Pre-SUSY 2021: The Summer School on Supersymmetry & Unification of Fundamental Interactions

Introduction to GUT

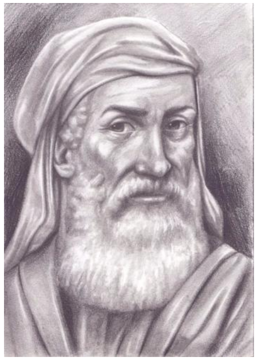
Ilia Gogoladze

University of Delaware
Newark, USA

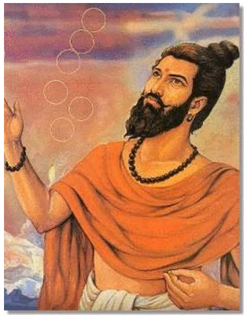
August 19 , 2021

Concept of Composition of Matter

The concept that matter is composed of discrete units & cannot be divided into arbitrarily tiny quantities has been around for millennia



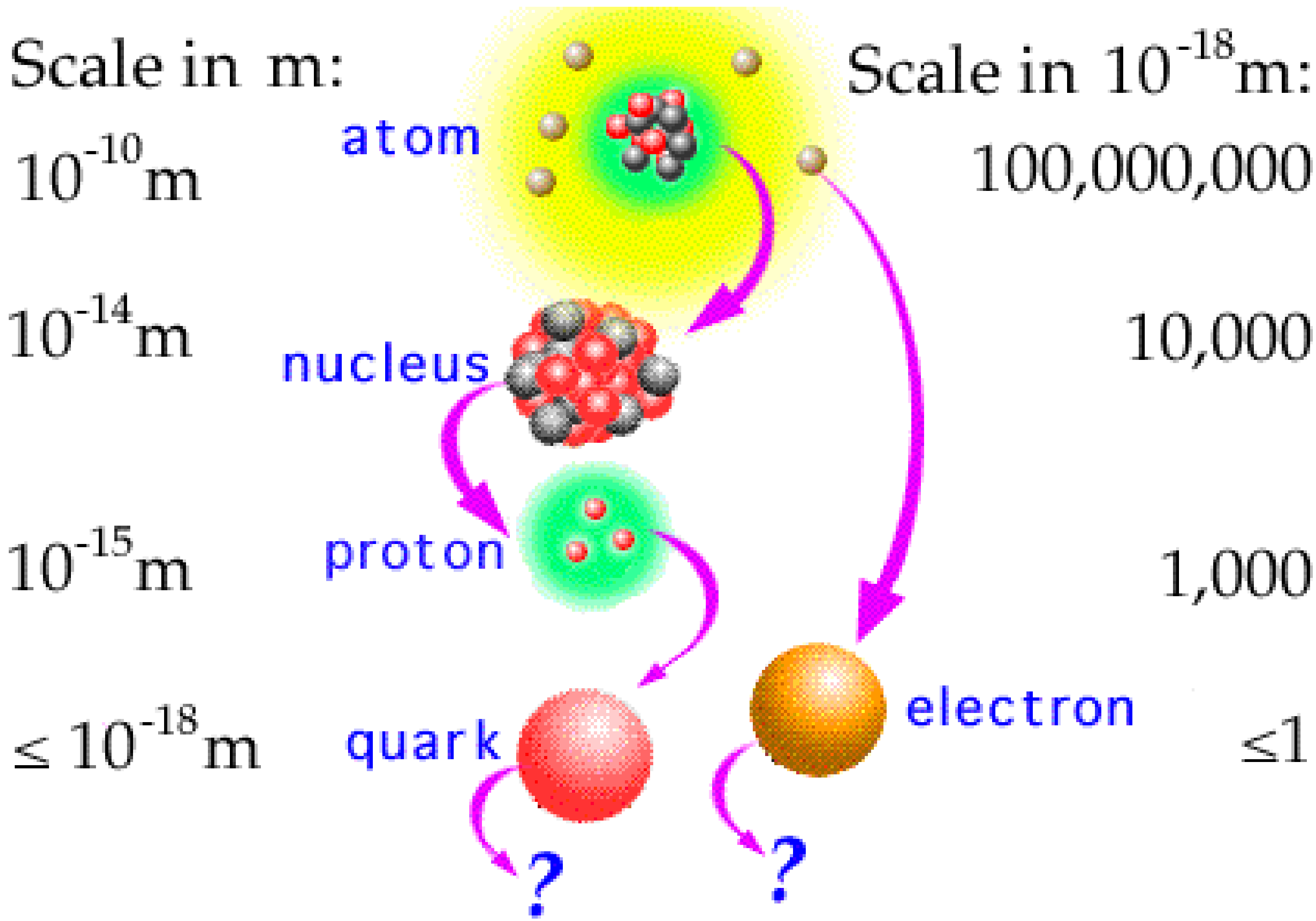
Empedocles, 490–430 BC



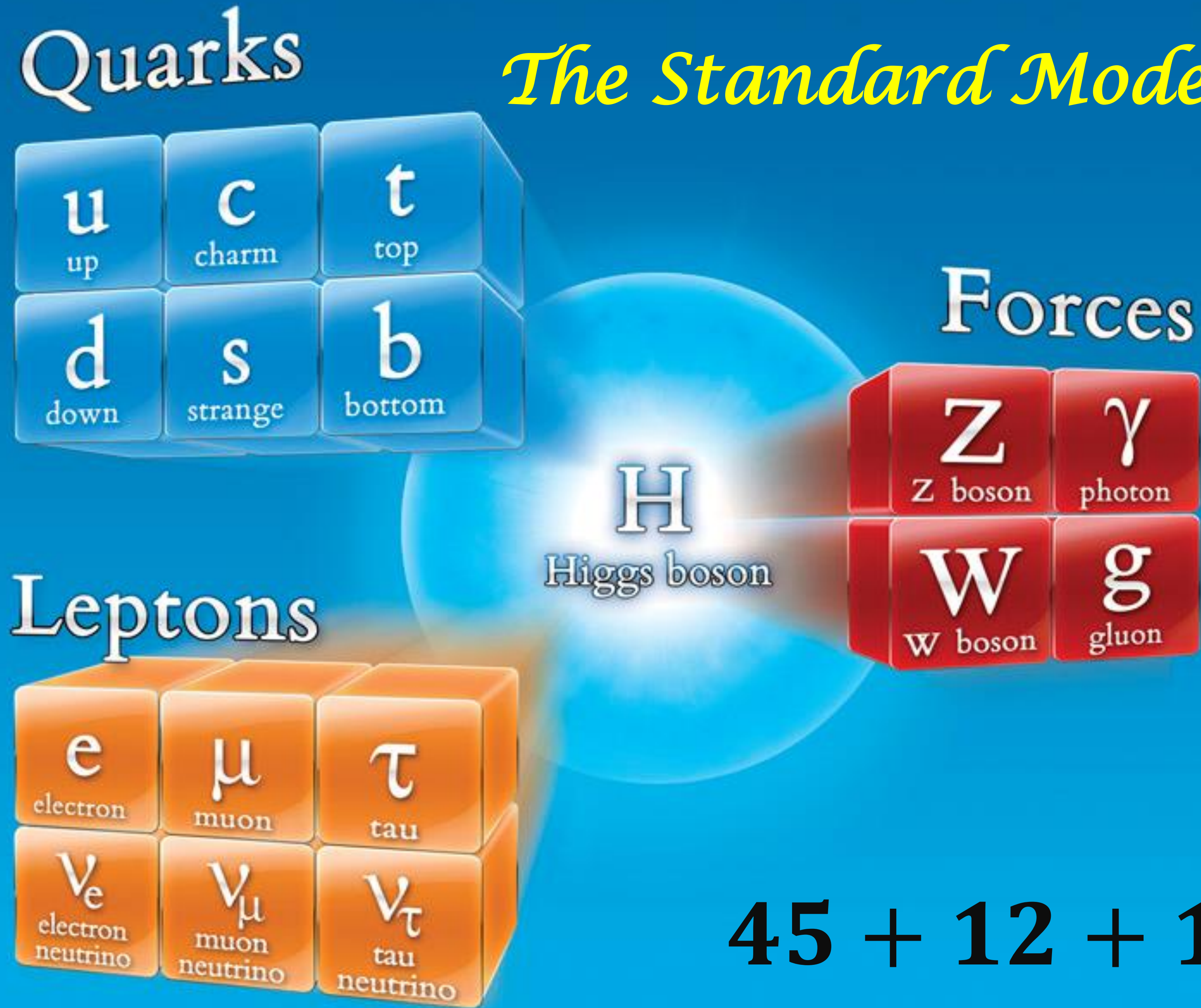
Kanada, ~500 BC, India

In ancient China, it was believed that all matter was composed of the 5 elements: Water, Wood, Metal, Fire, & Earth.

