Higher order kernels in Dipole Showers: Dire

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Taming the accuracy of event generators

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Part I: Collinear limit

How to make parton showers more precise?

- Formulate parton-shower algorithm at NLO [Nagy,Soper] arXiv:1705.08093
  Naturally, NLO DGLAP evolution must be part of the full solution
- NLO DGLAP splitting kernels known since long
  [Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
  [Floratos,Kounnas,Lacaze] NPB192(1981)417
- So far not implemented in parton showers because
  - NLO-calculation 4-2ε dimensional, but parton showers 4D
  - Overlap with soft-gluon resummation must be treated at NLO
- Focus on purely collinear corrections for a start
  Flavor-changing case is simplest but requires all the technology:
  - Redefine time-like Sudakovs to recover NLO DGLAP evolution
    [Jadach,Skrzypek] hep-ph/0312355
  - Phase-space factorization and kinematics for $2 \to 4$ transitions
    [Prestel,SH] arXiv:1705.00742
  - Negative NLO corrections $\to$ weighted veto algorithm
    [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204

Time-like parton showers and the DGLAP equation

- DGLAP equation for fragmentation functions

$$\frac{\mathrm{d}x\, D_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g} \int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\, \frac{\alpha_s}{2\pi} \big[zP_{ab}(z)\big]_+ \,\tau D_b(\tau,t)\,\delta(x-\tau z)$$

- Define plus prescription $\big[zP_{ab}(z)\big]_+ = \lim_{\varepsilon\to 0} zP_{ab}(z,\varepsilon)$

$$P_{ab}(z,\varepsilon) = P_{ab}(z)\,\Theta(1-z-\varepsilon) - \delta_{ab} \sum_{c\in\{q,g\}} \frac{\Theta(z-1+\varepsilon)}{\varepsilon} \int_0^{1-\varepsilon} \mathrm{d}\zeta\,\zeta\, P_{ac}(\zeta)$$

- Rewrite for finite ε

$$\frac{\mathrm{d}\ln D_a(x,t)}{\mathrm{d}\ln t} = -\sum_{c=q,g} \int_0^{1-\varepsilon} \mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi} P_{ac}(\zeta) + \sum_{b=q,g} \int_x^{1-\varepsilon} \frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}$$

- First term is logarithmic derivative of Sudakov factor

$$\Delta_a(t_0,t) = \exp\left\{ -\int_{t_0}^{t} \frac{\mathrm{d}\bar{t}}{\bar{t}} \sum_{c=q,g} \int_0^{1-\varepsilon} \mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi} P_{ac}(\zeta) \right\}$$

Time-like parton showers and the DGLAP equation

- Use generating function $\mathcal{D}_a(x,t,\mu^2) = D_a(x,t)\Delta_a(t,\mu^2)$ to write

$$\frac{\mathrm{d}\ln\mathcal{D}_a(x,t,\mu^2)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}\;.$$

- A similar probability density is used to generate initial-state emissions
But final-state showers are typically unconstrained (hadrons not identified)
In this case the probability density is modified to

$$\frac{\mathrm{d}}{\mathrm{d}\ln t}\ln\left(\frac{\mathcal{D}_a(x,t,\mu^2)}{D_a(x,t)}\right) = \sum_{b=q,g}\int_0^{1-\varepsilon}\mathrm{d}z\,z\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\;.$$

- **Net result**: Unitarity implies that forward-branching Sudakovs must include a 'symmetry factor' z [Jadach,Skrzypek] hep-ph/0312355
- Convenient interpretation as "tagging" of evolving parton
- Equivalent to standard technique at LO due to symmetry of $P_{ab}(z)$
More care is needed at NLO [Prestel,SH] arXiv:1705.00742

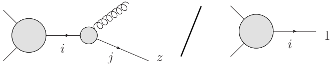
Collinear parton evolution at NLO

[Curci,Furmanski,Petronzio] NPB175(1980)27, [Floratos,Kounnas,Lacaze] NPB192(1981)417

