hull not guaranteed to contain the BH region. This motivates a stronger version of the WGC known as Tower/Sub-Lattice WGC **[Andriolo,Junghans, Noumi, GS];[Heidenreich, Reece, Rudelius];[Montero, GS, Soler]**
- Modern perspective on the magnetic WGC (emergence) also motivates this tower version **[Grimm, Palti, Valenzuela]**

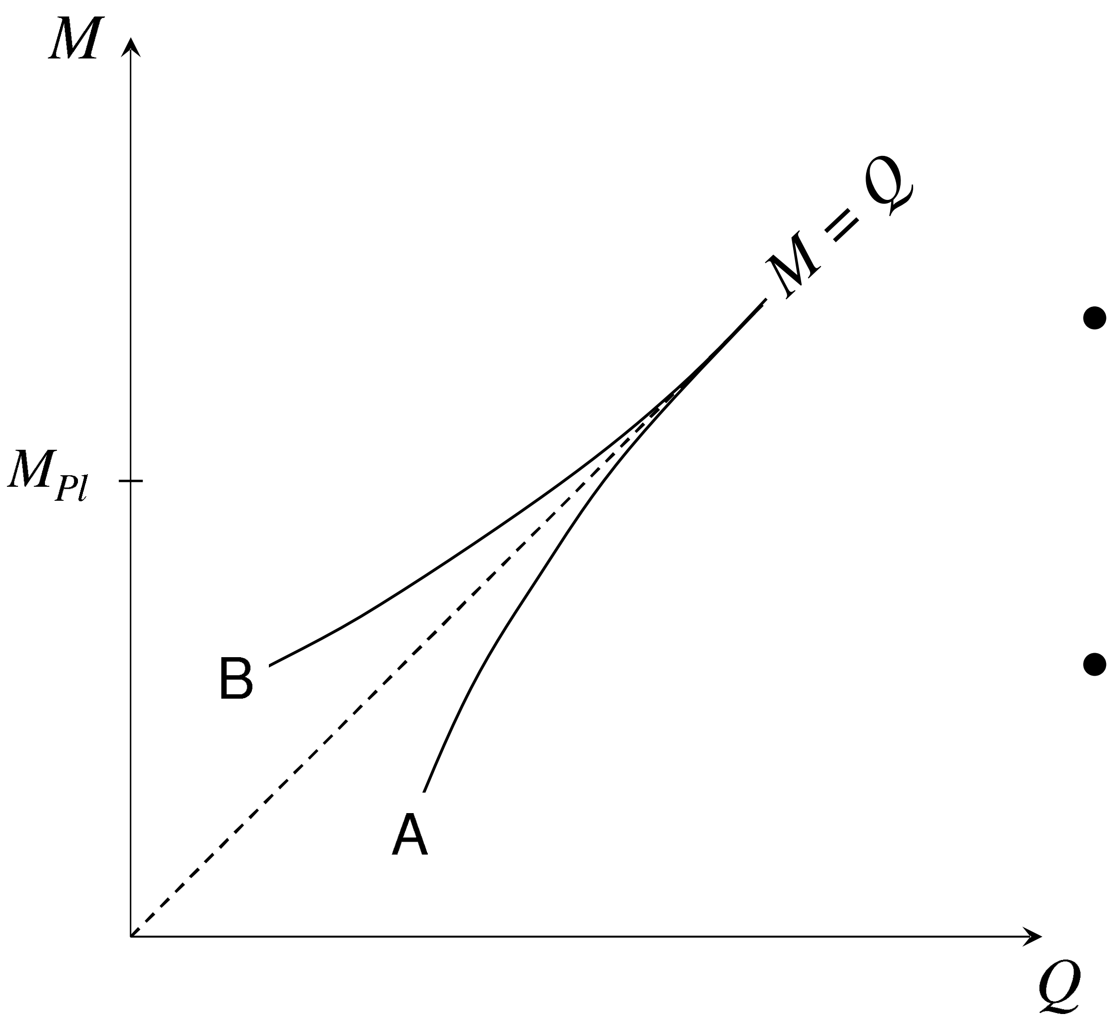
Evidence for the WGC

Evidence for the WGC



Extremality of Black Holes

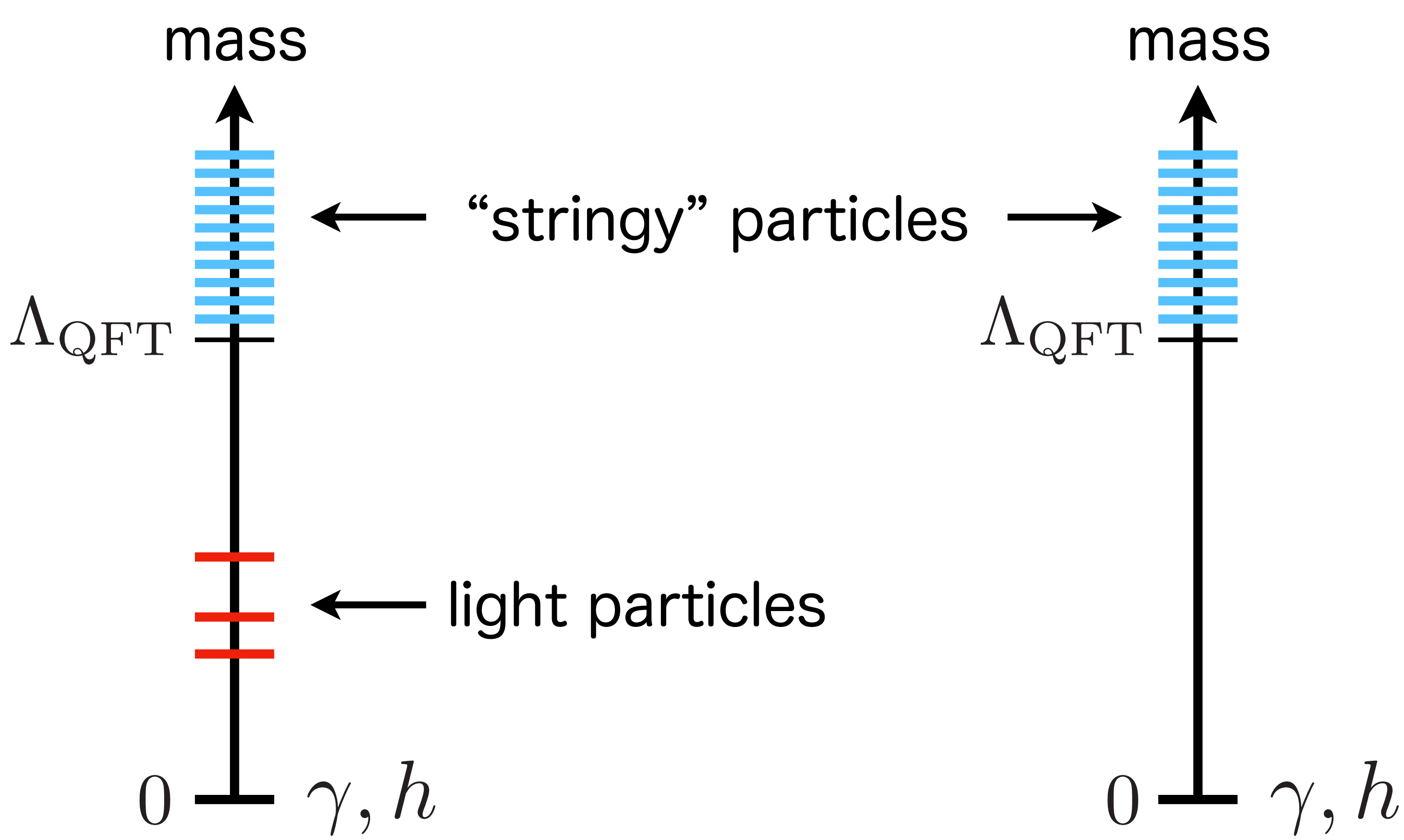
- The mild form of the WGC requires only ***some*** state for an extremal BH to decay to.
- **Can an extremal BH satisfy the WGC?**



- Higher derivative corrections can make extremal BHs lighter than the **classical bound** Q=M
- Demonstrated to be the case for 4D heterotic extremal BHs. **[Kats, Motl, Padi, '06]**
- We showed that this behavior (A) follows from unitarity (at least for some classes of theories). **[Hamada, Noumi, GS]**

WGC from Unitarity and Causality

- We assume a ***weakly coupled UV completion*** at scale Λ_{QFT}. Our proof for the strict WGC bound applies to at least two classes of theories:



- Theories with ***light*** (compared with Λ_{QFT})***, neutral i) parity-even scalars*** (e.g., dilaton, moduli), or **ii) spin ≥ 2 particles**
- ***UV completion*** where the photon & the graviton are accompanied by different sets of Regge states (as in open string theory).