Unification of Matter

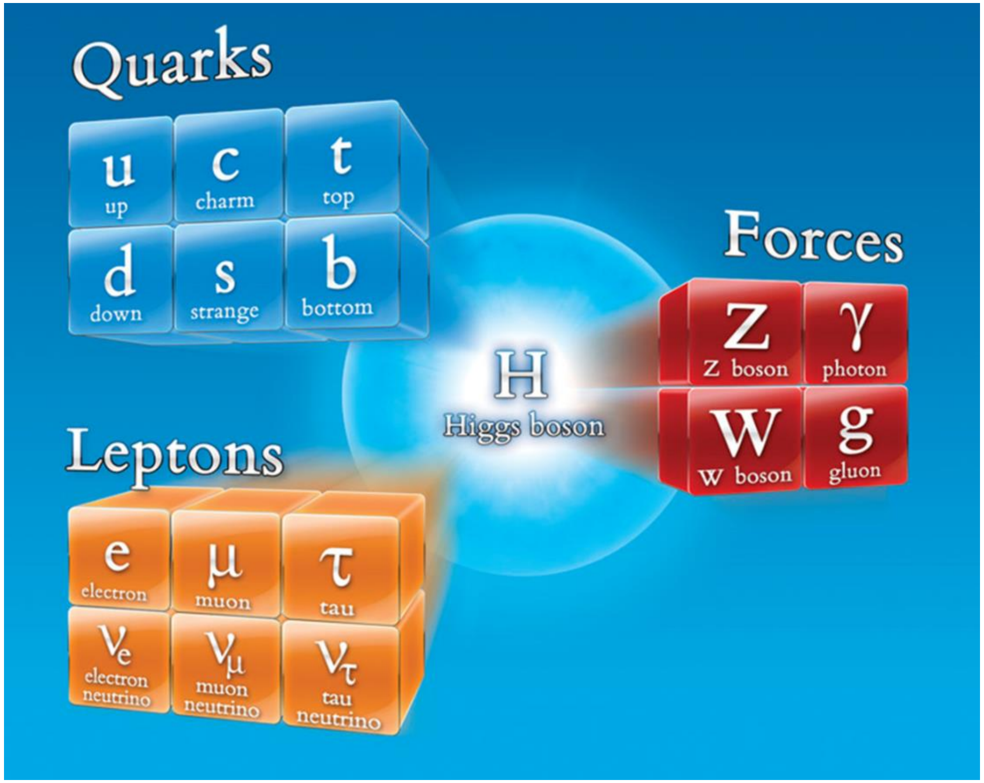


The Standard Model



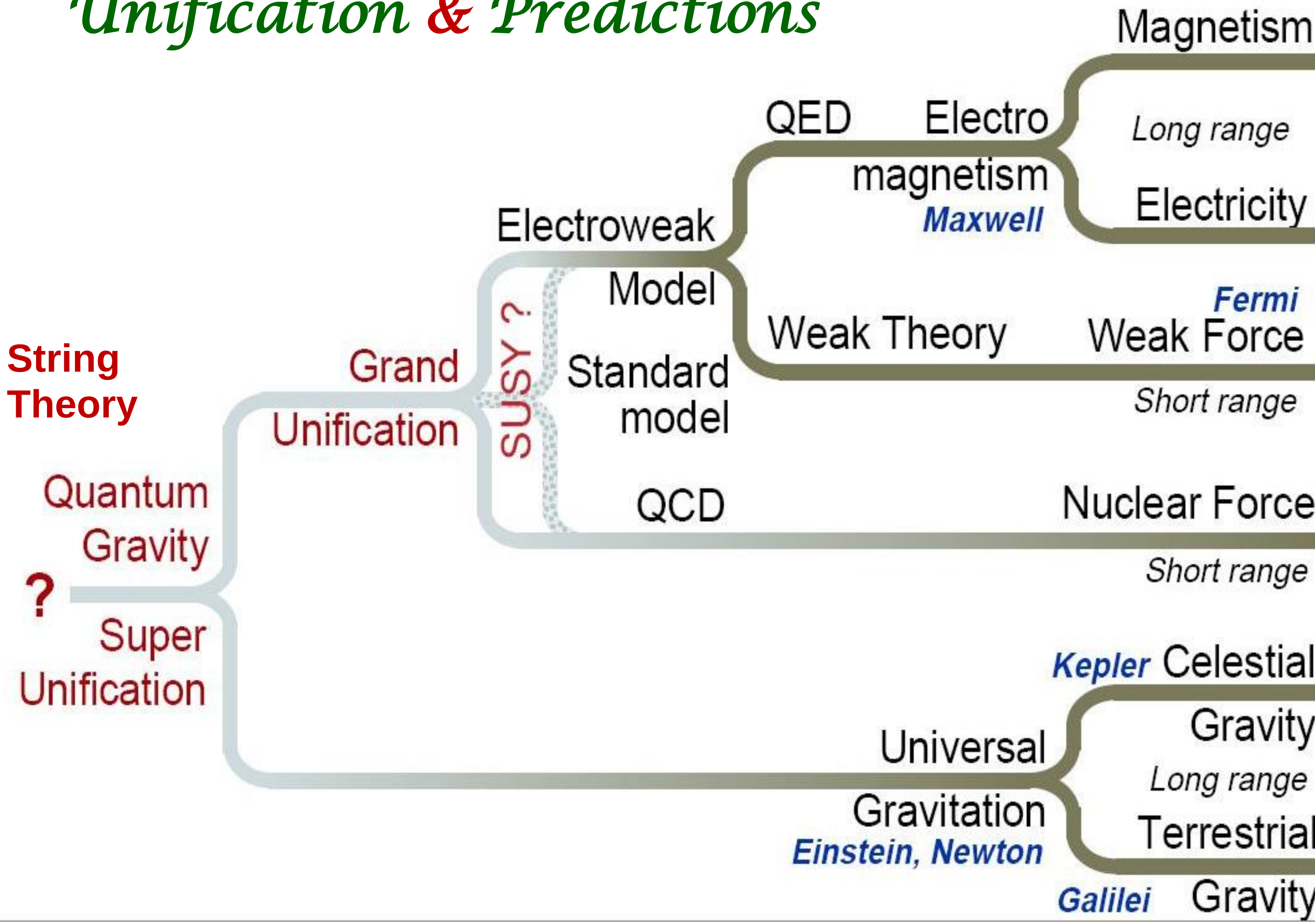
$$45 + 12 + 1$$

Welcome to the Particle ZOO



4 *vs* **58**

Unification & Predictions



The SM

The Standard Model (SM) is the theory governing fundamental particles & interaction (except Gravity)

$$\boldsymbol{For\ L \geq 10^{-18}\, m \quad \Leftrightarrow \quad E \leq 10^{3}\ GeV}$$

SM is the Theory of Forces & the Particles

Forces

$$\textbf{Strong} \times \boldsymbol{Weak} \times \boldsymbol{Hypercharge}$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

8 gluons $\times\ A^{\pm}, A^{3} \times\ B \rightarrow$ **Spin 1 bosons**

$$\alpha_3 \approx \frac{1}{8.6} \times \alpha_2 \approx \frac{1}{29.6} \times \alpha_1 \approx \frac{1}{98.3}$$

Measured at stale of $\approx 90\ GeV$

"Chiral Fermions"

- **Fermions: Dirac bispinor ψ**
- **Chiral: definition of handedness:**

$$\psi_L = \frac{1}{2}(1-\gamma_5)\psi \quad \rightarrow \quad \textit{Left}$$

$$\psi_R = \frac{1}{2}(1+\gamma_5)\psi \quad \rightarrow \quad \textit{Right}$$

each has only two components

Particle content of the SM consists of three generations of chiral fermions

The SM Particles are "Chiral Fermions"

Left: Electroweak (EW) Doublets

$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ d \end{pmatrix} \rightarrow$ ***Quarks: each comes in 3 colors*** *(R,G,B)*

$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \rightarrow$ ***Leptons: No colors***

Right: all components are EW singlets

$(u), (c), (t)$
$(d), (s), (b)$ $\rightarrow$ ***Quarks: each comes in 3 colors*** *(R,G,B)*

$(e), (\mu), (\tau) \rightarrow$ ***Leptons: No colors***

Particles are "Chiral Fermions"

Let's adapt a common notation to describe transformation properties of the particles under the SM gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L : (3,\ 2)_{\frac{1}{3}}$; $\quad d_R : (3, 1)_{-\frac{2}{3}}$; $\quad U_R : (3, 1)_{\frac{4}{3}}$;

$L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L : (1,\ 2)_{-1} \quad e_R : (1, 1)_{-2}$

$\nu_R : (1, 1)_0 \rightarrow$ **if it exist**

Note: d_R is 3 under $SU(3)_c$, *not* $\bar{3}$: different handedness of the same down quark!

