The Standard Model of Electroweak & Strong Interactions

Theory and Practice

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Prof. Steven Weinberg, R.I.P.

(may 3, 1933 – July 23, 2021)

Steven Weinberg

From Wikipedia, the free encyclopedia

Steven Weinberg (/ˈwaɪnbɜːrg/; May 3, 1933 – July 23, 2021) was an American theoretical physicist and Nobel laureate in physics for his contributions with Abdus Salam and Sheldon Glashow to the unification of the weak force and electromagnetic interaction between elementary particles.

He held the Josey Regental Chair in Science at the University of Texas at Austin, where he was a member of the Physics and Astronomy Departments. His research on elementary particles and physical cosmology was honored with numerous prizes and awards, including in 1979 the Nobel Prize in physics and 1991 the National Medal of Science. In 2004, he received the Benjamin Franklin Medal of the American Philosophical Society, with a citation that said he was "considered by many to be the preeminent theoretical physicist alive in the world today." He was elected to the US National Academy of Sciences, Britain's Royal Society, the American Philosophical Society, and the American Academy of Arts and Sciences.

Steven Weinberg

These Lectures:

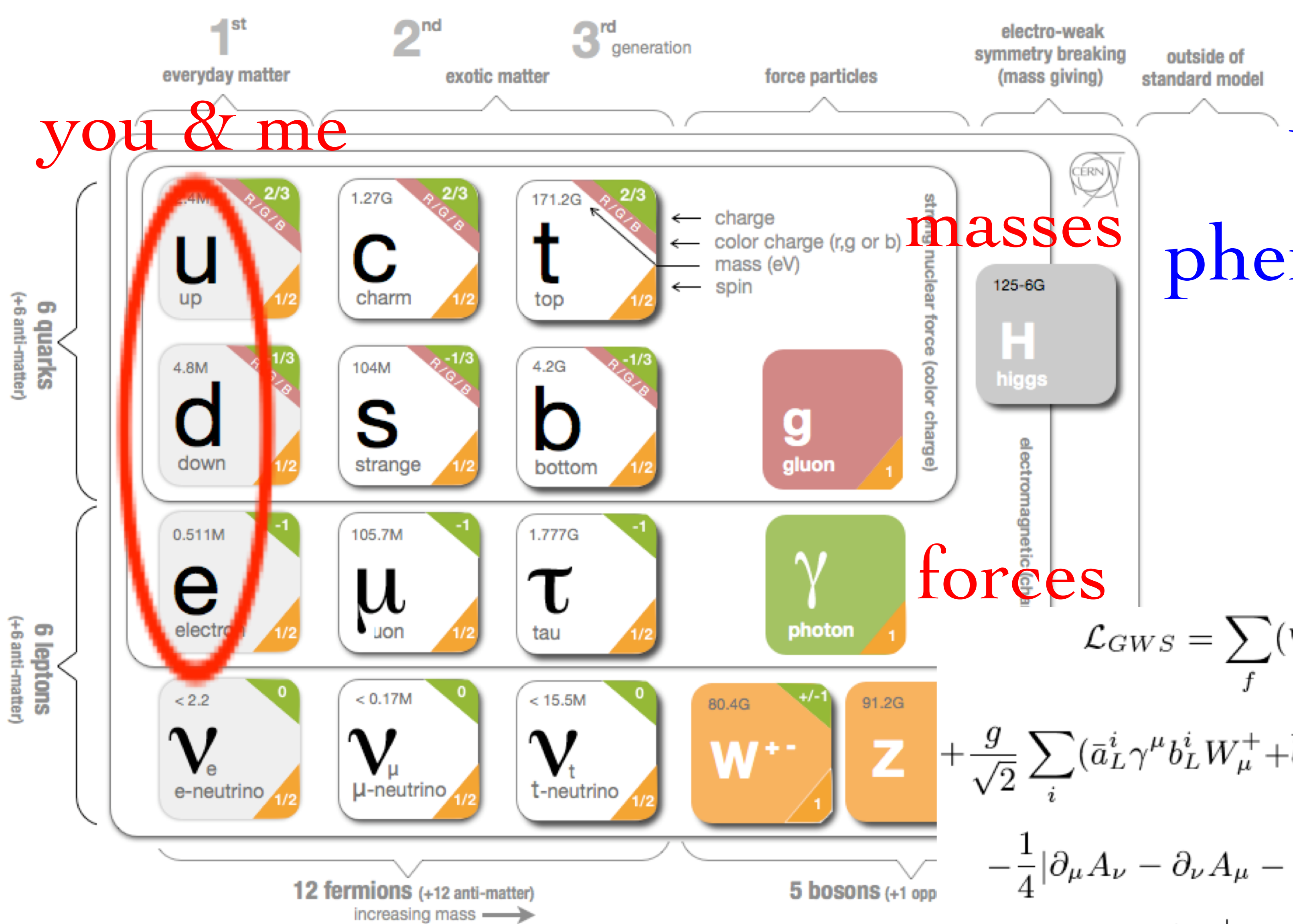
The Standard Model

On a nut-shell:

you & me

masses

forces



Will take a historical, phenomenological approach:

Not from here!

$$
\begin{aligned}
\mathcal{L}_{GWS} = \sum_f (\bar{\Psi}_f(i\gamma^\mu \partial \mu - m_f)\Psi_f - eQ_f\bar{\Psi}_f\gamma^\mu\Psi_f A_\mu) + \\
+\frac{g}{\sqrt{2}}\sum_i(\bar{a}^i_L\gamma^\mu b^i_L W^+_\mu + \bar{b}^i_L\gamma^\mu a^i_L W^-_\mu) + \frac{g}{2c_w}\sum_f \bar{\Psi}_f\gamma^\mu(I^3_f - 2s^2_w Q_f - I^3_f\gamma_5)\Psi_f Z_\mu + \\
-\frac{1}{4}|\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 - \frac{1}{2}|\partial_\mu W^+_\nu - \partial_\nu W^+_\mu + \\
-ie(W^+_\mu A_\nu - W^+_\nu A_\mu) + ig' c_w(W^+_\mu Z_\nu - W^+_\nu Z_\mu|^2 + \\
-\frac{1}{4}|\partial_\mu Z_\nu - \partial_\nu Z_\mu + ig' c_w(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 + \\
-\frac{1}{2}M^2_\eta\eta^2 - \frac{gM^2_\eta}{8M_W}\eta^3 - \frac{g'^2 M^2_\eta}{32M_W}\eta^4 + |M_W W^+_\mu + \frac{g}{2}\eta W^+_\mu|^2 + \\
+\frac{1}{2}|\partial_\mu\eta + iM_Z Z_\mu + \frac{ig}{2c_w}\eta Z_\mu|^2 - \sum_f \frac{g}{2}\frac{m_f}{M_W}\bar{\Psi}_f\Psi_f\eta
\end{aligned}
$$

Outline

Lecture I: The Making of the SM

A. Deep Root in E&M → QED
B. The Strong Nuclear Force → QCD
C. The Weak Nuclear Force
D. Electro-Weak Unification→ The SM

Lecture II: Story of Mass-generation

A. Spontaneous Symmetry Breaking
B. The Nambu-Goldstone Theorem
   & the Higgs Mechanism
C. Fermion Mass Generation
D. The Higgs Boson Interactions

Lect I. The Making of the SM

A. Deep Root in QED

Maxwell Equations ➔

Lorentz invariance, U(1) Gauge Invariance

Electromagnetic fields can be treated by

E(x,t), **B**(x,t)

the introduction of co-variant vector potential

$A_\mu(x,t)$

makes the symmetries manifest (but redundant)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

1). Lorentz/Local Gauge invariance manifest.

2). Classically, geometrical interpretation: fiber bundles...

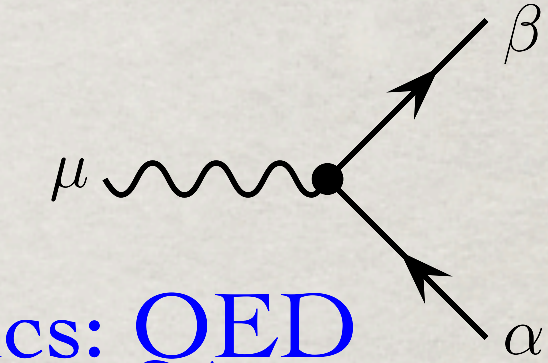
3). Quantum-mechanically, wave function for the EM field.

Dirac's relativistic theory:

Lorentz/Local gauge invariant ➔ antiparticle e^+

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m_e)\psi$$

$$D_\mu = \partial_\mu + ieA_\mu$$



$$= (+ie\gamma^\mu)_{\beta\alpha}$$

Quantum Electro-Dynamics: QED

Feynman/Schwinger/Tomonaga ➔ Renormalization



Anomalous magnetic moment:

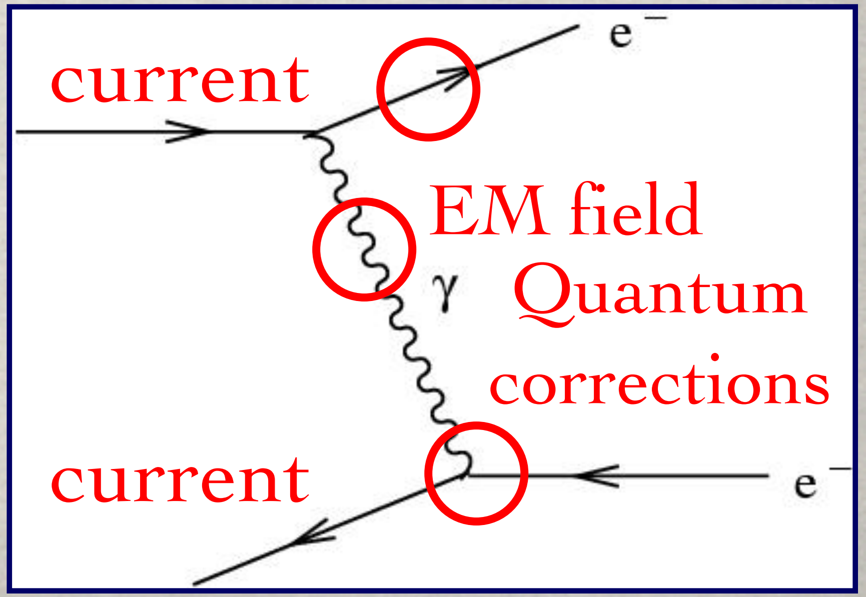
$(g\text{-}2)_e$

$$a_e(\text{Schwinger}) \approx \frac{\alpha}{2\pi} \approx 0.0011614$$

$$a_e^{theo} = 0.001159652181643(763)$$

$$a_e^{exp} = 0.00115965218073(28)$$

QED becomes the most accurate theory in science!



Warmup Exercise 1:

For charge scalar field $\phi^{\pm}$, construct the locally $U(1)_{em}$ gauge invariant Lagranian and derive the Feynman rules for its EM interactions.

Sketch a calculation for the differential and total cross section for the process:

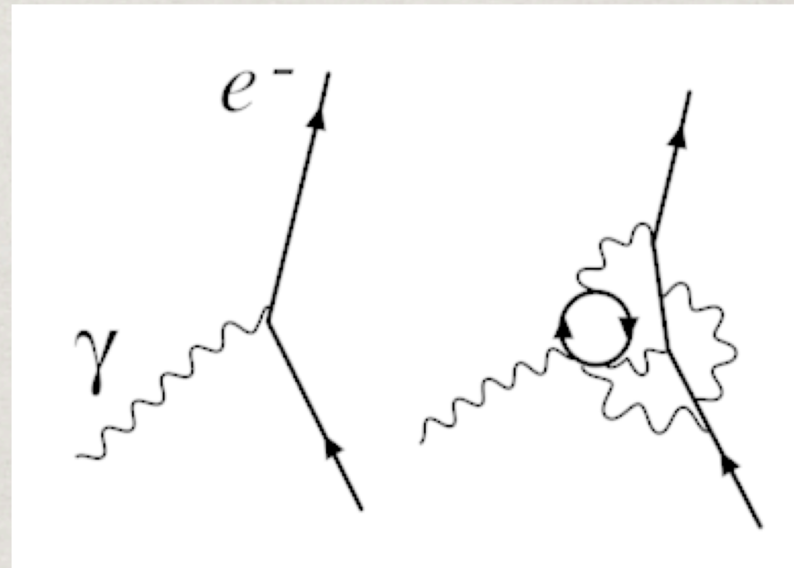
$$e^+ e^- \rightarrow \phi^+ \phi^-$$

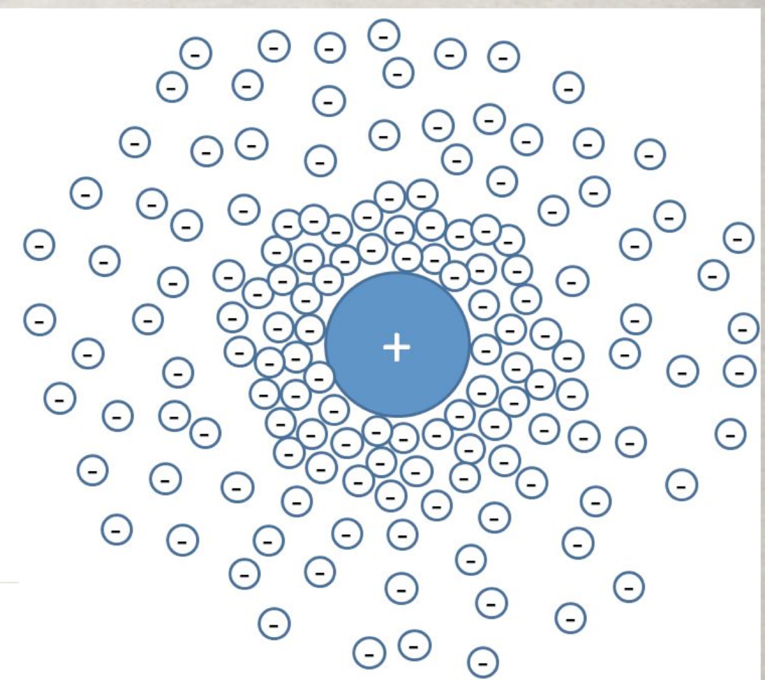
Warmup Exercise 2:

Find the value for a muon anomalous magnetic moment $(g\text{-}2)_{\mu}$ and compare with that for the electron. Where is the difference from?.