- Higher-order DGLAP evolution kernels obtained from factorization

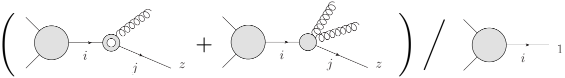
$$D^{(0)}_{ji}(z,\mu) = \delta_{ij}\delta(1-z) \qquad \leftrightarrow$$

$$D^{(1)}_{ji}(z,\mu) = -\frac{1}{\varepsilon}P^{(0)}_{ji}(z) \qquad \leftrightarrow$$



$$D^{(2)}_{ji}(z,\mu) = -\frac{1}{2\varepsilon}P^{(1)}_{ji}(z) + \frac{\beta_0}{4\varepsilon^2}P^{(0)}_{ji}(z) + \frac{1}{2\varepsilon^2}\int_z^1 \frac{\mathrm{d}x}{x} P^{(0)}_{jk}(x)P^{(0)}_{ki}(z/x)$$

$\leftrightarrow$

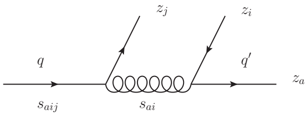


- $P^{(n)}_{ji}$ not probabilities, but sum rules hold ($\leftrightarrow$ unitarity constraint)
  In particular: Momentum sum rule identical between LO & NLO
- **Goal**: Perform the NLO computation of $P^{(1)}_{ji}$ fully differentially
  using modified dipole subtraction [Catani,Seymour] hep-ph/9605323

Collinear parton evolution at NLO

[Prestel,SH] arXiv:1705.00742

- Simulation of exclusive states requires computing splitting functions on the fly using differential NLO calculation & collinear factorization
- Schematically very similar to Catani-Seymour dipole subtraction
- Simplest example: Flavor-changing configuration $q \to q'$



Tree-level expression[1] $\leftrightarrow$ real-emission correction in CS

$$P_{qq'} = \frac{1}{2} C_F T_R \frac{s_{aij}}{s_{ai}} \left[-\frac{t_{ai,j}^2}{s_{ai} s_{aij}} + \frac{4\, z_j + (z_a - z_i)^2}{z_a + z_i} + (1 - 2\varepsilon) \left(z_a + z_i - \frac{s_{ai}}{s_{aij}} \right) \right]$$

Subtraction term (q→g)⊗(g→q') $\leftrightarrow$ differential subtraction term in CS

$$\tilde{P}_{qq'} = C_F T_R \frac{s_{aij}}{s_{ai}} \left(\frac{1 + \tilde{z}_j^2}{1 - \tilde{z}_j} - \varepsilon (1 - \tilde{z}_j) \right) \left(1 - \frac{2}{1 - \varepsilon} \frac{\tilde{z}_a \tilde{z}_i}{(\tilde{z}_a + \tilde{z}_i)^2} \right) + \ldots$$

[1] $(z_a + z_i)\, t_{ai,j} = 2(z_a s_{ij} - z_i s_{aj}) + (z_a - z_i) s_{ai}$

Collinear parton evolution at NLO

[Prestel,SH] arXiv:1705.00742

- Complete NLO result schematically given by

$$P^{(1)}_{qq'}(z) = \mathrm{C}_{qq'}(z) + \mathrm{I}_{qq'}(z) + \int \mathrm{d}\Phi_{+1}\Big[\mathrm{R}_{qq'}(z,\Phi_{+1}) - \mathrm{S}_{qq'}(z,\Phi_{+1})\Big]$$

- Real correction $R_{qq'}$ and subtraction terms $S_{qq'}$ $\nearrow$ previous slide
  Difference finite in 4 dimensions $\to$ amenable to MC simulation
- Integrated subtraction term and factorization counterterm given by

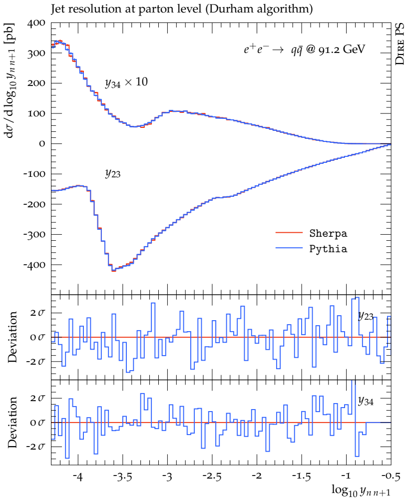
$$\mathrm{I}_{qq'}(z) = \int \mathrm{d}\Phi_{+1} S_{qq'}(z,\Phi_{+1})$$