Charge Conjugate

Recall ”charge conjugate” operation (particle ↔ antipartocle) $\psi^c \equiv i\,\gamma^2\psi^*$ **Since** $\gamma_5^* = \gamma_5$

$$(\psi_R)^c = i\gamma^2 \left(\frac{1}{2}(1+\gamma_5)\psi\right)^* = \frac{i}{2}\gamma^2(1+\gamma_5)\psi^* =$$

since $\{\gamma^\mu,\ \gamma^5\} = 0$

$$= \frac{1}{2}(1-\gamma_5)\left[i\gamma^2\psi^*\right] = (\psi^c)_L$$

The conjugate of a right-handed component of a fermion is the left-handed component of the conjugate fermion!

$$\psi^c \equiv i\,\gamma^2\psi^*$$

Left Handed Base

It is more convenient to work in left (or right) handed bases. We can just drop all "*L*" subscripts & write all field in terms of left-handed components

$Q: (3,\ 2)_{\frac{1}{3}}$; $d^c: (\overline{3}, 1)_{\frac{2}{3}}$; $U^c: \left(\overline{\overline{3}}, 1\right)_{-\frac{4}{3}}$;

$L: (1,\ 2)_{-1}$ $e^c: (1, 1)_2$

$\nu^c: (1, 1)_0 \rightarrow$ **if it exist**

First generation only, others just repeat

The SM Higgs Sector

$$SU(3)_c \times \underbrace{SU(2)_L \times U(1)_Y}_{U(1)_{EM}}$$

$<\varphi>$ Higgs VEV

φ - Higgs field is $SU(2)_L$ doublet, complex scalar field

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}_{y=1}$$

⟵ Four degree of freedom

$$V(\varphi) = -\mu^2 \varphi^+ \varphi + \lambda(\varphi^+ \varphi)^2$$

Minimum at $v = \sqrt{\frac{\mu^2}{\lambda}} \approx 246\ GeV$ ⇨ $< \varphi > = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$$\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

φ^+ ⟵ $Q_{EM} = +1$

φ^0 ⟵ $Q_{EM} = 0$

$$Q_{EM} = T_3 + Y$$

The component which gets VEV must be Electrically neutral ($Q_{EM} = 0$). So that EM is the remaining unbroken symmetry

The SM Higgs Sector

Parametrize Higgs in terms of direction relative to new vacuum. Using the polar variables for the scalar fields

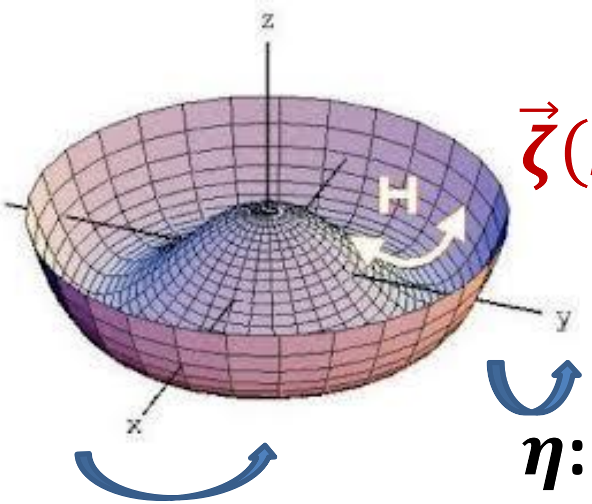
$$\varphi = U^{-1}(\zeta)\begin{pmatrix} 0 \\ (v+\eta(x))/\sqrt{2} \end{pmatrix}$$

$$U(\zeta) = exp[i\vec{\zeta}(x)\cdot\vec{\tau}/v]$$

Higgs degrees of freedom are now

$\vec{\zeta}(x)$: $(\xi^{\pm},\xi^{3})$ **would be Goldstone boson**

$\eta(x)$: **the physical Higgs**



η: **massive**

$\xi^{i}: massles$

Higgs Mechanism (Schematically)

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

$(A^{\pm},\ A^3)$ (B)

Gluons Unaffected

Eats $\xi^{\pm}$

eats ξ^3

$B \rightarrow$ " Unaffected"

$W^{\pm} \rightarrow masive$

$(A')^3 \rightarrow massive$

$$M_{W^{\pm}}^2 = \frac{g_2^2\, v^2}{4} \approx 83\ GeV$$

$$\begin{pmatrix} Z^0 \\ \gamma \end{pmatrix} = \begin{pmatrix} cos\theta_W & -sin\theta_W \\ sin\theta_W & cos\theta_W \end{pmatrix} \begin{pmatrix} (A')^3 \\ B \end{pmatrix}$$

$$M_{Z^o}^2 = \frac{M_{W^{\pm}}^2}{cos^2\theta} \approx 90\ GeV$$

$$\alpha_y^{-1} = \alpha_{EM}^{-1} cos^2\theta_w$$

$$sin^2\theta = \frac{g_y^2}{g_y^2 + g_2^2} \approx 0.23$$

$$\alpha_2^{-1} = \alpha_{EM}^{-1} sin^2\theta_w$$

$$\alpha_{EM}^{-1} = 127$$

Yukawa Sector

$$\mathcal{L} = Y_d \overline{Q}_L \varphi \, d_R + Y_u \overline{Q}_L (i\,\tau_2 \varphi^*) u_R + Y_e L_L \varphi \, e_R + h.c.$$

The fermions gain Dirac masses

$$m_i = Y_i < \varphi > = \frac{Y_i v}{\sqrt{2}} \qquad v \approx 264\ GeV$$

$\nu^c : (1,1)_0 \rightarrow$ **if it exist, then** $\rightarrow Y_\nu L (i\,\tau_2 \varphi^*) \nu^c$

We have three generation quarks & leptons.

We have mixing between generation.

The SM Summary

	$SU(3)_c \times SU(2)_L \times U(1)_Y$
Gauge bosons **Spin: 1**	Gluons: $(8,1)_0$; $A^{\pm}, A^3: (1,3)_0$; $B: (1,1)_0$
Matter (Left handed base) **Spin: 1/2**	$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L : (3,\ 2)_{\frac{1}{3}}$; $d^c: (\bar{3},1)_{\frac{2}{3}}$; $U^c: (\bar{3},1)_{-\frac{4}{3}}$ $L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L : (1,2)_{-1}$; $e^c: (1,1)_2$; ν_e^c; $(1,1)_0$
The SM Higgs **Spin: 0**	$\varphi: (1,2)_1$

$$Q_{EM} = T_3 + \frac{Y}{2}$$

What sets Values of Y?

Note: **$U(1)_Y$ is abelian group, so any normalization is allowed.**

From fermions content:

$$Q_{EM} = T_3 + \frac{Y}{2}$$

We have measured Q_{EM} experimentally. So, relative hypercharge assignment are fixed by experimental observations!

But is there a theoretical reason for these relative values of Y?

Chiral Adler-Bell-Jackiw (ABJ) Anomaly

The chiral *ABJ* anomaly spoils the renormalizability of a gauge theory

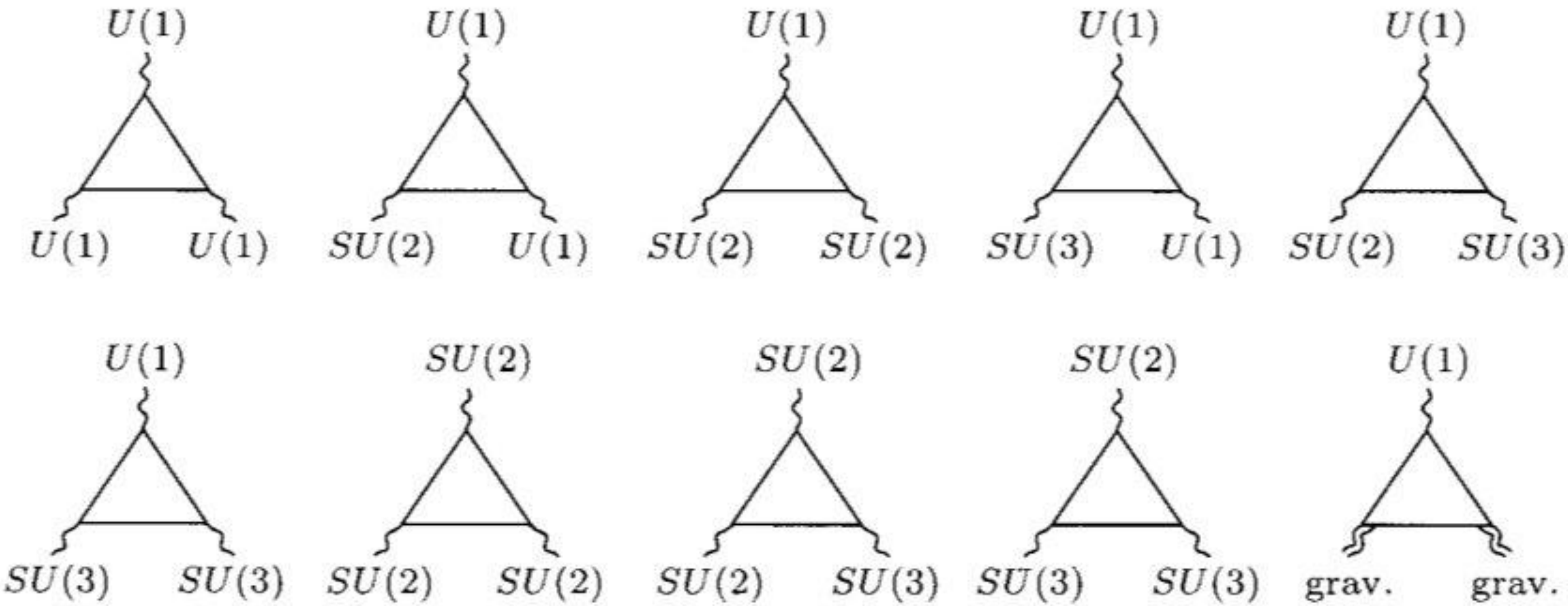


Figure 20.2. Possible gauge anomalies of weak interaction theory. All of these anomalies must vanish for the Glashow-Weinberg-Salam theory to be consistent.