Higher Derivative Corrections

- In the IR, the BH dynamics is described by an EFT of photon & graviton.
- In D=4, the general effective action up to 4-derivative operators (assume parity invariance for simplicity):

$$S = \int d^4x\sqrt{-g}\left[\frac{2M_{\rm Pl}^2}{4}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \Delta\mathcal{L}\right]$$

where

$$\begin{aligned}\Delta\mathcal{L} = \quad & c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\ & + c_4 R F_{\mu\nu}F^{\mu\nu} + c_5 R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + c_6 R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} \\ & + c_7 F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + c_8 F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}.\end{aligned}$$

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- In D=4, the general effective action up to 4-derivative operators (assume parity invariance for simplicity):

$$S = \int d^4x\sqrt{-g}\Big[\frac{2M_{\rm Pl}^2}{4}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_1}{4M_{\rm Pl}^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{\alpha_2}{4M_{\rm Pl}^4}(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2 + \frac{\alpha_3}{2M_{\rm Pl}^2}F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma}\Big]$$

by field redefinition. Here, $W^{\mu\nu\rho\sigma}$ is the **Weyl tensor**:

$$R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma} + \frac{1}{2}\left(g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}\right) - \frac{1}{3}Rg_{\mu[\rho}g_{\sigma]\nu}$$

Extremality Condition

- The higher derivative operators modify the BH solutions, so the charge-to-mass ratio of an extremal BH is corrected:

$$z = \frac{\sqrt{2} M_{\mathrm{Pl}} |Q|}{M} = 1 + \frac{2}{5} \frac{(4\pi)^2}{Q^2} (2\alpha_1 - \alpha_3)$$

[Kats, Motl, Padi, '06]

applicable when the BH is sufficiently heavy: $M^2 \sim Q^2 M_{\mathrm{Pl}}^2 \gg \alpha_i M_{\mathrm{Pl}}^2$

because extremal BHs in Einstein-Maxwell theory satisfy:

$$R \sim M_{\mathrm{Pl}}^4 / M^2 \text{ and } F^2 \sim M_{\mathrm{Pl}}^6 / M^2$$

- Proving the WGC (mild form) amounts to showing:

$$2\alpha_1 - \alpha_3 \geq 0\,.$$

so large extremal BHs can decay into smaller extremal BHs.

Sketch of the Proof: Step 1

[Hamada, Noumi, GS]

- We first show that for the aforementioned theories, **causality** implies

$$|\alpha_1| \gg |\alpha_3|$$

because α_3 leads to causality violation and an infinite tower of massive higher spin states is required to UV complete the EFT at tree-level **[Camanho, Edelstein, Maldacena, Zhiboedov].**