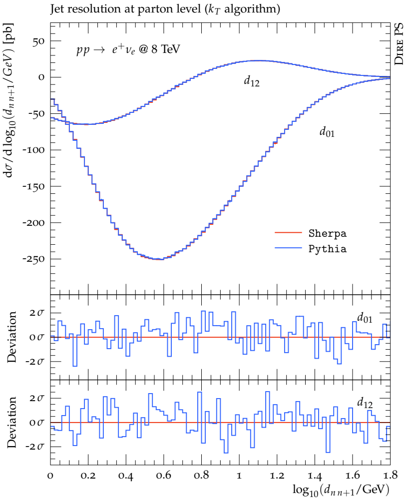
$$\mathrm{C}_{qq'}(z) = \int_z \frac{\mathrm{d}x}{x}\left(P^{(0)}_{qg}(x) + \varepsilon\mathcal{J}^{(1)}_{qg}(x)\right)\frac{1}{\varepsilon}P^{(0)}_{gq}(z/x)$$

$$\mathcal{J}^{(1)}_{qg}(z) = 2C_F\left(\frac{1+(1-x)^2}{x}\ln(x(1-x)) + x\right)$$

- Analytical computation of I not needed, as $\mathrm{I} + \mathcal{P}/\varepsilon$ finite
  generate as endpoint at $s_{ai} = 0$, starting from integrand at $\mathcal{O}(\varepsilon)$
- All components of $P^{(1)}_{qq'}$ eventually finite in 4 dimensions
  Can be simulated fully differentially in parton shower

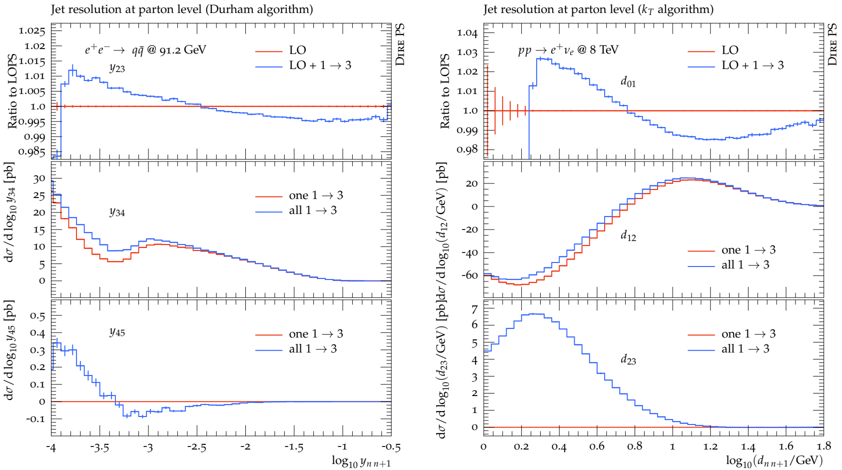
Validation





- Effect of single $1 \to 3$ emission on leading and next-to-leading jet rate

Impact relative to leading-order prediction



- Effect of $1 \to 3$ emissions on leading jet rate
- Impact of multiple $1 \to 3$ emissions

Part II: Soft limit

Soft evolution at the next-to-leading order

[Marchesini,Korchemsky] PLB313(1993)433, hep-ph/9210281

- Soft-gluon resummed expression of Drell-Yan or DIS cross section

$$\frac{1}{\sigma}\frac{d\sigma(z,Q^2)}{d\log Q^2} = \mathcal{H}(Q^2)\,\widetilde{W}(z,Q^2)$$

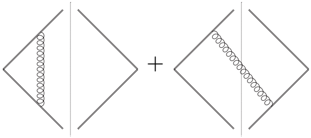
RGE governed by Wilson loop $\widetilde{W}$ ($Q(1-z)$ - total soft gluon energy)

