Parton showers beyond leading logarithmic accuracy

Taming the accuracy of event generators, CERN, 29 June 2020

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Core questions on event generators

- Why is improving event generators important for precision measurements at the LHC?
- What decisions and physics go into designing a parton shower?
- How do we assess the accuracy of a parton shower?
- Can we systematically achieve next-to-leading logarithmic accuracy for broad range of observables?
Why are event generators important?

- Provide realistic and generic all-purpose simulations used in broad range of analyses with various cuts and observable choices.
- LHC collisions probe wide range of scales at which physics needs to be described accurately.
- Event generators are relied on for many different purposes, e.g.
  - background and signal estimates for new physics searches
  - uncertainty estimates for standard model measurements
  - phenomenology studies of new tools and observables
  - training data for machine learning models
What goes into simulating a high-energy collision?

- LHC collisions probe physics across scales, from hard process at the TeV scale to non-perturbative modelling below the GeV scale.

- Parton showers span several orders of magnitude to provide crucial link between hard interaction and observable particles.

- Multi-scale evolution lead to large logarithms of ratio of scales: to what accuracy are they under control?
What information is hidden in an event?

▶ There has been much progress in using jet substructure to design efficient taggers for boosted $H/W/Z$ bosons.

▶ Recent state-of-the-art tools use machine learning models trained on Monte Carlo data.

▶ These methods are only as good as the training data they rely on.
Consider the angle $\Delta \psi_{12}$ between the hardest emissions in a jet.
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Large deviations from NLL in current dipole showers.

Difference between quark and gluon distributions might lead to models that learn to discriminate based on a feature that doesn’t exist in real data!
Basic picture of dipole showers

- Many showers are dipole/antenna showers where gluon emissions correspond to dipole splittings.
- Squared amplitudes obtained from recursive chain of emissions.

Two key ingredients:
- Kinematic mapping $\tilde{p}_i, \tilde{p}_j \rightarrow p_i, p_j, p_k$.
- Evolution variable $\nu$ defining order of emissions.
Dipole shower evolution

Evolution from state with \( n \) particles to state with \( n + 1 \) is described by

\[
\frac{d\mathcal{P}_{n \to n+1}}{d \ln \nu} = \sum_{\text{dipoles } \{i,j\}} \int d\tilde{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s(k_t) + K\alpha_s^2(k_t)}{\pi} \left[ g(\tilde{\eta}) a_k P_{\tilde{i} \to ik}(a_k) + g(-\tilde{\eta}) b_k P_{\tilde{j} \to jk}(b_k) \right],
\]

- \( \nu \) is the evolution variable (e.g. \( k_t \) in dipole c.o.m. frame)
- \( g(\tilde{\eta}) \) is a function partitioning the dipole using the rapidity of the emission within the dipole (with \( g(\tilde{\eta}) + g(-\tilde{\eta}) = 1 \))
- \( P_{\tilde{i} \to ik}(z) \) are first-order splitting functions
What is the accuracy of a parton shower?

- Parton showers are often referred to as leading logarithmic accurate.
- This means that it generates the correct squared amplitude in limit where both energy and angle of emissions are strongly ordered.
- Distributions can be compared to analytic resummations

For example, Thrust, defined as

\[
T = \max_{\tilde{n}_T} \frac{\sum_i |\vec{p}_i \cdot \tilde{n}_T|}{\sum_i |\vec{p}_i|}
\]

we have, for \(\alpha_s L \sim 1\)

\[
\sigma(1 - T < e^{-L}) = \sigma_0 \exp\left[ Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots \right]
\]

\(\text{LL} \quad \text{NLL} \quad \text{NNLL}\)
What is the accuracy of a parton shower?

- Are existing dipole showers strictly LL accurate for all observables or better in some contexts?
- For what observables do we achieve a given accuracy with a given parton shower?
- Can we design a parton shower that can systematically achieve NLL accuracy for broad range of observables?
  - global event shapes (Thrust, jet rates, angularities, broadening, ...)
  - non-global observables (e.g. energy in a rapidity slice)
  - multiplicity
NLL accuracy requires that the shower generates correct squared amplitude in a limit where every pair of emissions is strongly ordered for at least one logarithmic variable $k_t$ and $\theta$.

I.e., should reproduce correct effective matrix element squared when all emissions are well separated in Lund diagram ($d_{12}, d_{23}, \ldots \gg 1$)
Achieving NLL accuracy

- NLL accuracy requires that the shower generates correct squared amplitude in a limit where every pair of emissions is strongly ordered for at least one logarithmic variable $k_t$ and $\theta$.

- I.e., should reproduce correct effective matrix element squared when all emissions are well separated in Lund diagram ($d_{12}, d_{23}, \cdots \gg 1$)

- allowed to make $O(1)$ mistake when pair of emissions is close ($d_{23} \sim 1$)
Ingredients of a shower

There are two key ingredients in the design of a parton shower

- How to associate colour and transverse recoil to dipoles?
- The choice of evolution variable (transverse momentum, angle, ...)