The key effect of renormalization: Running of coupling with energies

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} = \frac{\partial g}{\partial \ln \mu},$$





$$\beta(e) = \frac{e^3}{12\pi^2}$$

$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \frac{\alpha(Q_0^2)}{3\pi} \ln(Q^2/Q_0^2)}$$

$$\alpha_{QED}(keV) = 1/137$$
$$\alpha_{QED}(M_Z) = 1/128$$

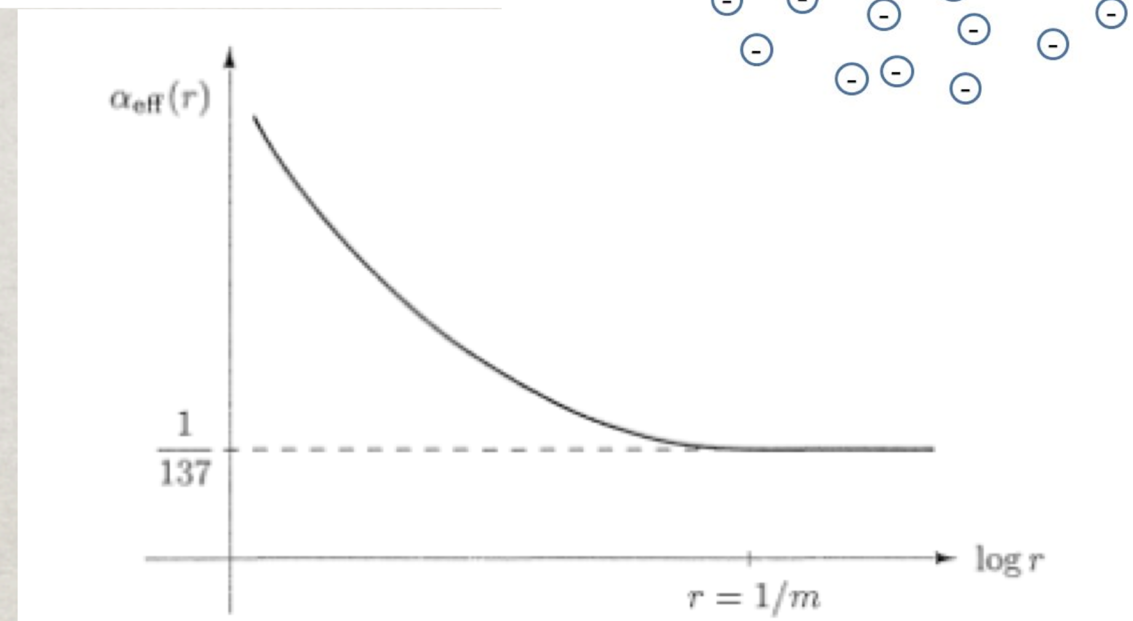


Figure 7.10. A qualitative sketch of the effective electromagnetic coupling constant generated by the one-loop vacuum polarization diagram, as a function of distance. The horizontal scale covers many orders of magnitude.

The Landau pole:

It blows up at high energies! Must be modified at UV.

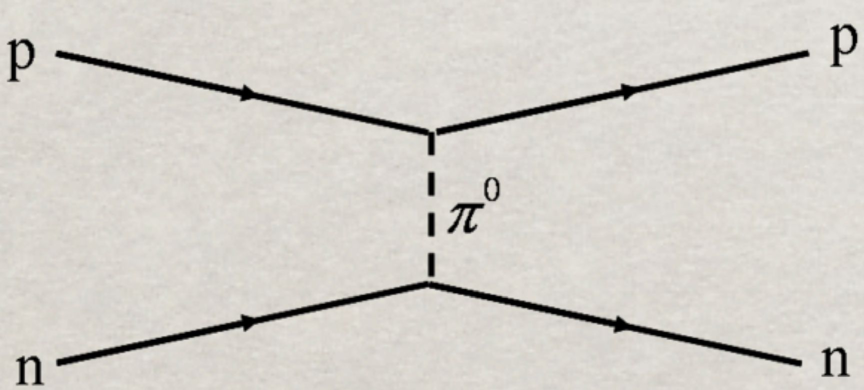
B. There Is a strong force!

Ever since Rutherford established the atomic nuclear model → a new force to bound p^+ to a nucleus: proton, "the 1st"

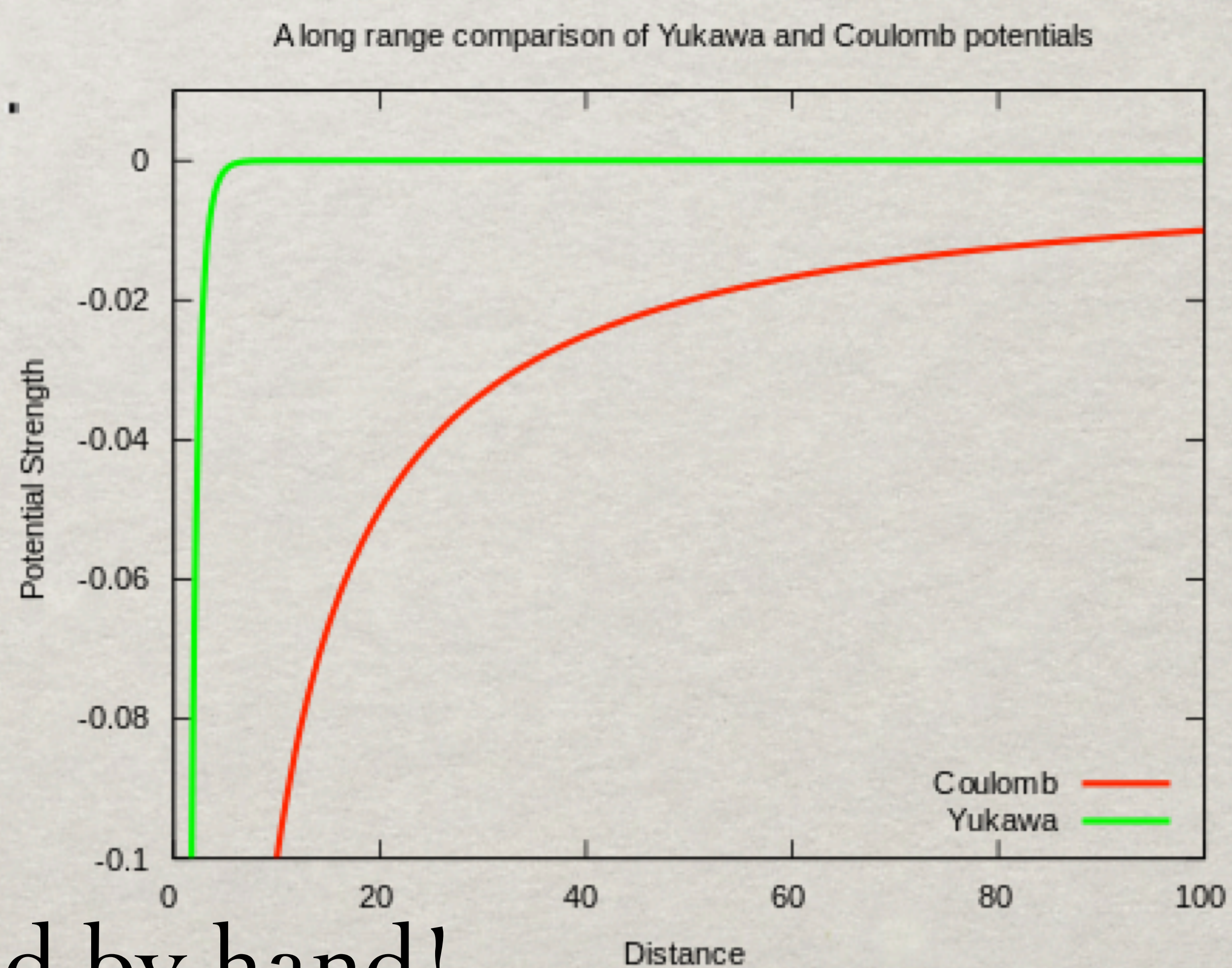
The discovery neutron (1932) → a charge-independent force: Heisenberg → (p^+, n^0) "iso-spin" doublet

Yukawa (1935) → a universal attractive force via pions

$$\mathcal{L}_{\text{int}}(x) = g\overline{\psi}(x)\phi(x)\psi(x).$$



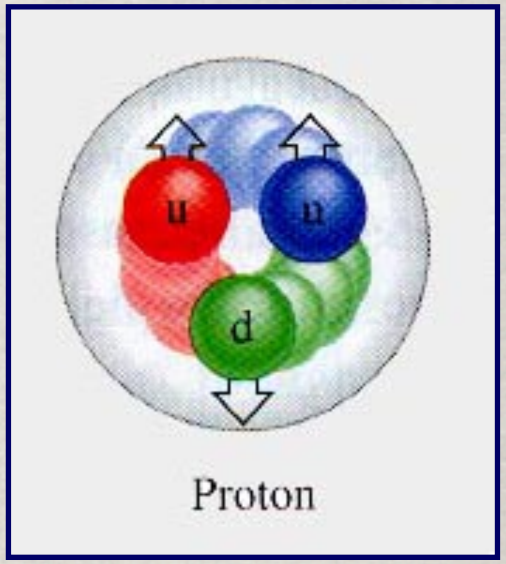
$$V_{\text{Yukawa}}(r) = -g^2 \frac{e^{-mr}}{r}$$



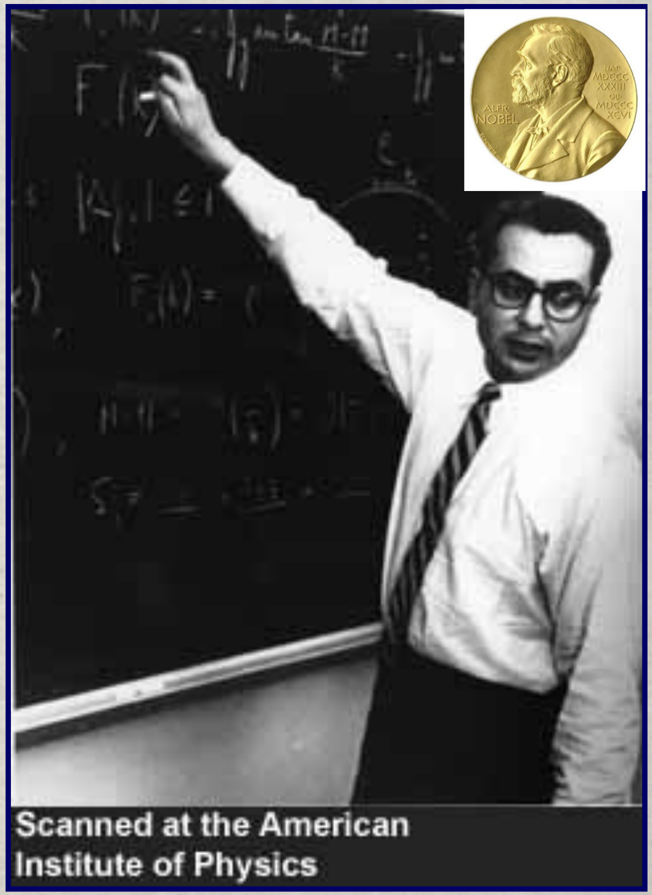
Discoveries & theory hand by hand!

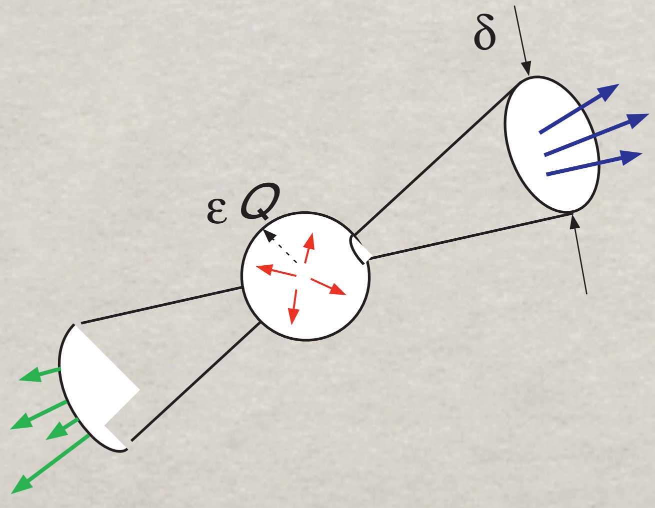
What IS the strong force?

Numerous "hadrons" discovered (50's):
Mesons: π, η, ρ, ω … Baryons: p, n, Δ, Λ, Σ …
How to understand/describe them?
→ Hadronic string theory developed.



- Gell-Mann – Zweig's "quarks" (1963)
- $\pi^0 \to \gamma\,\gamma$ 3 colors (1964)
- Proton structure by DIS (1969)
- 2 or 3-jet structure (q: 1975, g: 1979)





SU(3)$_C$ gauge theory established (1973)

Quantum-Chromo-Dynamics (QCD)

H. Fritzsch, M. Gell-Mann, H. Leutwyler (1973)

$$\mathcal{L} = \sum_f^{n_f} \bar{q}_f (i\gamma^\mu \partial_\mu - g_s \gamma^\mu A_\mu + m_f) q_f - \frac{1}{2} \mathrm{Tr} F_{\mu\nu}^2$$

Direct analogue of QED

$$F_{\mu\nu} = \partial \mu A_\nu - \partial \mu A_\nu + i g_s [A_\mu, A_\nu]$$

← Non-Abelian

$$A^\mu(x) = \sum_1^8 A(x)_a^\mu \; T^a, \qquad [T^a, T^b] = i f_{abc} T^c.$$

QED analogue:

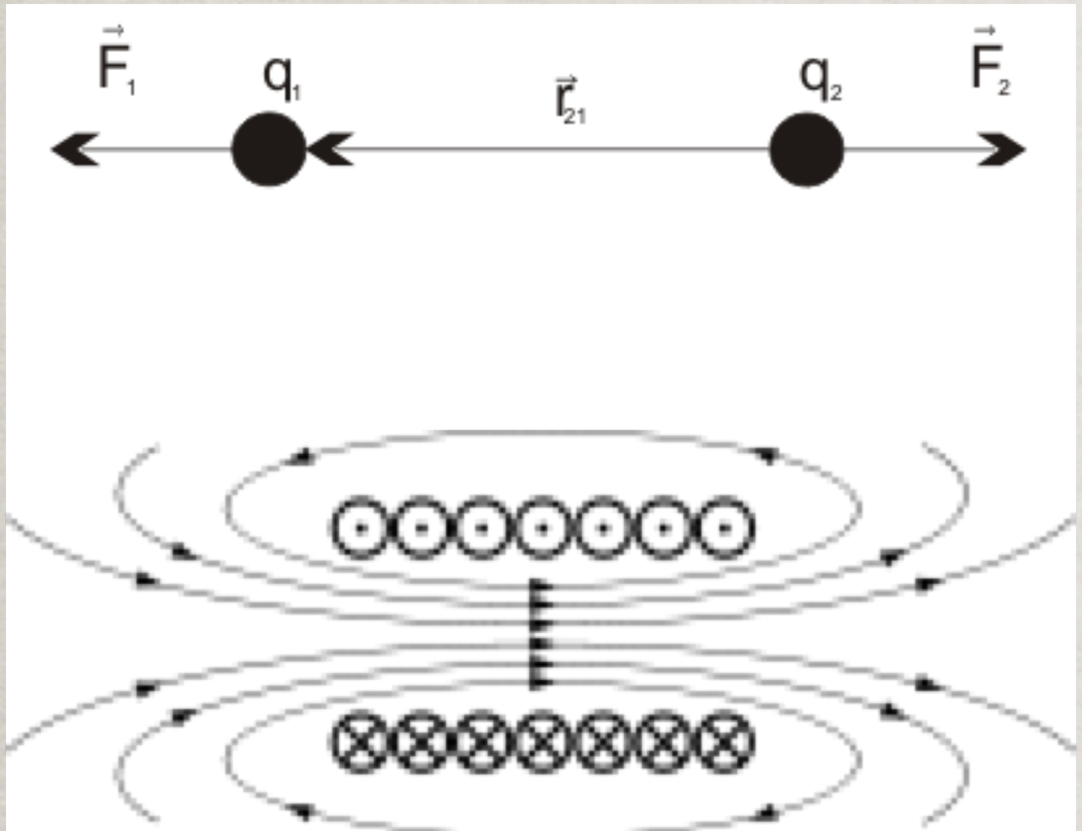
- Similar gauge principles;
- Tempting for perturbative renormalization calculations

