Building a consistent parton shower

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in collaboration with Jeff Forshaw and Simon Plätzer

Based on arXiv:2003.06400

Outline 1

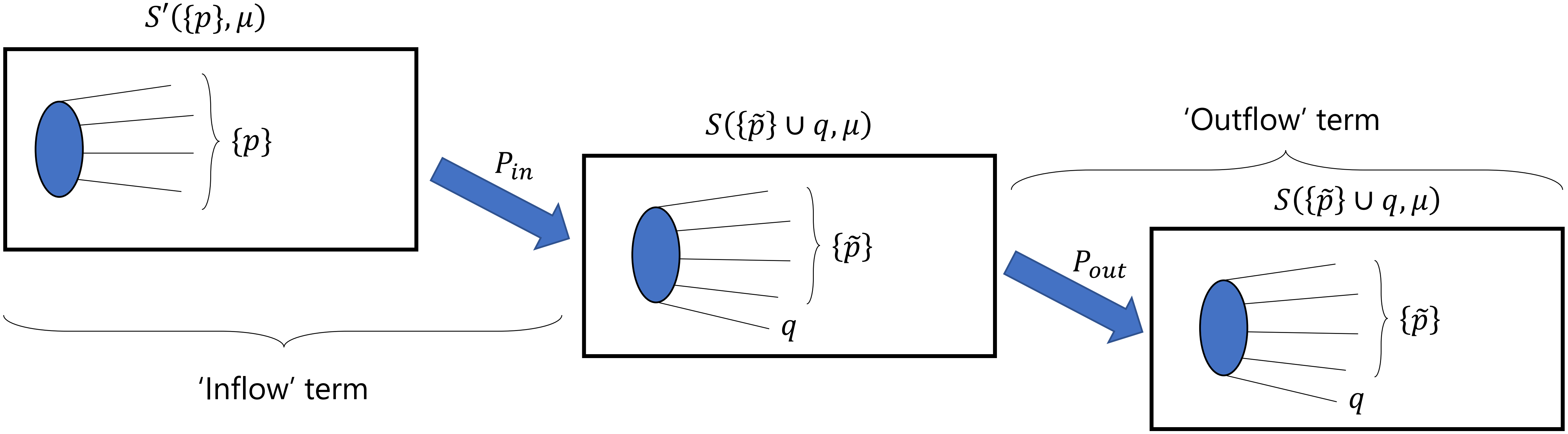
- In previous work:
  - “Parton branching at amplitude level” J. Forshaw, JH, S. Plätzer arXiv:1905.08686
  - “Soft gluon evolution and non-global logarithms”
    R. Ángeles Martínez, M. De Angelis, J. Forshaw, S. Plätzer, M. Seymour arXiv:1802.08531
- The outcome was a Markovian algorithm (the PB algorithm) which generates the leading infra-red singularity structure of amplitudes dressed with arbitrary numbers of partons. It is full colour.

Outline 2

- PB algorithm has one point of ambiguity – momentum conservation.
- PB algorithm should be able to derive coherent branching and dipole showers.
- What do these say about momentum conservation?

Setting up the problem 1

$$\frac{\mathrm{d}S(\{\tilde{p}\}\cup q,\mu)}{\mathrm{d}\ln\mu} = \underbrace{\prod_{p\in\{p\}}\int \mathrm{d}^4p\,\delta^4\left(p-g_p(\{\tilde{p}\})\right)P_{in}(q)\cdot S'(\{p\},\mu)}_{\text{'Inflow' term}} - \underbrace{P_{out}}_{\text{'Outflow' term}}\cdot S(\{\tilde{p}\}\cup q,\mu\,)$$



Setting up the problem 2

$$\frac{\mathrm{d}S(\{\tilde{p}\} \cup q, \mu)}{\mathrm{d}\ln\mu} = \underbrace{\prod_{p\in\{p\}} \int \mathrm{d}^4 p\, \delta^4\left(p - g_p(\{\tilde{p}\})\right)}_{=\int \mathrm{d}R} P_{in}(q) \cdot S'(\{p\}, \mu) - P_{out} \cdot S(\{\tilde{p}\} \cup q, \mu)$$

Define a measure to integrate out intermediate states. Always possible to do provided the Markovian system is without cycles.

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Setting up the problem 3

$$\frac{\mathrm{d}S(\{\tilde{p}\} \cup q, \mu)}{\mathrm{d}\ln\mu} = \prod_{p \in \{p\}} \int \mathrm{d}^4 p\, \delta^4\left(p - g_p(\{\tilde{p}\})\right) P_{in}(q) \cdot S'(\{p\}, \mu) - P_{out} \cdot S(\{\tilde{p}\} \cup q, \mu)$$

We're dealing with systems of evolving soft or collinear partons.
Full form of $\mathrm{d}R$ is not obvious.
We know the limits it must satisfy.

$$\int \mathrm{d}R = \prod_{p \in \{p\}} \int \mathrm{d}^4 p\, \Delta(\{p\}, \{\tilde{p}\}) \begin{cases} \xrightarrow{q \text{ is soft}} \prod_{p \in \{p\}} \int \mathrm{d}^4 p\, \delta^{4n}(\{p\} - \{\tilde{p}\}) \\ \xrightarrow{q || k \in \{p\}} \prod_{p \in \{p\}} \int \mathrm{d}^4 p\, \delta^4(k - \tilde{\mathrm{k}}/z)\delta^{4n-4}(\{p\}\backslash \mathrm{k} - \{\tilde{p}\}\backslash \tilde{\mathrm{k}}) \end{cases}$$

$$z \approx \frac{E_k}{E_q + E_k}$$

The PB algorithm starting point 1

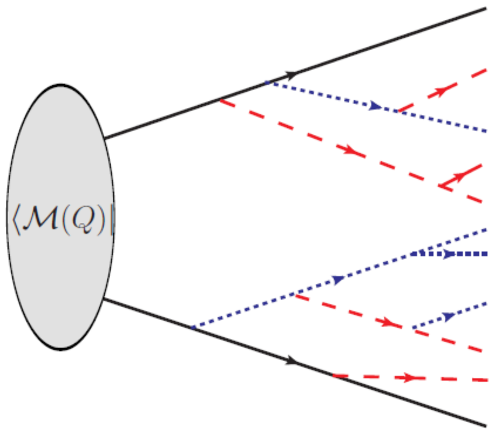
$$\boxed{\begin{aligned} q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} &= -\,\boldsymbol{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\boldsymbol{\Gamma}_n^\dagger(q_\perp) \\ &\quad + \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\,\perp})\,\mathbf{A}_{n-1}(q_{n\,\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\,\perp})\,q_\perp\,\delta(q_\perp - q_{n\,\perp}) \end{aligned}}$$

The PB algorithm starting point 2

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = -\,\mathbf{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\mathbf{\Gamma}_n^\dagger(q_\perp) + \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\perp})\,\mathbf{A}_{n-1}(q_{n\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\perp})\,q_\perp\,\delta(q_\perp - q_{n\perp})$$

The system we're evolving gives rise to a change of amplitude density matrices:

$$\mathbf{A}_0(q_{0\perp};\{p\}_0) \mapsto \mathbf{A}_1(q_{1\perp};\{p\}_1) \mapsto \ldots \mapsto \mathbf{A}_n(q_{n\perp};\{p\}_n)$$
$$\mathbf{A}_0(Q;\{p\}_0) = \mathbf{H}(Q;P_1,\ldots,P_{n_\mathrm{H}}),\ \text{where}\ \mathbf{H} \equiv |\mathcal{M}\rangle\langle\mathcal{M}|$$

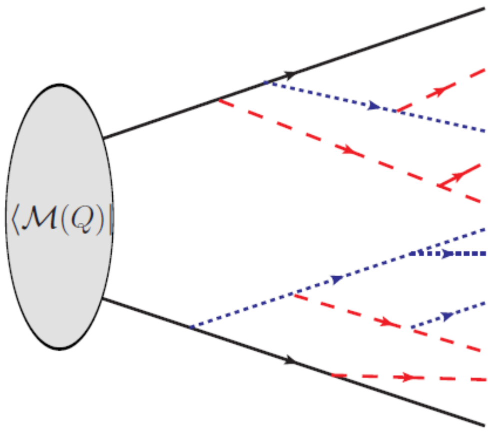


$$\frac{\mathrm{d}S(\{\tilde{p}\}\cup q,\mu)}{\mathrm{d}\ln\mu} = \prod_{p\in\{p\}} \int \mathrm{d}^4p\,\delta^4\big(p - g_p(\{\tilde{p}\})\big)\,P_{in}(q)\cdot S'(\{p\},\mu') - P_{out}\cdot S(\{\tilde{p}\},\mu)$$

The PB algorithm starting point 3

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = -\,\boldsymbol{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\boldsymbol{\Gamma}_n^\dagger(q_\perp)$$
$$+ \int \mathrm{d}R_n\, \mathbf{D}_n(q_{n\,\perp})\,\mathbf{A}_{n-1}(q_{n\,\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\,\perp})\; q_\perp\; \delta(q_\perp - q_{n\,\perp})$$

The "inflow" terms are amplitude level emission operators.



$$\frac{\mathrm{d}S(\{\tilde{p}\} \cup q, \mu)}{\mathrm{d}\ln\mu} = \prod_{p\in\{p\}} \int \mathrm{d}^4p\, \delta^4\big(p - g_p(\{\tilde{p}\})\big)\, P_{in}(q) \cdot S'(\{p\},\mu') - P_{out} \cdot S(\{\tilde{p}\},\mu)$$

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The PB algorithm starting point 4

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = \overbrace{-\,\boldsymbol{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\boldsymbol{\Gamma}_n^\dagger(q_\perp)} + \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\,\perp})\,\mathbf{A}_{n-1}(q_{n\,\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\,\perp})\,q_\perp\,\delta(q_\perp - q_{n\,\perp})$$

The "outflow" terms dress amplitudes with soft and collinear loops.
When integrated these are equivalent to anomalous dimension matrices.

$$\frac{\mathrm{d}S(\{\tilde{p}\}\cup q,\mu)}{\mathrm{d}\ln\mu} = \prod_{p\in\{p\}} \int \mathrm{d}^4p\,\delta^4\big(p - g_p(\{\tilde{p}\})\big)\,P_{in}(q)\cdot S'(\{p\},\mu') - P_{out}\cdot S(\{\tilde{p}\},\mu)$$

The PB algorithm starting point 5

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = -\,\boldsymbol{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\boldsymbol{\Gamma}_n^\dagger(q_\perp) + \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\perp})\,\mathbf{A}_{n-1}(q_{n\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\perp})\,q_\perp\,\delta(q_\perp - q_{n\perp})$$

$$\mathbf{D}_n(q_{n\perp}; q_n \cup \{\tilde{p}\}_{n-1})\,\mathbf{O}\,\mathbf{D}_n^\dagger(q_{n\perp}; q_n \cup \{\tilde{p}\}_{n-1}) = \sum_{i_n,j_n} \int \delta q_{n\perp}^{(i_n,j_n)}(q_{n\perp})\,\mathbf{S}_n^{i_n}\,\mathbf{O}\,\mathbf{S}_n^{j_n\,\dagger} + \sum_{i_n} \int \delta q_{n\perp}^{(i_n,\vec{n})}(q_{n\perp})\,\mathbf{C}_n^{i_n}\,\mathbf{O}\,\mathbf{C}_n^{i_n\,\dagger}$$

Operators for the generation of soft gluons. These terms are interference like.

Operators for the generation of collinear partons. These terms are self-energy like.