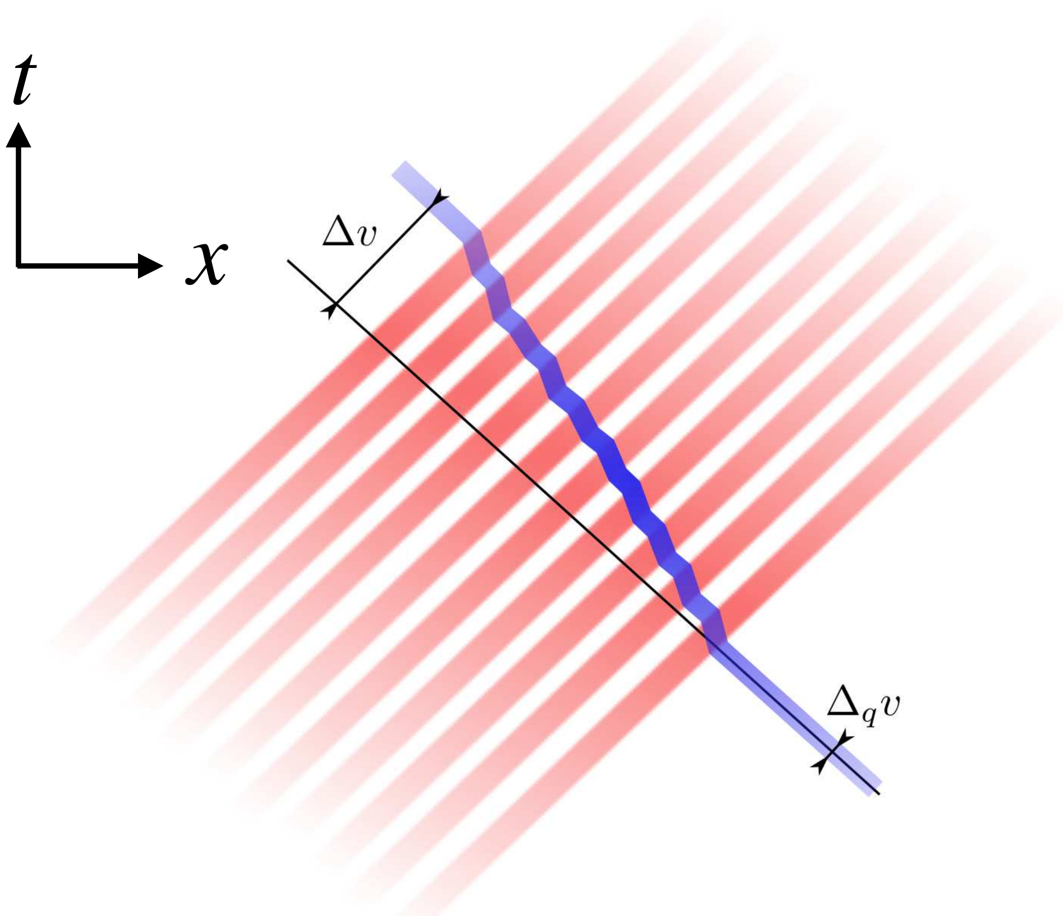


fig: Camanho et al '14

phase shift of photon propagation:

$$\delta \sim s\left(\ln(L_{\rm IR}/b) \pm \frac{|\alpha_3|}{b^2} + \ldots\right)$$

time delay in GR

helicity dependent phase shit

b : impact parameter $L_{\rm IR}$: IR cutoff

Sketch of the Proof: Step 1

[Hamada, Noumi, GS]

- **Time advancement** if $b^2 \ln(L/b) \ll |\alpha_3|$
- Phase shift generated by spin J is $\delta \sim s^{J-1}$. A finite # of higher spin particles does not help → infinite tower of higher spin states.
- **Causality violation** above energy scale $\Lambda_{\rm QFT} \lesssim \dfrac{M_P}{\sqrt{|\alpha_3|}}$
- Integrating out light neutral scalars does not give significant contributions to α_3 and so $|\alpha_1| \gg |\alpha_3|$
- If there are different Regge towers as in theories with open strings:

$$\alpha_{1,2,3}^{\rm closed} \sim \frac{M_{\rm Pl}^2}{M_s^2} \quad \ll \quad \alpha_{1,2}^{\rm open} \sim \frac{M_{\rm Pl}^2}{g_s M_s^2}\,, \qquad g_{\rm open} \sim \sqrt{g_s} \gg g_s$$

- If there are light fields or different Regge towers, α_3 is **subdominant** compared with the causality preserving terms α_1 and α_2.

Sketch of the Proof: Step 2

[Hamada, Noumi, GS]

- The forward limit t→0 of γγ scattering for the aforementioned theories:

$$\mathscr{M}^{1234}(s) = \sum_n \left[\frac{g_{h_1h_2n}\, g_{\bar{h}_3\bar{h}_4n}}{m_n^2 - s} P^{1234}_{\mathbf{s}_n}(1) + \frac{g_{h_1h_4n}\, g_{\bar{h}_3\bar{h}_2n}}{m_n^2 + s} P^{1432}_{\mathbf{s}_n}(1) \right] + \text{analytic}$$

Spinning polynomials

[Arkani-Hamed, Huang, Huang, ’17]

Froissart bound $a_n + b_n s$

- The higher derivative operator parametrized by α_1 leads to:

$$\alpha_1 (F_{\mu\nu} F^{\mu\mu})^2 \quad \Rightarrow \quad \mathscr{M} \sim \alpha_1 s^2$$

Unitarity ⇒ $\alpha_1 > 0$

[to be precise , gravitational positivity bound]

extremal

$Q = M$

BH

$Q - q \leq M - m$

● a state $q \geq m$ **can be an extremal BH!**

More General Black Holes

- We found an **entropy-extremality relation** **[Hamada, Noumi, GS]** which implies that in theories satisfying the WGC, $z_{ext}>1 \Leftrightarrow \Delta S>0$.
- However, for Einstein-Maxwell-dilaton theory, positivity bounds alone do not ensure that $z_{ext}>1$ **[Loges, Noumi, GS, '19a].**

$$I=\int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R-\frac{M_{\mathrm{Pl}}^2}{2}(\partial\phi)^2-\frac{1}{4}e^{-2\lambda\phi}\big(F^2\big)\right]$$

