The MiNNLO$_{PS}$ method

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LAPTh Annecy

Taming the accuracy of event generators

CERN, 1 July 2020

* in collaboration with P. Monni, P. Nason, M. Wiesemann, G. Zanderighi

Plan of the talk

1. Brief recap on MiNLO and NNLO+PS with reweighting

   Hamilton,Nason,Zanderighi+Oleari: 1212.4504,1206.3572
   Astill,Bizon,Hamilton,Karlberg,Nason,ER,Wiesemann,Zanderighi: ’12-’18

2. The MiNNLOPS method

   Monni,Nason,ER,Wiesemann,Zanderighi: 1908.06987
   Monni,ER,Wiesemann: 2006.04133

What we want to achieve

- NNLO accuracy for observables inclusive on radiation.
- NLO(LO) accuracy for $F + 1(2)$ jet observables (in the hard region).
  - appropriate scale choice for each kinematics regime
- no merging scale required.
- when combined with p_T-ordered parton showers, leading-log accuracy of the parton shower preserved.

Multiscale Improved NLO: a-priori choose scales in multijet NLO computation

[Hamilton,Nason,Zanderighi '12]

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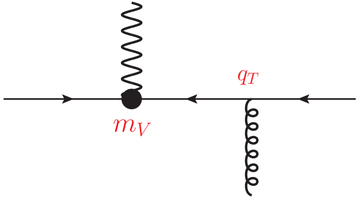

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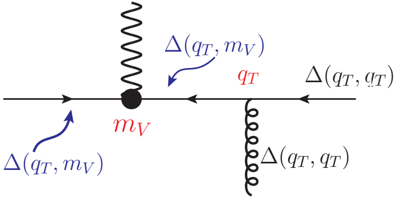
$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}} = \frac{\alpha_{\mathrm{S}}(q_{\mathrm{T}})}{2\pi}\left[\Delta^2_{\mathrm{f}}(q_{\mathrm{T}})\left[B^{(\mathrm{FJ})}\left(1 + \frac{\alpha_{\mathrm{S}}}{2\pi}\tilde{S}^{(1)}_{\mathrm{f}}(q_{\mathrm{T}})\right) + \frac{\alpha_{\mathrm{S}}}{2\pi}V^{(\mathrm{FJ})}(\bar{\mu}_R)\right] + \frac{\alpha_{\mathrm{S}}}{2\pi}\int d\Phi_{\mathrm{r}}\Delta^2_{\mathrm{f}}(q_{\mathrm{T}})R^{(\mathrm{FJ})}\right]$$



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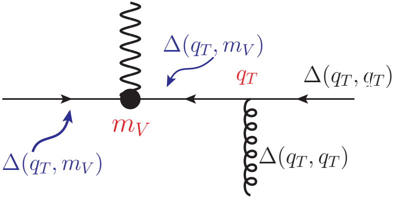

- $\bar{\mu}_R = q_{\mathrm{T}}$
- $\Delta^2_{\mathrm{f}}(q_{\mathrm{T}}) = \exp(-\tilde{S}_{\mathrm{f}}(q_{\mathrm{T}}))$
- $\tilde{S}_{\mathrm{f}}(q_{\mathrm{T}}) = \int_{q_{\mathrm{T}}^2}^{m_{\mathrm{F}}^2}\frac{dq^2}{q^2}\left[A_{\mathrm{f}}(\alpha_{\mathrm{S}}(q^2))\log\frac{m_{\mathrm{F}}^2}{q^2} + B_{\mathrm{f}}(\alpha_{\mathrm{S}}(q^2))\right]$
- $\frac{\alpha_{\mathrm{S}}}{2\pi}\tilde{S}^{(1)}_{\mathrm{f}}(q_{\mathrm{T}}) = \frac{\alpha_{\mathrm{S}}}{2\pi}\left[\frac{1}{2}A_{1,\mathrm{f}}\log^2\frac{m_{\mathrm{F}}^2}{q_{\mathrm{T}}^2} + B_{1,\mathrm{f}}\log\frac{m_{\mathrm{F}}^2}{q_{\mathrm{T}}^2}\right]$
- $\mu_F = q_{\mathrm{T}}$