- Non-abelian exponentiation theorem allows to expand as

$$\widetilde{W} = \exp\left\{\sum_{i=1}^{\infty} w^{(\mathrm{n})}\right\}$$

- One-loop result given by

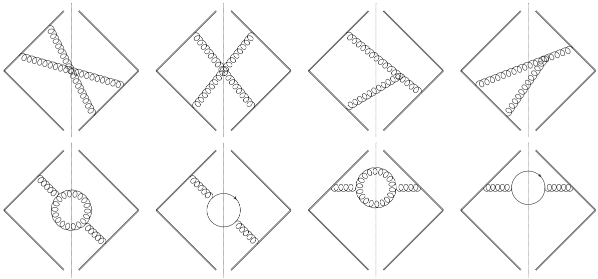
$$w^{(1)} = C_F\,\frac{\alpha_s(\mu)}{2\pi}\left[\ln^2 L + \frac{\pi^2}{6}\right] \qquad \leftrightarrow$$



where $L = -b_+ b_- / b_0^2$ and $b_0 = 2\,e^{-\gamma_E}/\mu$

Soft evolution at the next-to-leading order

- 2-loop contribution $w^{(2)}$ computed from (reals only) [Belitsky] hep-ph/9808389



- Renormalized result in position space

$$w^{(2)} = C_F \, \frac{\alpha_s^2(\mu)}{(2\pi)^2} \left[-\frac{\beta_0}{6} \ln^3 L + \Gamma^{(2)}_{\rm cusp} \ln^2 L + 2 \ln L \left(\Gamma^{(2)}_{\rm soft} + \frac{\pi^2}{12} \beta_0 \right) + \ldots \right]$$

$$\Gamma^{(2)}_{\rm cusp} = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{10}{9} T_R \, n_f \, , \qquad \beta_0 = \frac{11}{6} C_A - \frac{2}{3} T_R n_f$$

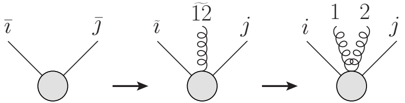
$$\Gamma^{(2)}_{\rm soft} = \left(\frac{101}{27} - \frac{11}{72} \pi^2 - \frac{7}{2} \zeta_3 \right) C_A - \left(\frac{28}{27} - \frac{\pi^2}{18} \right) T_R \, n_f$$

- This is the benchmark to be reproduced by exclusive MC simulation

Separation of soft and collinear sectors

[Dulat,Prestel,SH] arXiv:1805.03757

- Phase space parametrized in terms of total soft momentum $q = p_1 + p_2$



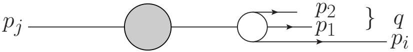
- Momentum space result expanded in Laurent series using

$$\frac{1}{q_\pm^{1+\epsilon}} = -\frac{1}{\epsilon}\,\delta(q_\pm) + \sum_{i=0}^{\infty} \frac{\epsilon^n}{n!} \left(\frac{\ln^n q_\pm}{q_\pm} \right)_+$$

- Unitarity implies that factorized plus distributions like $[1/q_+]_+[1/q_-]_+$ have no PS analogue $\rightarrow$ define double-plus distributions instead

$$\big[f(q_+, q_-)\big]_{++}\, g(q_+, q_-) = f(q_+, q_-)\Big(g(q_+, q_-) - g(0,0)\Big)$$

- Re-organize entire calculation in terms of pure soft & collinear terms
  Key observation: $q_\pm = 0$ implies collinear limit for 1 & 2 emissions



Soft evolution at the next-to-leading order

[Catani,Grazzini] hep-ph/9908523

- Real-emission corrections can be written in convenient form

$$\mathcal{S}_{ij}^{(q\bar{q})}(1,2) = -\frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\frac{T_R}{s_{12}}\Big(1-4\,z_1 z_2\cos^2\phi_{12,ij}\Big)$$

$$\mathcal{S}_{ij}^{(gg)}(1,2) = \mathcal{S}_{ij}^{(\mathrm{s.o.})}(1,2)\,\frac{C_A}{2}\left(1+\frac{s_{i1}s_{j1}+s_{i2}s_{j2}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\right) + \frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\frac{C_A}{s_{12}}\Big(-2+4\,(1-\epsilon)\,z_1 z_2\cos^2\phi_{12,ij}\Big)$$

- Strongly ordered and spin correlation components

$$\mathcal{S}_{ij}^{(\mathrm{s.o.})}(1,2) = \frac{s_{ij}}{s_{i1}s_{12}s_{j2}} + \frac{s_{ij}}{s_{j1}s_{12}s_{i2}} - \frac{s_{ij}^2}{s_{i1}s_{j1}s_{i2}s_{j2}}$$