Non-Abelian gauge theory: Yang-Mills $SU(3)_C$

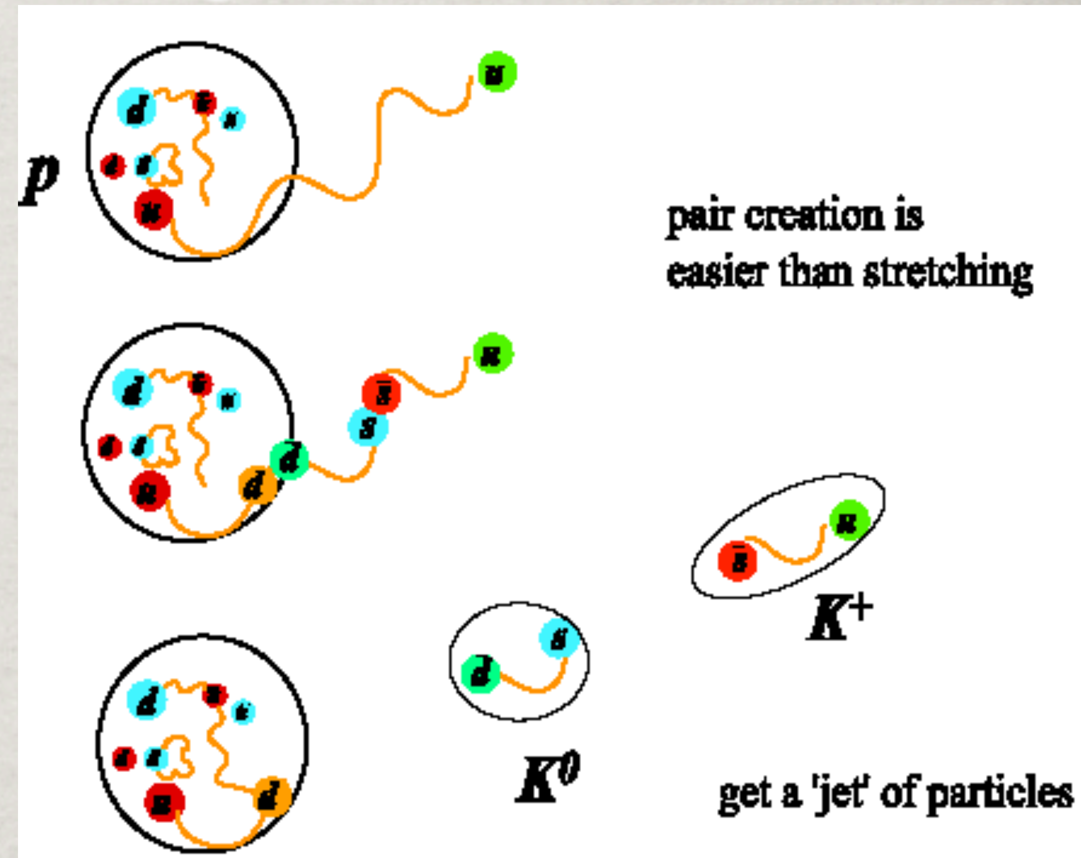
- Self gauge interactions among 8 gluons;
- Coupling rather strong, may invalidate perturbation theory

QED versus QCD

Electromagnetism vs. Strong force



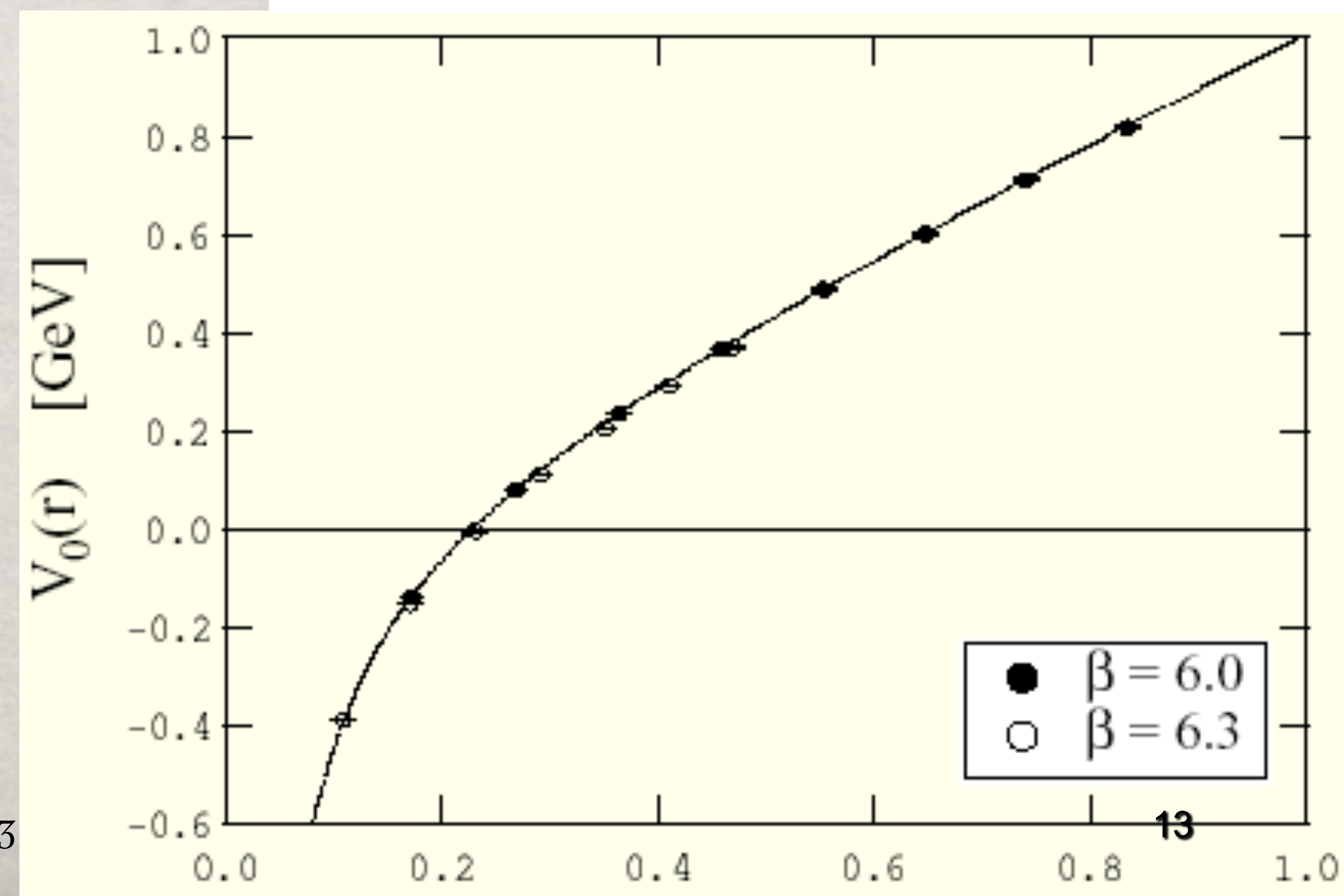
Photons vs. gluons



QED: $V(r) = -\alpha_{em}/r$

QCD: $V(r) = -\alpha_s/r + k\,r$

In long distances, we see charged particles, but not colored particles!



QCD at Low Energies: Quark condensation

Consider the two-flavor massless QCD

$$-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a - \sum_{u,d}(\bar{q}_L\gamma^\mu D_\mu q_L + \bar{q}_R\gamma^\mu D_\mu q_R)$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \to \left(U_L\,\gamma_L + U_R\,\gamma_R\right)\begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow SU(2)_L \otimes SU(2)_R$$

QCD exhibits a L-R chiral symmetry.

Below Λ_{QCD}, QCD becomes strongly interacting and forms condensate: $\langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \sim v^3$

$$SU(2)_L \otimes SU(2)_R \Rightarrow SU(2)_V, \text{thus } U_L = U_R.$$

Chiral symmetry is broken to iso-spin.

The Spontaneous Symmetry Breaking

“ The Lagrangian of the system may display an symmetry, but the ground state does not respect the same symmetry.”

The concept of SSB: profound, common.

Known Example: Ferromagnetism

Domains Before Magnetization

Above a critical temperature, the system is symmetric, magnetic dipoles randomly oriented. Below a critical temperature, the ground state is a completely ordered configuration in which all dipoles are ordered in some arbitrary direction,

Domains After Magnetization

SO(3) → SO(2)

In QCD chiral symmetry breaking,

(3+3) - 3 = 3 Nambu-Goldstone bosons:

π^+, π^-, π^0 (u, d bound states)

In the non-linear formulation of the Chiral Lagrangian for the Goldstone bosons:

$$\phi = \frac{v}{\sqrt{2}} \exp(i\vec{\tau}\cdot\vec{\pi}/v) \equiv \frac{v}{\sqrt{2}} U, \quad \mathcal{L} = \frac{v^2}{4} Tr(\partial^\mu U \partial_\mu U)$$

necessarily derivative coupling.

Exercise 3: Linearize the Chiral Lagrangian for π-π interaction and calculate one scattering amplitude.¶

¶ J. Donoghue et al., Dynamics of the SM.

Y. Nambu was the first one to have formulated the spontaneous symmetry breaking in a relativistic quantum field theory (1960).

He is the one to propose the understanding of the nucleon mass by dynamical chiral symmetry breaking: The Nambu-Jona-Lasinio Model.

2008 Nobel Prize in physics: "for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

Be aware of the difference between the dynamical mass for baryons (you and me) and that of elementary particles by the Higgs mechanism.

"Pseudo-Nambu-Goldstone Bosons"

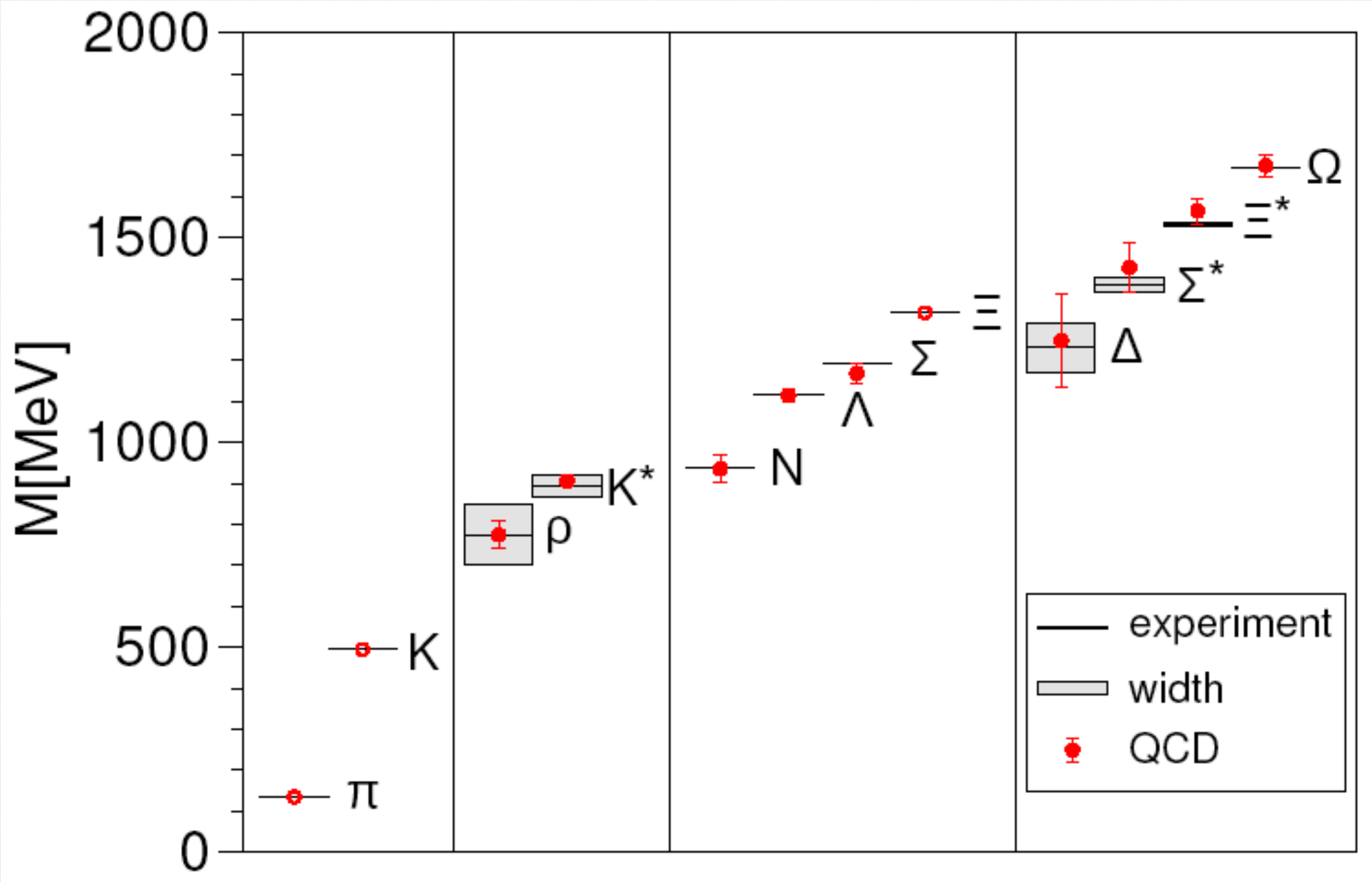
When a continuous symmetry is broken both explicitly AND spontaneously, and if the effect of the explicit breaking is much smaller than the SSB, then the Goldstone are massive, governed by the explicit breaking, thus called:
"Pseudo-Nambu-Goldstone bosons".

The pions are NOT massless, due to explicit symmetry breaking. They are "Pseudo-Nambu-Goldstone bosons".

Except the photon, no massless boson (a long-range force carrier) has been seen in particle physics!

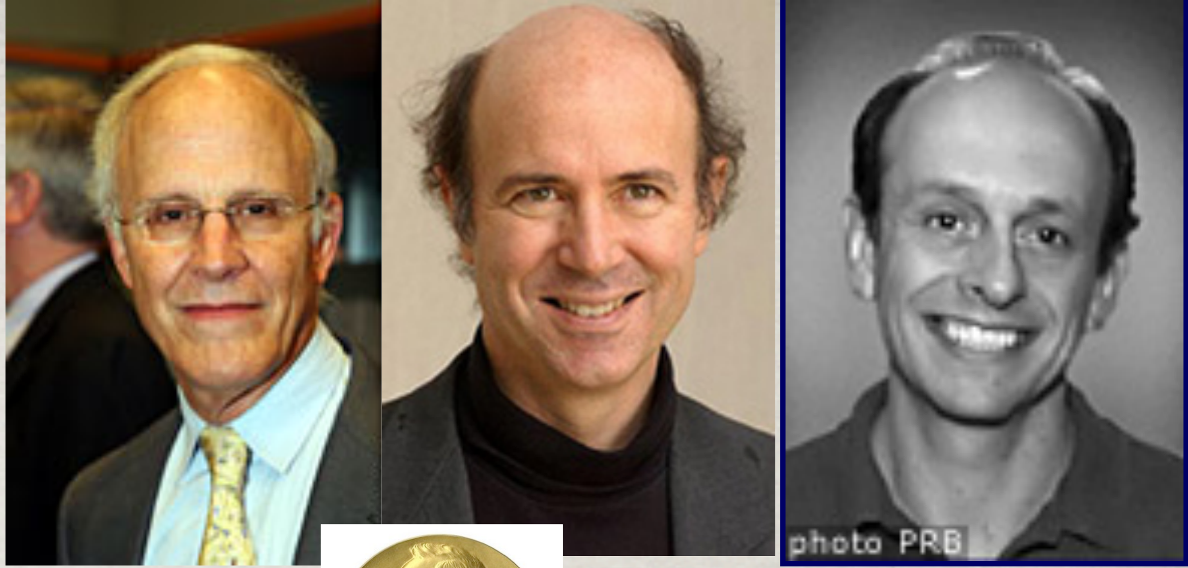
Most Mass due to QCD:

From quark constituents to hadrons:
(From PDG, based on lattice QCD)