From Peskin & Schroeder

Miraculous Cancellation of Anomalies

- $SU(3)_C^2 \times U(1)_Y$: $\frac{1}{2}\left[2\times(\frac{1}{6})+1\times(\frac{-2}{3})+1\times(\frac{1}{3})\right]=0$

- $SU(2)_L^2 \times U(1)_Y$: $\frac{1}{2}\left[3\times(\frac{1}{6})+1\times(\frac{-1}{2})\right]=0$

- $(\text{gravity})^2 \times U(1)_Y$:

  $\left[3\times2\times(\frac{1}{6})+3\times(\frac{-2}{3})+3\times(\frac{1}{3})+2\times(\frac{-1}{2})+1\times1\right]=0$

- $U(1)_Y^3$:

$\left[3\times2\times(\frac{1}{6})^3+3\times(\frac{-2}{3})^3+3\times(\frac{1}{3})^3+2\times(\frac{-1}{2})^3+1\times(1)^3\right]=0$

Relative *Y*-values are fixed → charge quantization

But overall normalization still is not fixed

The SM: Things to Remember

1) **Lots of clearly disconnected representations for gauge boson & particle content**
2) **3 independent gauge couplings:** $(g_1, \; g_2, g_Y)$
3) **Yukawa sector is unconstrained.**
4) **Particle representations are chiral**

$$Q_L = \left(3, 2, \frac{1}{6}\right), \qquad but \qquad NO \quad \left(\bar{3}, 2, -\frac{1}{6}\right)$$

5) **Overall normalization for hypercharge unfixed, (since $U(1)_Y$ Abelian), Even thought relative *Y*-values are fixed**

The SM: Things to Remember

6) Higgs mechanism breaks

$$SU(2)_L \times U(1)_Y \xrightarrow{<\phi> \neq 0} U(1)_{EM}$$

In general, the subgroup which survives is the subgroup with respect the field getting the non zero VEV is neutral

7) In the SM: Baryon # (B) conserved

Lepton # (L) conserved

Thus, the lightest baryon proton is stable!

Note:* *B* – *is actually broken by instanton effects (very small)

***L* – *can be broken by RH neutrino Majorana mass,* $m\nu^c\nu^c$**

Elements of Group Theory

A group G is a set of elements (A, B, C, ..) with the following properties

- ***Closure*: if A and B are in G, C = AB is also in G;**
- ***Association*: A (B C) = (A B) C**
- ***Identity*: There exists an element E such that EA = AE = A for every A in G**
- ***Inverse:* For every A in G, there exists an element A^{-1} such that: A A^{-1}= A^{-1} A = E**

If multiplication is commutative A B = B A for all A & B in G, G is *Abelian* group

Elements of Group Theory

Unitary group U(N), is the set of $N \times N$ unitary matrices: $U\,U^{\dagger} = U^{\dagger}U = 1$

It is Non – *Abelian* for N > 1.

The group of $N \times N$ unitary matrices with a unit determinant is called the *special unitary group* SU(N).

Unitary matrix can be written in terms of a hermitian matrix $(H^{\dagger} = H)$: $U = e^{iH}$

$det(e^{A}) = e^{trA}$ & det(U) = 1 $\Longrightarrow$ tr(H) = 0

Since there are $(n^2 - 1)$ traceless hermitian $N \times N$ matrices, an element of SU(N) is $U = \exp\{\sum_{a=1}^{n^2-1} \theta_a \lambda_a\}$

θ_a is (real) group parameter. λ_a is group generator. Rank of SU(N) group is $(N - 1)$

Toward Unification

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

The rank, $(N-1)$, of The SM gauge symmetry is

$$2+1+1=4$$

The SM gauge symmetry can be subgroup of bigger group. No restriction from group theory point of view.

What are the physics constraints?

What Groups G can we Choose?

$SU(3)_c \times SU(2)_L \times U(1)_Y$ **SM gauge symmetry rank is: $2+1+1=4$**	**group G must be rank ≥ 4 & contain SM as subgroup**
SM has chiral (complex) reps. $(\bar{3}, 1, 2/3)$ but not $(3, 1, -2/3)$	**Group G must also have chiral reps**
SM is free of chiral anomaly	**Group G must have reps for which chiral anomalies are canceled**
If we wont to relate the gauge couplings to each other	**G should be a simple group**

Classification of Lie Groups					
Rank =1	U(1), SU(2)	SO(3)	Sp(2)		
Rank=2	SU(3)	SO(5)	Sp(4)	SO(4)	G_2
Rank=3	SU(4)	SO(7)	Sp(6)	SO(6)	
Rank=4	SU(5)	SO(9)	Sp(8)	SO(8)	F4
Rank=5	SU(6)	SO(11)	Sp(10)	SO(10)	
Rank=6	SU(7)	SO(13)	Sp(12)	SO(12)	E_6 ....

Blue color indicates that group has complex representation

Does $SU(5)$ symmetry have the Potential for a Successful Unification ?

SU(5) symmetry has the following representations:

1, 5, 10, 15, 24, 45, 50, 78 etc.

Recall each SM generation contains 15 states and 3 generations. $(3 \times 15 = 45)$

$$\mathbf{SU(5)} \supset \mathbf{SU(2)} \times \boldsymbol{SU(3)} \times \boldsymbol{U(1)}$$

$$\mathbf{15} = (3,1)_6 + (2,3)_1 + (1,6)_{-4}$$

$$\mathbf{45} = (2,1)_3 + (1,3)_1 + (3,3)_{-2} + (1,3)_8 + (2,3)_{-7} +$$

$$+(1,6)_{-2} + (2,8)_3$$

Here all $\boldsymbol{U(1)}$ charges are normalized to avoid fractions

$SU(5)$ Unification

But let's look at $\bar{5}$ and 10 dimensional representation

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

$$\bar{5} = (\bar{3}, 1)_2 + (1, 2)_{-3}$$

$$10 = (3, 1)_{-4} + (3, 2)_1 + (1, 1)_6$$

we have to rescale $U(1)$ quantum numbers by 1/6

$$10_{[\alpha\beta]} = \underbrace{(\bar{3}, 1)}_{u^c}{}_{-\frac{2}{3}} + \underbrace{(3, 2)}_{Q}{}_{\frac{1}{6}} + \underbrace{(1, 1)}_{e^c}{}_{1}$$

$$\bar{5} = \underbrace{(\bar{3}, 1)}_{d^c}{}_{\frac{1}{3}} + \underbrace{(1, 2)}_{L}{}_{-\frac{1}{2}}$$

Nothing left over & no exotics!

Matter Multiples in $SU(5)$ Unification

An Entire SM generation fits into: $\bar{\mathbf{5}} + \mathbf{10}$

In matrix notation, we have

$$\bar{\mathbf{5}} : (d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

$$\mathbf{10} : \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}$$

Chiral ABJ Anomaly

Since we have not added new exotic fermions, the anomaly cancelation still it is OK

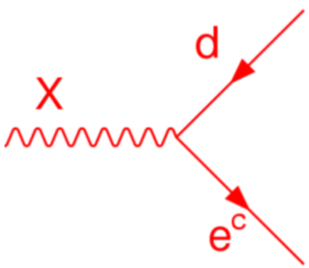
$SU(5)$ *Gauge Bosons*

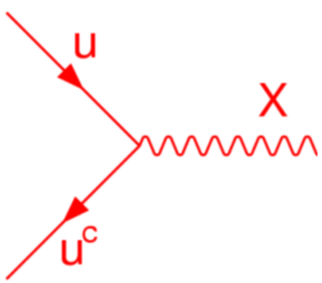
$$\mathbf{24} \rightarrow (\mathbf{8},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{1},\mathbf{1})_0 + (\mathbf{3},\mathbf{2})_{-5/6} + (\bar{\mathbf{3}},\mathbf{2})_{5/6}$$

gluons — $A^{\pm}, A^0$ — B — X & Y bosons

All SM gauge bosons are successfully embedded

X **&** ***Y*** **gauge bosons carry both color & electroweak charges simultaneously.**
They can connect quarks ⟷ leptons!
They can also turn quark directly to antiquark!