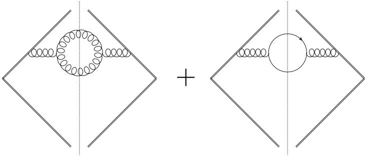
$$4\,z_1 z_2\cos^2\phi_{12,ij} = \frac{(s_{i1}s_{j2}-s_{i2}s_{j1})^2}{s_{12}s_{ij}(s_{i1}+s_{i2})(s_{j1}+s_{j2})}$$

- Apparently simple structure, but unlike collinear NLO results not reflected by iterated leading-order splitting kernels
  $\to$ not all denominators can be composed from LO expressions

NLO subtraction: Dipole approach

[Dulat,Prestel,SH] arXiv:1805.03757

- Nearly ok subtraction obtained from spin correlated parton shower



$$= \sum_{b=q,g} J_{ij,\mu}(p_{12}) J_{ij,\nu}(p_{12}) \, \frac{P_{gb}^{\mu\nu}(z_1)}{s_{12}}$$

- Building blocks are eikonal currents

$$J_{ij}^{\mu}(q) = \frac{p_i^{\mu}}{2p_i q} - \frac{p_j^{\mu}}{2p_j q}$$

and collinear splitting functions

$$P_{gq}^{\mu\nu}(z) = T_R \left(-g^{\mu\nu} + 4\, z(1-z)\, \frac{k_\perp^{\mu} k_\perp^{\nu}}{k_\perp^2} \right)$$

$$P_{gg}^{\mu\nu}(z) = C_A \left(-g^{\mu\nu} \left(\frac{z}{1-z} + \frac{1-z}{z} \right) - 2\,(1-\varepsilon) z(1-z)\, \frac{k_\perp^{\mu} k_\perp^{\nu}}{k_\perp^2} \right)$$

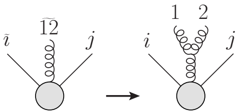
- Finite remainder has integrable singularities $\rightarrow$ not suitable for MC
problem arises from interference of abelian & non-abelian diagrams

NLO subtraction: Antenna approach – Kinematics

[Dulat,Prestel,SH] arXiv:1805.03757

- In iterated emission $\bar{\imath}\,\bar{\jmath} \to \widetilde{\imath}\,\widetilde{12}\,j \to ij12$ emission probability of first step written in terms of momenta after second step is

$$\frac{\tilde{p}_i p_j}{2(\tilde{p}_i\tilde{p}_{12})(\tilde{p}_{12}p_j)} = \frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2}) - s_{ij}s_{12}}$$



- Not identical to desired "eikonal" $s_{ij}/((s_{i1}+s_{i2})(s_{j1}+s_{j2}))$ in soft$\otimes$collinear terms of $\mathcal{S}_{ij}$ but easily corrected by weight

$$w_{ij}^{12} = 1 - \frac{s_{ij}s_{12}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})} = \left(\frac{p_{\perp,12}^{(ij)}}{m_{\perp,12}^{(ij)}}\right)^2$$

- Iterated eikonals of type $s_{ij}/(s_{i1}s_{j1})$, $s_{j1}/(s_{12}s_{j2})$ in $\mathcal{S}_{ij}^{(\mathrm{s.o.})}$ reconstructed by partial fractioning & matching to LO^2 $\to$ additional weight

$$\bar{w}_{ij}^{12} = \frac{(s_{i1}+s_{i2})(s_{j1}+s_{j2}) - s_{ij}s_{12}}{s_{i1}s_{j1}+s_{i2}s_{j2}} = \frac{\left(p_{\perp,12}^{(ij)}\right)^2}{\left(p_{\perp,1}^{(ij)}\right)^2 + \left(p_{\perp,2}^{(ij)}\right)^2}$$

- These weights lie between zero and one and reduce emission rates

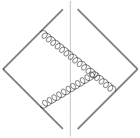
Leading color fully differential soft evolution at NLO

[Dulat,Prestel,SH] arXiv:1805.03757

- Squared LO eikonal and negative term in $\mathcal{S}_{ij}^{(\mathrm{s.o.})}$ both have no parton-shower analogue $\rightarrow$ correct for both mismatches by adding sub-leading color contribution to $i1$-collinear splitting functions

$$P_{ij,A}^{(\mathrm{slc})}(1,2) = \frac{2\,s_{ij}}{s_{i1}+s_{j1}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2}\left(\bar{C}_{ij}-C_A\right)\,,\quad \bar{C}_{ij} = \begin{cases} 2C_F & \text{if } i \;\&\; j \text{ quarks} \\ C_A & \text{else} \end{cases}$$

