Recoil schemes in angular ordered parton showers

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Taming the accuracy of event generators

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Based on Bewick, S.F.R., Richardson and Seymour [arxiv:1904.11866]

Log accuracy in AO PS

- Angular ordered parton showers cannot reproduce the matrix elements of several emissions ordered in energy but with commensurate angles: they cannot describe **non-global observables** like the away-from-jet energy flow. [Banfi, Corcella, Dasgupta '06]
- There are some classes of infrared-safe observables where an improved (+CMW and spin corr) coherent branching formalism leads to full **next-to-leading log accuracy** (e.g. in semi-inclusive hard processes such as deep inelastic scattering and Drell-Yan at large x [Catani, Webber, Marchesini '90]) ...
- ...but only if we are able to design a **recoil scheme** such that soft emissions do not modify the kinematic of previous ones! [Dagupta, Dreyer, Hamilton, Monni, Salam '18]
  $\Rightarrow$ Let's consider the case of double soft gluon emission in $e^+e^- \to q\bar{q}$ with $\Delta\eta \gg 1$, so that

$$dP_2^{\rm soft} = \frac{1}{2!}\prod_{i=1}^{2} \frac{2C_F\alpha_s(k_{Ti})}{\pi}\frac{dk_{Ti}}{k_{Ti}}d\eta_i = \frac{1}{2!}\prod_{i=1}^{2} dP_1^{\rm soft}(k_{Ti},\eta_i)$$

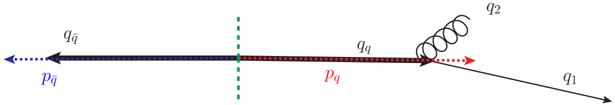
Herwig7 angular-ordered parton shower

- The (anti-)quark is identified as **shower progenitor** and the anti-quark (quark) is its colour partner: each shower progenitor is showered independently in the frame where it is anti-collinear with the colour partner.

  AO PS use colour correlations only among the shower progenitor: this approx is sufficient to reach **NLL** accuracy in many shape observables, like the thrust [Catani, Trentadue, Turnock, Webber '92]

Herwig7 angular-ordered parton shower

- The (anti-)quark is identified as **shower progenitor** and the anti-quark (quark) is its colour partner: each shower progenitor is showered independently in the frame where it is anti-collinear with the colour partner.
- Single emission from the quark:



$$\begin{cases} q_1 &= z p_q + \beta_1 p_{\bar{q}} + p_T \\ q_2 &= (1-z) p_q + \beta_2 p_{\bar{q}} - p_T \\ q_q &= p_q + (\beta_1 + \beta_2)\, p_{\bar{q}} \end{cases}$$

$$\tilde{q}^2 = \frac{q_q^2}{z(1-z)} = \frac{2 q_1 \cdot q_2}{z(1-z)} = \frac{p_T^2}{z^2(1-z)^2} \sim E_q^2 \theta^2$$

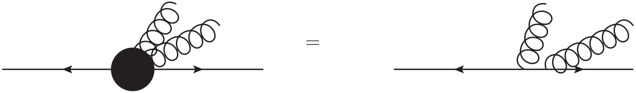
- **Soft limit**: $1 - z \equiv \epsilon \to 0$, $\boxed{p_T \to \epsilon \tilde{q}, \quad \eta \to \log\left(\frac{Q}{\tilde{q}}\right)}$

$$dP^{\text{herwig}} \to 2C_F \frac{\alpha_s(\epsilon\tilde{q})}{\pi} \frac{d\tilde{q}}{\tilde{q}} \frac{d\epsilon}{\epsilon} = \boxed{2C_F \frac{\alpha_s(p_T)}{\pi} \frac{dp_T}{p_T} d\eta} \quad ✓$$

Herwig7 angular-ordered parton shower

- two emissions, first $\eta_1 > 0$:
  1. $\boxed{\eta_2 < 0}$: two "indepent" emissions