The PB algorithm starting point 6

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = -\,\mathbf{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\mathbf{\Gamma}_n^\dagger(q_\perp) + \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\,\perp})\,\mathbf{A}_{n-1}(q_{n\,\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\,\perp})\,q_\perp\,\delta(q_\perp - q_{n\,\perp})$$

$$\mathbf{D}_n(q_{n\,\perp};q_n\cup\{\tilde{p}\}_{n-1})\,\mathbf{O}\,\mathbf{D}_n^\dagger(q_{n\,\perp};q_n\cup\{\tilde{p}\}_{n-1}) = \sum_{i_n,j_n}\int \delta q_{n\,\perp}^{(i_n,j_n)}(q_{n\,\perp})\,\mathbf{S}_n^{i_n}\,\mathbf{O}\,\mathbf{S}_n^{j_n\,\dagger} + \sum_{i_n}\int \delta q_{n\,\perp}^{(i_n,\vec{n})}(q_{n\,\perp})\,\mathbf{C}_n^{i_n}\,\mathbf{O}\,\mathbf{C}_n^{i_n\,\dagger}$$

We separate the recoil measure between interference and self-energy terms to allow for them to differ.

$$\mathrm{d}R_n\,\mathbf{S}_n^{i_n}\,\mathbf{O}\,\mathbf{S}_n^{j_n\,\dagger} \equiv \left(\prod_{i_n}\mathrm{d}^4 p_{i_n}\right)\mathfrak{R}_{i_n j_n}^{\mathrm{soft}}\,\mathbf{S}_n^{i_n}\,\mathbf{O}\,\mathbf{S}_n^{j_n\,\dagger}$$

$$\mathrm{d}R_n\,\mathbf{C}_n^{i_n}\,\mathbf{O}\,\mathbf{C}_n^{i_n\,\dagger} \equiv \left(\prod_{i_n}\mathrm{d}^4 p_{i_n}\right)\mathfrak{R}_{i_n}^{\mathrm{coll}}\,\mathbf{C}_n^{i_n}\,\mathbf{O}\,\mathbf{C}_n^{i_n\,\dagger}$$

The PB algorithm starting point 7

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = -\,\boldsymbol{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\boldsymbol{\Gamma}_n^\dagger(q_\perp) + \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\,\perp})\,\mathbf{A}_{n-1}(q_{n\,\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\,\perp})\,q_\perp\,\delta(q_\perp - q_{n\,\perp})$$

$$\mathbf{D}_n(q_{n\,\perp};q_n\cup\{\tilde{p}\}_{n-1})\,\mathbf{O}\,\mathbf{D}_n^\dagger(q_{n\,\perp};q_n\cup\{\tilde{p}\}_{n-1}) = \sum_{i_n,j_n}\int \delta q_{n\,\perp}^{(i_n,j_n)}(q_{n\,\perp})\,\mathbf{S}_n^{i_n}\,\mathbf{O}\,\mathbf{S}_n^{j_n\,\dagger} + \sum_{i_n}\int \delta q_{n\,\perp}^{(i_n,\vec{n})}(q_{n\,\perp})\,\mathbf{C}_n^{i_n}\,\mathbf{O}\,\mathbf{C}_n^{i_n\,\dagger}$$

$$\mathrm{d}R_n\,\mathbf{S}_n^{i_n}\,\mathbf{O}\,\mathbf{S}_n^{j_n\,\dagger} \equiv \left(\prod_{i_n}\mathrm{d}^4p_{i_n}\right)\mathfrak{R}^{\mathrm{soft}}_{i_nj_n}\;\mathbf{S}_n^{i_n}\,\mathbf{O}\,\mathbf{S}_n^{j_n\,\dagger}$$

$$\mathrm{d}R_n\,\mathbf{C}_n^{i_n}\,\mathbf{O}\,\mathbf{C}_n^{i_n\,\dagger} \equiv \left(\prod_{i_n}\mathrm{d}^4p_{i_n}\right)\mathfrak{R}^{\mathrm{coll}}_{i_n}\;\mathbf{C}_n^{i_n}\,\mathbf{O}\,\mathbf{C}_n^{i_n\,\dagger}$$

Phase-space for the ith parton

$$\mathrm{d}\sigma_n(\mu) = \left(\prod_{i=1}^{n}\mathrm{d}\Pi_i\right)\mathrm{Tr}\,\mathbf{A}_n(\mu)$$

Coherent branching as a flow equation 1

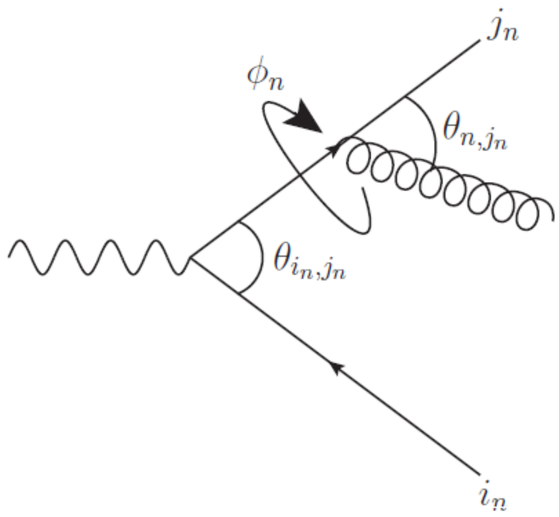
$$\zeta \frac{\partial \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\dots,n}}{\partial \zeta} \approx$$
$$-\sum_{j_{n+1}} \sum_{v} \frac{\alpha_s}{\pi} \int \mathrm{d}z\, \mathcal{P}_{vv_{j_{n+1}}}(z) \left\langle \Theta_{\text{on shell}} \right\rangle_{n+1} \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\dots,n} + \sum_{v} \frac{\alpha_s}{\pi} \mathcal{P}_{vv_{j_n}}(z_n)$$
$$\times \left\langle \Theta_{\text{on shell}} \right\rangle_n \int \mathrm{d}^4 p_{j_n}\, \delta^4(p_{j_n} - z_n^{-1} \tilde{p}_{j_n}) \left\langle |\mathcal{M}_{n-1}(\zeta_{n,j_n})|^2 \right\rangle_{1,\dots,n-1} \zeta_{n,j_n}\, \delta(\zeta - \zeta_{n,j_n})$$

Coherent branching as a flow equation 2

$$\zeta \frac{\partial \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\dots,n}}{\partial \zeta} \approx$$

$$-\sum_{j_{n+1}} \sum_{v} \frac{\alpha_s}{\pi} \int \mathrm{d}z\, \mathcal{P}_{vv_{j_{n+1}}}(z)\, \langle \Theta_{\text{on shell}} \rangle_{n+1} \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\dots,n} + \sum_{v} \frac{\alpha_s}{\pi} \mathcal{P}_{vv_{j_n}}(z_n)$$

$$\times \langle \Theta_{\text{on shell}} \rangle_n \int \mathrm{d}^4 p_{j_n}\, \delta^4(p_{j_n} - z_n^{-1} \tilde{p}_{j_n}) \left\langle |\mathcal{M}_{n-1}(\zeta_{n,j_n})|^2 \right\rangle_{1,\dots,n-1} \zeta_{n,j_n}\, \delta(\zeta - \zeta_{n,j_n})$$



$$\langle f \rangle_{1,\dots,n} = \int \frac{\mathrm{d}\phi_n}{2\pi} \cdots \int \frac{\mathrm{d}\phi_1}{2\pi} f(\phi_1, \dots, \phi_n)$$

$$\zeta_{n,j_n} = 1 - \cos\theta_{n,j_n}$$

$$\frac{\mathrm{d}S(\{\tilde{p}\} \cup q, \mu)}{\mathrm{d}\ln\mu} = \prod_{p \in \{p\}} \int \mathrm{d}^4 p\, \delta^4\left(p - g_p(\{\tilde{p}\})\right) P_{in}(q) \cdot S'(\{p\}, \mu') - P_{out} \cdot S(\{\tilde{p}\}, \mu)$$

Coherent branching as a flow equation 3

$$\zeta \frac{\partial \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\ldots,n}}{\partial \zeta} \approx$$
$$-\sum_{j_{n+1}} \sum_{v} \frac{\alpha_s}{\pi} \int \mathrm{d}z\, \mathcal{P}_{vv_{j_{n+1}}}(z) \left\langle \Theta_{\text{on shell}} \right\rangle_{n+1} \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\ldots,n} + \sum_{v} \frac{\alpha_s}{\pi} \mathcal{P}_{vv_{j_n}}(z_n)$$
$$\times \left\langle \Theta_{\text{on shell}} \right\rangle_n \int \mathrm{d}^4 p_{j_n}\, \delta^4(p_{j_n} - z_n^{-1} \tilde{p}_{j_n}) \left\langle |\mathcal{M}_{n-1}(\zeta_{n,j_n})|^2 \right\rangle_{1,\ldots,n-1} \zeta_{n,j_n}\, \delta(\zeta - \zeta_{n,j_n})$$

"Full" colour, $C_F = \frac{4}{3}$, collinear splitting functions, e.g.

$$P_{qq}(z_n) = C_F \frac{1 + z_n^2}{1 - z_n}$$

$$\frac{\mathrm{d}S(\{\tilde{p}\} \cup q, \mu)}{\mathrm{d}\ln\mu} = \prod_{p \in \{p\}} \int \mathrm{d}^4 p\, \delta^4\left(p - g_p(\{\tilde{p}\})\right) P_{in}(q) \cdot S'(\{p\}, \mu') - P_{out} \cdot S(\{\tilde{p}\}, \mu)$$

Coherent branching as a flow equation 4

$$\zeta \frac{\partial \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\dots,n}}{\partial \zeta} \approx$$
$$- \sum_{j_{n+1}} \sum_{v} \frac{\alpha_s}{\pi} \int \mathrm{d}z\, \mathcal{P}_{v v_{j_{n+1}}}(z) \left\langle \Theta_{\text{on shell}} \right\rangle_{n+1} \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\dots,n} + \sum_{v} \frac{\alpha_s}{\pi} \mathcal{P}_{v v_{j_n}}(z_n)$$
$$\times \left\langle \Theta_{\text{on shell}} \right\rangle_n \underbrace{\int \mathrm{d}^4 p_{j_n}\, \delta^4(p_{j_n} - z_n^{-1} \tilde{p}_{j_n})} \left\langle |\mathcal{M}_{n-1}(\zeta_{n,j_n})|^2 \right\rangle_{1,\dots,n-1} \zeta_{n,j_n}\, \delta(\zeta - \zeta_{n,j_n})$$

$$\int \mathrm{d}R$$

$$\frac{\mathrm{d}S(\{\tilde{p}\} \cup q, \mu)}{\mathrm{d}\ln\mu} = \overbrace{\prod_{p \in \{p\}} \int \mathrm{d}^4 p\, \delta^4\left(p - g_p(\{\tilde{p}\})\right)} P_{in}(q) \cdot S'(\{p\}, \mu') - P_{out} \cdot S(\{\tilde{p}\}, \mu)$$