- The leading 4-derivative operators:

$$\int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{\alpha_1}{4}e^{-6\lambda\phi}\big(F^2\big)^2+\frac{\alpha_2}{4}e^{-6\lambda\phi}\big(F\widetilde{F}\big)^2+\frac{\alpha_3}{2}e^{-4\lambda\phi}\big(FFW\big)+\frac{\alpha_4}{2}e^{-2\lambda\phi}(R_{\mathrm{GB}})\right.$$
$$\left.+\frac{\alpha_5}{4}e^{-2\lambda\phi}(\partial\phi)^4+\frac{\alpha_6}{4}e^{-4\lambda\phi}(\partial\phi)^2\big(F^2\big)+\frac{\alpha_7}{4}e^{-4\lambda\phi}\big(\partial\phi\partial\phi FF\big)\right],$$

modifies the extremal bound for a general dyonic black hole:

$$z_{\mathrm{ext}}=1+\frac{2}{5q_eq_m}\alpha_i\mathcal{M}_i(\zeta) \qquad\qquad \zeta\to 1 \text{ extremal}$$

The Role of Symmetries

[Loges, Noumi, GS, '20b] (also [Andriolo, Noumi, Huang, Ooguri, GS, '20])

- Unitarity requires $\alpha_7 > 0$ but $M_7(\zeta) < 0$ for all ζ so $\Delta z_{ext} < 0$.
- Such operator does not appear in isolation. In some well motivated UV complications, the set of α_i combine to give an overall $\Delta z_{ext} > 0$.
- In **[Loges, Noumi, GS, '20b]**, we examine how **symmetries** can impose additional structure on the EFT to ensure that $\Delta z_{ext} > 0$.
- Consider Einstein-Maxwell-dilaton-axion theory. We found that extra symmetries (e.g. **SL(2,R)** (S-duality) and **O(d,d)** (T-duality)) when combined with either **scattering positivity bounds** or **null energy condition**, are strong enough to ensure that $\Delta z_{ext} > 0$.
- We also explore implications of $N \geq 2$ **SUSY.** We found that the puzzling terms which give $\Delta z_{ext} < 0$ are needed to make corrections to extremality identically zero, as expected for BPS states.

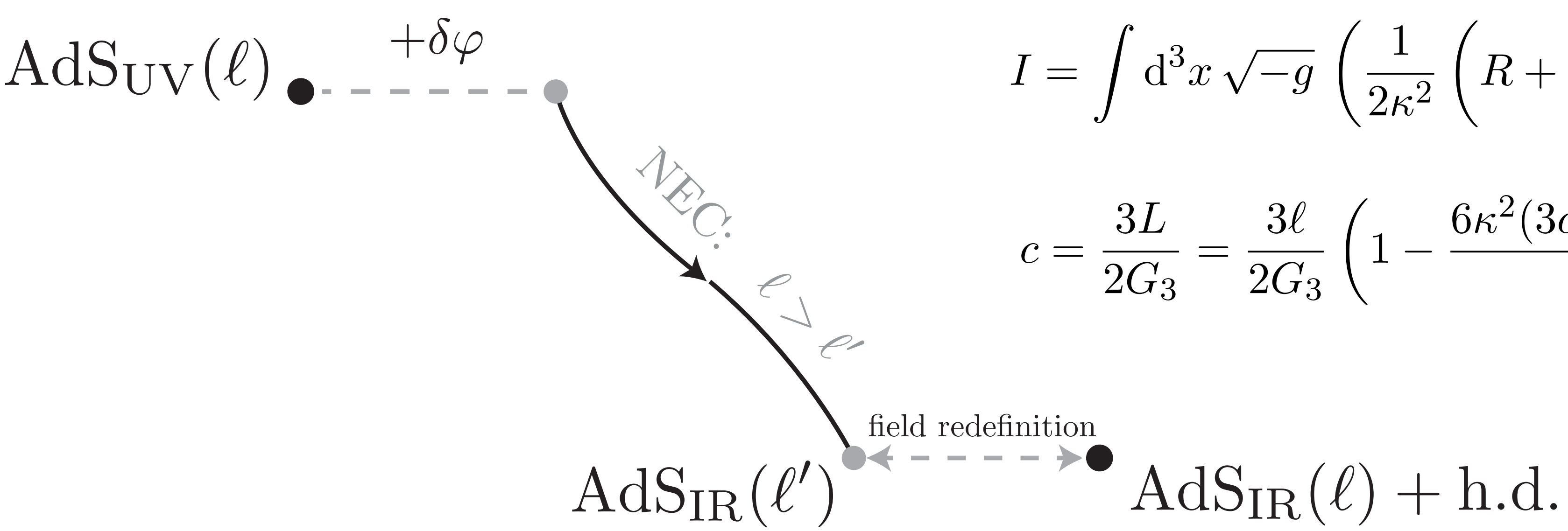
A New Spin on the WGC

[Aalsma, Cole, Loges, GS, '20] (see Loges's talk)