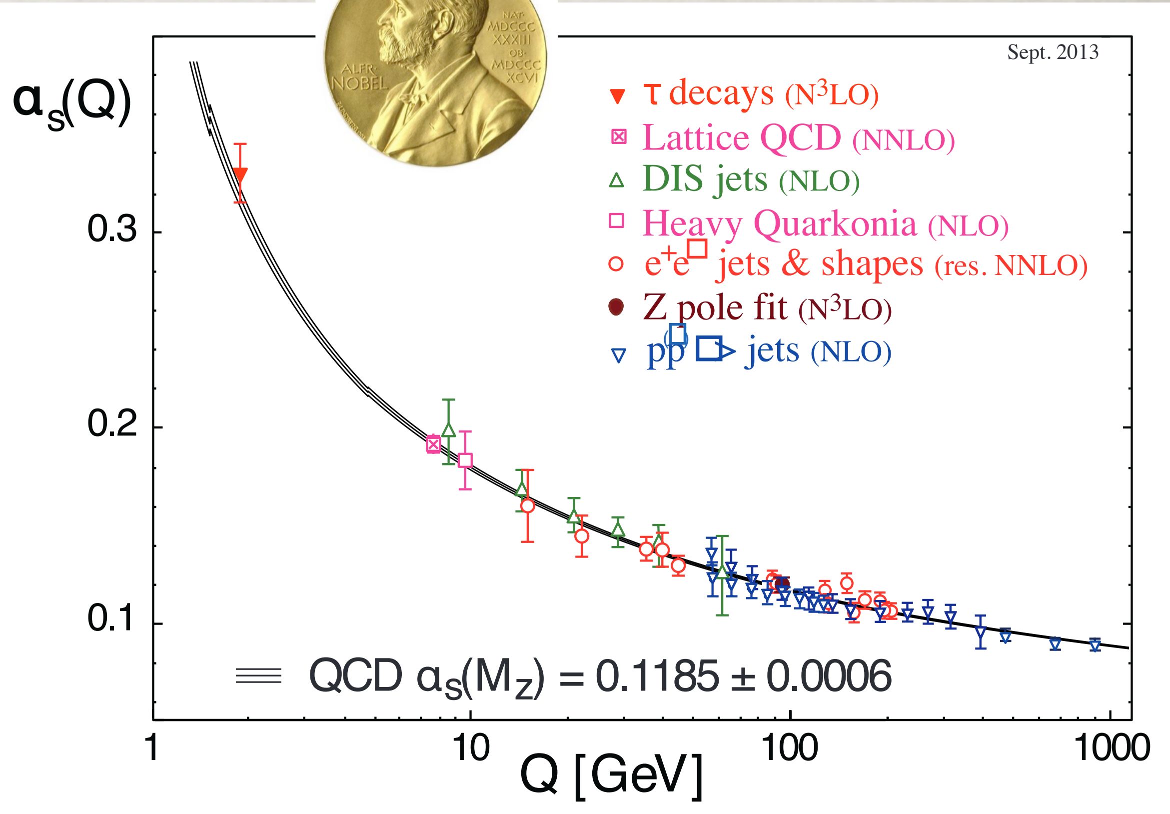
Majority of the (luminous) mass around us is of dynamical origin, from strong interactions (u, d quarks + gluons).
It is not from the Higgs mechanism!.

QCD at High Energies



Interaction strength changes fast with energy/distance scale:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{(33-2n_f)\alpha_s(Q_0^2)}{12\pi}\ln(Q^2/Q_0^2)}$$



At high energies, $E >> \Lambda_{QCD}$, QCD is weakly interacting! "Asymptotic freedom"

- Perturbative → prediction for high energy experiments (ee, ep, pp etc. LHC …)
- Think about higher energy physics at M_{GUT}, M_{PL}
- Early universe cosmology at high T.

QCD Factorization Theorem:

J. Collins, D. Soper, G. Sterman (1985)

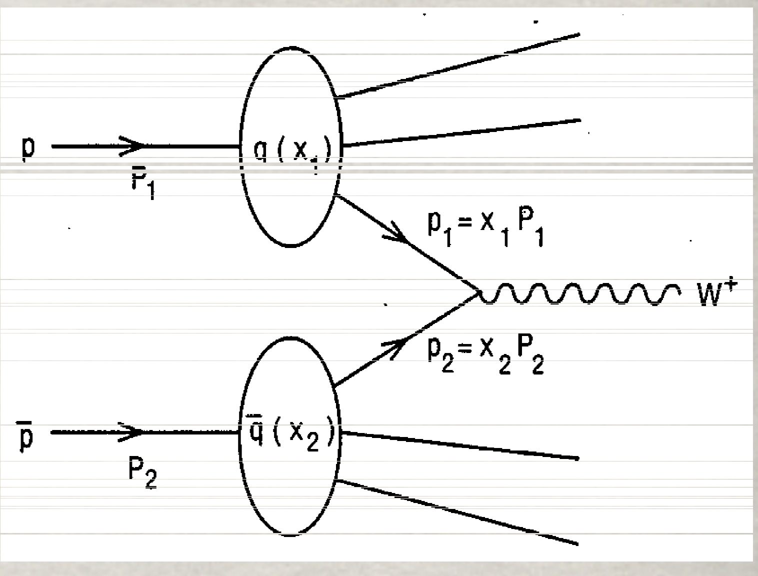
In high energy collisions involving a hadron, the total cross sections can be factorized into two factors:

(1). hard subprocess of parton scattering with a large scale $\mu^2 \gg \Lambda^2_{QCD}$;

(2). "parton distribution functions" (hadronic structure with $Q^2 < \mu^2$.)

Observable cross sections at hadron level:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2)\ \hat{\sigma}_{parton}(s).$$

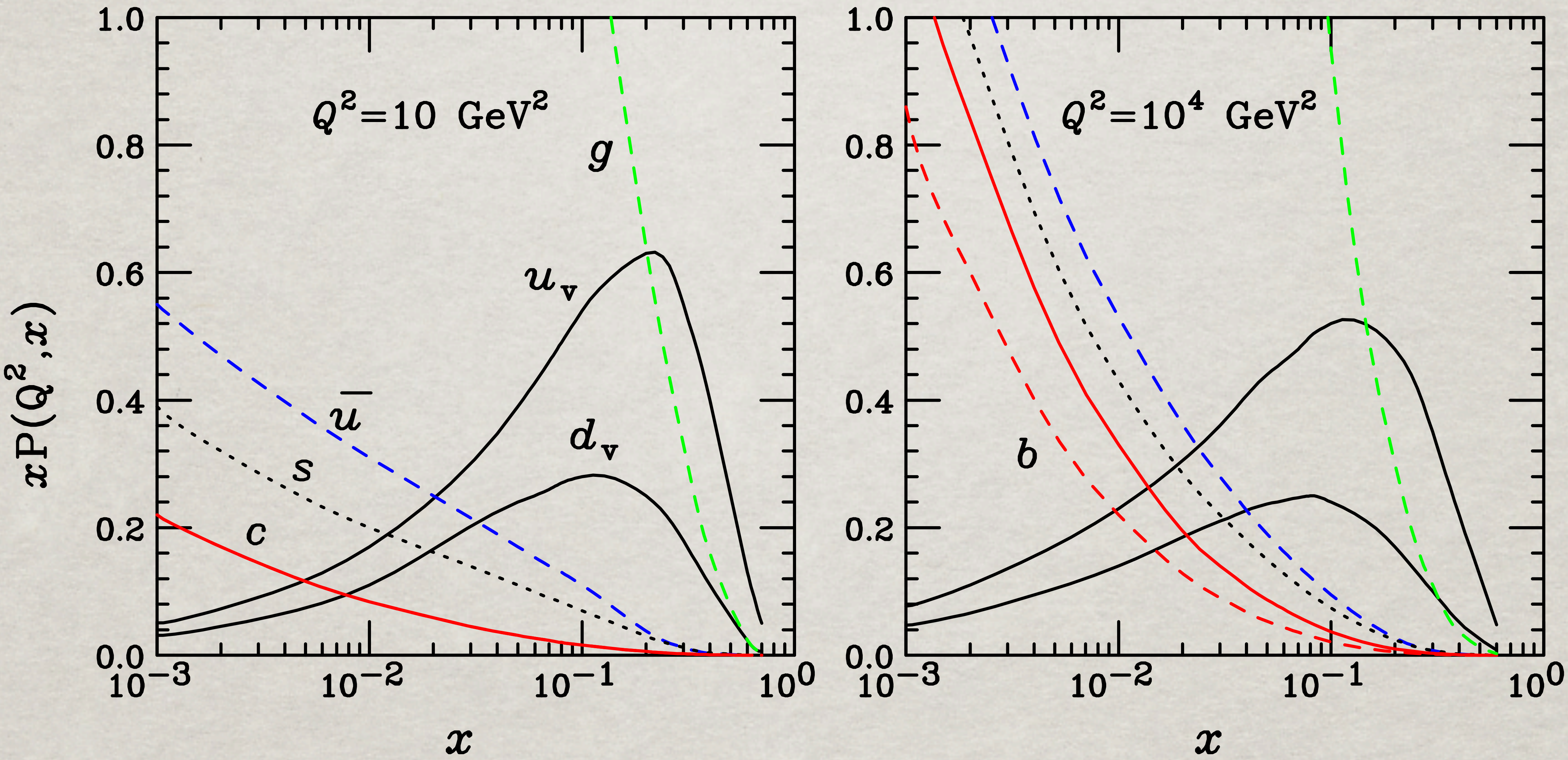


† $\hat{\sigma}_{parton}(s)$ is theoretically calculated by perturbation theory (in the SM or models beyond the SM).

Ultra violet (UV) divergence (beyond leading order) is renormalized;
Infra-red (IR) divergence is cancelled by soft gluon emissions;
Co-linear divergence (massless) is factorized into PDF

PDF's: $q(x, Q^2)$, $g(x, Q^2)$, ...

Typical quark/gluon parton distribution functions:



(CTEQ-5)

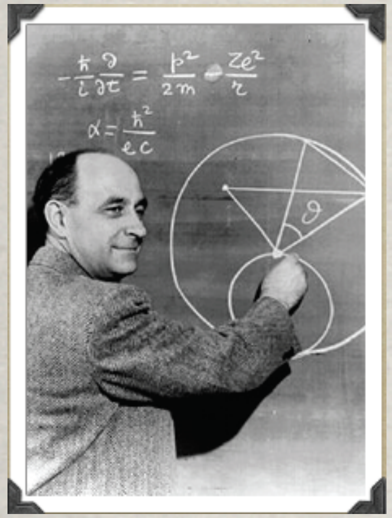
Quarks carry ½ momentum; gluons carry the other ½!

CTEQ, MRS (Durham), NLOPDF etc.

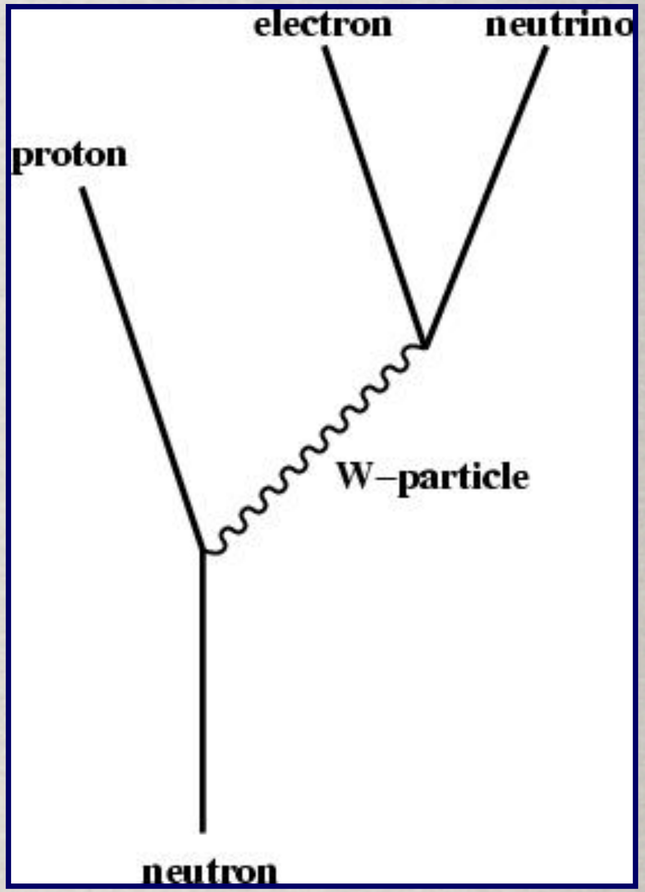
C. The Weak Nuclear Force

beta decay n → $p^+ e^- \nu$ ➔ Charged current interaction: $W^{\pm}$

ν N → ν N ➔ Neutral current interaction



via: Z^0 (1973)



$$-\mathcal{L}^{cc}_{eff} = \frac{G_F}{\sqrt{2}} J^{\mu}_{W} J^{\dagger}_{W\mu}, \quad -\mathcal{L}^{NC}_{eff} = \frac{G_F}{\sqrt{2}} J^{\mu}_{Z} J_{Z\mu}.$$

$$J^{(\pm)}_{\lambda} = \sum_i \bar{\psi}_i \tau_{\pm} \gamma_{\lambda} (1-\gamma_5) \psi_i,$$

- Beyond E&M, Fermi was inspired by the current-current interactions to construct the weak interaction (1934).
- parity violation ➔ V-A interactions (1957).

The fact $G_F = (300\ \text{GeV})^{-2}$ implies that:

1. A new mass scale to show up at O(100 GeV).
2. Partial-wave Unitarity requires new physics below

$E < 300$ GeV

Exercise 4:

Assume that the $\nu\, e \rightarrow \nu\, e$ scattering amplitude to be

$$\mathcal{M} = G_F\, E_{cm}^2$$

estimate the unitarity bound on the c.m. energy.

Partial wave expansion:

$$a_{I\ell}(s) \;=\; \frac{1}{64\pi}\int_{-1}^{1} d\cos\theta\;\; P_\ell(\cos\theta)\;\; \mathcal{M}^I(s,t)$$

Partial wave unitarity:

$$Im\,(a_{I\ell}) = |a_{I\ell}|^2 < 1, \quad Re\,(a_{I\ell}) < \frac{1}{2}$$

D. The Idea of Unification:

Within a frame work of relativistic, quantum, gauge field theory

PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

SHELDON L. GLASHOW †

Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

Received 9 September 1960

Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.

The birth of the Standard Model:

VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

[11] In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.

[12]M. Ademollo and R. Gatto, Nuovo Cimento 44A, 282

bra is slightly larger than that (0.23%) obtained from the ρ-dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta \to \pi^+\pi^-\gamma)/$) calculated in Refs. 12 and 14.

. M. Brown and P. Singer, Phys. Rev. Letters 8, (1962).