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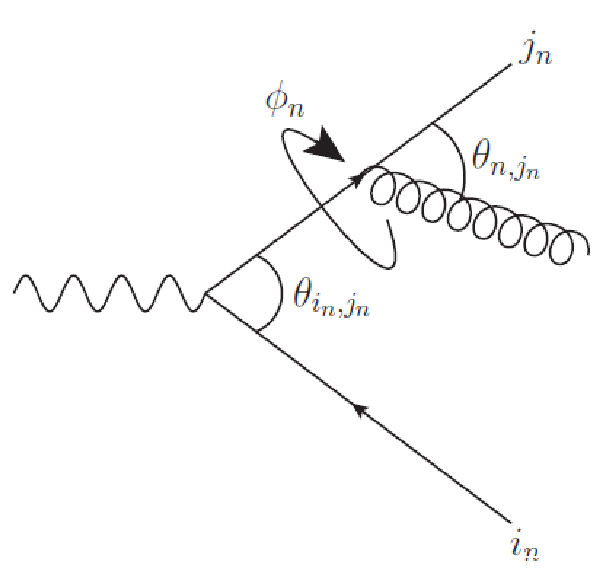
Deriving CB 1

$$d\sigma_n(\mu) = \left(\prod_{i=1}^{n} d\Pi_i\right) \mathrm{Tr}\, \mathbf{A}_n(\mu)$$

$$\Sigma(\mu; \{p\}_0, \{v\}) = \int \sum_n d\sigma_n(\mu)\, u(\{p\}_n, \{v\})$$



$|M_n|^2 \propto \mathrm{Tr}\, \mathbf{A}_n$

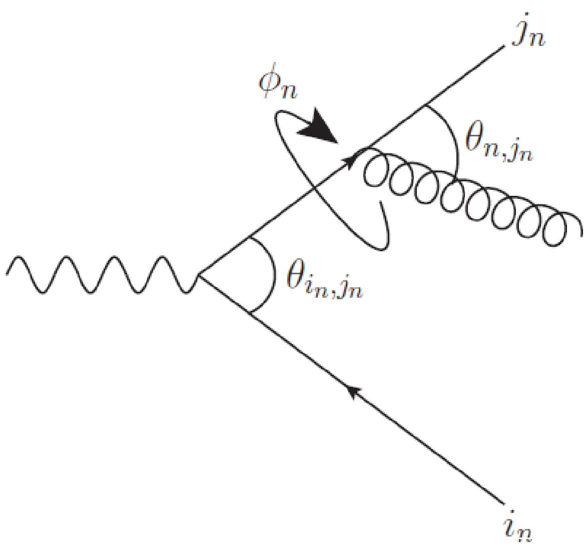
$$\langle |M_n|^2\, u(\{p\}_n)\rangle_{1,\dots,n} \approx \int \prod_{i=1}^{n} \frac{d\phi_i}{2\pi} \langle |M_n|^2\rangle_{1,\dots,n}\, u(\{p\}_n) = \langle |M_n|^2\rangle_{1,\dots,n} \langle u(\{p\}_n)\rangle_{1,\dots,n} \,.$$

$$\begin{aligned}\langle |M_n|^2\, u(\{p\}_n)\rangle_n &= \langle |M_n|^2\rangle_n \, \langle u(\{p\}_n)\rangle_n \\ &\quad + \sigma_n(|M_n|^2)\, \sigma_n(u(\{p\}_n)\, \mathrm{Cor}_n(|M_n|^2, u(\{p\}_n))\end{aligned}$$

Issues with non-global logarithms.
A. Banfi, G. Corcella, M. Dasgupta hep-ph/0612282

Deriving CB 2

$$\int \frac{\mathrm{d}S_2^{(q_n)}}{4\pi} \frac{1}{2} \mathbf{S}_n^{j_n} \cdot \mathbf{S}_n^{i_n} \propto \int \frac{\mathrm{d}S_2^{(q_n)}}{4\pi} \int \frac{\delta q_{n\,\perp}^{(i_n,j_n)}(q_\perp)}{q_\perp}\, 2\,\Theta_{\text{on shell}}$$
$$= \int \frac{\mathrm{d}\Omega_{q_n}}{4\pi} \int \frac{\mathrm{d}E_{q_n}}{E_{q_n}} E_{q_n}^2 \frac{\tilde{p}_{i_n} \cdot \tilde{p}_{j_n}}{\tilde{p}_{i_n} \cdot q_n \tilde{p}_{j_n} \cdot q_n} \Theta_{\text{on shell}}\; \delta(q_{n\,\perp}^{(i_n,j_n)} - q_\perp)$$



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Deriving CB 3

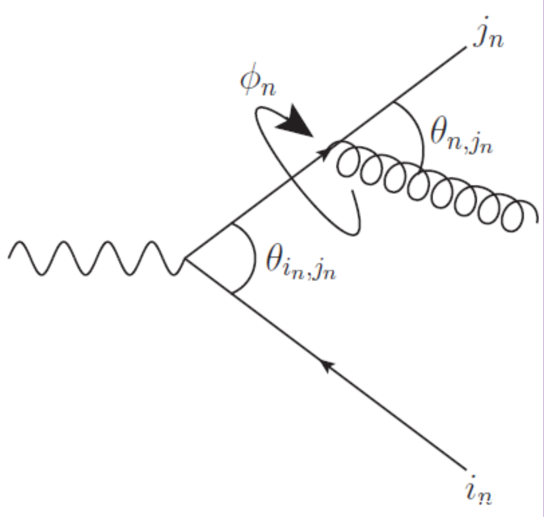
$$\int \frac{\mathrm{d}S_2^{(q_n)}}{4\pi} \frac{1}{2} \mathbf{S}_n^{j_n} \cdot \mathbf{S}_n^{i_n} \propto \int \frac{\mathrm{d}S_2^{(q_n)}}{4\pi} \int \frac{\delta q_{n\perp}^{(i_n,j_n)}(q_\perp)}{q_\perp} \, 2\,\Theta_{\text{on shell}}$$

$$= \int \frac{\mathrm{d}\Omega_{q_n}}{4\pi} \int \frac{\mathrm{d}E_{q_n}}{E_{q_n}} E_{q_n}^2 \frac{\tilde{p}_{i_n} \cdot \tilde{p}_{j_n}}{\tilde{p}_{i_n} \cdot q_n \tilde{p}_{j_n} \cdot q_n} \Theta_{\text{on shell}} \; \delta(q_{n\perp}^{(i_n,j_n)} - q_\perp)$$

$$= \int \frac{\mathrm{d}\Omega_{q_n}}{4\pi} \int \frac{\mathrm{d}E_{q_n}}{E_{q_n}} \left(P_{i_n j_n} + P_{j_n i_n}\right) \Theta_{\text{on shell}} \; \delta(q_{n\perp}^{(i_n,j_n)} - q_\perp)$$

$$2P_{i_n j_n} = \frac{n_{i_n} \cdot n_{j_n} - n_{i_n} \cdot n}{n_{i_n} \cdot n \; n_{j_n} \cdot n} + \frac{1}{n_{i_n} \cdot n}$$

$$\langle P_{i_n j_n} \rangle_{\phi_{n,i_n}} = \frac{\Theta(\theta_{j_n,i_n} - \theta_{n,i_n})}{1 - \cos\theta_{n,i_n}}$$



Deriving CB 4

$$\int \frac{\mathrm{d}\Omega_{q_n}}{4\pi} \int \frac{\mathrm{d}E_{q_n}}{E_{q_n}} \left(P_{i_n j_n} + P_{j_n i_n}\right) \Theta_{\text{on shell}}\ \delta(q_{n\perp}^{(i_n,j_n)} - q_\perp)$$

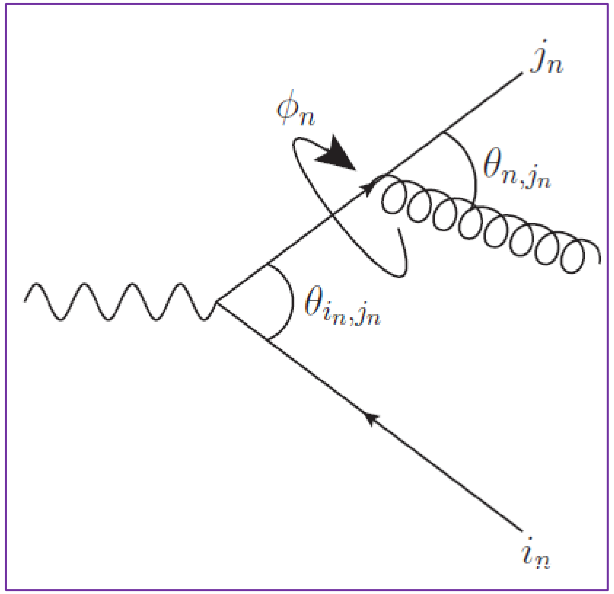
$$2P_{i_n j_n} = \frac{n_{i_n} \cdot n_{j_n} - n_{i_n} \cdot n}{n_{i_n} \cdot n\ n_{j_n} \cdot n} + \frac{1}{n_{i_n} \cdot n}$$

$$\mathfrak{R}^{\text{soft}}_{i_n j_n} = \frac{(q_{n\perp}^{(i_n j_n)})^2}{2E_n^2} \left(P_{i_n j_n} \mathfrak{R}_{i_n} + P_{j_n i_n} \mathfrak{R}_{j_n}\right)$$

$$\mathfrak{R}_{j_n} = \delta^4(p_{j_n} - z_n^{-1} \tilde{p}_{j_n}) \prod_{i_n \neq j_n} \delta^4(p_{i_n} - \tilde{p}_{i_n}) + \mathcal{O}(q_\perp/Q)$$

$$\mathrm{d}R_n\ \mathbf{S}_n^{i_n}\, \mathbf{O}\, \mathbf{S}_n^{j_n\,\dagger} \equiv \left(\prod_{i_n} \mathrm{d}^4 p_{i_n}\right) \mathfrak{R}^{\text{soft}}_{i_n j_n}\ \mathbf{S}_n^{i_n}\, \mathbf{O}\, \mathbf{S}_n^{j_n\,\dagger}$$

$$\mathrm{d}R_n\ \mathbf{C}_n^{i_n}\, \mathbf{O}\, \mathbf{C}_n^{i_n\,\dagger} \equiv \left(\prod_{i_n} \mathrm{d}^4 p_{i_n}\right) \mathfrak{R}^{\text{coll}}_{i_n}\ \mathbf{C}_n^{i_n}\, \mathbf{O}\, \mathbf{C}_n^{i_n\,\dagger}$$



Outcome

$$\zeta \frac{\partial \langle |\mathcal{M}_n(\zeta)|^2 \rangle_{1,\dots,n}}{\partial \zeta} \approx$$
$$- \sum_{j_{n+1}} \sum_{v} \frac{\alpha_s}{\pi} \int \mathrm{d}z \, \mathcal{P}_{vv_{j_{n+1}}}(z) \, \langle \Theta_{\text{on shell}} \rangle_{n+1} \, \langle |\mathcal{M}_n(\zeta)|^2 \rangle_{1,\dots,n} + \sum_{v} \frac{\alpha_s}{\pi} \mathcal{P}_{vv_{j_n}}(z_n)$$
$$\times \langle \Theta_{\text{on shell}} \rangle_n \int \mathrm{d}^4 p_{j_n} \, \delta^4(p_{j_n} - z_n^{-1} \tilde{p}_{j_n}) \, \langle |\mathcal{M}_{n-1}(\zeta_{n,j_n})|^2 \rangle_{1,\dots,n-1} \, \zeta_{n,j_n} \, \delta(\zeta - \zeta_{n,j_n})$$

$$\langle |M_n|^2 \rangle_{1,\dots,n} = \left(\frac{2\alpha_s}{\pi} \right)^n \prod_{i=1}^{n} (1 - z_i)^{-1} \mathrm{Tr} \, \langle \mathbf{A}_n(\zeta) \rangle_{1,\dots,n}$$

$$\mathfrak{R}^{\text{soft}}_{i_n j_n} = \frac{(q_{n\perp}^{(i_n j_n)})^2}{2E_n^2} \left(P_{i_n j_n} \mathfrak{R}_{i_n} + P_{j_n i_n} \mathfrak{R}_{j_n} \right) + h$$

where $\langle h \rangle_n \approx$ NNL

$$\mathfrak{R}_{j_n} = \delta^4(p_{j_n} - z_n^{-1} \tilde{p}_{j_n}) \prod_{i_n \neq j_n} \delta^4(p_{i_n} - \tilde{p}_{i_n}) + \mathcal{O}(q_\perp / Q)$$

Deriving a dipole shower 1

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = -\,\boldsymbol{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\boldsymbol{\Gamma}_n^\dagger(q_\perp) + \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\perp})\,\mathbf{A}_{n-1}(q_{n\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\perp})\,q_\perp\,\delta(q_\perp - q_{n\perp})$$

In overview, a two step process:

1. Expand the PB evolution equation in powers of the number of colours, N_c. This reduces colour structures to being dipole (soft physics) or Casimir (hard-collinear physics).