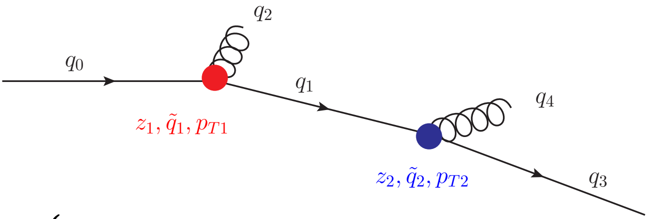
  2. $\boxed{\eta_2 > 0,\ |\eta_1 - \eta_2| \gg 1}$:



$|\eta_1 - \eta_2| \gg 1$: this suppress the gluon splitting;
angular ordering $\mathbf{z_1^2 \tilde{q}_1^2} > \mathbf{\tilde{q}_2^2}$ imposes that the one with smallest rapidity comes first;

Correct colour factor, we need to check just the recoil

Herwig7 angular-ordered parton shower



$$
\begin{cases}
q_0 &= p_q + (\beta_2 + \beta_3 + \beta_4) p_{\bar{q}} \\
q_1 &= z_1 p_q + (\beta_3 + \beta_4) p_{\bar{q}} + p_{T1} \\
q_2 &= (1 - z_1) p_q + \beta_2 p_{\bar{q}} - p_{T1} \\
q_3 &= z_2 z_1 p_q + \beta_3 p_{\bar{q}} + z_2 p_{T1} + p_{T2} \\
q_4 &= (1 - z_2) z_1 p_q + \beta_4 p_{\bar{q}} + (1 - z_2) p_{T1} - p_{T2}
\end{cases}
$$

$$\boxed{\mathbf{q_0^2} = \frac{\mathbf{p_{T1}^2}}{z_1(1-z_1)} + \frac{q_1^2}{z_1}} \quad \Rightarrow \quad \text{Impossible to preserve simultaneously } q_0^2 \text{ and } p_{T1}^2$$

The choice of the preserved quantity determines the **recoil scheme**

p_T-preserving scheme

The original (and simplest) choice of [Gieseke, Stephens and Webber '03] is to preserve the **transverse momentum**:

$$\tilde{q}_i^2 = \frac{p_{Ti}^2}{z_i^2(1-z_i)^2}$$

- $\boxed{p_{Ti}^2 = z_i^2(1-z_i)^2\tilde{q}_i^2 \to \epsilon_i^2\tilde{q}_i^2}$ $\boxed{\eta_i \to \log\left(\frac{Q}{\tilde{q}_i}\right)}$

  $\Rightarrow$ correct soft limit by construction.

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- $\boxed{q_0^2 = z_1(1-z_1)\tilde{q}_1^2 + \frac{z_2(1-z_2)\tilde{q}_2^2}{z_1}}$
- Migration of events toward the hard region of the spectrum after multiple emissions leads to very small $\alpha_s^{\text{CMW}} = 0.1074$ value to fit the data. ($\alpha^{\overline{\text{MS}}} = 0.118 \to \alpha_s^{\text{CMW}} = 0.1256$)

q^2-preserving scheme

In Ref. [Reichelt, Richardson and Siodmok, '17] the **virtuality**-preserving scheme is introduced:

$$\tilde{q}_i^2 = \frac{q_i^2}{z_i(1-z_i)}$$

- The transverse momentum of the first emission is reduced

$$\boxed{p_{T1}^2} = \max\left[0, (1-z_1)\left[z_1^2(1-z_1)\tilde{q}_1^2 - z_2(1-z_2)\tilde{q}_2^2\right]\right]$$

$$\boxed{\to \max\left[0, \epsilon_1\left[\epsilon_1\tilde{q}_1^2 - \epsilon_2\tilde{q}_2^2\right]\right]}$$

$$\boxed{\eta_1 \to \frac{1}{2}\log\left(\frac{Q^2}{\tilde{q}_1^2 - \frac{\epsilon_2}{\epsilon_1}\tilde{q}_2^2}\right)}$$

the p_T is set to 0 and the virtuality increases if the reconstruction is not possible; we need $\epsilon_2 \ll \epsilon_1$ to be sure the soft limit is ok;

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- Better description of the tail of the distributions and in general better agreement with data ($\alpha_s^{\text{CMW}} = 0.1244$).