A MODEL OF LEPTONS*

Steven Weinberg†

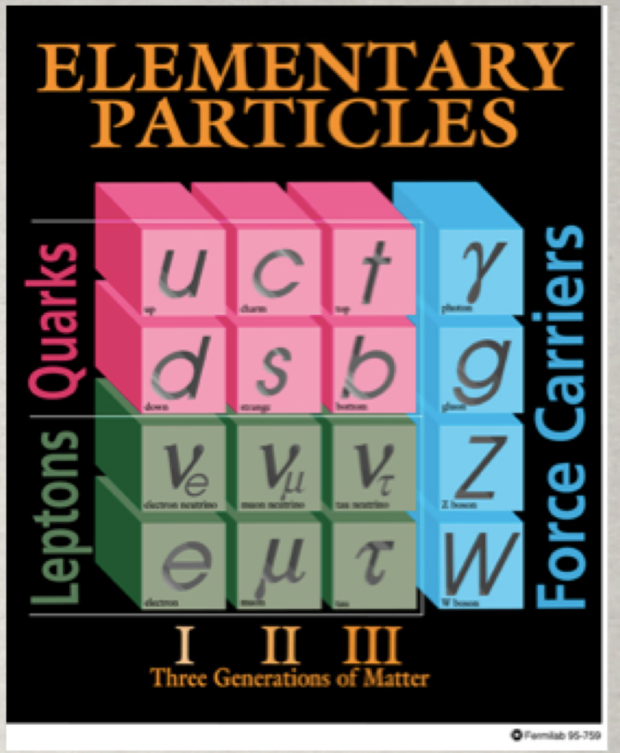
Physics Department, ambridge, Massachusetts 1967)

on a right-handed singlet

$$R = [\tfrac{1}{2}(1-\gamma_5)]e.$$

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite[1] these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences

The EW Unification I:
Particle representation



- Simple structure and particle contents:

Leptons:

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L, \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L, \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L, \quad e_R,\ \mu_R,\ \tau_R, \quad (\nu_R' s\ ?)$$

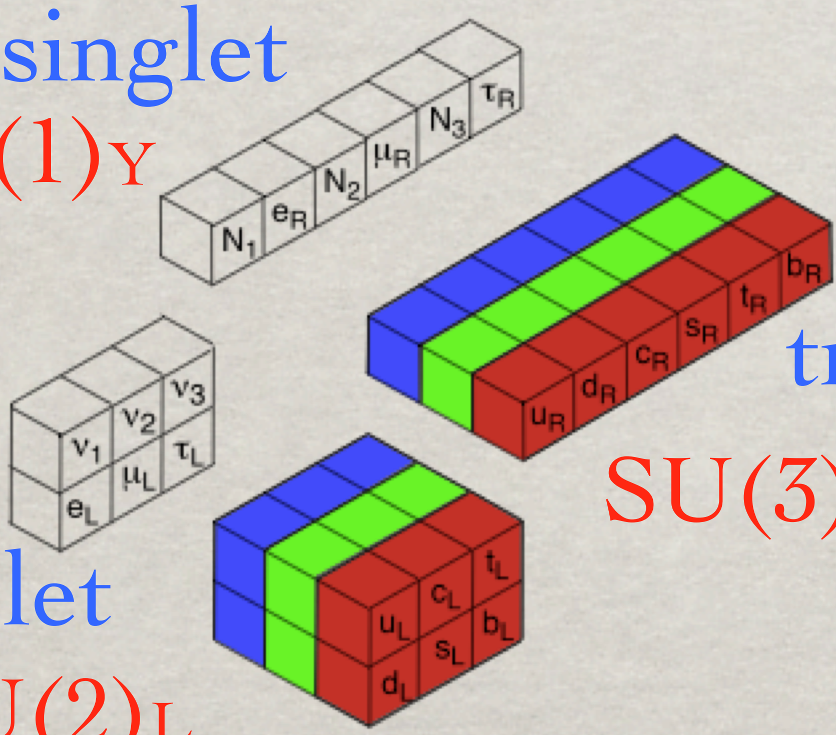
Quarks:

$$\begin{pmatrix}u\\ d\end{pmatrix}_L, \quad \begin{pmatrix}c\\ s\end{pmatrix}_L, \quad \begin{pmatrix}t\\ b\end{pmatrix}_L, \quad u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R$$

(1979 Nobel)

singlet

U(1)Y



triplet

SU(3)c

doublet

SU(2)L

The EW Unification II:

The gauge interactions

$$
-\frac{g}{2\sqrt{2}}\sum_i \overline{\Psi}_i\,\gamma^\mu\,(1-\gamma^5)(T^+\,W_\mu^+ + T^-\,W_\mu^-)\,\Psi_i
$$

$$
-e\sum_i q_i\,\overline{\psi}_i\,\gamma^\mu\,\psi_i\,A_\mu \quad -\frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}
$$

$$
-\frac{g}{2\cos\theta_W}\sum_i \overline{\psi}_i\,\gamma^\mu(g_V^i - g_A^i\gamma^5)\,\psi_i\,Z_\mu \ .
$$

$$
M_W = \frac{1}{2}\,g\,v = \frac{e\,v}{2\sin\theta_W},
$$

$$
M_Z = \frac{1}{2}\sqrt{g^2+g'^2}\,v = \frac{e\,v}{2\sin\theta_W\cos\theta_W} = \frac{M_W}{\cos\theta_W},
$$

$$
M_\gamma = 0.
$$

$$
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu
$$

$$
W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu
$$

$SU(2)_L$: Non-Abelian gauge theory, asymptotically free

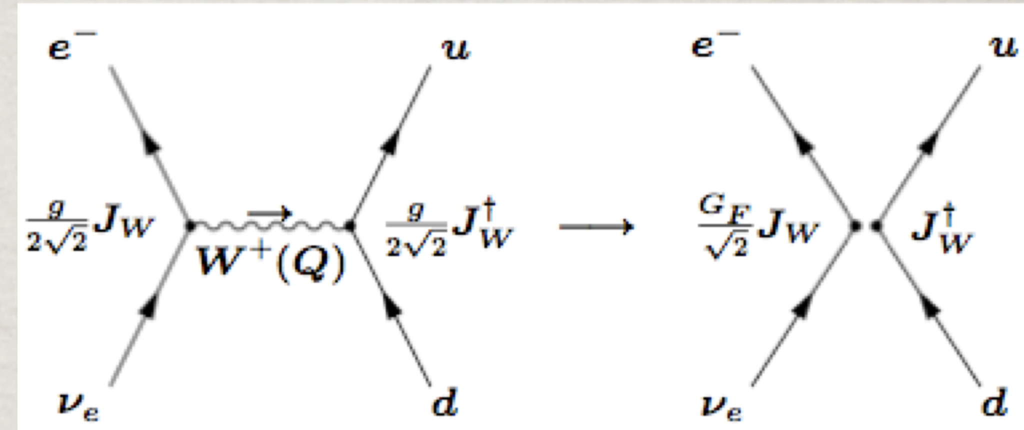
$U(1)_Y$: Non-asymptotically free → Landau pole!

“Weak force” NOT Weak!

$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ interactions.

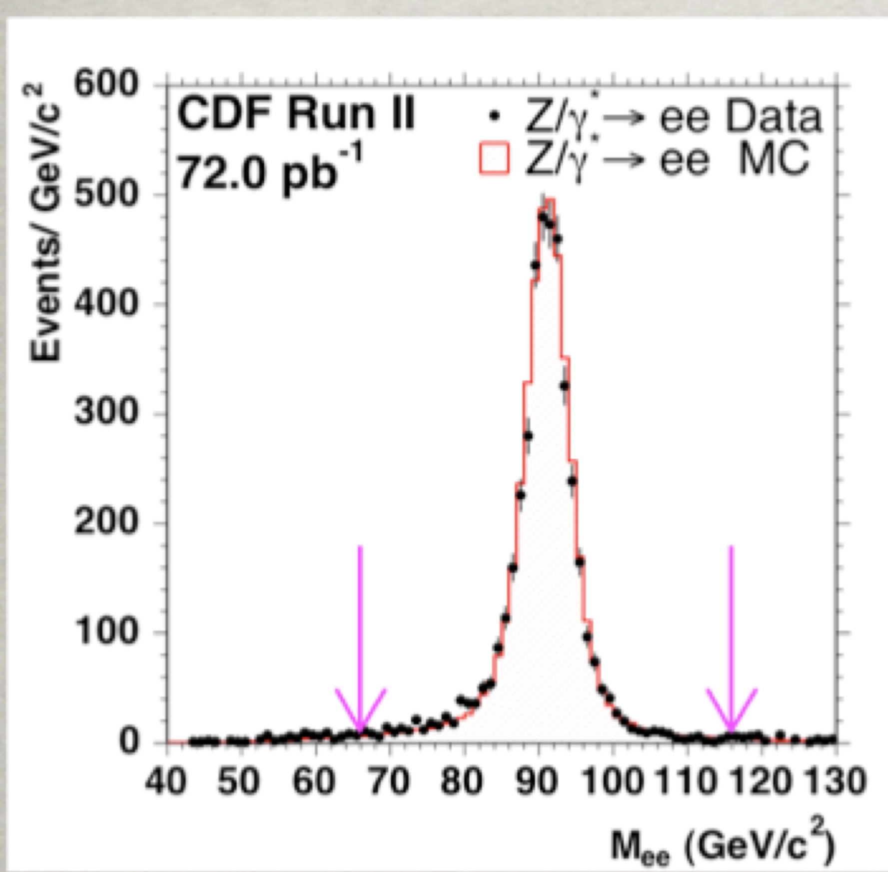
$e = g \sin\theta_W$ coupling unification

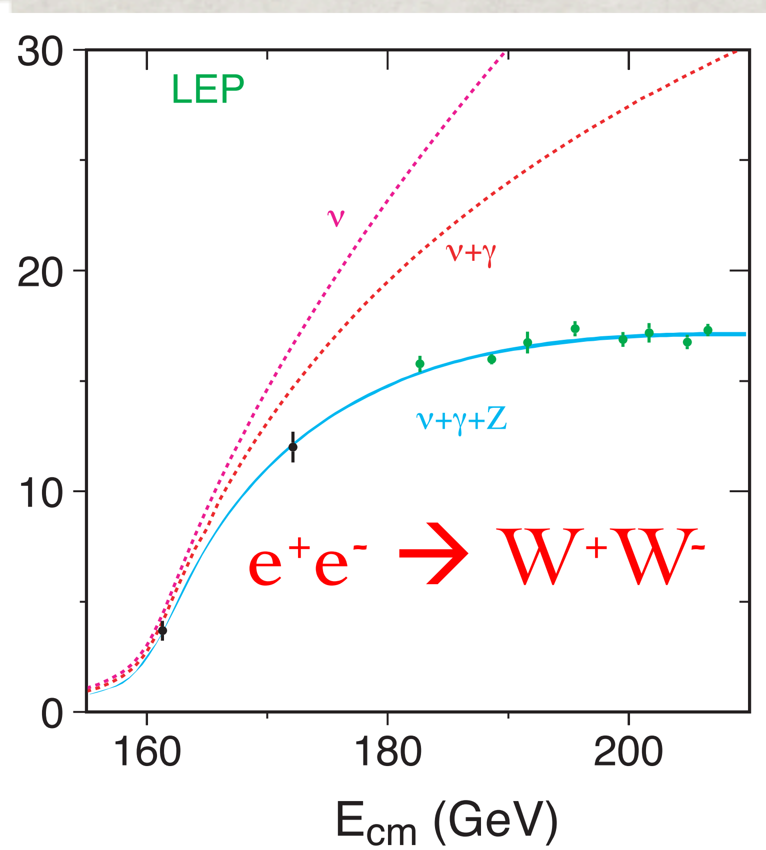
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$ short – range scale.

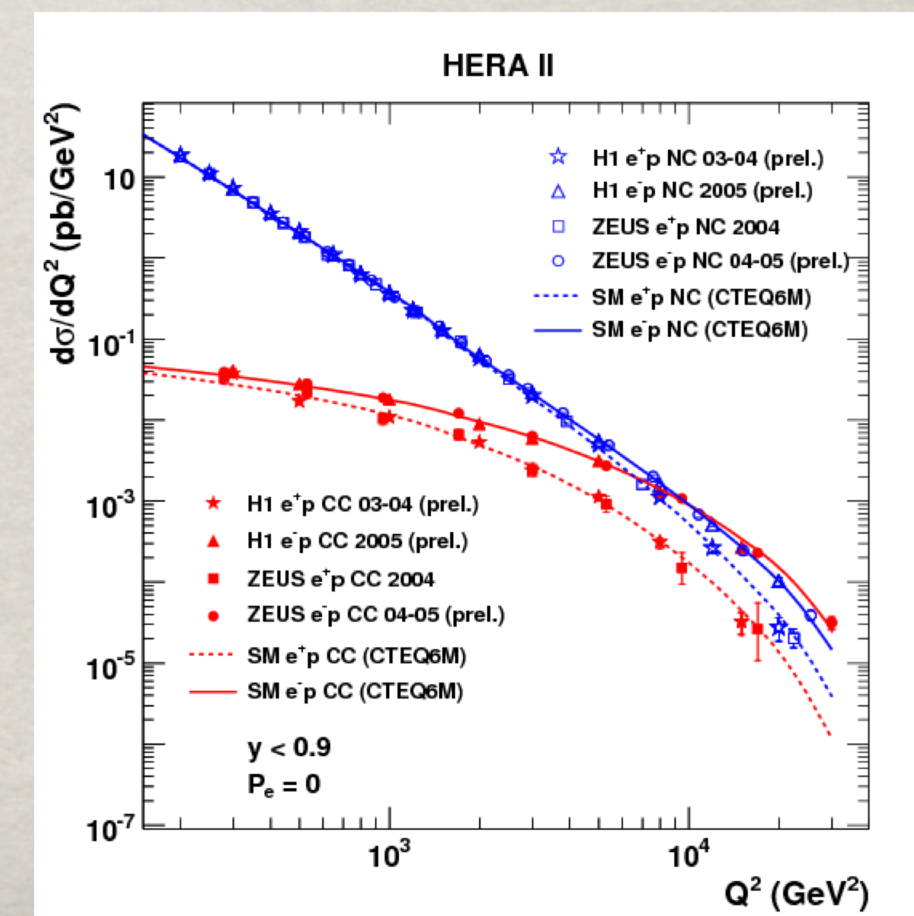


The EW couplings merging:

The EW scale is fully open up:







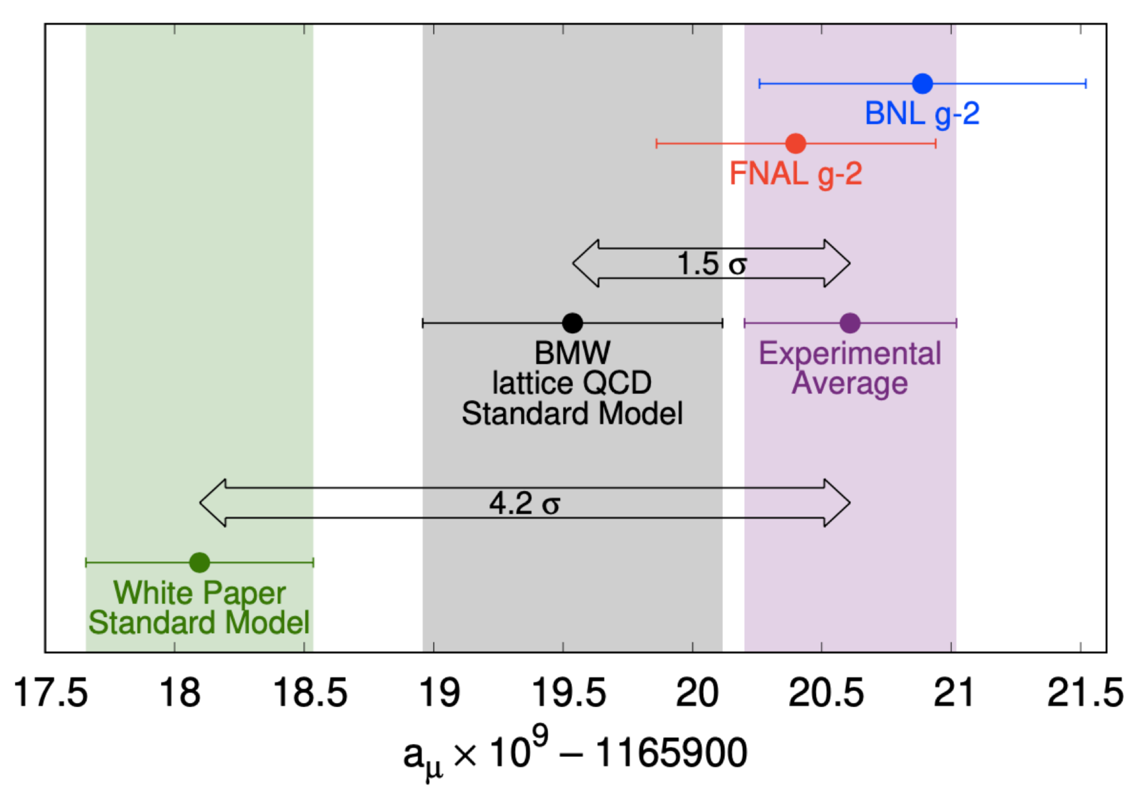
Quantity	Value	Standard Model	Pull	Dev.
M_Z [GeV]	91.1876 ± 0.0021	91.1874 ± 0.0021	0.1	0.0
Γ_Z [GeV]	2.4952 ± 0.0023	2.4961 ± 0.0010	−0.4	−0.2
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7426 ± 0.0010	—	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.69 ± 0.06	—	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	84.005 ± 0.015	—	—
σ_{had} [nb]	41.541 ± 0.037	41.477 ± 0.009	1.7	1.7
R_e	20.804 ± 0.050	20.744 ± 0.011	1.2	1.3
R_μ	20.785 ± 0.033	20.744 ± 0.011	1.2	1.3
R_τ	20.764 ± 0.045	20.789 ± 0.011	−0.6	−0.5
R_b	0.21629 ± 0.00066	0.21576 ± 0.00004	0.8	0.8
R_c	0.1721 ± 0.0030	0.17227 ± 0.00004	−0.1	−0.1
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01633 ± 0.00021	−0.7	−0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.4	0.6
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5	1.6
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1034 ± 0.0007	−2.6	−2.3
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0739 ± 0.0005	−0.9	−0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1035 ± 0.0007	−0.5	−0.5
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	0.2324 ± 0.0012	0.23146 ± 0.00012	0.8	0.7
	0.23200 ± 0.00076		0.7	0.6
	0.2287 ± 0.0032		−0.9	−0.9
A_e	0.15138 ± 0.00216	0.1475 ± 0.0010	1.8	2.1
	0.1544 ± 0.0060		1.1	1.3
	0.1498 ± 0.0049		0.5	0.6
A_μ	0.142 ± 0.015		−0.4	−0.3
A_τ	0.136 ± 0.015		−0.8	−0.7
	0.1439 ± 0.0043		−0.8	−0.7
A_b	0.923 ± 0.020	0.9348 ± 0.0001	−0.6	−0.6
A_c	0.670 ± 0.027	0.6680 ± 0.0004	0.1	0.1
A_s	0.895 ± 0.091	0.9357 ± 0.0001	−0.4	−0.4

Some tension?

(nearly) perfect agreement between SM theory & experiments!

Coupling universality: Strong evidence for gauge force!

Tension on $(g\text{-}2)_\mu$:



Lecture II: Story of Mass-generation

A. Spontaneous Symmetry Breaking
B. The Nambu-Goldstone Theorem
   & the Higgs Mechanism
C. Fermion Mass Generation
D. The Higgs Boson Interactions

A Problem! Pauli's Criticism:

An Anecdote by Yang: SU(2) gauge symmetry

Wolfgang Pauli (1900-1958) was spending the year in Princeton, and was deeply interested in symmetries and interactions.... Soon after my seminar began, when I had written on the blackboard,

Wolfgang Pauli and C. N. Yang

$(\partial_\mu - i\epsilon \mathbf{B}_\mu)\psi$

Pauli asked, "What is the mass of this field $\mathbf{B}_\mu$?" I said we did not know. Then I resumed my presentation but soon Pauli asked the same question again. I said something to the effect that it was a very complicated problem, we had worked on it and had come to no definite conclusions. I still remember his repartee: "That is not sufficient excuse". I was so taken aback that I decided, after a few moments' hesitation, to sit down. There was general embarrassment. Finally Oppenheimer, who was chairman of the seminar, said "We should let Frank proceed". I then resumed and Pauli did not ask any more questions during the seminar.

The local gauge symmetry prevents gauge bosons masses!

$$\frac{1}{2}M_A^2 A_\mu A^\mu \to \frac{1}{2}M_A^2\left(A_\mu - \frac{1}{e}\partial_\mu\alpha\right)\left(A^\mu - \frac{1}{e}\partial^\mu\alpha\right) \neq \frac{1}{2}M_A^2 A_\mu A^\mu$$

Even worse:

"The Left- and right-chiral electrons carry different Weak charges" (Lee & Yang)

$$J_\lambda^{(\pm)} = \sum_i \bar{\psi}_i \tau_\pm \gamma_\lambda (1-\gamma_5)\psi_i,$$

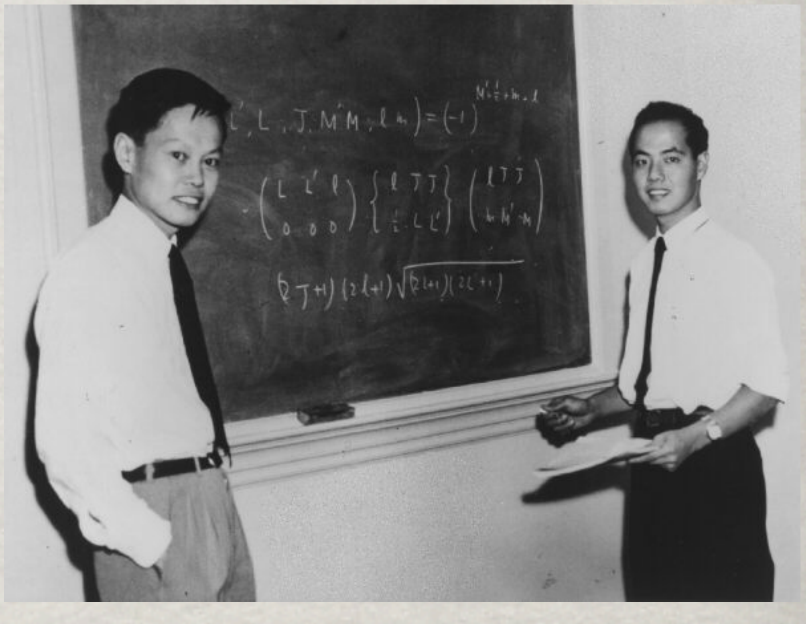
- Simple structure and particle contents:

Leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad e_R,\ \mu_R,\ \tau_R, \quad (\nu'_R\ s\ ?)$$

Quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R$$



Fermion masses also forbidden by gauge symmetry!

$$-m_e \bar{e}e = -m_e \bar{e}\left(\frac{1}{2}(1-\gamma_5) + \frac{1}{2}(1+\gamma_5)\right)e = -m_e(\bar{e}_R e_L + \bar{e}_L e_R)$$

Electroweak gauge theory → massless!

A. The Spontaneous Symmetry Breaking

-- Nature May Not be THAT Symmetric:

"The Lagrangian of the system may display an symmetry, but the ground state does not respect the same symmetry."

Recall the earlier example in low-energy QCD:

$$\langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \sim v^3$$

Exercise 5:

Find (or make up) other examples for spontaneous symmetry breaking.

Also, think about the relations between the fundamental theoretical formalisms (Newton's Law; Maxwell Equations; Einstein Equation; Lagrangians...) and specific states for a given system (initial and boundary conditions of a system).

B. The Nambu-Goldstone Theorem
-- A show stopper or helper?

"If a continuous symmetry of the system is spontaneously broken, then there will appear a massless degree of freedom, called the Nambu-Goldstone boson."

Symmetry: $[Q, H] = QH - HQ = 0$

Vacuum state: $H|0\rangle = E_{min}|0\rangle$ But: $Q|0\rangle \neq 0 = |0'\rangle$

$(QH - HQ)|0\rangle = 0 = (E_{min} - H)|0'\rangle$,

thus: $H|0'\rangle = E_{min}|0'\rangle$

There is a new, non-symmetric state $|0'\rangle$,
that has a degenerate energy with vacuum $|0\rangle$,
thus massless: the Nambu-Goldstone boson.

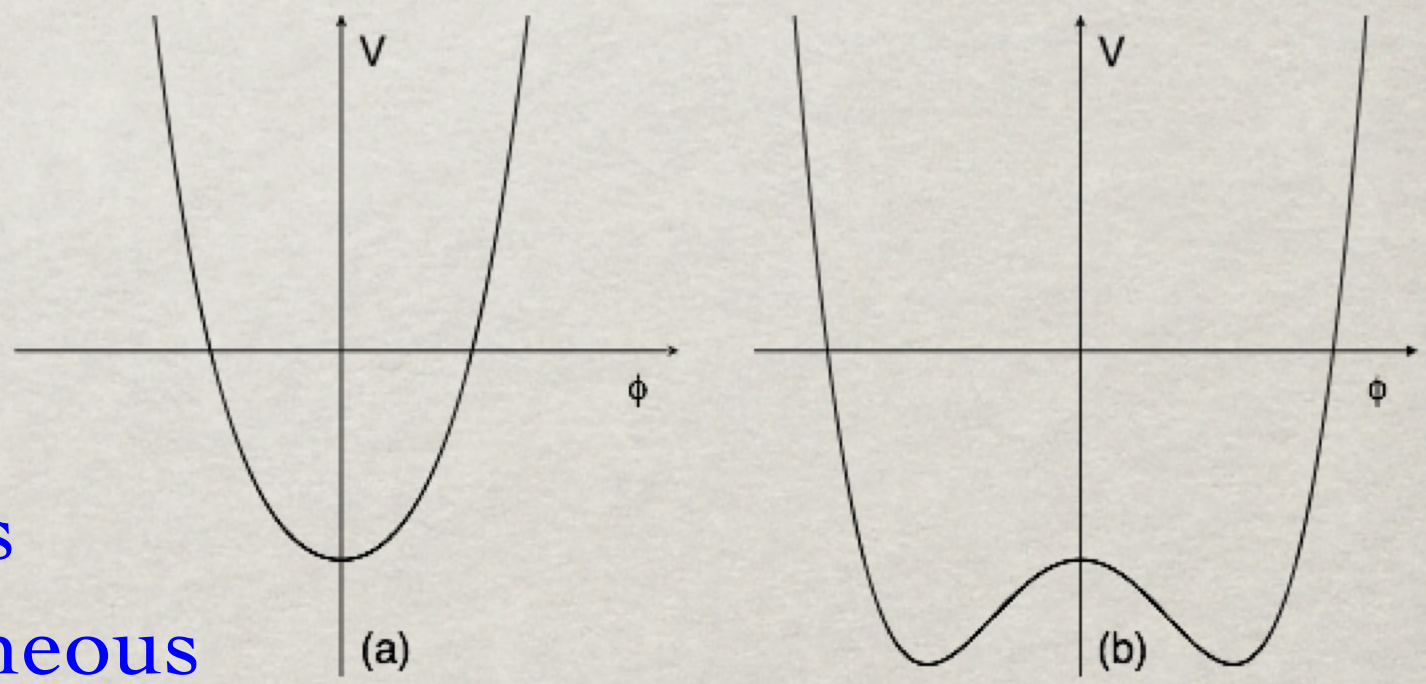
An illustrative (Goldstone's original) Model:

(a). Background complex scalar field Φ:

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi^* \phi) \qquad V = \frac{\lambda}{4}\left(\phi^*\phi - \frac{\mu^2}{\lambda}\right)^2$$

Invariant under a U(1) global transformation:

$$\phi \to e^{i\alpha} \phi$$



For $\mu^2 > 0$, the vacuum is shifted, and thus spontaneous symmetry breaking.

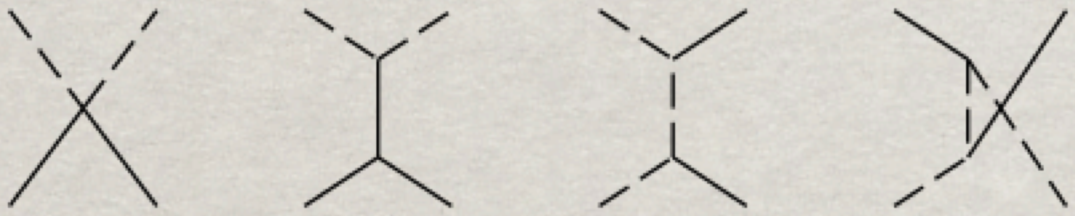
$$v = \langle 0|\phi|0\rangle = \mu/\sqrt{\lambda}.$$

Particle spectrum:[§]

Shift: $$R = \sqrt{2}Re(\phi - v), \quad I = \sqrt{2}\ Im\phi,$$

Then: $$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial^\mu R \partial_\mu R - \frac{\lambda v^2}{2}R^2 - \frac{\lambda\mu}{2\sqrt{2}}R^3 - \frac{\lambda}{16}R^4 \\ &+ \frac{1}{2}\partial^\mu I \partial_\mu I - \frac{\lambda\mu}{2\sqrt{2}}RI^2 - \frac{\lambda}{16}(R^2I^2 + I^4)\end{aligned}$$

* R is a massive scalar: $M_R = \sqrt{\lambda}\, v$.
* I is massless, interacting.
* Though not transparent, it can be verified:[§]



$$\mathcal{M}(RI \to RI)|_{p\to 0} \to 0!$$

I does decouple at low energies!

Exercise 6: Show this result by an explicit calculation.

[§] C. Burgges, hep-ph/9812468

(b). Field Φ Re-definition:

[+] C. Burgges, hep-ph/9812468

Weinberg's 1st Law of Theoretical Physics[+]:

"You can use whatever variables you like. But if you used the wrong one, you'd be sorry."

Define:

$$\phi(x) = \chi(x)\, e^{i\theta(x)},$$

$$\mathcal{L} = -\partial_\mu\chi\partial^\mu\chi - \chi^2\partial_\mu\theta\partial^\mu\theta - V(\chi^2).$$

(this is like from the rectangular *form* to the *polar form*.)

We then see that:

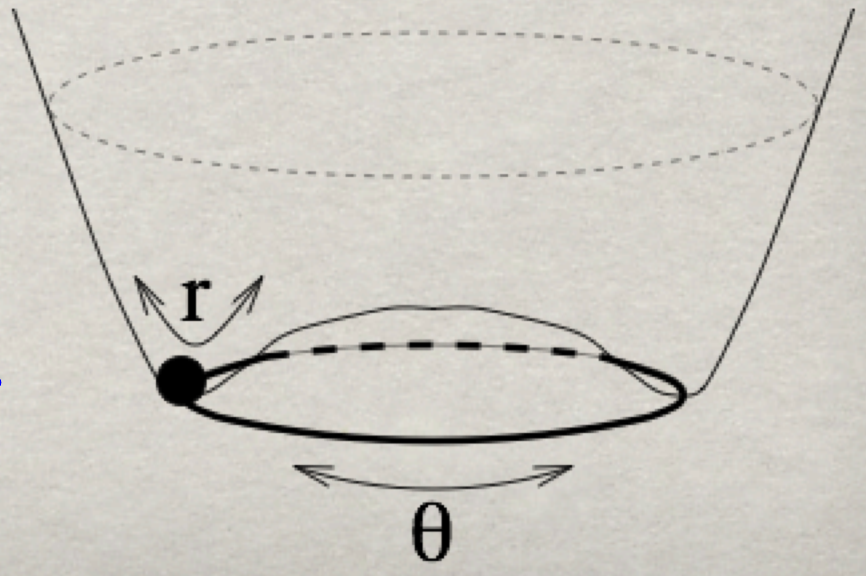
* the θ field is only derivatively coupled, and thus decoupled at low energies

* the θ field respects an inhomogeneous transformation

$$\theta \to \theta + \alpha, \quad \phi = ve^{i\theta(x)}$$

a phase rotation from the vacuum:

* the $\chi(x)$ is massive radial excitation.