Deriving a dipole shower 2

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = -\,\mathbf{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\mathbf{\Gamma}_n^\dagger(q_\perp)$$
$$+ \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\perp})\,\mathbf{A}_{n-1}(q_{n\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\perp})\,q_\perp\,\delta(q_\perp - q_{n\perp})$$

In overview, a two step process:

1. Expand the PB evolution equation in powers of the number of colours, N_c. This reduces colour structures to being dipole (soft physics) or Casimir (hard-collinear physics).

2. Use the form of $\int \mathrm{d}R_n$ inherited from CB to partition the dipoles. This allows soft and collinear physics to be combined into a single emission kernel per colour line.

Dipole shower 1

$$
\begin{aligned}
&q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp} \\
&\approx -\frac{\alpha_s}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z\, \Theta_{\text{on shell}}\, P_{v_{i_{n+1}} v_{i_{n+1}}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2 \\
&+ \frac{\alpha_s}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n} \right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{i_n} v_{i_n}}(z_n)\; q_\perp \delta(q_{n\,\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\,\perp}^{(i^c_n,\overline{i^c}_n)})|^2
\end{aligned}
$$

Dipole shower 2

$$q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp}$$
$$\approx -\frac{\alpha_s}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z\, \Theta_{\text{on shell}}\, P_{v_{i_{n+1}} v_{i_{n+1}}}(z) \, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2$$
$$+ \frac{\alpha_s}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n} \right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{i_n} v_{i_n}}(z_n)\, q_\perp \delta(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2$$

$$|\mathcal{M}_n^{(\sigma)}(q_\perp; \{P_1, ..., P_{n_H}, (z_1, q_{1\perp}^{(i^c_1,\overline{i^c}_1)}, \phi_1), ..., (z_n, q_{n\perp}^{(i^c_n,\overline{i^c}_n)}, \phi_n)\})|^2$$

$$\mathfrak{R}^{\text{dipole}}_{i^c_n} + \mathfrak{R}^{\text{dipole}}_{j^c_n} = \mathfrak{R}^{\text{soft}}_{i^c_n j^c_n}$$

$$P_{qq}(z_n) = C_F \frac{1+z_n^2}{1-z_n}, \qquad P_{gg}(z_n) = \frac{C_A}{2} \frac{1+z_n^3}{1-z_n}.$$

Leading colour $C_F = \frac{3}{2}$

Dipole shower 3

$$\begin{aligned} q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp} \\ \approx -\frac{\alpha_\mathrm{s}}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z \, \Theta_{\text{on shell}} \, P_{v_{i_{n+1}} v_{i_{n+1}}}(z) \, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2 \\ + \frac{\alpha_\mathrm{s}}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n} \right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{i_n} v_{i_n}}(z_n) \, q_\perp \delta(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2 \end{aligned}$$

$$\mathfrak{R}^{\text{dipole}}_{i^c_n} = \left(\frac{1}{2} + \mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n) \right) \mathfrak{R}_{i^c_n}$$

$$\mathfrak{R}_{i^c_n} = \delta^4(p_{i_n} - z_n^{-1} \tilde{p}_{i_n}) \prod_{i_n \neq j_n} \delta^4(p_{j_n} - \tilde{p}_{j_n}) + \mathcal{O}(q_\perp/Q)$$

$$\mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n) = \left[\frac{T \cdot p_{i^c_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{i^c_n} \cdot q_n} - \frac{T \cdot p_{\overline{i^c}_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{\overline{i^c}_n} \cdot q_n} \right]$$

$$T = \sum_{i_n} p_{i_n}$$

When $T = p_{i^c_n} + p_{\bar{i}^c_n}$ this partitioning is equivalent to CS dipole factorisation.

This happens only when event ZMF (lab) frame coincides with the dipole ZMF.

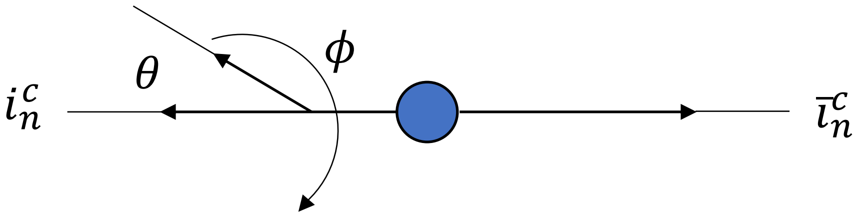
Dipole shower 4

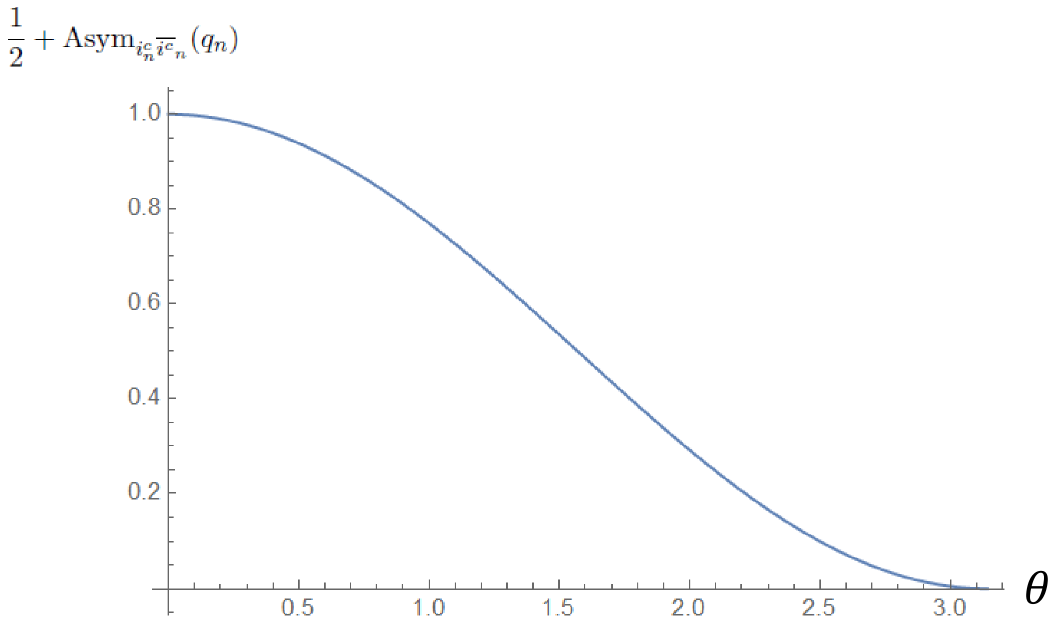
$$q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp}$$
$$\approx -\frac{\alpha_s}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z\, \Theta_{\text{on shell}} P_{v_{i_{n+1}} v_{i_{n+1}}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2$$
$$+ \frac{\alpha_s}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n}\right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{i_n} v_{i_n}}(z_n)\, q_\perp \delta(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2$$

$$\mathfrak{R}^{\text{dipole}}_{i^c_n} = \left(\frac{1}{2} + \text{Asym}_{i^c_n \overline{i^c}_n}(q_n)\right) \mathfrak{R}_{i^c_n}$$

$$\mathfrak{R}_{i^c_n} = \delta^4(p_{i_n} - z_n^{-1} \tilde{p}_{i_n}) \prod_{i_n \neq j_n} \delta^4(p_{j_n} - \tilde{p}_{j_n}) + \mathcal{O}(q_\perp / Q)$$

$$\text{Asym}_{i^c_n \overline{i^c}_n}(q_n) = \left[\frac{T \cdot p_{i^c_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{i^c_n} \cdot q_n} - \frac{T \cdot p_{\overline{i^c}_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{\overline{i^c}_n} \cdot q_n}\right], \qquad T = \sum_{i_n} p_{i_n}$$





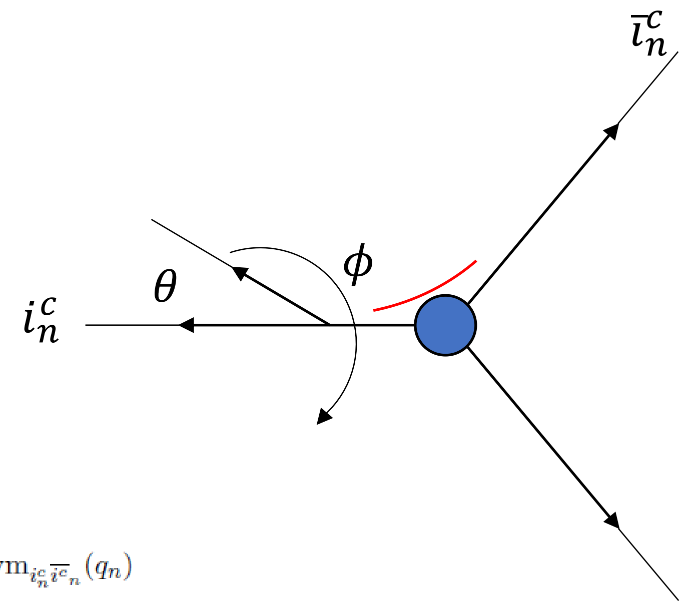
Dipole shower 5

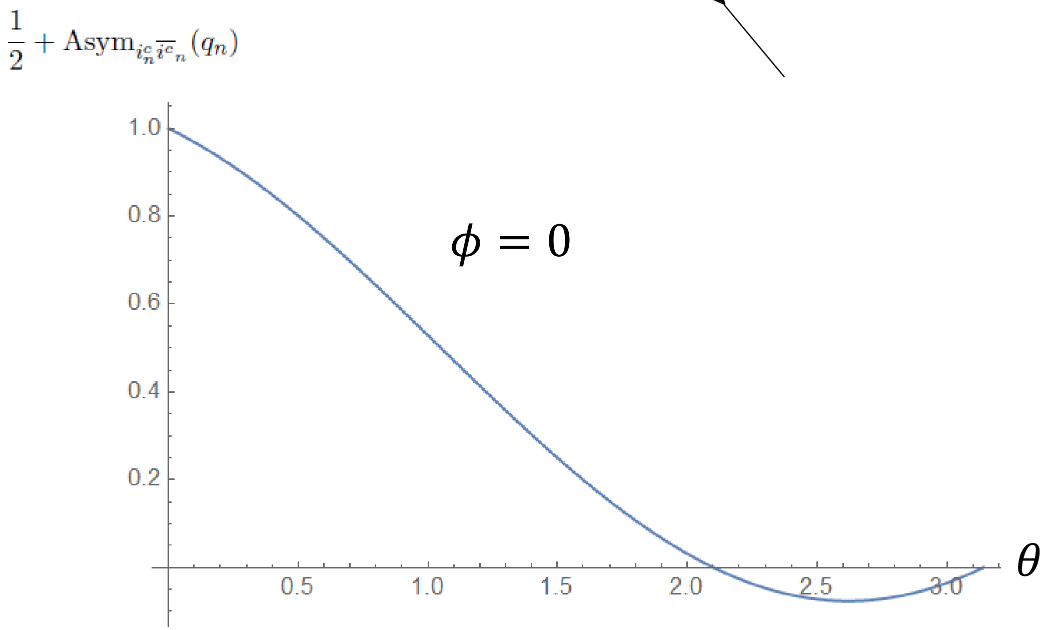
$$
\begin{aligned}
&q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp} \\
&\approx -\frac{\alpha_s}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z\, \Theta_{\text{on shell}}\, P_{v_{i_{n+1}} v_{i_{n+1}}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2 \\
&+ \frac{\alpha_s}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n} \right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{i_n} v_{i_n}}(z_n)\, q_\perp \delta(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2
\end{aligned}
$$