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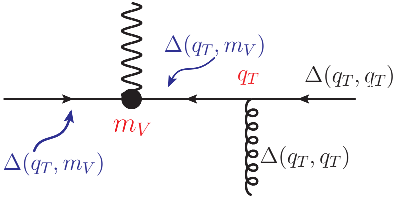

☞ Sudakov FF included on $F+j$ Born kinematics

- ▶ MiNLO-improved FJ yields finite results also when 1st jet is unresolved ($q_{\mathrm{T}} \to 0$)
- ▶ $\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}}$ allows to extend the validity of FJ−POWHEG [called "FJ−MiNLO" hereafter]

MiNLO′ → NNLO+PS

- formal accuracy of FJ-MiNLO for inclusive observables carefully investigated.

  [Hamilton et al. 1212.4504]

- possible to improve FJ-MiNLO such that inclusive NLO is recovered (NLO$^{(\mathrm{F})}$), without spoiling NLO accuracy of F+j (NLO$^{(\mathrm{FJ})}$):

  MiNLO′ : NLO+PS merging of F and F+j, without merging scale

- accurate control of subleading small-p_T logarithms is needed:
  - include B_2 (NNLL) coefficient in MiNLO-Sudakov.
  - set scales in R, V and subtraction terms equal to q_T.
  - without the above requirements, spurious $\alpha_\mathrm{S}^{3/2}$ terms show up in $\sigma_{\mathrm{NLO}}^{(\mathrm{F})}$ after integration over q_T.

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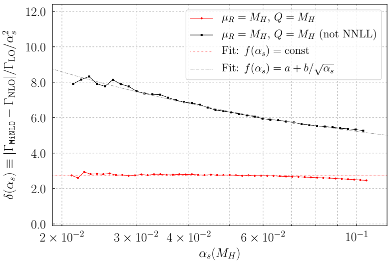

- $H \to b\bar{b}$ with MiNLO′ [Bizon,ER,Zanderighi, '19]
  - NNLL resummation of 3-jet resolution parameter (CA) from 1607.03111 ($\delta\mathcal{F}_{\mathrm{clust.}}$ and $\delta\mathcal{F}_{\mathrm{correl.}}$)
- possible to explicitly verify that an error in B_2 leads to spurious $\alpha_{\mathrm{S}}^{3/2}$ terms

$$\delta(\alpha_{\mathrm{S}}) = \frac{1}{\alpha_{\mathrm{S}}^2} \frac{|\Gamma_{\mathtt{MiNLO}} - \Gamma_{\mathrm{NLO}}|}{\Gamma_{\mathrm{LO}}}$$

- plot: error = no $\delta\mathcal{F}_{\mathrm{clust.}}$ and $\delta\mathcal{F}_{\mathrm{correl.}}$

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	F (inclusive)	$F+j$ (inclusive)	$F+2j$ (inclusive)
✓ F-FJ @ NLOPS	NLO	NLO	LO
F @ NNLOPS	NNLO	NLO	LO

- for color-singlet production F, the above procedure is general, and (almost) process independent.
  - a generalization of the MiNLO′ approach for processes with jets at LO has also been proposed.

  [Frederix,Hamilton '15, see also Carrazza et al. '18]

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- reweighting (differential on Φ_{F}) of “MiNLO-generated” events allows one to achieve NNLO+PS accuracy

MiNNLO$_{\text{PS}}$

- Albeit formally correct, reweighting is a bottleneck for complex processes
  - approximations needed
  - discrete binning $\rightarrow$ delicate in less populated regions
  - it remains very CPU intensive
  - for complicated processes, it's not user friendly
- In 1908.06987, we developed a new method: NNLOPS accuracy without reweighting
- Through a precise connection of the MiNLO′ method and $p_{\rm T}$ resummation, possible to isolate the missing ingredients and reach NNLO accuracy
- In the follow-up paper 2006.04133 we refined some implementational aspects

[Notation: From this point, $X = \sum_k \left(\frac{\alpha_{\rm S}}{2\pi}\right)^k [X]^{(k)}$]