- Using the Iyer-Wald formalism, we reformulate the WGC **covariantly** as:

$$\int_\Sigma \mathrm{d}^{d-1}x\,\sqrt{h}\,\delta T^{\mathrm{eff}}_{ab}\,\xi^a n^b \le 0$$

- For BTZ black hole, a **spinning WGC** follows from the **c-theorem**:



$$I = \int \mathrm{d}^3x\,\sqrt{-g}\left(\frac{1}{2\kappa^2}\left(R + \frac{2}{\ell^2}\right) + \alpha_1 \ell R^2 + \alpha_2 \ell R_{ab}R^{ab}\right)$$

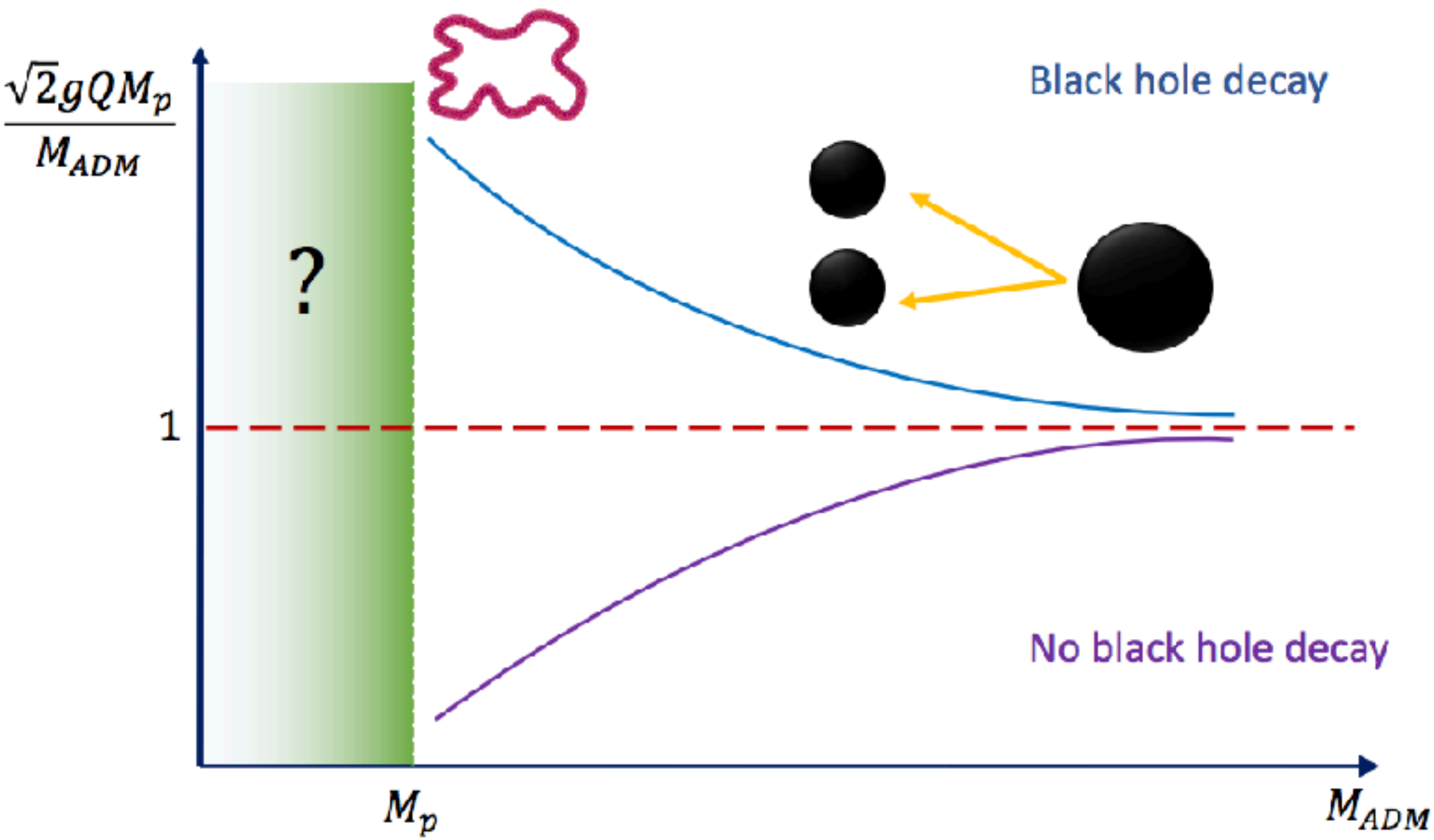
$$c = \frac{3L}{2G_3} = \frac{3\ell}{2G_3}\left(1 - \frac{6\kappa^2(3\alpha_1 + \alpha_2)}{\ell}\right)$$