$$
\mathfrak{R}^{\text{dipole}}_{i^c_n} = \left(\frac{1}{2} + \mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n) \right) \mathfrak{R}_{i^c_n}
$$

$$
\mathfrak{R}_{i^c_n} = \delta^4(p_{i_n} - z_n^{-1} \tilde{p}_{i_n}) \prod_{i_n \neq j_n} \delta^4(p_{j_n} - \tilde{p}_{j_n}) + \mathcal{O}(q_\perp/Q)
$$

$$
\mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n) = \left[\frac{T \cdot p_{i^c_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{i^c_n} \cdot q_n} - \frac{T \cdot p_{\overline{i^c}_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{\overline{i^c}_n} \cdot q_n} \right], \qquad T = \sum_{i_n} p_{i_n}
$$





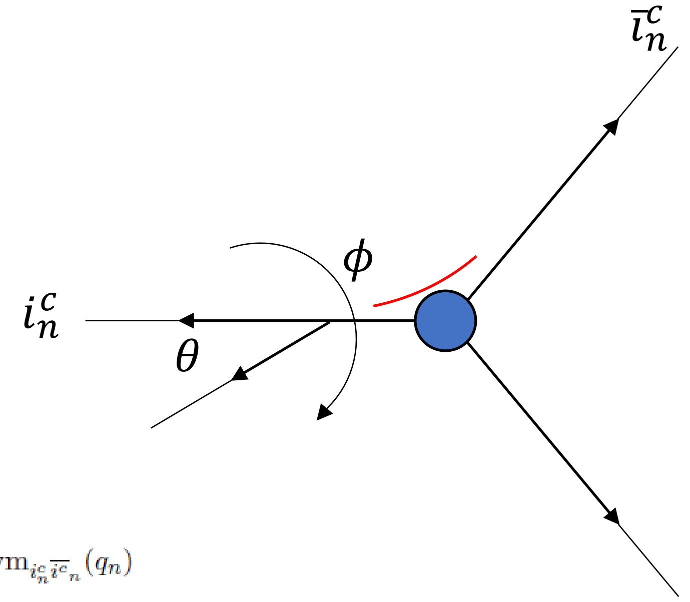
Dipole shower 6

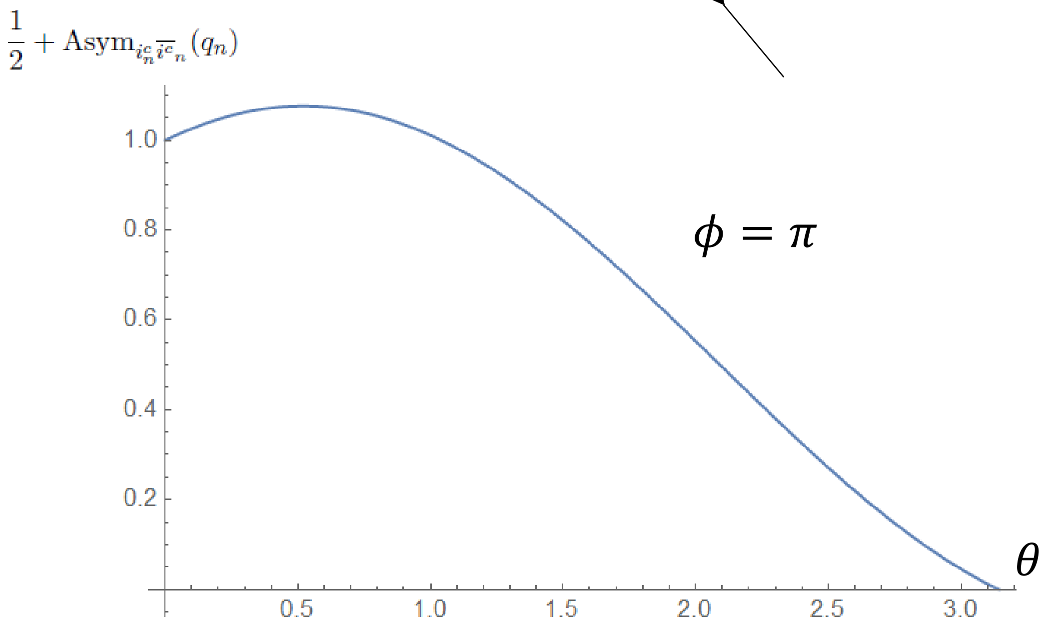
$$
\begin{aligned}
q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp}
&\approx -\frac{\alpha_s}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})}\, \delta\big(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp\big) \int \mathrm{d}z\, \Theta_{\text{on shell}}\, P_{v_{i_{n+1}} v_{i_{n+1}}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2 \\
&+ \frac{\alpha_s}{\pi} \int \Big(\prod_{j_n} \mathrm{d}^4 p_{j_n}\Big)\, \mathfrak{R}^{\text{dipole}}_{i^c_n}\, P_{v_{i_n} v_{i_n}}(z_n)\, q_\perp \delta\big(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp\big) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2
\end{aligned}
$$

$$
\mathfrak{R}^{\text{dipole}}_{i^c_n} = \left(\frac{1}{2} + \text{Asym}_{i^c_n \overline{i^c}_n}(q_n)\right) \mathfrak{R}_{i^c_n}
$$

$$
\mathfrak{R}_{i^c_n} = \delta^4(p_{i_n} - z_n^{-1} \tilde{p}_{i_n}) \prod_{i_n \neq j_n} \delta^4(p_{j_n} - \tilde{p}_{j_n}) + \mathcal{O}(q_\perp/Q)
$$

$$
\text{Asym}_{i^c_n \overline{i^c}_n}(q_n) = \left[\frac{T \cdot p_{i^c_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{i^c_n} \cdot q_n} - \frac{T \cdot p_{\overline{i^c}_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{\overline{i^c}_n} \cdot q_n}\right], \qquad T = \sum_{i_n} p_{i_n}
$$





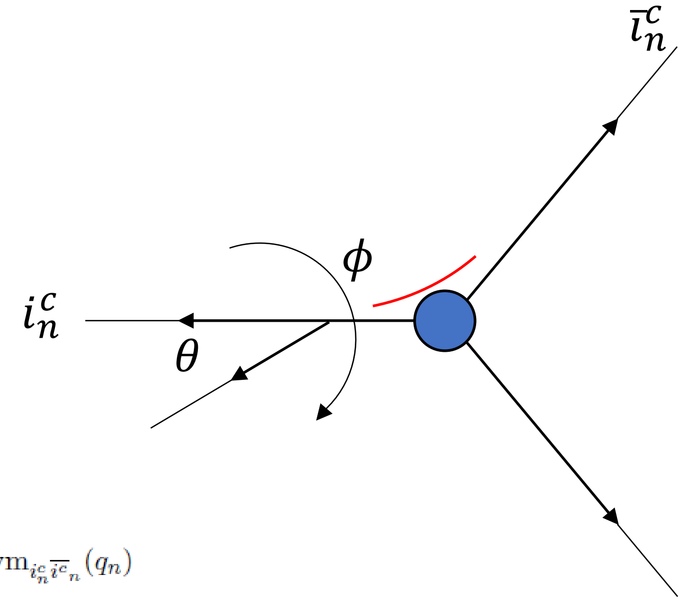
Dipole shower 7

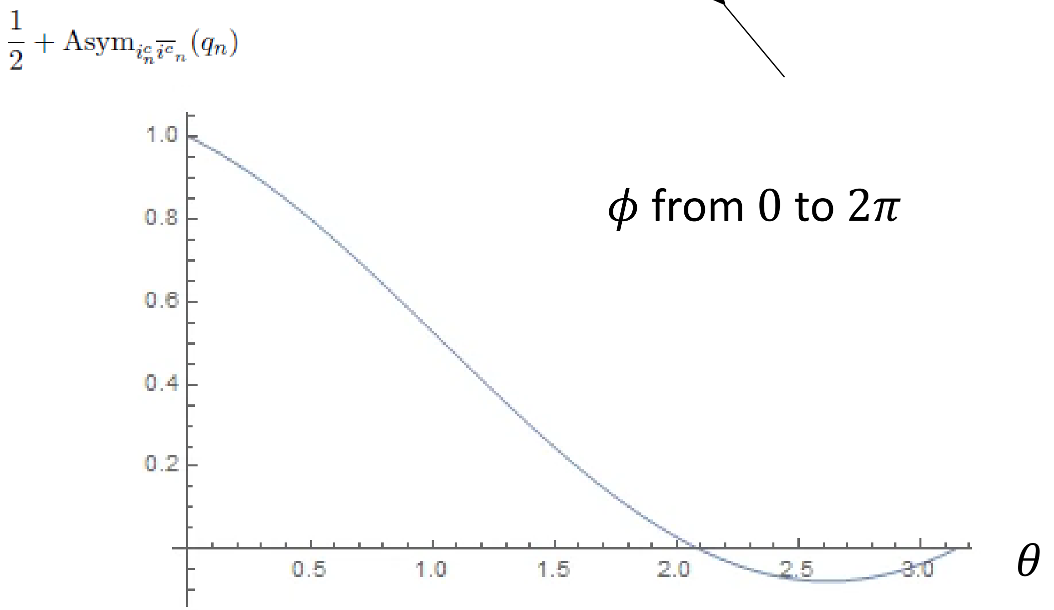
$$
q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp}
\approx -\frac{\alpha_s}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta\!\left(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp\right) \int \mathrm{d}z\, \Theta_{\text{on shell}}\, P_{v_{i_{n+1}} v_{i_{n+1}}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2
+ \frac{\alpha_s}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n} \right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{i_n} v_{i_n}}(z_n)\, q_\perp \delta\!\left(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp\right) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2
$$

$$
\mathfrak{R}^{\text{dipole}}_{i^c_n} = \left(\frac{1}{2} + \text{Asym}_{i^c_n \overline{i^c}_n}(q_n) \right) \mathfrak{R}_{i^c_n}
$$

$$
\mathfrak{R}_{i^c_n} = \delta^4(p_{i_n} - z_n^{-1} \tilde{p}_{i_n}) \prod_{i_n \neq j_n} \delta^4(p_{j_n} - \tilde{p}_{j_n}) + \mathcal{O}(q_\perp / Q)
$$

$$
\text{Asym}_{i^c_n \overline{i^c}_n}(q_n) = \left[\frac{T \cdot p_{i^c_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{i^c_n} \cdot q_n} - \frac{T \cdot p_{\overline{i^c}_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{\overline{i^c}_n} \cdot q_n} \right], \qquad T = \sum_{i_n} p_{i_n}
$$