Dot-product preserving scheme

In [Bewick et al. '19] we suggested something with intermediate properties

$$\tilde{q}^2 = \frac{2q_1 \cdot q_2}{z_i(1-z_i)}$$

- The transverse momentum of the first emission is reduced by subsequent emissions

$$p_{T1}^2 = (1-z_1)^2 \left[z_1^2 \tilde{q}_1{}^2 - \sum_{i=2}^{n} z_i(1-z_i)\tilde{q}_i^2 \right]$$

  but $\tilde{q}_{i+1} < z_i \tilde{q}_i$ implied $p_{T1} > 0$ even for infinite emissions; the **double-soft** limit is correct

$$\boxed{p_{T1}^2 \to \epsilon_1^2 \left[\tilde{q}_1^2 - \epsilon_2 \tilde{q}_2^2\right] \to \epsilon_1^2 \tilde{q}_1^2}, \qquad \boxed{\eta_1 \to \log\left(\frac{Q}{\tilde{q}_1}\right)}$$

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- However, the virtuality still increases ...

$$\boxed{q_0^2 = z_1(1-z_1)\tilde{q}_1^2 + z_2(1-z_2)\tilde{q}_2^2}$$

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- The non-log enhanced region of the phsp is still overpopulated, leading $\alpha_s^{\text{CMW}} = 0.1136$.

Phase space

- In dipole-showers, the phase-space factorization is exact;
- In the `Herwig7` angular-ordered parton shower, the phase-space factorization is correct only in the soft or collinear limit. The exact formula for the case under analysis

$$\frac{d\Phi_n(q,\bar{q},\ldots)}{d\Phi_2(q,\bar{q})} = \lambda\left(1, \frac{q_q^2}{s}, \frac{q_{\bar{q}}^2}{s}\right) \prod_{i=1}^{n} \frac{d\tilde{q}_i^2}{(4\pi)^2} z_i(1-z_i)dz_i$$

where $\lambda(1,a,b) = \sqrt{1-2(a+b)+(a^2-b^2)^2}$.

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where $\lambda(1,a,b) = \sqrt{1-2(a+b)+(a^2-b^2)^2}$.
- $\lambda \approx 1$ if the emissions are all soft or collinear and is far from 1 in the hard region of the spectrum.
- We can accept the event with probability λ to improve the description of the tail of the distributions, (large virtualities) without spoiling the soft-collinear region (small virtualities).
- This improves the tail of the distributions and leads to $\alpha_s^{\mathrm{CMW}} = 0.1186$.

Discussion point

Is possible to have a solid phase-space factorization (dipole inspired?) for AO showers?

Global recoil

- Prior the shower $p_i = \{\sqrt{m_i^2 + |\vec{q}_i|}, \vec{p}_i\}$ that satisfy

$$\sum_i \sqrt{m_i^2 + |\vec{p}_i|} = \sqrt{s}, \quad \sum_i \vec{p}_i = \vec{0}.$$

- After the parton shower, the shower progenitors have acquired some virtuality and their three-momentum changed: $q_i = \{\sqrt{q_i^2 + |\vec{q}_i|^2}, \vec{q}_i\}$

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- To achieve three-momentum conservation we can define for each particle a boost so that

$$q_i \xrightarrow{\beta_i} q_i' = \{\sqrt{q_i^2 + \lambda^2 |\vec{p}_i|^2}, \lambda \vec{p}_i\} \quad \Rightarrow \sum_i \vec{q}_i' = \lambda \sum_i \vec{p}_i = \vec{0}$$

  and the daughters are boosted along the direction of the progenitor
- λ is found by solving

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If only soft-emissions take place $q_i^2 = m_i^2 + \mathcal{O}(\epsilon)$, thus this boost gives subleading contributions

Double soft-emission kinematics at LO

- Phase-space veto and global recoil at the end give subleading corrections if only soft emissions take place.
- For two soft-collinear emissions we thus have the following Lund variables