X **&** ***Y*** **bosons have electric charge** $\left\{\pm\frac{4}{3}, \pm\frac{1}{3}\right\}$

Hypercharge Normalization

Overall hypercharge Y normalization finally fixed

$$\mathbf{SU(5)} \rightarrow \mathbf{SU(3)_c \times SU(2)_w \times U(1)_Y}$$

Hypercharge is one of the non-Abelian generator

$$Q_{EM} = T_3 + Y = T_3 + c\,T_0$$

$$\bar{5} : (d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

$$Y(5) = \left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right)$$

$$T_0 = \frac{1}{\sqrt{60}}((2,\ 2\ , 2, -3, -3)$$

$$\mathbf{c} = -\sqrt{\frac{3}{5}}$$

$$Y_{SU(5)} = \sqrt{\frac{3}{5}}\, Y_{SM}$$

$$[D_\mu = \partial_\mu + i\frac{g_y Y}{2} B_\mu]$$

The product ($g_y\ Y$) must be preserved $g_Y{}^{SU5} = \sqrt{\frac{5}{3}}\, g_Y{}^{SM}$

$SU(5)$ $\mathcal{GUT}$

So, unification into a single GUT group such as SU(5) requires all generators to act with a common couplings

$$g_5 \equiv \left(g_3 = g_2 = g_1 = \sqrt{\frac{5}{3}}\ g_y \right) \textbf{ or } \quad \alpha_5 \equiv (\alpha_3 = \alpha_2 = \alpha_1 = \frac{5}{3}\alpha_Y)$$

Unification does not fix overall values of coupling but fix the ratios between couplings

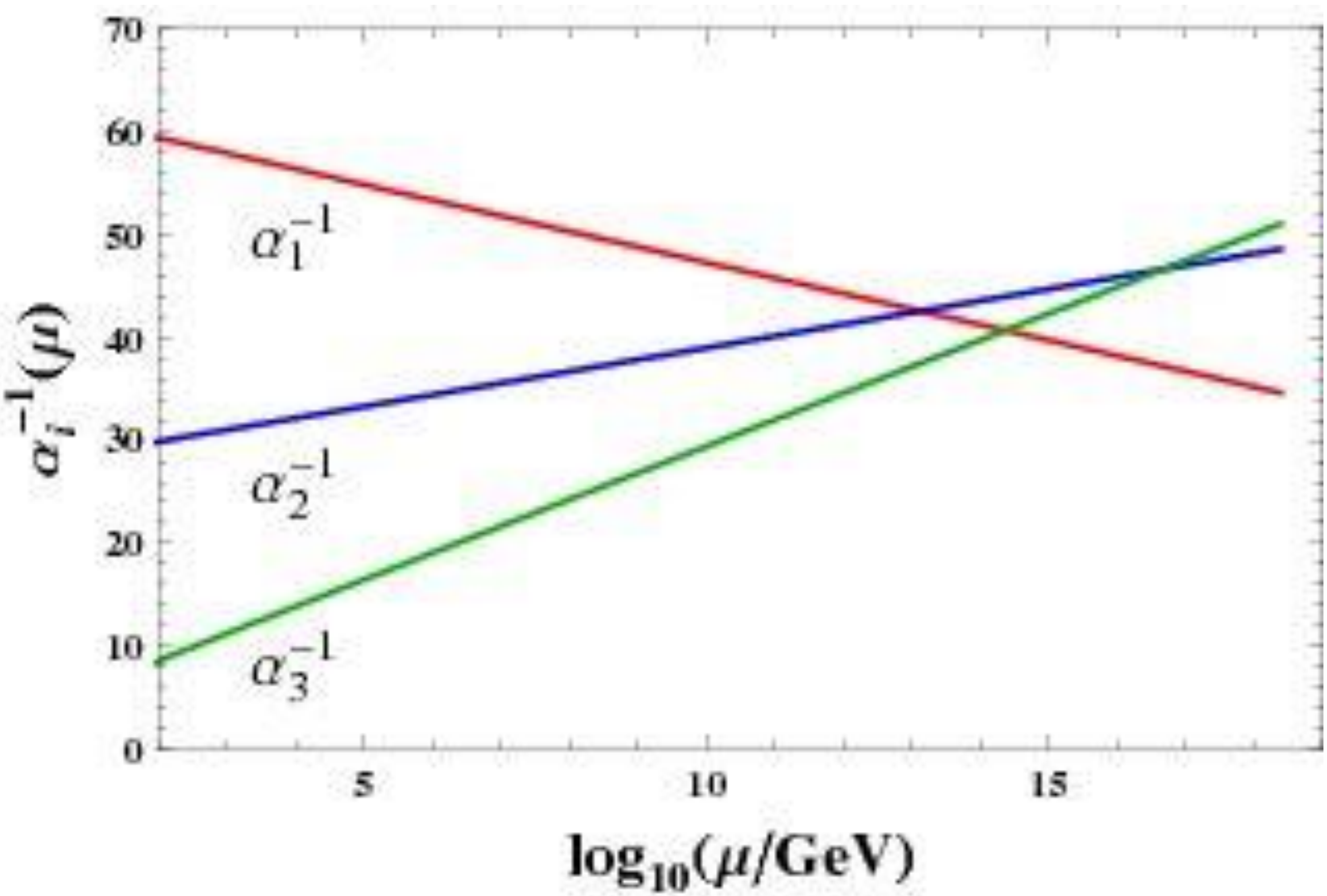
$$\frac{g_3}{g_2} = 1 \qquad \sin^2\theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{3}{8} = 0.375$$

But at electroweak scale we have

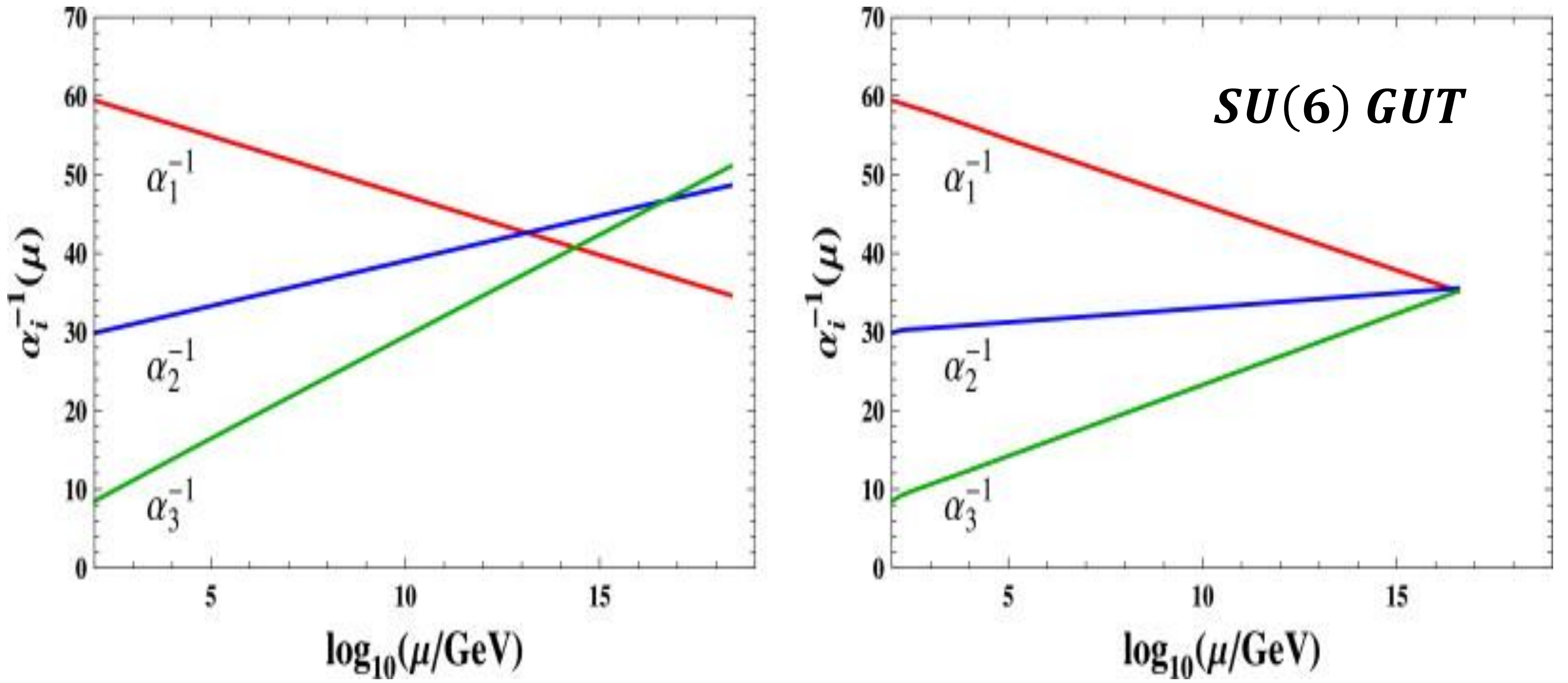
$$\alpha_3^{-1} \approx 8.5$$
$$\alpha_2^{-1} \approx 29.6$$
$$\alpha_1^{-1} \approx 59.1$$