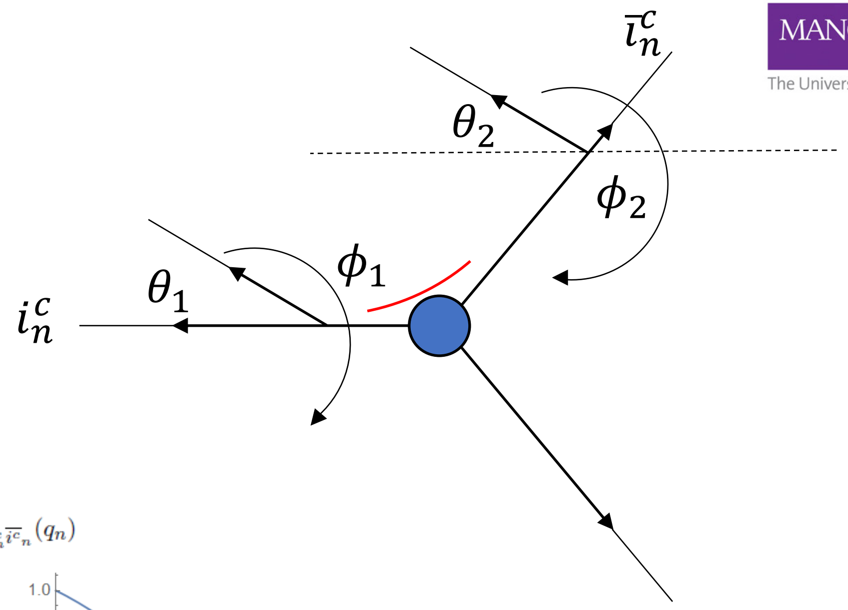
Dipole shower 8

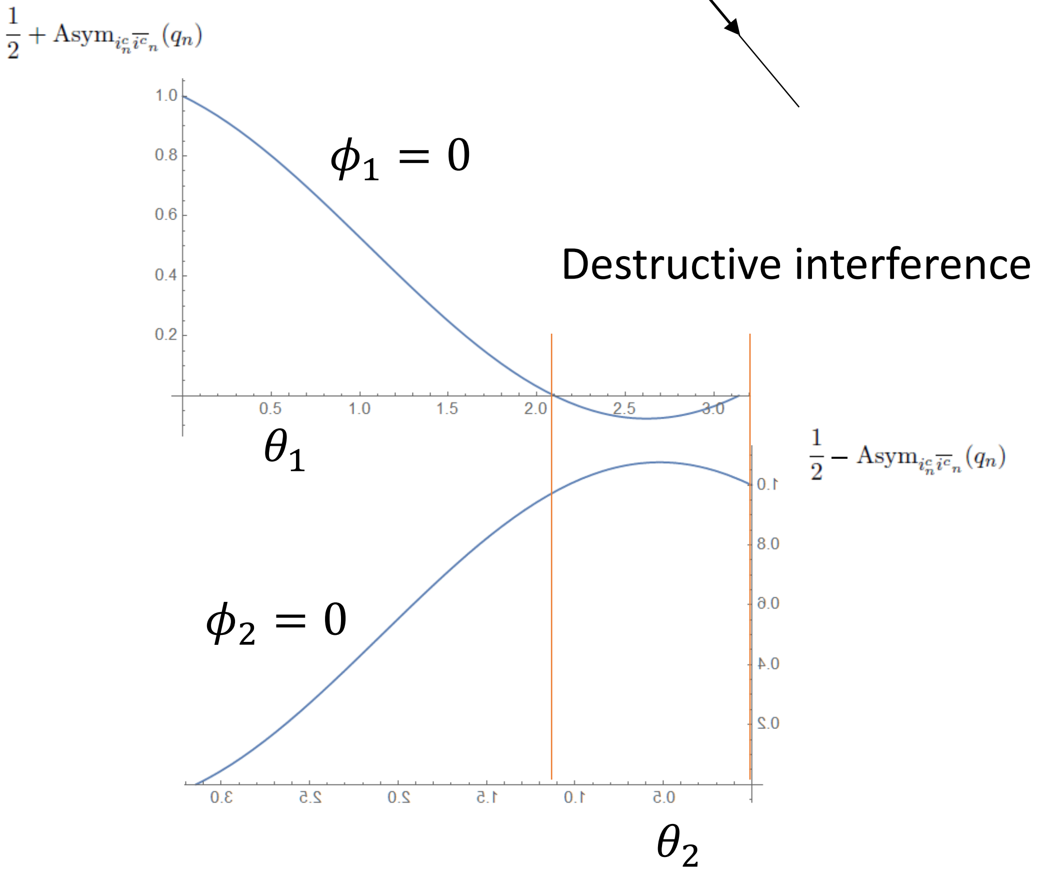
$$q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp}$$
$$\approx -\frac{\alpha_s}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z\, \Theta_{\text{on shell}} P_{v_{i_{n+1}} v_{i_{n+1}}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2$$
$$+ \frac{\alpha_s}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n} \right) \underbrace{\mathfrak{R}^{\text{dipole}}_{i^c_n}} P_{v_{i_n} v_{i_n}}(z_n)\, q_\perp \delta(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2$$

$$\underbrace{\mathfrak{R}^{\text{dipole}}_{i^c_n}} = \left(\frac{1}{2} + \underbrace{\mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n)} \right) \underbrace{\mathfrak{R}_{i^c_n}}$$

$$\underbrace{\mathfrak{R}_{i^c_n}} = \delta^4(p_{i_n} - z_n^{-1} \tilde{p}_{i_n}) \prod_{i_n \neq j_n} \delta^4(p_{j_n} - \tilde{p}_{j_n}) + \mathcal{O}(q_\perp / Q)$$

$$\underbrace{\mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n)} = \left[\frac{T \cdot p_{i^c_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{i^c_n} \cdot q_n} - \frac{T \cdot p_{\overline{i^c}_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{\overline{i^c}_n} \cdot q_n} \right], \qquad T = \sum_{i_n} p_{i_n}$$





Dipole shower 9

$$q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp} \approx -\frac{\alpha_s}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\bar{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\bar{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z\, \Theta_{\text{on shell}}\, P_{v_{i_{n+1}} v_{i_{n+1}}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2 + \frac{\alpha_s}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n}\right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{i_n} v_{i_n}}(z_n)\, q_\perp \delta(q_{n\perp}^{(i^c_n,\bar{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\bar{i^c}_n)})|^2$$

$$\mathfrak{R}^{\text{dipole}}_{i^c_n} = \left(\frac{1}{2} + \mathrm{Asym}_{i^c_n \bar{i^c}_n}(q_n)\right) \mathfrak{R}_{i^c_n}$$

$$\mathfrak{R}_{i^c_n} = \delta^4(p_{i_n} - z_n^{-1}\tilde{p}_{i_n}) \prod_{i_n \neq j_n} \delta^4(p_{j_n} - \tilde{p}_{j_n}) + \mathcal{O}(q_\perp/Q)$$

$$\mathrm{Asym}_{i^c_n \bar{i^c}_n}(q_n) = \left[\frac{T\cdot p_{i^c_n}}{4T\cdot q_n} \frac{(q_{n\perp}^{(i^c_n \bar{i^c}_n)})^2}{p_{i^c_n}\cdot q_n} - \frac{T\cdot p_{\bar{i^c}_n}}{4T\cdot q_n} \frac{(q_{n\perp}^{(i^c_n \bar{i^c}_n)})^2}{p_{\bar{i^c}_n}\cdot q_n}\right], \qquad T = \sum_{i_n} p_{i_n}$$

Many important properties shared with the Panscales showers (M. Dasgupta et al. 2002.11114).

Partitions the dipole symmetrically in the event zero momentum frame if the parent partons are equal energy.

Both halves of the partitioning always sum to one and have rapid rise to 1 (and 0) in the collinear (anti-collinear) regions.

The main difference is the destructive interference. Destructive interference has strong azimuthal dependence.

$$\mathfrak{R}^{\text{dipole}}_{i^c_n} + \mathfrak{R}^{\text{dipole}}_{j^c_n} = \mathfrak{R}^{\text{soft}}_{i^c_n j^c_n}$$

Dipole shower 10

$$
q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp}
\approx -\frac{\alpha_\mathrm{s}}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z\, \Theta_{\text{on shell}}\, P_{v_{i_{n+1}} v_{i_{n+1}}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2
+ \frac{\alpha_\mathrm{s}}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n} \right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{i_n} v_{i_n}}(z_n)\, q_\perp \delta(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2
$$

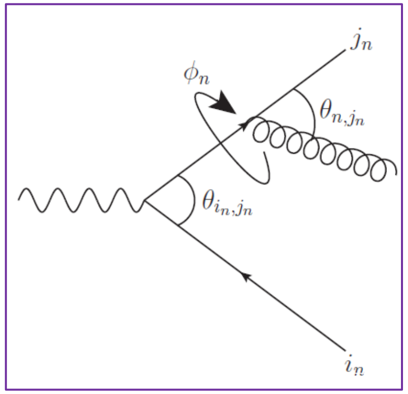
$$
\mathfrak{R}^{\text{dipole}}_{i^c_n} = \left(\frac{1}{2} + \mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n) \right) \mathfrak{R}_{i^c_n}
$$

$$
\mathfrak{R}_{i^c_n} = \delta^4(p_{i_n} - z_n^{-1}\tilde{p}_{i_n}) \prod_{i_n \neq j_n} \delta^4(p_{j_n} - \tilde{p}_{j_n}) + \mathcal{O}(q_\perp/Q)
$$

$$
\mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n) = \left[\frac{T \cdot p_{i^c_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{i^c_n} \cdot q_n} - \frac{T \cdot p_{\overline{i^c}_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{\overline{i^c}_n} \cdot q_n} \right], \qquad T = \sum_{i_n} p_{i_n}
$$

$$
|\mathcal{M}_n^{(\sigma)}(q_\perp; \{P_1, \ldots, P_{n_\mathrm{H}}, (z_1, q_{1\perp}^{(i^c_1,\overline{i^c}_1)}, \phi_1), \ldots, (z_n, q_{n\perp}^{(i^c_n,\overline{i^c}_n)}, \phi_n)\})|^2
$$

$$
P_{qq}(z_n) = \mathcal{C}_\mathrm{F} \frac{1+z_n^2}{1-z_n}, \qquad P_{gg}(z_n) = \frac{\mathcal{C}_\mathrm{A}}{2} \frac{1+z_n^3}{1-z_n}.
$$



Leading colour

$$
\zeta \frac{\partial \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\ldots,n}}{\partial \zeta} \approx
- \sum_{j_{n+1}} \sum_{v} \frac{\alpha_\mathrm{s}}{\pi} \int \mathrm{d}z\, \mathcal{P}_{v v_{j_{n+1}}}(z) \left\langle \Theta_{\text{on shell}} \right\rangle_{n+1} \left\langle |\mathcal{M}_n(\zeta)|^2 \right\rangle_{1,\ldots,n} + \sum_{v} \frac{\alpha_\mathrm{s}}{\pi} \mathcal{P}_{v v_{j_n}}(z_n)
\times \left\langle \Theta_{\text{on shell}} \right\rangle_n \int \mathrm{d}^4 p_{j_n}\, \delta^4(p_{j_n} - z_n^{-1} \tilde{p}_{j_n}) \left\langle |\mathcal{M}_{n-1}(\zeta_{n,j_n})|^2 \right\rangle_{1,\ldots,n-1} \zeta_{n,j_n}\, \delta(\zeta - \zeta_{n,j_n})
$$