- Corrected extremality bound: $\dfrac{J_3}{\ell M_3} \le 1 + \dfrac{48\pi G_3(3\alpha_1 + \alpha_2)}{\ell}$

- Rotating BHs are naturally unstable via the Penrose process; this WGC bound is a consequence of **quantum gravity (holography)**.

WGC and Modular Invariance

- In **[Aalsma, Cole, GS, '19],** we argued that for extremal BHs with a near horizon AdS_3 geometry, we can use modular invariance and anomalies to infer that there is a tower of superextremal states interpolating between perturbative string states and BHs.



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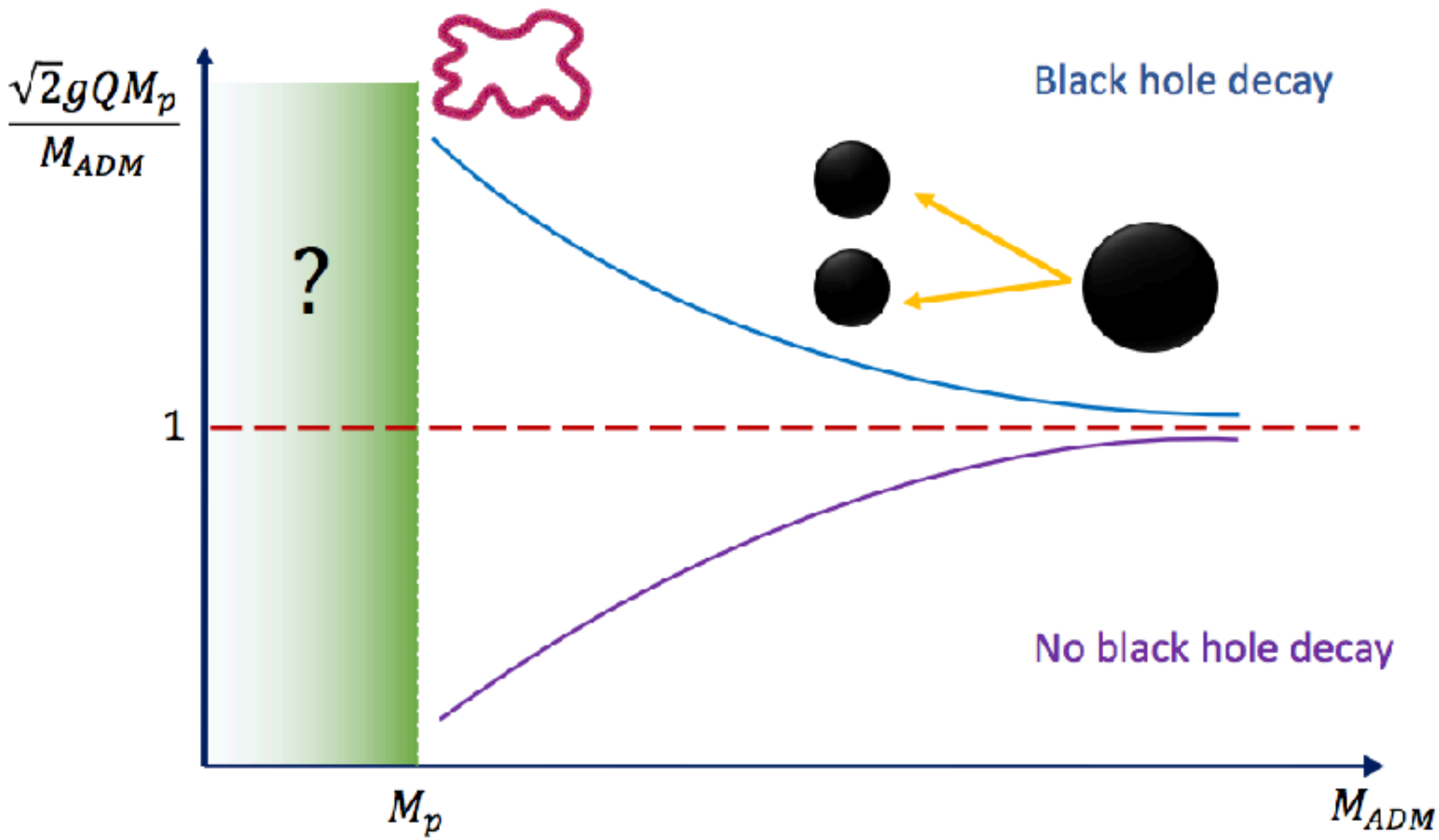
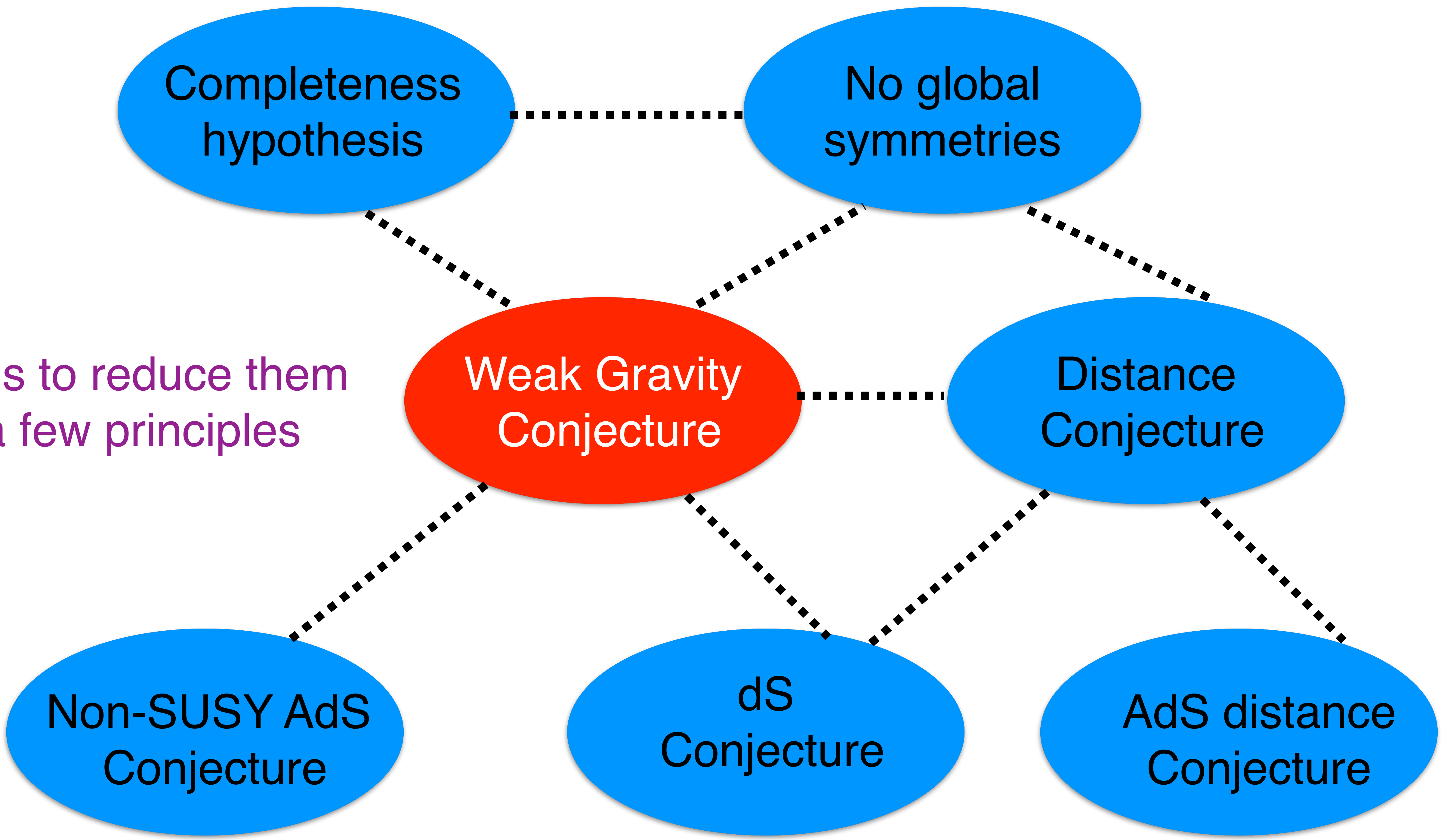


Summary

Web of Interconnected Swampland Criteria



Goal is to reduce them to a few principles