- Couplings are not equal
- $\sin^2\theta_W \approx 0.23$, $NOT\ 0.375$

Unification of Gauge couplings



Unification of Gauge Couplings



$$Q\left(3,\ 2,\frac{1}{6}\right)+\overline{Q}\left(\overline{3},2,-\frac{1}{6}\right)+D\left(3,1,\frac{1}{3}\right)+\overline{D}\left(\overline{3},1,-\frac{1}{3}\right)$$

$$m_h = 125\ GeV$$

J.L. Chkareuli, I. Gogoladze, A. Kobakhidze, Phys.Lett.B 340 (1994) 63

Spontaneous GUT Symmetry Breaking

We can use Higgs mechanism just as in the SM to break GUT symmetry

$$\mathrm{SU}(5) \xrightarrow{<\Sigma>} \mathrm{SU}(3)_c \times SU(2)_w \times U(1)_Y \xrightarrow{<\phi>} \mathrm{SU}(3)_c \times U(1)_{EM}$$

In order to preserve $\mathrm{SU}(3)_c \times SU(2)_w \times U(1)_Y$ subgroup of SU(5) symmetry the Higgs rep must contain a component (1, 1, 0) neutral under SM !

The smallest rep. that can contains (1, 1, 0) under the SM is the 24-plet = adjoin. –> traceless, 5×5 matrix.

$$< \Sigma > = V_{GUT}\ diag\,(2, 2\ \ 2, -3, -3)$$

This is the analogue of demanding in the SM :

$$< \varphi > = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

Spontaneous GUT Symmetry Breaking

Can 24-plet (Σ) develop this minima

$$< \Sigma > = V_{GUT}\ diag\,(2, 2\ \ 2, -3, -3)$$

$$V(\Sigma, \phi) = -m_1^2 tr\Sigma^2 + \lambda_1 \left(tr\Sigma^2\right)^2 + \lambda_2 \left(tr\Sigma^4\right)$$
$$- m_2^2(\phi^\dagger \phi) + \lambda_3(\phi^\dagger \phi)^2$$
$$+ \lambda_4 \left(tr\Sigma^2\right) \left(\phi^\dagger \phi\right) + \lambda_5(\phi^\dagger\, \Sigma^2 \phi)$$

Discrete Z_2 symmetry is imposed to eliminate cubic terms.
Here ϕ is 5-plet o0f SU(5) containing the SM Higgs .

For $\lambda_2 > 0, \lambda_2 > -\frac{7}{30}\lambda_2,$ the potential has desired minima with

$$V_{GUT}^2 = \frac{m_1^2}{60\lambda_1 + 14\lambda_2}$$

GUT Scale Spectrum

$$\mathbf{24}\ (\boldsymbol{\Sigma}) \rightarrow (\mathbf{8},\mathbf{1},)_{\mathbf{0}} + (\mathbf{1},\mathbf{3})_{\mathbf{0}} + (\mathbf{1},\mathbf{1})_{\mathbf{0}} + (\mathbf{3},\mathbf{2})_{-\mathbf{5}/\mathbf{6}} + (\overline{\mathbf{3}},\mathbf{2})_{\mathbf{5}/\mathbf{6}}$$

$(8,1,)_0 + (1,3)_0 + (1,1)_0$ **become massive physical Higgs field with mass** $O(M_{GUT})$. **The** X **&** Y **gauge boson "eat"** $(3,2)_{-5/6} + (\overline{3},2)_{5/6}$ **massless goldstone bosons from 24-plet & get masses** $m_{x,y} \sim g_{GUT} V_{GUT}$

So, no additional new particle from gauge & 24 Higgs multiples at EW scale!

Doublet-Triplet Splitting Problem

$$\phi(5) = \phi_3(3,1)_{1/3} + \phi_2(1,2)_{-1/2}$$

$$V' = -m_2^2(\phi^\dagger\phi) + \lambda_4\left(tr\Sigma^2\right)\left(\phi^\dagger\phi\right) + \lambda_5(\phi^\dagger\,\Sigma^2\phi)$$

Color triple ϕ_3 get mass:

$$-m_2^2 + (30\lambda_4 + 4\lambda_5)V_{GUT}$$

The SM Higgs doublet ϕ_2 get mass:

$$-m_2^2 + (30\lambda_4 + 9\lambda_5)V_{GUT}$$

We can make the SM Higgs doublet $O(M_Z)$ at tree level light while having color triplet $O(M_{GUT})$. But there are radiative corrections …

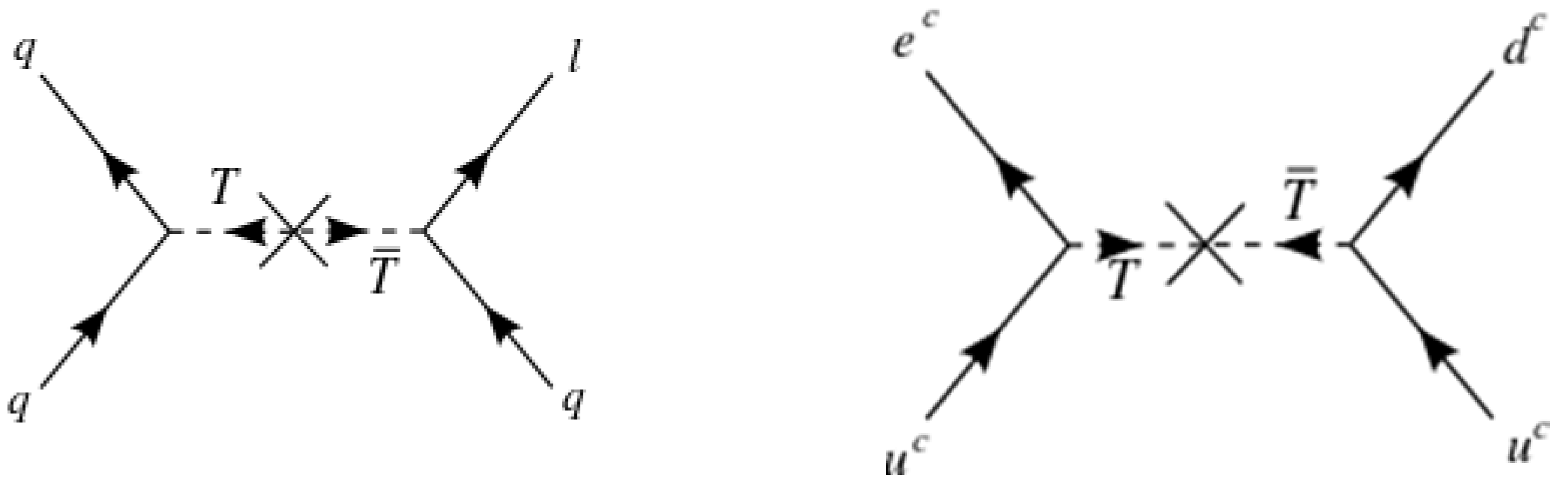
Gauge Hierarchy Problem

$$m_{\phi_2} = \left(m_{\phi_2}\right)_0 + \frac{\alpha_2}{4\pi}\Lambda^2 + \cdots$$

Proton Decay

Color Higgs triplet mediate Proton decay

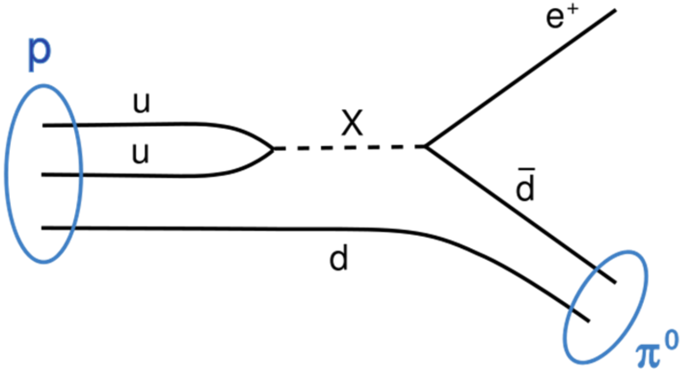
$$\mathbf{10}_f\ \mathbf{10}_f\ \mathbf{5}_\phi + \bar{\mathbf{5}}_f \mathbf{10}_f \mathbf{5}^*_\phi$$



Color triplet $\phi_3 \equiv T$ mass needs to be more then 10^{13} GeV to satisfy current experimental constrain

Proton Decay

X & *Y* **gauge boson mediate Proton decay**



$$p \rightarrow e^{+}\pi^{0},\ \tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 \approx [10^{35\pm1} yr]^{-1}$$

The current experimental limit is:

$$\tau(p \rightarrow e^{+}\pi^{0}) > 1.4 \times 10^{34}\mathrm{yr}$$

Yukawa Sector

$$Y_5\,\bar{5}_f 10_f\; 5^* + \; Y_{10} 10_f\, 10_f\; 5_H$$

$$Y_5\,\bar{5}_f 10_f\; 5^* \Rightarrow Y_5\,(d^c\,Q\,H^* \; + L\,e^c\,H^*) \Rightarrow \; Y_d \;=\; Y_E^T$$

Yukawa (mass) matrix for down quarks is just the transpose of the Yukawa (mass) matrix for the charge leptons.

$$m_e = m_d, \qquad m_\mu \;= m_s\,, \qquad m_\tau = m_b$$

Extrapolating down to EW scale using RGE equation it contradict experimental observations.