Dipole shower 11

$$
q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp}
\approx -\frac{\alpha_\mathrm{s}}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z \, \Theta_{\text{on shell}} \, P_{v_{i_{n+1}} v_{i_{n+1}}}(z) \, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2
+ \frac{\alpha_\mathrm{s}}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n} \right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{i_n} v_{i_n}}(z_n) \, q_\perp \delta(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2
$$

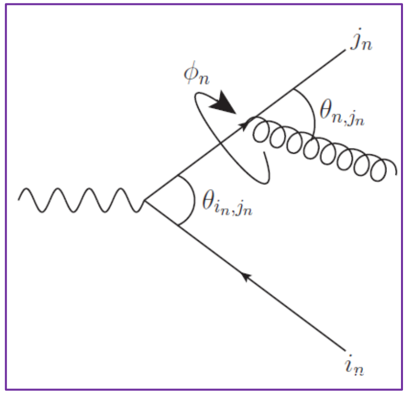
$$
\mathfrak{R}^{\text{dipole}}_{i^c_n} = \left(\frac{1}{2} + \mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n) \right) \mathfrak{R}_{i^c_n}
$$

$$
\mathfrak{R}_{i^c_n} = \delta^4(p_{i_n} - z_n^{-1} \tilde{p}_{i_n}) \prod_{i_n \neq j_n} \delta^4(p_{j_n} - \tilde{p}_{j_n}) + \mathcal{O}(q_\perp / Q)
$$

$$
\mathrm{Asym}_{i^c_n \overline{i^c}_n}(q_n) = \left[\frac{T \cdot p_{i^c_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{i^c_n} \cdot q_n} - \frac{T \cdot p_{\overline{i^c}_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{\overline{i^c}_n} \cdot q_n} \right], \qquad T = \sum_{i_n} p_{i_n}
$$

$$
|\mathcal{M}_n^{(\sigma)}(q_\perp; \{P_1, \ldots, P_{n_\mathrm{H}}, (z_1, q_{1\perp}^{(i^c_1,\overline{i^c}_1)}, \phi_1), \ldots, (z_n, q_{n\perp}^{(i^c_n,\overline{i^c}_n)}, \phi_n)\})|^2
$$

$$
P_{qq}(z_n) = \mathcal{C}_\mathrm{F} \frac{1+z_n^2}{1-z_n}, \qquad P_{gg}(z_n) = \frac{\mathcal{C}_\mathrm{A}}{2} \frac{1+z_n^3}{1-z_n}.
$$



“Full” colour $C_\mathrm{F} = \frac{4}{3}$

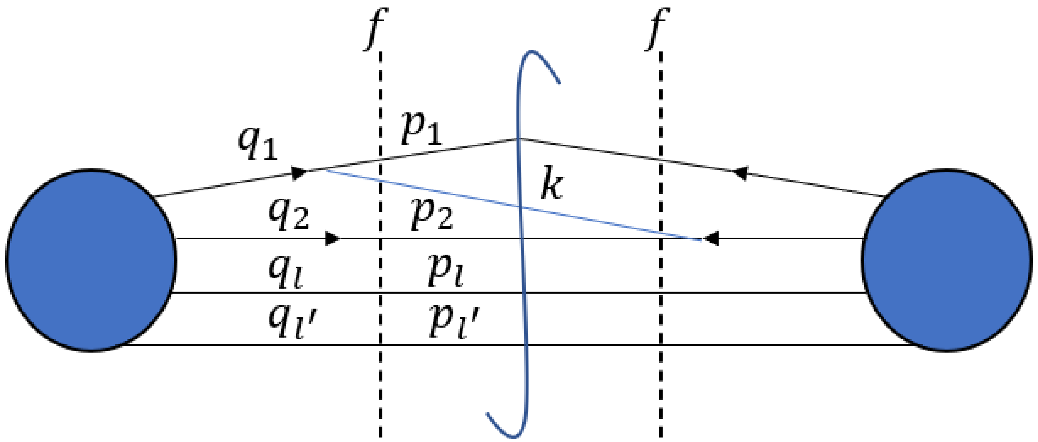
$$
\zeta \frac{\partial \, \langle |\mathcal{M}_n(\zeta)|^2 \rangle_{1,\ldots,n}}{\partial \zeta} \approx
- \sum_{j_{n+1}} \sum_{v} \frac{\alpha_\mathrm{s}}{\pi} \int \mathrm{d}z \, \mathcal{P}_{v v_{j_{n+1}}}(z) \, \langle \Theta_{\text{on shell}} \rangle_{n+1} \, \langle |\mathcal{M}_n(\zeta)|^2 \rangle_{1,\ldots,n} + \sum_{v} \frac{\alpha_\mathrm{s}}{\pi} \mathcal{P}_{v v_{j_n}}(z_n)
\times \langle \Theta_{\text{on shell}} \rangle_n \int \mathrm{d}^4 p_{j_n} \, \delta^4(p_{j_n} - z_n^{-1} \tilde{p}_{j_n}) \, \langle |\mathcal{M}_{n-1}(\zeta_{n,j_n})|^2 \rangle_{1,\ldots,n-1} \, \zeta_{n,j_n} \, \delta(\zeta - \zeta_{n,j_n})
$$

What about transverse recoil? 1

- Here we lack analytic insight, so instead we looked at the successes of other approaches.
- First choice: local or global?

$$p_1 = a_1 q_1 + b_1 q_2 - (1-g) k_T$$
$$p_2 = a_2 q_1 + b_2 q_2 - g k_T$$
$$k = a_k q_1 + b_k q_2 + k_T$$

$$p_1 = f \cdot (a_1 q_1 + b_1 q_2 - c(1-g) k_T)$$
$$p_2 = f \cdot (a_2 q_1 + b_2 q_2 - c g k_T)$$
$$k = f \cdot (a_k q_1 + b_k q_2 + k_T)$$
$$p_{l'}^{\mu} = f_{\nu,l'l}^{\mu} q_l^{\nu}$$



What about transverse recoil? 2

- Here we lack analytic insight, so instead we looked at the successes of other approaches.
- First choice: local or global?

$$\begin{gathered} p_1 = a_1 q_1 + b_1 q_2 - (1-g)k_T \\ p_2 = a_2 q_1 + b_2 q_2 - g k_T \\ k = a_k q_1 + b_k q_2 + k_T \end{gathered}$$

$$\begin{gathered} p_1 = f \cdot (a_1 q_1 + b_1 q_2 - c(1-g)k_T) \\ p_2 = f \cdot (a_2 q_1 + b_2 q_2 - c g k_T) \\ k = f \cdot (a_k q_1 + b_k q_2 + k_T) \\ p_{l'}^{\mu} = f_{\nu,l'l}^{\mu} q_l^{\nu} \end{gathered}$$

$$a_1 + a_2 + a_k = 1 \;\&\; b_1 + b_2 + b_k = 1$$

$$\sum_i p_i + k = \sum_i q_i \;\&\; \left(\sum_i p_i + k\right)^2 = \left(\sum_i q_i\right)^2$$

$k||p_1 \rightarrow p_1 = a_1 q_1 \;\&\; k = (1-a_1)q_1$ other momenta unaffected

k soft $\rightarrow p_1 = q_1 \;\&\; p_2 = q_2 \;\&\; k = a_k q_1 + b_k q_2 + k_T$ other momenta unaffected

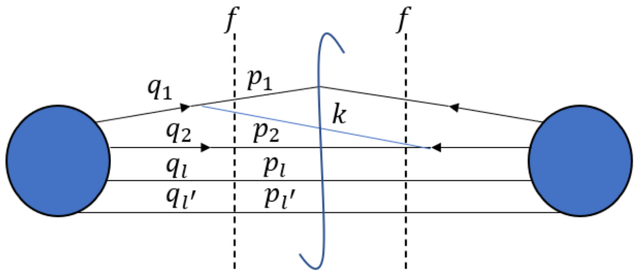
What about transverse recoil? 3

- Here we lack analytic insight, so instead we looked at the successes of other approaches.
- First choice: local or global?

$$
\begin{aligned}
p_1 &= a_1 q_1 + b_1 q_2 - (1 - g) k_T \\
p_2 &= a_2 q_1 + b_2 q_2 - g k_T \\
k &= a_k q_1 + b_k q_2 + k_T
\end{aligned}
$$

$$
\begin{aligned}
p_1 &= f \cdot (a_1 q_1 + b_1 q_2 - c(1 - g) k_T) \\
p_2 &= f \cdot (a_2 q_1 + b_2 q_2 - c g k_T) \\
k &= f \cdot (a_k q_1 + b_k q_2 + k_T) \\
p_{l'}^{\mu} &= f^{\mu}_{\nu,l'l} q_l^{\nu}
\end{aligned}
$$

g must be not be constant. Must partition the dipole (for $c \not\approx 0$).



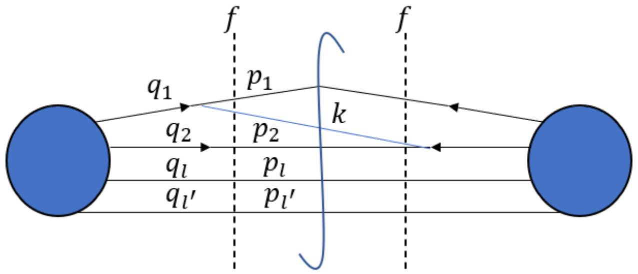
Dasgupta et al. 1805.09327

What about transverse recoil? 4

- Here we lack analytic insight, so instead we looked at the successes of other approaches.
- First choice: ~~local~~ or global?

$$\cancel{\begin{aligned} p_1 &= a_1 q_1 + b_1 q_2 - (1-g)k_T \\ p_2 &= a_2 q_1 + b_2 q_2 - g k_T \\ k &= a_k q_1 + b_k q_2 + k_T \end{aligned}}$$

$$\begin{aligned} p_1 &= f \cdot (a_1 q_1 + b_1 q_2 - c(1-g)k_T) \\ p_2 &= f \cdot (a_2 q_1 + b_2 q_2 - c g k_T) \\ k &= f \cdot (a_k q_1 + b_k q_2 + k_T) \\ p_{l'}^{\mu} &= f^{\mu}_{\nu, l' l} q_l^{\nu} \end{aligned}$$



$$c = 0$$

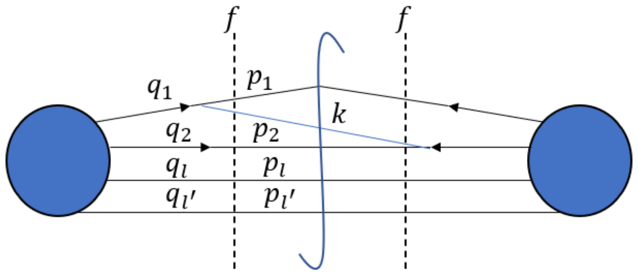
$$f^{\mu}_{\nu, l' l} = \kappa\, \Lambda^{\mu}_{\nu}\, \delta_{l' l} \text{ where } \Lambda^{\mu}_{\nu} \in SO(1,3)$$

What about transverse recoil? 5

- Here we lack analytic insight, so instead we looked at the successes of other approaches.
- First choice: ~~local~~ or global?