Design two new showers with different recoil: PanLocal and PanGlobal
Transverse recoil boundaries

\[ \log k_t \]

\[ \eta = \log \tan(\theta/2) \]

- \( q \) side
- \( \bar{q} \) side

- \( E_k < m_{q\bar{q}} \)

\( \bar{q} \) recoils
- \( g \) recoils
- \( q \) recoils

Expected
Transverse recoil boundaries

Transverse recoil assigned to end that is closer in angle in dipole c.o.m. frame
Transverse recoil boundaries

- Transverse recoil assigned to end that is closer in angle in event c.o.m. frame.
Choice of ordering variable

\[ \eta = \log \tan(\theta/2) \]

\[ k_t \text{ ordering} \]

\[ k_t \text{ recoil from } q: \text{ OK} \]
Choice of ordering variable

$\log k_t$

$\eta = \log \tan(\theta/2)$

$q$ side

$\bar{q}$ side

$k_t$ ordering

$k_t$ recoil from 1: not OK

$E_k \lesssim m_q \bar{q}$

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Choice of ordering variable

- Use ordering variable intermediate between transverse momentum and angle $0 < \beta < 1$.
- Ensures that emissions with commensurate $k_t$ are produced at successively smaller angles.

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Kinematic map of the PanLocal shower

dipole \( \tilde{p}_i, \tilde{p}_j \rightarrow p_i, p_j, p_k \).

\[
\begin{align*}
  p_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp, \\
  p_i &= a_i \tilde{p}_i + b_i \tilde{p}_j - f k_\perp, \\
  p_j &= a_j \tilde{p}_i + b_j \tilde{p}_j - (1 - f) k_\perp,
\end{align*}
\]

where \( f \rightarrow 1 \) for \( k \rightarrow i \) and \( f \rightarrow 0 \) for \( k \rightarrow j \).

\[
\begin{align*}
  k_t &= \rho v e^{\beta|\bar{\eta}|}, \\
  \rho &= \left( \frac{s_i s_j}{Q^2 s_{ij}} \right)^{\frac{1}{2}}, \\
  a_k &= \sqrt{\frac{s_j}{s_{ij} s_i}} k_t e^{+\bar{\eta}}, \\
  b_k &= \sqrt{\frac{s_i}{s_{ij} s_j}} k_t e^{-\bar{\eta}}
\end{align*}
\]

where \( s_{ij} = 2 \tilde{p}_i \cdot \tilde{p}_j, s_i = 2 \tilde{p}_i \cdot Q \)

Partitioning of the dipole occurs at equal angles between the emission and the dipole ends in the event c.o.m. frame.
Alternatively, we can formulate a global recoil scheme, which defines the PanGlobal shower.

Longitudinal recoil is handled by dipole-local map

\[
\text{dipole } \{\tilde{p}_i, \tilde{p}_j\} \implies \begin{align*}
\tilde{p}_k &= a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp, \\
\tilde{p}_i &= (1 - a_i) \tilde{p}_i, \\
\tilde{p}_j &= (1 - b_k) \tilde{p}_j.
\end{align*}
\]

But transverse recoil is distributed across whole event by boost and rescaling.

- This scheme works for \(0 \leq \beta < 1\), i.e. including for \(k_t\) ordering.
How to probe the accuracy of a shower

- Run full shower for smaller and smaller values of $\alpha_s$, keeping $\alpha_s L$ constant.
- Ratio to NLL of each distribution deviates from one: because of residual NNLL term or because of NLL mistake?

![Graph showing $\Delta\psi_{12}$ vs. $|\Delta\psi_{12}|$ for different $\alpha_s$ values.](image)
How to probe the accuracy of a shower

- Run full shower for smaller and smaller values of $\alpha_s$, keeping $\alpha_sL$ constant
- Ratio to NLL of each distribution deviates from one: because of residual NNLL term or because of NLL mistake?
- Extrapolation $\alpha_s \to 0$ proves agreement with NLL, here for $\Delta \psi_{12}$ observable considered earlier
How to probe the accuracy of a shower

▶ Run full shower for smaller and smaller values of $\alpha_s$, keeping $\alpha_sL$ constant

▶ Ratio to NLL of each distribution deviates from one: because of residual NNLL term or because of NLL mistake?

▶ Extrapolation $\alpha_s \to 0$ proves agreement with NLL, here for $\Delta\psi_{12}$ observable considered earlier
What tests should a NLL shower pass?

NLL accurate shower should pass numerical tests showing agreement with analytic NLL resummation for broad range of observables:

▶ event shapes sensitive to transverse momentum, e.g. jet broadenings
▶ event shapes probing $k_t e^{-|\eta|/2}$, e.g. fractional moment $FC_{1/2}$
▶ event shapes probing $k_t e^{-|\eta|}$, e.g. thrust
▶ non-global observables e.g. transverse momentum in a rapidity slice
▶ jet multiplicity probing the recursive structure of the shower
Systematic assessment of parton shower accuracy

- Pythia 8 deviates from NLL, while PanLocal $0 < \beta < 1$ and PanGlobal $0 \leq \beta < 1$ correctly reproduces global observables, non-global observables and multiplicities.

Orange coding indicates NLL issues at fixed order that are masked at all orders.

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CONCLUSIONS
Conclusions

- Parton showers and their accuracies is an essential component for the theoretical and experimental HEP program.

- Formal accuracy of a shower can be defined in terms of comparison with analytic resummations, using systematic numerical checks.

- From simple building blocks, possible to design shower that is NLL accurate, i.e. controlling terms $O(\alpha_s^n L^n)$, for both global and non-global observables.

- Paves the way for further progress: subleading colour, spin correlations, ISR, NNLL, ...