For third generation we find $m_b \approx 3\, m_\tau$, This ratio becomes much better for SUSY scenario.

We also have: $\left(\frac{m_e}{m_\mu} = \frac{m_d}{m_s}\right)$ **which is obviously contradictive**

Are the Solution for $m_e = m_d,\ m_\mu = m_s$?

Yes, we need to expand the Higgs sector. Introduce a 45 rep as another new Higgs: $Y_{45}\ \bar{5}_f 10_f\ 45^*$

$$\left(\frac{m_e}{m_\mu} = \frac{m_d}{m_s}\right) \quad \textbf{becomes} \longrightarrow \quad \left(\frac{m_e}{m_\mu} = \frac{1}{9}\frac{m_d}{m_s}\right)$$

Or consider effective Non-renormalizable couplings

$$Y_5\ \bar{5}_f 10_f 5_H^* + Y'_5\ \bar{5}_f 10_f \left(\frac{\Sigma}{\mathrm{M}}\right)^n 5^*$$

Yukawa Sector, Up Quarks

$$Y_{10} \mathbf{10}_f \ \mathbf{10}_f \ \mathbf{5}_H$$

$$\mathbf{10} = \ u^c \ (3,1)_{-2/3} + \ Q \ (3,2)_{1/6} + \ e^c \ (1,1)_1$$

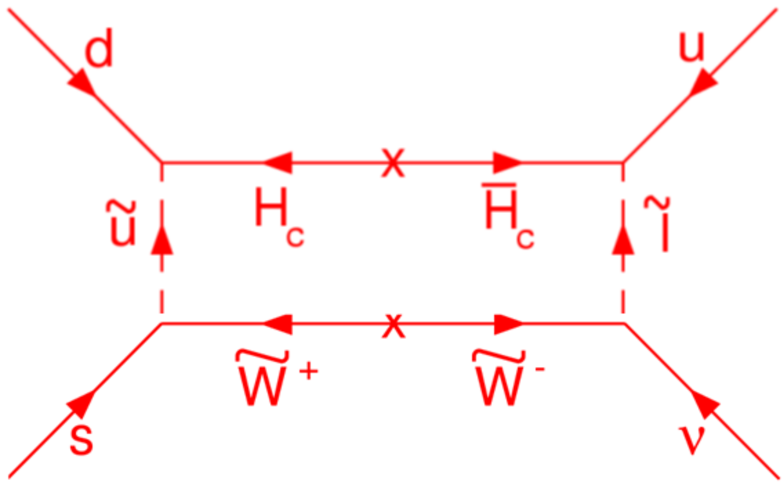
$$\mathbf{10}_f \ \mathbf{10}_f \ \mathbf{5}_H \quad \Longrightarrow \quad Y_U \ = \ Y_U^T$$

The Yukawa(mass) matrix for the up quarks is symmetric

Low Energy Supersymmetry

- **Resolves the gauge hierarchy problem;**
- **Provides cold dark matter candidate (LSP);**
- **Implements radiative electroweak symmetry breaking;**
- **Predicts new particles accessible at the collaider;**
- **Improves unification of the SM gauge and Yukawa couplings.**

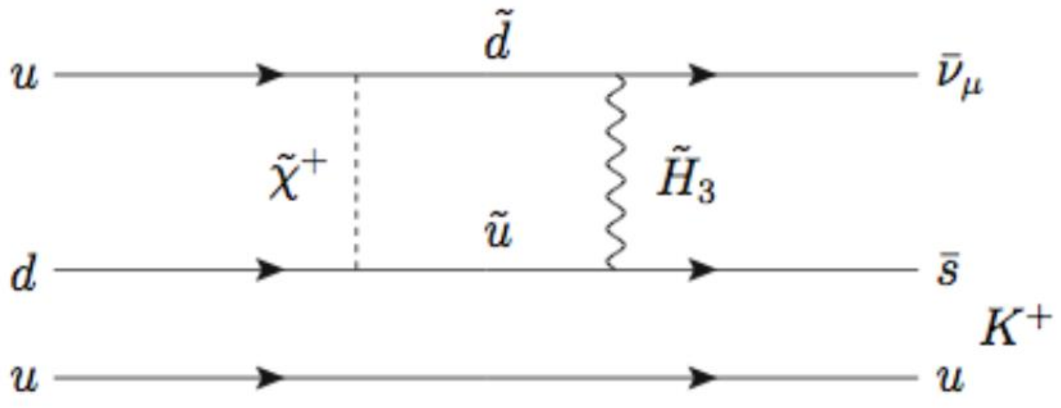
$d = 5$ Proton Decay



$$\Gamma^{-1}_{d=5}(p \to \bar{\nu} K^+) \simeq 1.2 \cdot 10^{31}\,\mathrm{yrs} \times \left(\frac{0.012\,\mathrm{GeV}^3}{\beta_H}\right)^2 \left(\frac{7}{\bar{A}_S^\alpha}\right)^2 \left(\frac{1.25}{R_L}\right)^2$$

$$\times \left(\frac{M_T}{2 \cdot 10^{16}\,\mathrm{GeV}}\right)^2 \left(\frac{m_{\tilde{q}}}{1.5\,\mathrm{TeV}}\right)^4 \left(\frac{190\,\mathrm{GeV}}{M_{\tilde{W}}}\right)^2 ,$$

$d = 5$ Proton Decay



Very crudely: $\tau(p \to \bar{\nu}\, K^+) \sim \frac{m^4{}_{\tilde{q}}}{M^2_{\chi^\pm}} \tan^2\beta$

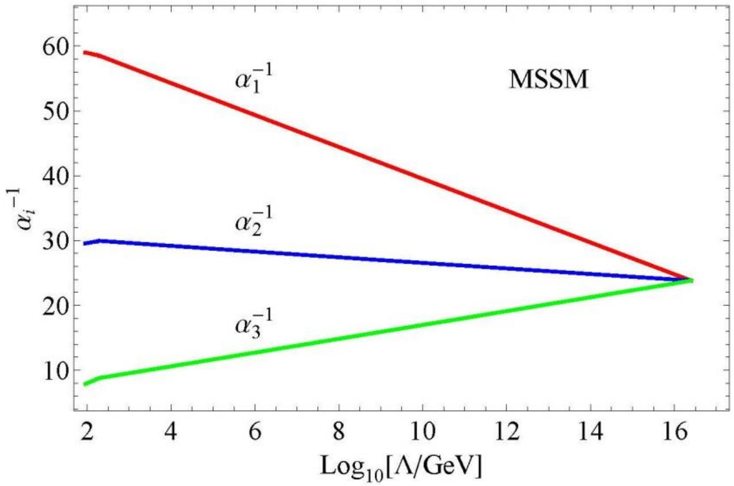
1) Reduce effective $d\, \tilde{u}\, H_c$ by non renormilizable couplings $d\, \tilde{u} \left(\frac{\Sigma}{M}\right)^n H_c$

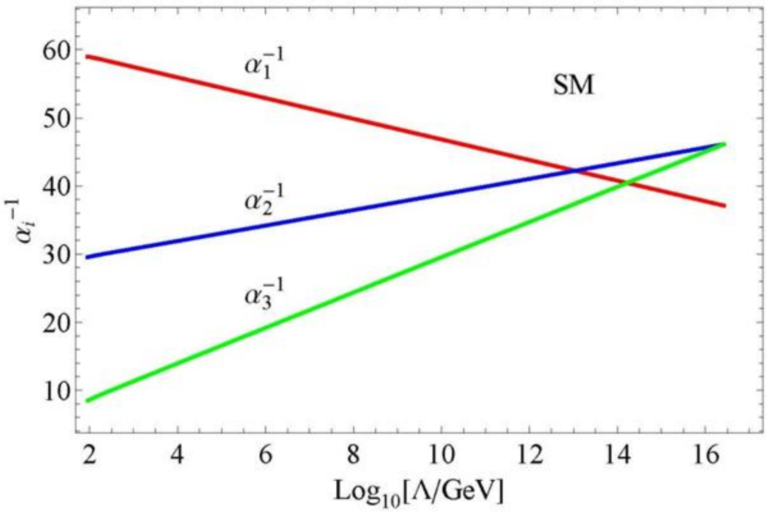
2) Affectively increase mass $m\, H_c \overline{H_c}$ *by introdusing additional* $(H'_c + \overline{H_c}')$