$$\xcancel{\begin{aligned} p_1 &= a_1 q_1 + b_1 q_2 - (1-g)k_T \\ p_2 &= a_2 q_1 + b_2 q_2 - g k_T \\ k &= a_k q_1 + b_k q_2 + k_T \end{aligned}}$$

$$\begin{aligned} p_1 &= \kappa\Lambda \cdot (a_1 q_1 + b_1 q_2) \\ p_2 &= \kappa\Lambda \cdot (a_2 q_1 + b_2 q_2) \\ k &= \kappa\Lambda \cdot (a_k q_1 + b_k q_2 + k_T) \\ p^\mu_{l'} &= \kappa\Lambda^\mu_\nu q^\nu_l \end{aligned}$$



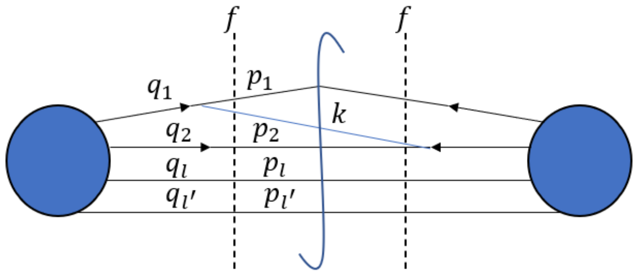
$$c = 0$$

$$f^\mu_{\nu,l'l} = \kappa\, \Lambda^\mu_\nu\, \delta_{l'l} \text{ where } \Lambda^\mu_\nu \in \mathrm{SO}(1,3)$$

What about transverse recoil? 6

- Here we lack analytic insight, so instead we looked at the successes of other approaches.
- First choice: ~~local~~ or global?

$$p_1 = \kappa\Lambda \cdot (a_1 q_1 + b_1 q_2)$$
$$p_2 = \kappa\Lambda \cdot (a_2 q_1 + b_2 q_2)$$
$$k = \kappa\Lambda \cdot (a_k q_1 + b_k q_2 + k_T)$$
$$p_{l'}^{\mu} = \kappa\Lambda^{\mu}_{\nu} q_l^{\nu}$$

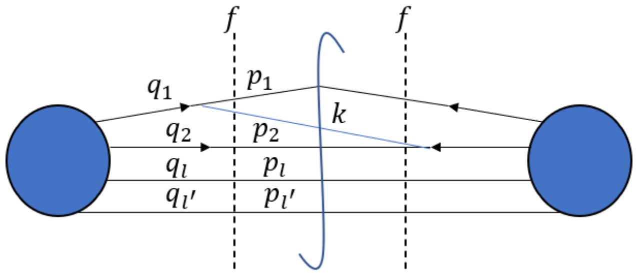


Bewick et al 1904.11866 found evidence that a global recoil scheme was sufficient to resum at NLL in an angular ordered shower.

What about transverse recoil? 7

- Here we lack analytic insight, so instead we looked at the successes of other approaches.
- First choice: ~~local~~ or global?

$$p_1 = \kappa\Lambda \cdot (a_1 q_1 + b_1 q_2)$$
$$p_2 = \kappa\Lambda \cdot (a_2 q_1 + b_2 q_2)$$
$$k = \kappa\Lambda \cdot (a_k q_1 + b_k q_2 + k_T)$$
$$p_{l'}^{\mu} = \kappa\Lambda^{\mu}_{\nu} q_l^{\nu}$$



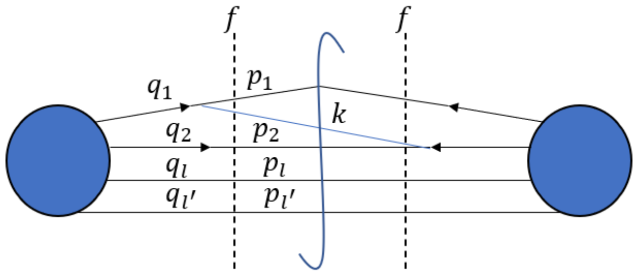
On the leg 1 side of the dipole we let $a_1 = z$, and $a_k = 1 - z$, and $a_2 = b_1 = b_2 = 0$.
b_k is determined by requiring $k^2 = 0$.
Vice versa on the leg 2 side of the dipole.

Bewick et al 1904.11866

What about transverse recoil? 8

- Here we lack analytic insight, so instead we looked at the successes of other approaches.
- First choice: ~~local~~ or global?

$$p_1 = \kappa\Lambda \cdot (a_1 q_1 + b_1 q_2)$$
$$p_2 = \kappa\Lambda \cdot (a_2 q_1 + b_2 q_2)$$
$$k = \kappa\Lambda \cdot (a_k q_1 + b_k q_2 + k_T)$$
$$p_{l'}^{\mu} = \kappa\Lambda^{\mu}_{\nu} q_l^{\nu}$$



However not unique!

Let $a_1 = 1 - a_k$, and $a_2 = 0$, and $b_2 = 1 - b_k$, and $b_1 = 0$.
This gives a shower with a momentum map identical to the Panglobal shower with $\beta = 0$.

Dasgupta et al. 2002.11114

What about transverse recoil? 9

A word on accuracy.

NLL accuracy in a shower is dependent on how transverse recoil is handled and on the partitioning. In our paper we perform various fixed order, and all orders, analytic crosschecks of NLL accuracy. However more certainly need doing, particularly numerical checks.

Concluding…

- We have managed to derive a coherent branching/angular ordered shower and a dipole shower from the PB algorithm.
- A requirement for consistency constrained some ambiguities in the PB algorithm: our initial goal.
- It has also instructed us on how to construct a dipole shower that inherits the successes of coherent branching.

- We are now implementing various generalisations of this dipole shower into HERWIG and are beginning broader pheno studies.

Concluding…

Work to come!

$$\mathfrak{R}^{\text{soft}}_{i_n j_n} = \frac{(q^{(i_n j_n)}_{n\perp})^2}{2E_n^2}\left(P_{i_n j_n}\mathfrak{R}_{i_n} + P_{j_n i_n}\mathfrak{R}_{j_n}\right) + h$$

where $\langle h\rangle_n \approx \text{NNL}$

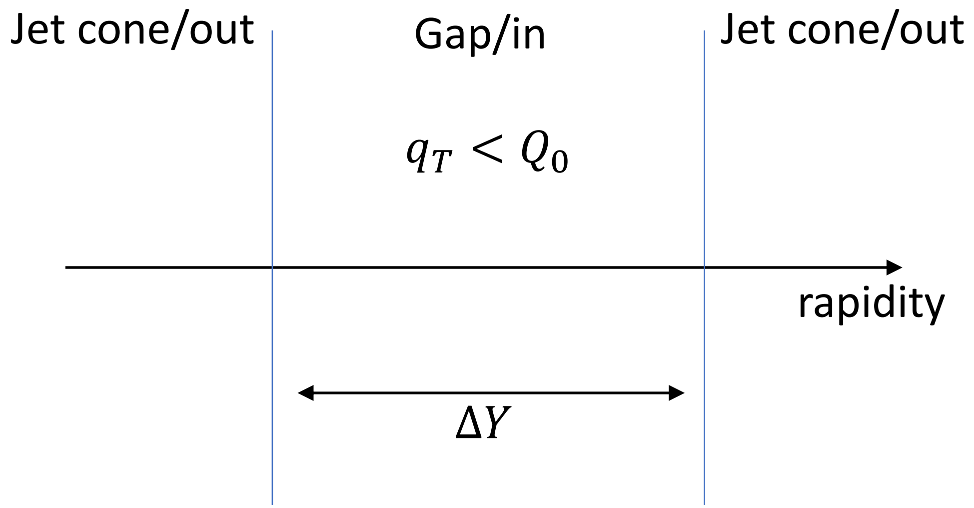
$$\mathfrak{R}_{j_n} = \delta^4(p_{j_n} - z_n^{-1}\tilde{p}_{j_n}) \prod_{i_n \neq j_n} \delta^4(p_{i_n} - \tilde{p}_{i_n}) + \mathcal{O}(q_\perp/Q)$$

*Correlation terms 1

$$\begin{aligned}\left\langle |M_n|^2\, u(\{p\}_n)\right\rangle_{1,\ldots,n} &= \left\langle |M_n|^2\right\rangle_{1,\ldots,n} \langle u(\{p\}_n)\rangle_{1,\ldots,n} \\ &+ \sum_{m=1}^{n} \sigma_m(\langle |M_n|^2\rangle_{1,\ldots,n})\, \sigma_m(\langle u(\{p\}_n)\rangle_{1,\ldots,n})\, \mathrm{Cor}_m(\langle |M_n|^2\rangle_{1,\ldots,n}\,, \langle u(\{p\}_n)\rangle_{1,\ldots,n}) \\ &+ \text{higher order correlations.} \qquad \text{(A.20)}\end{aligned}$$

*Correlation terms 2

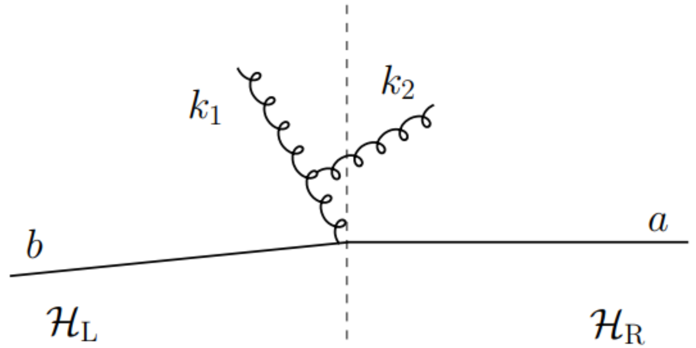
$$
\begin{aligned}
\left\langle |M_n|^2\, u(\{p\}_n)\right\rangle_{1,\ldots,n} &= \left\langle |M_n|^2\right\rangle_{1,\ldots,n} \left\langle u(\{p\}_n)\right\rangle_{1,\ldots,n} \\
&+ \sum_{m=1}^{n} \sigma_m(\langle |M_n|^2\rangle_{1,\ldots,n})\, \sigma_m(\langle u(\{p\}_n)\rangle_{1,\ldots,n})\, \mathrm{Cor}_m(\langle |M_n|^2\rangle_{1,\ldots,n}, \langle u(\{p\}_n)\rangle_{1,\ldots,n}) \\
&+ \text{higher order correlations.}
\end{aligned} \tag{A.20}
$$



$$
u_n(\{p\}_n) = \prod_{m=1}^{n} \left(\Theta_{\mathrm{out}}(q_m) + \Theta_{\mathrm{in}}(q_m)\Theta(Q_0 - q_{m,\perp})\right)
$$

$$
\begin{aligned}
\sigma_2(\langle u(\{p\}_2)\rangle_1) &= \sqrt{\langle u(\{p\}_2)\rangle_{1,2}\left(1 - \frac{\langle u(\{p\}_2)\rangle_{1,2}}{\langle u(\{p\}_1)\rangle_1}\right)}, \\
&= \left(\Theta_{\mathrm{out}}(q_1) + \Theta_{\mathrm{in}}(q_1)\Theta(Q_0 - q_{1,\perp})\right) \\
&\times \sqrt{\langle \Theta_{\mathrm{out}}(q_2) + \Theta_{\mathrm{in}}(q_2)\Theta(Q_0 - q_{2,\perp})\rangle_2 \left(1 - \langle \Theta_{\mathrm{out}}(q_2) + \Theta_{\mathrm{in}}(q_2)\Theta(Q_0 - q_{2,\perp})\rangle_2\right)} \neq 0
\end{aligned}
$$

$\propto \Theta_{\mathrm{in}}\Theta_{\mathrm{out}}$



M. Dasgupta, G. Salam arXiv:hep-ph/0104277
A. Banfi, G. Corcella, M. Dasgupta hep-ph/0612282