“Nambu-Goldstone Bosons”

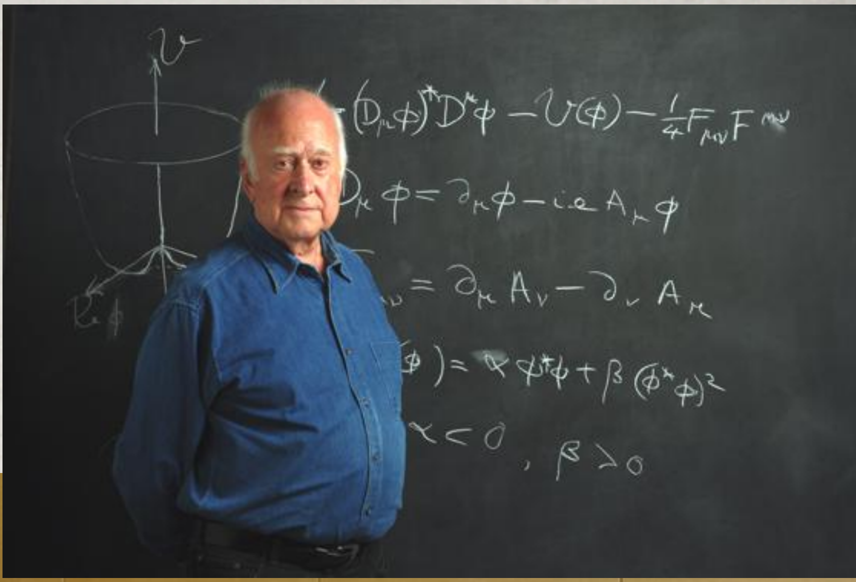
Except the photon, no massless boson (a long-range force carrier) has been seen in particle physics!

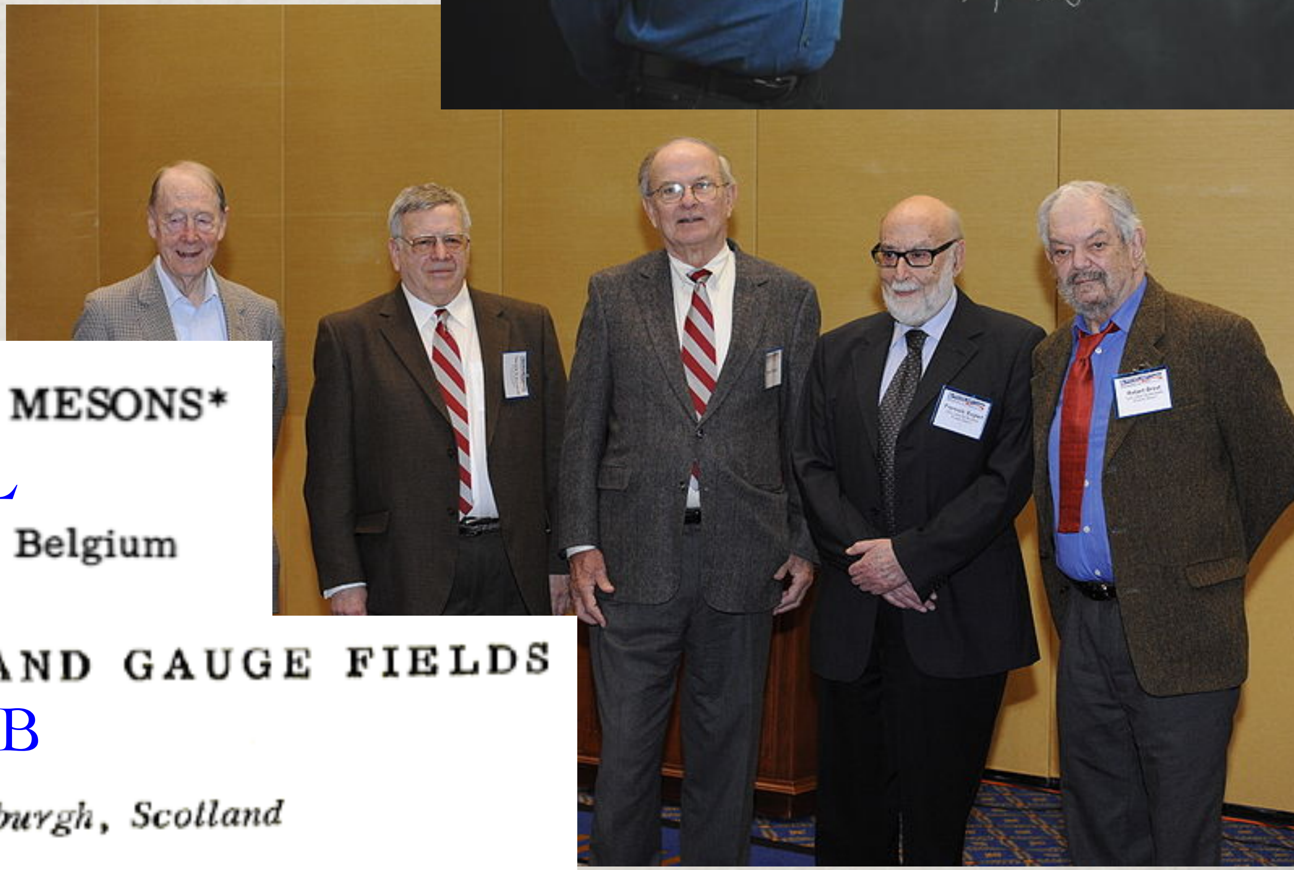
(Recall Pauli’s criticism)

The Spontaneous Symmetry Breaking:
Brilliant idea & common phenomena, confronts the Nambu-Goldstone theorem!

C. The Magic in 1964: The "Higgs Mechanism"

"If a LOCAL gauge symmetry is spontaneously broken, then the gauge boson acquires a mass by absorbing the Goldstone mode."





BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)

PRL

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

P. W. HIGGS
Tait Institute of Mathematical Physics, University of Edinburgh, Scotland
Received 27 July 1964

PLB

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

PRL

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England
(Received 12 October 1964)

PRL

An illustrative (original) Model:¶

$$\mathcal{L} = |\mathcal{D}^\mu \phi|^2 - \mu^2 |\phi|^2 - |\lambda| (\phi^* \phi)^2 - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

where

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$$

is a complex scalar field[4] and as usual

$$\mathcal{D}_\mu \equiv \partial_\mu + iqA_\mu$$

and

$$F_{\mu\nu} \equiv \partial_\nu A_\mu - \partial_\mu A_\nu .$$

The Lagrangian (5.3.1) is invariant under U(1) rotations

$$\phi \rightarrow \phi' = e^{i\theta} \phi$$

and under the local gauge transformations

$$\phi(x) \rightarrow \phi'(x) = e^{iq\alpha(x)} \phi(x),$$

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x).$$

¶ C. Quigg, Gauge Theories of the Strong ...

An illustrative (original) Model:¶

After the EWSB, parameterized in terms of

$$\langle \phi \rangle_0 = v/\sqrt{2}, \qquad \phi = e^{i\zeta/v}(v+\eta)/\sqrt{2} \approx (v+\eta+i\zeta)/\sqrt{2}.$$

Then the Lagrangian appropriate for the study of small oscillations is

$$\mathscr{L}_{\rm so} = \tfrac{1}{2}[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2\eta^2] + \tfrac{1}{2}[(\partial_\mu \zeta)(\partial^\mu \zeta)] - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + qvA_\mu(\partial^\mu \zeta) + \frac{q^2v^2}{2}A_\mu A^\mu + \cdots$$

The gauge field acquires a mass, mixes with the Goldstone boson.

Upon diagonalization: $$\frac{q^2v^2}{2}\left(A_\mu + \frac{1}{qv}\partial_\mu \zeta\right)\left(A^\mu + \frac{1}{qv}\partial^\mu \zeta\right),$$

a form that pleads for the gauge transformation

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{qv}\partial^\mu \zeta,$$

which corresponds to the phase rotation on the scalar field

$$\phi \to \phi' = e^{-i\zeta(x)/v}\phi(x) = (v+\eta)/\sqrt{2}.$$

the resultant Lagrangian is then:

$$\mathscr{L}_{so} = \tfrac{1}{2}[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2\eta^2] - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A'_\mu A'^\mu$$

- an η-field, with $(\text{mass})^2 = -2\mu^2 > 0$; the Higgs boson!
- a massive vector field A'_μ, with mass $= qv$
- no ζ-field.

- By virtue of a gauge choice - the unitary gauge, the ζ-field disappears in the spectrum: a massless photon "swallowed" the massless NG boson!

Degrees of freedom count:

Before EWSB: After:

2 (scalar)+2 (gauge pol.); 1 (scalar)+3 (gauge pol.)

- Two problems provide cure for each other!

massless gauge boson + massless NG boson

→ massive gauge boson + no NG boson

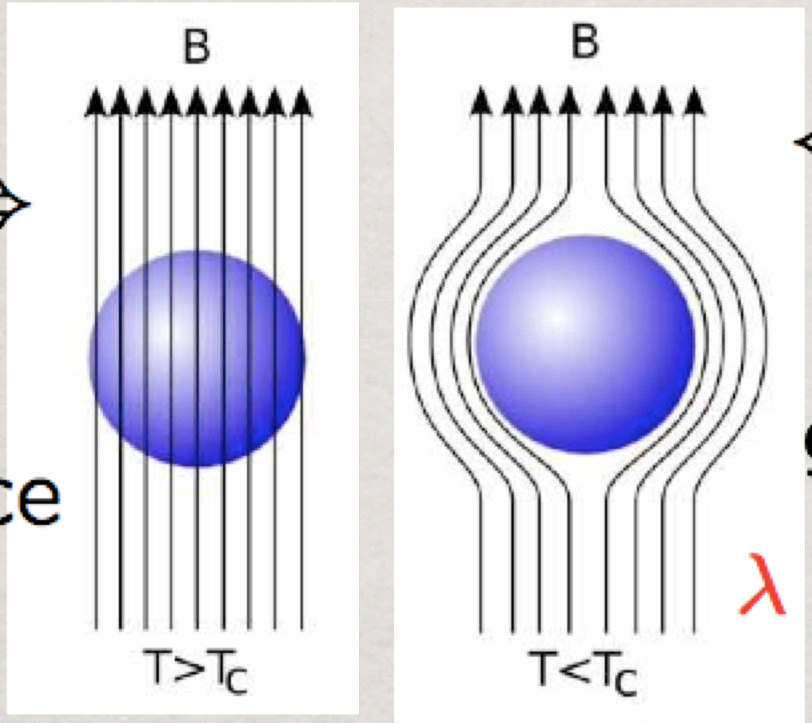
This is truly remarkable!

Known example: Superconductivity

Normal phase⇒

$E^2 = p^2c^2$

Long-range force



⇐Superconducting phase

$E^2 = p^2c^2 + m^2c^4$

gap leads to $\sim exp(-r/\lambda)$

$\lambda \sim m^{-1}$ penetration depth

In "conventional" electro-magnetic superconductivity:

$m_\gamma \sim m_e/1000, \quad T_c^{em} \sim \mathcal{O}(\text{few } K)$. BCS theory.

In "electro-weak superconductivity":

$m_w \sim G_F^{-\frac{1}{2}} \sim 100$ GeV, $T_c^w \sim 10^{15} K!$

As for the name …

1972: Ben Lee (Rochester Conf. at FNAL) named "Higgs boson" and the "Higgs mechanism".§

§ Peter Higgs: My Life as a Boson.

Font Size:

The New York Review of Books

The Crisis of Big Science

MAY 10, 2012

Steven Weinberg

As to my responsibility for the name "Higgs boson," because of a mistake in reading the dates on these three earlier papers, I thought that the earliest was the one by Higgs, so in my 1967 paper I cited Higgs first, and have done so since then. Other physicists apparently have followed my lead. But as Close points out, the earliest paper of the three I cited was actually the one by Robert Brout and François Englert. In extenuation of my mistake, I should note that Higgs and Brout and Englert did their work independently and at about the same time, as also did the third group (Gerald Guralnik, C.R. Hagen, and Tom Kibble). But the name "Higgs boson" seems to have stuck. ↩

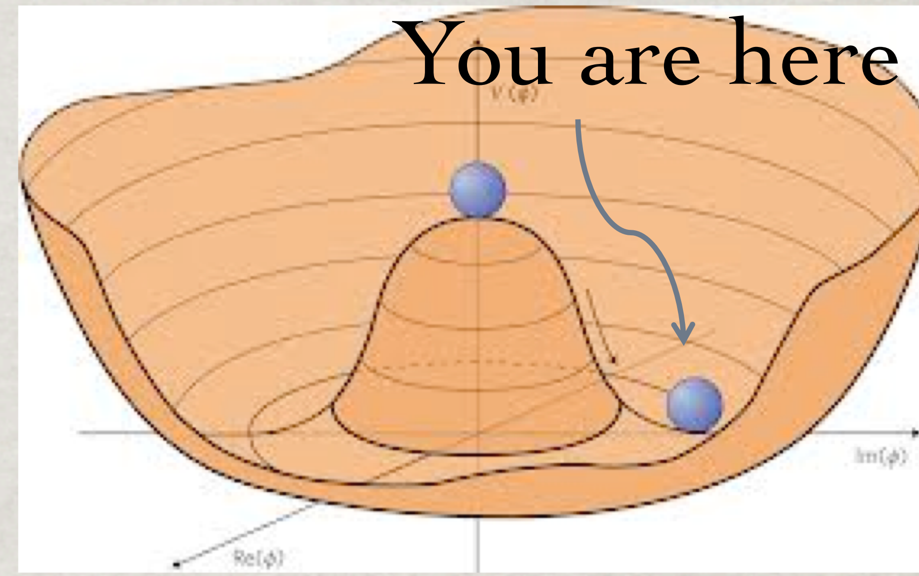
It's like Landau-Ginzburg
It's NOT Landau-Ginzburg

In the SM:

$$V(|\Phi|) \quad = \quad -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

$$<|\Phi|> = v = (\sqrt{2}G_F)^{-1/2} \approx 246 \text{ GeV}$$

$$m_H \approx 126 \text{ GeV}$$

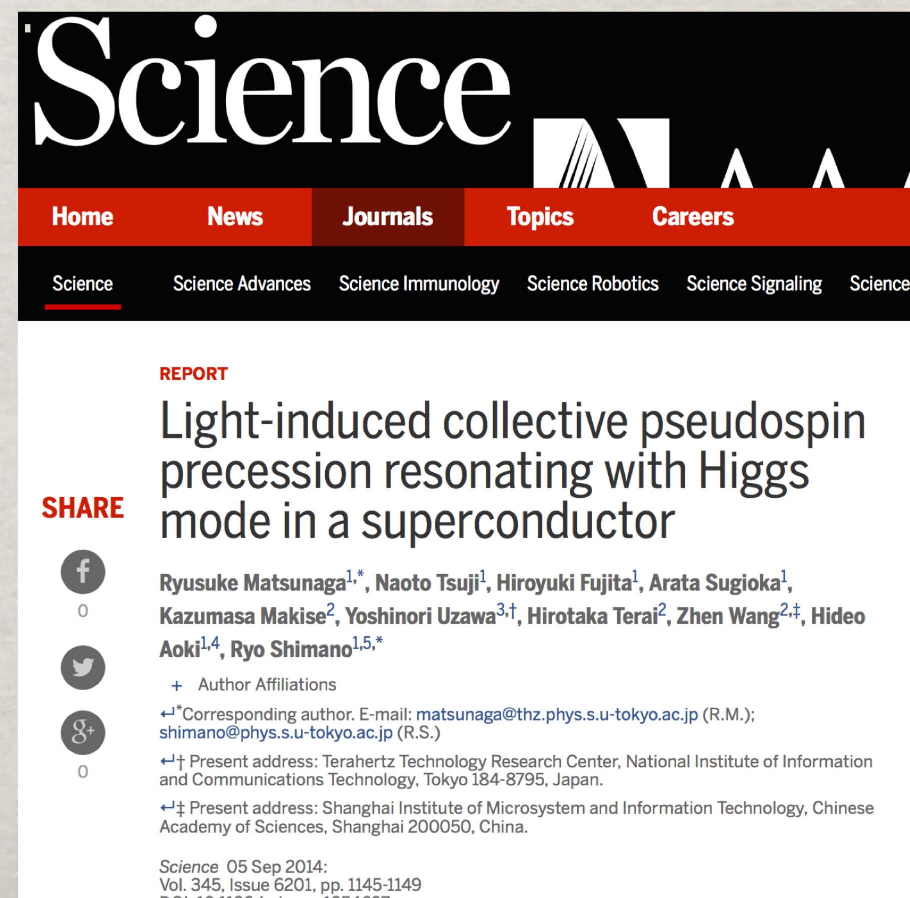


It is a weakly coupled, very narrow particle $(\Gamma/m \approx 10^{-5})$
elementary at a scale >1000 GeV!