3) Make squarks much heavier compare gauginos

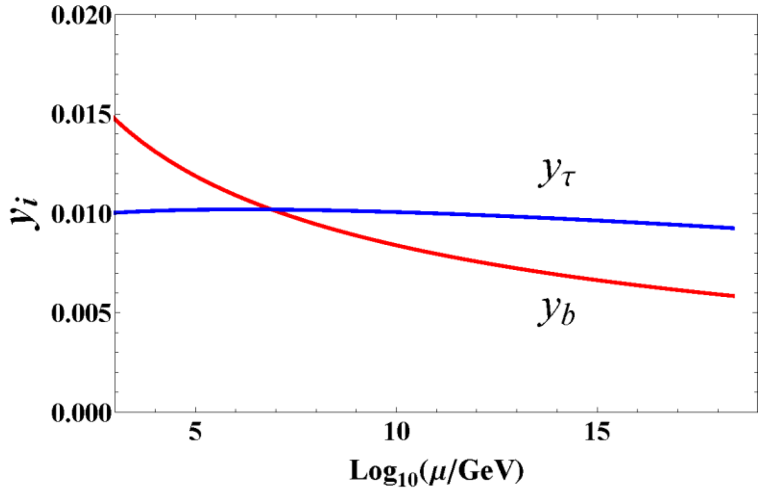
4) Consider possible structure for soft SSB masses and mixings

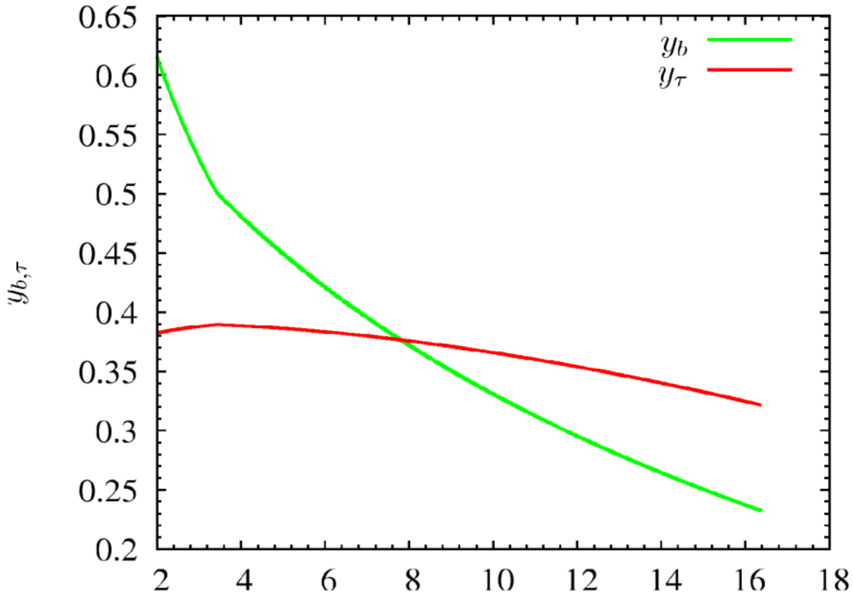
Improves Unification of the SM Gauge Couplings



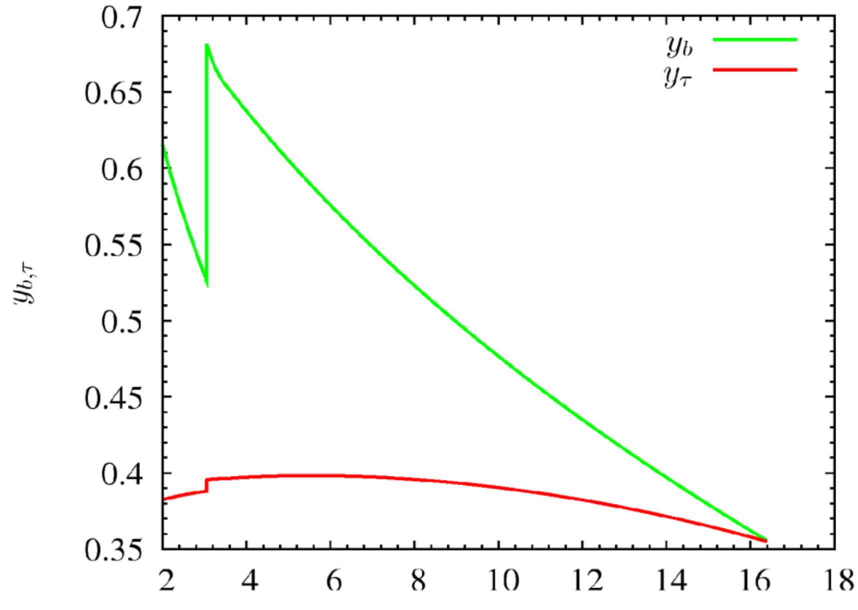


$b-\tau$ Yukawa coupling unification ♡ SUSY





(a) $\log_{10}(Q/\mathrm{GeV})$



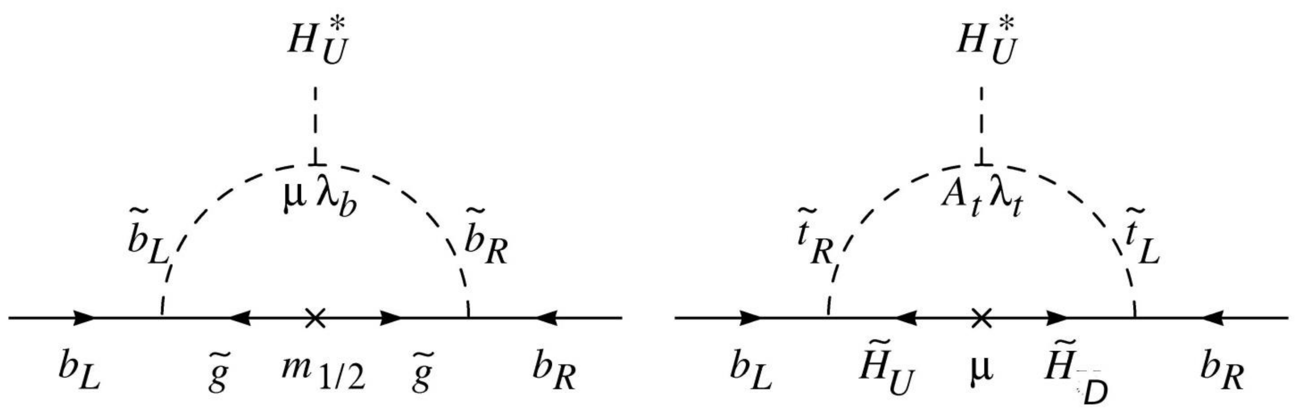
(b) $\log_{10}(Q/\mathrm{GeV})$

Finite SUSY threshold corrections

Dominant contributions to the bottom quark mass from the gluino and chargino loop

$$\delta y_b \approx \frac{g_3^2}{12\pi^2}\frac{\mu m_{\tilde{g}} \tan\beta}{m_{\tilde{b}}^2} + \frac{y_t^2}{32\pi^2}\frac{\mu A_t \tan\beta}{m_{\tilde{t}}^2} + \ldots$$

where $m_{\tilde{b}}$ and $m_{\tilde{t}}$ stands for sbottom and stop mass.



where $\lambda_b = y_b$ and $\lambda_t = y_t$

Doublet-Triplet splitting in SUSY SU(5)

$$W_{D-T} = \bar{\mathbf{5}}_H(\lambda \mathbf{24}_H + M)\mathbf{5}_H$$

$$\langle 24_H \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix} V$$

$$M_{H_c} = \lambda V + M \sim O(M_{GUT}) \quad M_H = -\frac{3}{2}\lambda V + M$$

Missing Partner Mechanismin SUSY SU(5)

$$W_H = M_{75} 75^2 + \lambda\, 75^3 + M_{50}\, 50 \cdot \overline{50} + \lambda_1\, 50 \cdot 75 \cdot \overline{5} + \lambda_2\, \overline{50} \cdot 75 \cdot 5$$

$$\mathbf{5} = (1,2,3) + (3,1,-2)$$
$$\mathbf{50} = (1,1,-12) + (3,1,-2) + (\bar{3},2,-7) + (\bar{6},3,-2) + (6,1,8) + (8,2,3)$$
$$\mathbf{75} = (1,1,0) + (3,1,10) + (3,2,-5) + (\bar{3},1,-10) + (\bar{3},2,5) + (\bar{6},2,-5) + (6,2,5) + (8,1,0) + (8,3,0)$$

The SM hypercharge $\frac{Y}{2}$ is $\frac{1}{6}$ times the charge quoted above.

Thank You

谢谢