- Second soft emission off Wilson lines occurs with color charge factor C_A due to interference with octet



$$P_{ij,B}^{(\mathrm{slc})}(1,2) = \frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2}\left(C_A-\bar{C}_{ij}\right)$$

- Combined effect on $i1$-collinear matched splitting function

$$P_{ij}^{(\mathrm{slc})}(1,2) = \left(C_A-\bar{C}_{ij}\right)\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}-\frac{2\,s_{ij}}{s_{i1}+s_{j1}}\right)\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2}$$

- Non-singular in $i1$-collinear limit $\rightarrow$ color charges of Wilson lines in soft-collinear limit are C_i and C_j, in agreement with DGLAP

Leading color fully differential soft evolution at NLO

[Dulat,Prestel,SH] arXiv:1805.03757

- Complete NLO-weighted LO splitting functions

$$(P_{qq})_i^k(1,2) = C_F\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ik}^{12}+\bar{w}_{ik}^{12}}{2}\right) + P_{ik}^{(\mathrm{slc})}(1,2)$$

$$(P_{gg})_{ij}(1,2) = C_A\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2} + w_{ij}^{12}\left(-1+z(1-z)\,2\cos^2\phi_{12}^{ij}\right)\right)$$

$$(P_{gq})_{ij}(1,2) = T_R\,w_{ij}^{12}\left(1-4z(1-z)\cos^2\phi_{12}^{ij}\right)$$

- Calculation completed by subtracted real correction, virtuals and factorization counterterms
- Counterterms are endpoint contributions, as in collinear limit

$$\tilde{\mathcal{S}}_{gq}^{(\mathrm{cusp})} = \delta(s_{12})\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\,T_R\Big[2z(1-z)+\big(1-2z(1-z)\big)\ln(z(1-z))\Big]$$

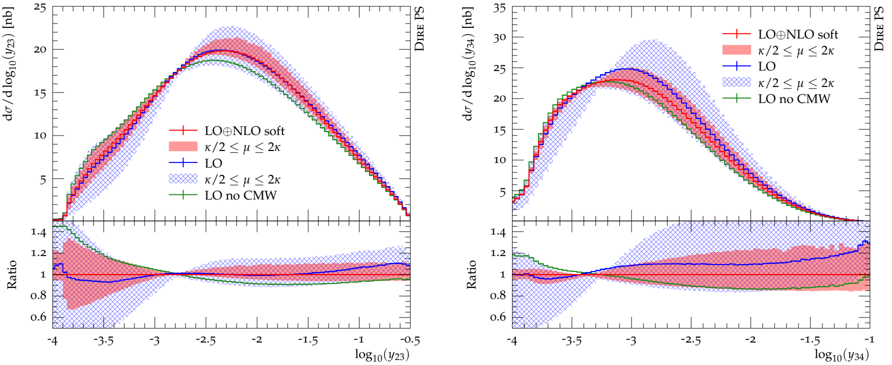
$$\tilde{\mathcal{S}}_{gg}^{(\mathrm{cusp})} = \delta(s_{12})\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\,2C_A\left[\frac{\ln z}{1-z}+\frac{\ln(1-z)}{z}+\big(-2+z(1-z)\big)\ln(z(1-z))\right]$$

$$\tilde{\mathcal{S}}_{wl}^{(\mathrm{cusp})} = -\,\delta(s_{i1})\,\frac{1}{2}\,\frac{C_A}{2}\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\left(\frac{\ln z_i}{1-z_i}+\frac{\ln(1-z_i)}{z_i}\right)+\big(\text{swaps}\big)$$

Sum integrates to CMW correction [Catani,Marchesini,Webber] NPB349(1991)635

Leading color fully differential soft evolution at NLO

[Dulat,Prestel,SH] arXiv:1805.03757



- Impact on $2 \to 3$ and $3 \to 4$ Durham jet rate at LEP I
- Uncertainty bands no longer just estimates
  but perturbative QCD predictions for the first time
- Fair agreement with CMW scheme

Thank you for your attention