Preserved quantity	p_T^2	q^2	$q_1 \cdot q_2$
p_{T1}^2	$\epsilon_1^2 \tilde{q}_1^2$	$\epsilon_1 \left[\epsilon_1 \tilde{q}_1^2 - \epsilon_2 \tilde{q}_2^2\right]$	$\epsilon_1^2 \tilde{q}_1^2$
η_1	$\frac{1}{2}\log\left(\frac{Q^2}{\tilde{q}_1^2}\right)$	$\frac{1}{2}\log\left(\frac{Q^2}{\tilde{q}_1^2 - \frac{\epsilon_2}{\epsilon_1}\tilde{q}_2^2}\right)$	$\frac{1}{2}\log\left(\frac{Q^2}{\tilde{q}_1^2}\right)$
p_{T2}^2	$\epsilon_2^2 \tilde{q}_2^2$		
η_2	$\frac{1}{2}\log\left(\frac{Q^2}{\tilde{q}_2^2}\right)$		

- The kinematics of the second emission is always correct;
- The kinematic of the first emission is correct in the p_T and dot-product preserving schemes.
- The p_T^2 and $q_1 \cdot q_2$ preserving schemes yield the correct double soft limit: what happens for the q^2 scheme?

Double soft-emission kinematics in the q^2 scheme

Ignoring the running of α_s

$$\frac{dP_2^{\text{soft}}}{dk_{Ta}^2\, d\eta_a\, dk_{Tb}^2\, d\eta_b} = \frac{C_F^2}{2!}\frac{\alpha_s^2}{\pi^2}\frac{1}{k_{Ta}^2\, k_{Tb}^2}$$

$$\frac{dP_2^{\text{herwig}}}{dk_{Ta}^2\, d\eta_a\, dk_{Tb}^2\, d\eta_b} = \int \frac{C_F^2}{2!}\frac{\alpha_s^2}{\pi^2}\prod_{i=1}^{2}\left[\frac{d\tilde{q}_i^2}{\tilde{q}_i^2}\frac{d\epsilon_i}{\epsilon_i}\right]\Theta\big(\tilde{q}_1^2 - \tilde{q}_2^2\big)$$
$$\times\Big[\delta(\eta_1-\eta_a)\delta(k_{T1}^2-k_{Ta}^2)\delta(\eta_2-\eta_b)\delta(k_{T2}^2-k_{Tb}^2) + a \leftrightarrow b\Big]$$

$$\boxed{R = \frac{1}{1+\frac{k_{Tb}}{k_{Ta}}e^{\eta_a-\eta_b}} \times \Theta\left(\frac{k_{Tb}}{k_{Ta}} - 2\sinh(\eta_a-\eta_b)\right) + a \leftrightarrow b}$$

Double soft-emission kinematics in the q^2 scheme

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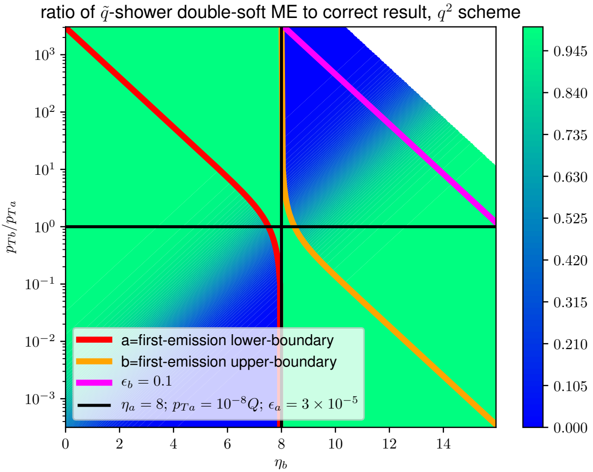


Log accuracy of the Thrust in the q^2 scheme

- **CAESAR** formalism: $\Sigma(L)$ probability that a shape observable is smaller than e^{-L}

$$\Sigma(L) = \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right] + \mathcal{O}(\alpha_s e^{-L}),$$

where g_1 cointains the LL, g_2 the NLL.