MiNNLO$_{\text{PS}}$

- from p_T resumption, differential cross section for F+X production can be written as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_\mathrm{T}\mathrm{d}\Phi_\mathrm{F}} = \frac{\mathrm{d}}{\mathrm{d}p_\mathrm{T}}\left\{\mathcal{L}(\Phi_\mathrm{F}, p_\mathrm{T})\exp(-\tilde{S}(p_\mathrm{T}))\right\} + R_f(p_\mathrm{T})$$

  - $R_f(p_\mathrm{T}) = \frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_\mathrm{F}\mathrm{d}p_\mathrm{T}} - \frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\Phi_\mathrm{F}\mathrm{d}p_\mathrm{T}}$
  - $\mathcal{L}(\Phi_\mathrm{F}, p_\mathrm{T})$: keep all the terms needed to obtain NNLO$^{(\mathrm{F})}$ accuracy: $H^{(1)}$, $H^{(2)}$, $C^{(1)}$, $C^{(2)}$ (and $[G^{(1)}\otimes f][G^{(1)}\otimes f]$ for $gg \to H$)
  - $\mu_{R/F} = p_\mathrm{T}$ in $\mathcal{L}(\Phi_\mathrm{F}, p_\mathrm{T})$ and $R_f(p_\mathrm{T})$
  - hard virtual corrections are evaluated at $\mu_R = p_\mathrm{T}$, while their scale should be $\mu_R \simeq m_\mathrm{F} \Rightarrow$ in $\tilde{S}(q_\mathrm{T})$, B_2 contains $H^{(1)} \equiv [V^{(\mathrm{F})}/B^{(\mathrm{F})}]$

- from p_T resummation, differential cross section for F+X production can be written as:

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} = \exp[-\tilde{S}(p_{\mathrm{T}})]\left\{D(p_{\mathrm{T}}) + \frac{R_f(p_{\mathrm{T}})}{\exp[-\tilde{S}(p_{\mathrm{T}})]}\right\}$$

$$D(p_{\mathrm{T}}) \equiv -\frac{\mathrm{d}\tilde{S}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}}\mathcal{L}(p_{\mathrm{T}}) + \frac{\mathrm{d}\mathcal{L}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}} \qquad \tilde{S}(p_{\mathrm{T}}) = \int_{p_{\mathrm{T}}}^{Q} \frac{dq^2}{q^2}\left[A_{\mathrm{f}}(\alpha_{\mathrm{S}}(q)) \log\frac{Q^2}{q^2} + B_{\mathrm{f}}(\alpha_{\mathrm{S}}(q))\right]$$

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- expand the above integrand in power of $\alpha_{\mathrm{S}}(p_{\mathrm{T}})$, keep the terms that are needed to get NLO$^{(\mathrm{F})}$ and, then, NNLO$^{(\mathrm{F})}$ accuracy, when integrating over p_{T}

MiNNLO$_{\text{PS}}$

- from p_T resummation, differential cross section for F+X production can be written as:

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- expand the above integrand in power of $\alpha_\mathrm{S}(p_\mathrm{T})$, keep the terms that are needed to get NLO$^{(\mathrm{F})}$ and, then, NNLO$^{(\mathrm{F})}$ accuracy, when integrating over p_T
- after expansion, all the terms with explicit logs will be of the type $\alpha_\mathrm{S}^m(p_\mathrm{T})L^n$, with $n = 0, 1$.

$$\int^{Q}\frac{dp_\mathrm{T}}{p_\mathrm{T}}L^n\alpha_\mathrm{S}^m(p_\mathrm{T})\exp(-\tilde{S}(p_\mathrm{T})) \sim (\alpha_\mathrm{S}(Q))^{m-(n+1)/2} \qquad L = \log Q/p_\mathrm{T}$$

MiNNLO$_{\text{PS}}$

$$\frac{d\sigma}{d\Phi_F dp_T} = \exp[-\tilde{S}(p_T)]\left\{ \frac{\alpha_S(p_T)}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(1)}\left(1+\frac{\alpha_S(p_T)}{2\pi}[\tilde{S}(p_T)]^{(1)}\right) + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(2)} + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^3[D(p_T)]^{(3)} + \text{regular terms}\right\}$$