Landau-Ginzburg:

Similar parameterization, but BCS as the underlying theory!

A collective mode of TeraHertz $(10^{-3}$ eV) vibration observed!



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REPORT

Light-induced collective pseudospin precession resonating with Higgs mode in a superconductor

SHARE

Ryusuke Matsunaga[1,*], Naoto Tsuji[1], Hiroyuki Fujita[1], Arata Sugioka[1], Kazumasa Makise[2], Yoshinori Uzawa[3,†], Hirotaka Terai[2], Zhen Wang[2,‡], Hideo Aoki[1,4], Ryo Shimano[1,5,*]

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D. Higgs Boson Interactions

1. The SM Lagrangian: $\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{Yuk}$.

The gauge part is **Pure gauge sector:**

$$\mathcal{L}_{gauge} = -\frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},$$

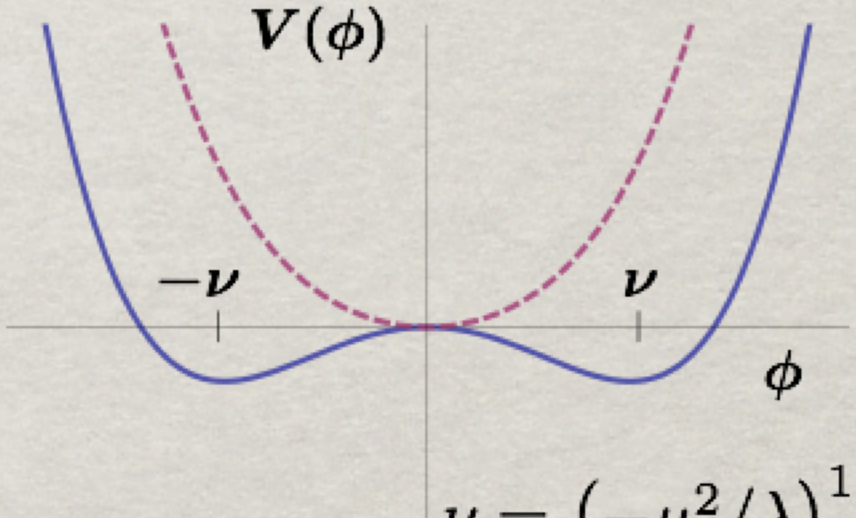
The scalar part of the Lagrangian is

The Higgs: $\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)$ $\quad D_\mu\phi = \left(\partial_\mu + ig\frac{\tau^i}{2}W^i_\mu + \frac{ig'}{2}B_\mu\right)\phi,$

$$V(\phi) = +\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2.$$

$$\phi = \frac{1}{\sqrt{2}}e^{i\sum\xi^i L^i}\begin{pmatrix}0\\ \nu + H\end{pmatrix}$$

$$\phi \rightarrow \phi' = e^{-i\sum\xi^i L^i}\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \nu + H\end{pmatrix}$$



$$\nu = (-\mu^2/\lambda)^{1/2}$$

$$\begin{aligned}\mathcal{L}_\phi &= (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)\\ &= \underline{M_W^2 W^{\mu+}W^-_\mu}\left(1+\frac{H}{\nu}\right)^2 + \underline{\frac{1}{2}M_Z^2 Z^\mu Z_\mu}\left(1+\frac{H}{\nu}\right)^2\\ &+ \frac{1}{2}(\partial_\mu H)^2 - V(\phi).\end{aligned}$$

$$M_H^2 = -2\mu^2 = 2\lambda v^2$$

$$V(\phi) = -\frac{\mu^4}{4\lambda} - \underline{\mu^2 H^2} + \lambda\nu H^3 + \frac{\lambda}{4}H^4.$$

The Fermions:§

$$\mathcal{L}_f = \sum_{m=1}^{F} \left(\bar{q}^0_{mL} i \not{D} q^0_{mL} + \bar{l}^0_{mL} i \not{D} l^0_{mL} + \bar{u}^0_{mR} i \not{D} u^0_{mR} \right.$$
$$\left. + \bar{d}^0_{mR} i \not{D} d^0_{mR} + \bar{e}^0_{mR} i \not{D} e^0_{mR} + \bar{\nu}^0_{mR} i \not{D} \nu^0_{mR}\right)$$

$$D_\mu q^0_{mL} = \left(\partial_\mu + \frac{ig}{2}\vec{\tau}\cdot\vec{W}_\mu + \frac{ig'}{6}B_\mu\right) q^0_{mL}$$
$$D_\mu l^0_{mL} = \left(\partial_\mu + \frac{ig}{2}\vec{\tau}\cdot\vec{W}_\mu - \frac{ig'}{2}B_\mu\right) l^0_{mL}$$

$$D_\mu u^0_{mR} = \left(\partial_\mu + \frac{2ig'}{3}B_\mu\right) u^0_{mR}$$
$$D_\mu d^0_{mR} = \left(\partial_\mu - \frac{ig'}{3}B_\mu\right) d^0_{mR}$$
$$D_\mu e^0_{mR} = \left(\partial_\mu - ig'B_\mu\right) e^0_{mR}$$
$$D_\mu \nu^0_{mR} = \partial_\mu \nu^0_{mR},$$

Gauge invariant, massless.

However, a fermion mass must flip chirality:

$$m_f(\bar{f}_L f_R + \bar{f}_R f_L)$$

and thus not SM gauge invariant L ≠ R !

Need something like a doublet:

$$y_f(\bar{f}_1, f_2)_L \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_L f_R$$

that's the Higgs doublet!

§ P. Langacker: TASI Lectures 2007.

The gauge invariant Yukawa interactions:

Need a doublet with a flip Y: $\tilde{\phi} = i\sigma_2\phi^*$

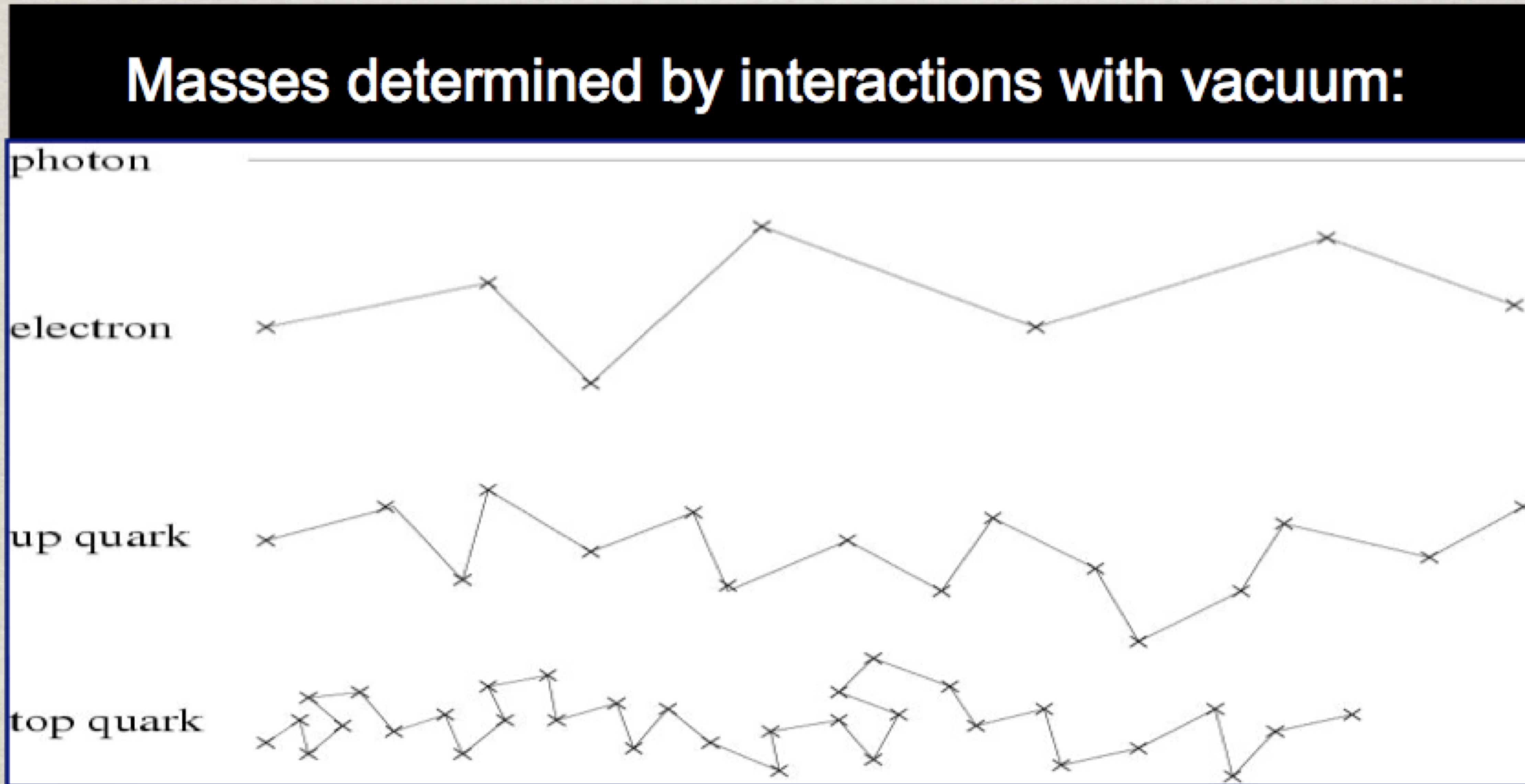
$$\mathcal{L}_{Yuk} = -\sum_{m,n=1}^{F}\Big[\Gamma^u_{mn}\bar{q}^{\,0}_{mL}\tilde{\phi}u^0_{nR} + \Gamma^d_{mn}\bar{q}^{\,0}_{mL}\phi d^0_{nR} + \Gamma^e_{mn}\bar{l}^{\,0}_{mn}\phi e^0_{nR} + \Gamma^\nu_{mn}\bar{l}^{\,0}_{mL}\tilde{\phi}\nu^0_{nR}\Big] + h.c.,$$

After the EWSB,

$$-\mathcal{L}_{Yuk} \to \sum_{m,n=1}^{F} \bar{u}^0_{mL}\Gamma^u_{mn}\left(\frac{\nu + H}{\sqrt{2}}\right)u^0_{mR} + (d,e,\nu)\text{ terms}$$
$$= \bar{u}^0_L\left(M^u + h^u H\right)u^0_R + (d,e,\nu)\text{ terms } + h.c.,$$

$$-\mathcal{L}_{Yuk} = \sum_i m_i\bar{\psi}_i\psi_i\left(1 + \frac{g}{2M_W}H\right) = \sum_i \underline{m_i\bar{\psi}_i\psi_i\left(1 + \frac{H}{\nu}\right)}$$

Higgs Boson Couplings:



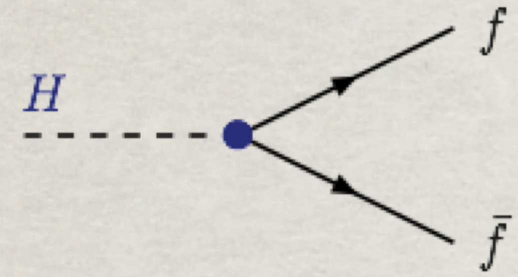
$$M_{W,Z} = \frac{1}{2} g_V v, \quad m_f = \frac{g_f}{\sqrt{2}} v, \quad v^{-2} = \sqrt{2}\, G_F.$$

Thus, where ever is mass, there will be H!

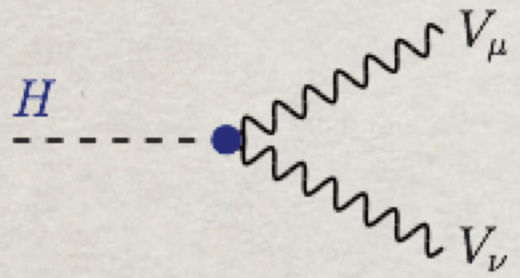
The Low-Energy-theorem:

$$m_i \to m_i \left(1 + \frac{H}{v}\right) \text{ for } p_H < v.$$

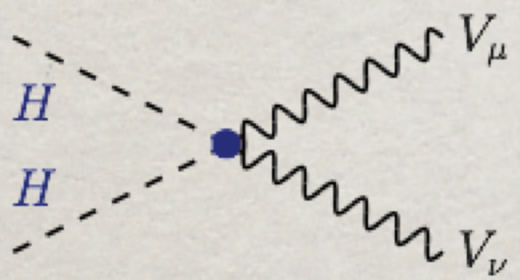
Feynman rules:



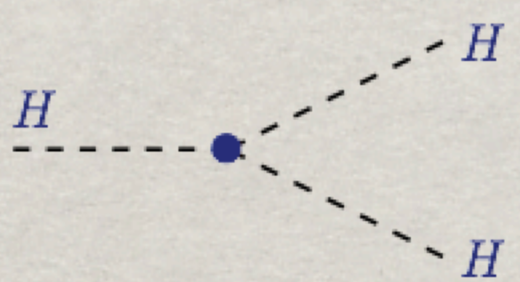
$$g_{Hff} \;=\; m_f/v \;=\; (\sqrt{2}G_\mu)^{1/2}\, m_f \qquad \times\ (i)$$



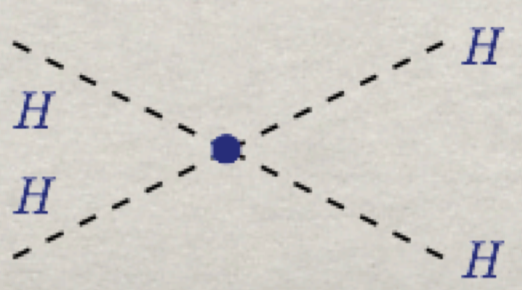
$$g_{HVV} \;=\; 2M_V^2/v \;=\; 2(\sqrt{2}G_\mu)^{1/2}\, M_V^2 \qquad \times\ (-ig_{\mu\nu})$$



$$g_{HHVV} \;=\; 2M_V^2/v^2 \;=\; 2\sqrt{2}G_\mu\, M_V^2 \qquad \times\ (-ig_{\mu\nu})$$



$$g_{HHH} \;=\; 3M_H^2/v \;=\; 3(\sqrt{2}G_\mu)^{1/2}\, M_H^2 \qquad \times\ (i)$$



$$g_{HHHH} = \;3M_H^2/v^2 \;=\; 3\sqrt{2}G_\mu\, M_H^2 \qquad \times\ (i)$$