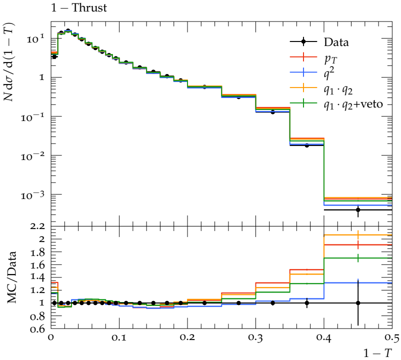
- Impact of the incorrect mapping at order α_s^2 for the **thrust**:

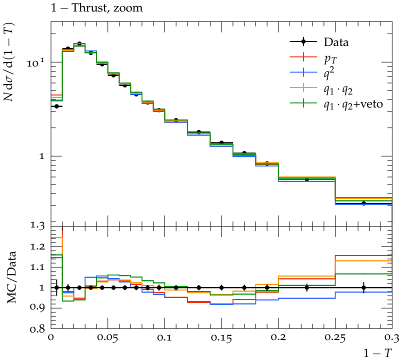
$$\begin{aligned}\delta\Sigma(L) =& 2\left(\frac{2\alpha_s C_F}{\pi}\right)^2 \int_0^{+\infty} d\eta_1 \int_{-\infty}^{-|\eta_1|} d\ell_1 \int_{\eta_1}^{+\infty} d\eta_2 \int_{-\infty}^{-|\eta_2|} d\ell_2 \\ & \times \left[\Theta\left(e^{-L} - V_{\rm ok}(\eta_1,\ell_1,\eta_2,\ell_2)\right) - \Theta\left(e^{-L} - V_{\rm PS}(\eta_1,\ell_1,\eta_2,\ell_2)\right)\right], \\ =& -\frac{C_F^2}{6}\alpha_s^2 L^2 + \mathcal{O}(\alpha_s^2 L)\end{aligned}$$

$\Rightarrow$ For dipole showers, the first NLL incorrect term is at α_s^3.

Selected LEP results

Thrust, DELPHI 1996

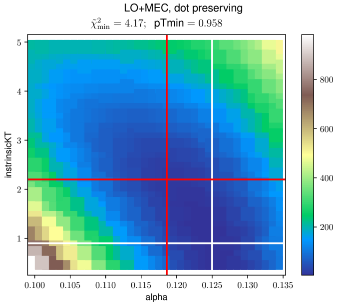


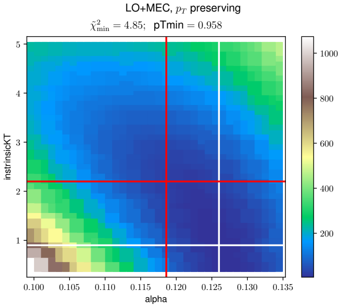


$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

Comments on ISR

- The current implementation of ISR preserves the p_T.
- Using the dot-product yields little difference
- The new tunes (white) suggest a better value for α_s^{CMW} rather than the old ones …(data: Z DY events at 7 TeV, p_T and ϕ^*).





$$\alpha_s^{\mathrm{ISR}} = 0.1247(1)$$
$$p_{T1}^2 = \epsilon_1^2 \left[\tilde{q}_1^2 + \epsilon_2 \tilde{q}_2^2\right]$$
$$-q_1^2 = \epsilon_1 \tilde{q}_1^2 + \epsilon_2 \tilde{q}_2^2$$

$$\alpha_s^{\mathrm{ISR}} = 0.1270(3)$$
$$p_{T1}^2 = \epsilon_1^2 \tilde{q}_1^2$$
$$-q_1^2 = \epsilon_1 \tilde{q}_1^2 + (1-\epsilon_1)\epsilon_2 \tilde{q}_2^2$$

Summary

- AO PS can reach NLL accuracy for many shape observables provided that the recoil scheme guarantees that a soft emission does not disturb the kinematic of other emissions;
- Preserving the p_T or the dot product meet the requirement but overpopulates the non-log enhanced region of the phase space when it comes to FSR;
- An implementation of the correct phase space factorization mitigates the problem: is it possible to have it in general at least for FSR?
- What else can we do when HO ME are not available?
- Can we tune α_s? $\alpha_s^{\text{ISR}} \neq \alpha_s^{\text{FSR}}$?
- AO PS cannot reproduce non global logs since the azimuth averaging washes away correlations. Can we build a consistent AO PS? [Forshaw, Holguin, Plätzer '20]