- as expected, for NLO$^{(F)}$ accuracy, we recovered `MiNLO'`, exactly
- $[D(p_T)]^{(3)}$ is the $\alpha_S^3(p_T)$ expansion of $D(p_T) = -\frac{d\tilde{S}(p_T)}{dp_T}\mathcal{L}(p_T) + \frac{d\mathcal{L}(p_T)}{dp_T}$
  - higher-order terms $\mathcal{O}(\alpha_S^4(p_T))$ will produce terms beyond accuracy, after integration on p_T
- "regular terms": $[R_f(p_T)/\exp[-\tilde{S}(p_T)]]^{(3)}$.
  - no $1/p_T$ factor, hence after integration they are of order $\mathcal{O}(\alpha_S^3)$.

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  - no $1/p_T$ factor, hence after integration they are of order $\mathcal{O}(\alpha_S^3)$.
- $[D(p_T)]^{(3)}$ contains many terms: symbolically rather compact, but numerically delicate
  - we used `HOPPET` and `hplog`
  - $\mu_{R/F} = K_{R/F}\, p_T$, and we must integrate down to $p_T \to 0$
  - out-of-the-box PDFs evolution from LHAPDF: cutoff $\Lambda_{PDF} \sim 1$ GeV

MiNNLO$_{\mathrm{PS}}$: implementation I

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} = \exp[-\tilde{S}(p_{\mathrm{T}})]\left\{ \frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}[\tilde{S}(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}}\right]^{(2)} + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)} \right\}$$

- $[D(p_{\mathrm{T}})]^{(3)}$: extracted from $p_{\mathrm{T}} \to 0$ limit, depends on $(\Phi_{\mathrm{F}}, p_{\mathrm{T}})$, not on Φ_{FJ}
  - to reach NNLO accuracy, singular region must be treated exactly
- in practice, we need to integrate over $\Phi_{\mathrm{FJ}} \Rightarrow$ mapping to evaluate $[D(p_{\mathrm{T}})]^{(3)}$:
  a) $\Phi_{\mathrm{FJ}} \to \Phi_{\mathrm{F}}$ smoothly when $p_{\mathrm{T}} \to 0$ [FKS ISR mapping (preserves rapidity of F)]
  b) recover the above equation, when integrating over Φ_{FJ} at fixed $(\Phi_{\mathrm{F}}, p_{\mathrm{T}})$

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$$\frac{d\sigma}{d\Phi_F dp_T} = \exp[-\tilde{S}(p_T)]\left\{\frac{\alpha_S(p_T)}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(1)}\left(1+\frac{\alpha_S(p_T)}{2\pi}[\tilde{S}(p_T)]^{(1)}\right) + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(2)} + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^3 [D(p_T)]^{(3)} F_\ell^{corr}(\Phi_{FJ})\right\}$$

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  b) recover the above equation, when integrating over Φ_{FJ} at fixed (Φ_F, p_T)

$$F^{corr}(\Phi_{FJ}) = \frac{J(\Phi_{FJ})}{\int d\Phi'_{FJ} J(\Phi'_{FJ})\delta(p_T - p'_T)\delta(\Phi_F - \Phi'_F)}$$

$$\int d\Phi'_{FJ} G(\Phi'_F, p'_T) F^{corr}(\Phi'_{FJ}) = \int d\Phi_F\, dp_T G(\Phi_F, p_T)$$

- to avoid spurious effects at large y_j: use rapidity of radiation
  . full matrix element: $J(\Phi_{FJ}) = |M^{FJ}(\Phi_{FJ})|^2 (f^{[a]} f^{[b]})$
  . compromise: $J(\Phi_{FJ}) = P(\Phi_{rad})(f^{[a]} f^{[b]})$

MiNNLO$_{\text{PS}}$: implementation II

Final master formula:

$$\frac{\mathrm{d}\bar{B}(\Phi_{\mathrm{FJ}})}{\mathrm{d}\Phi_{\mathrm{FJ}}} = \exp[-\tilde{S}(p_{\mathrm{T}})]\left\{\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(1)}\left(1+\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}[\tilde{S}(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(2)} + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)}F^{\mathrm{corr}}_{\ell}(\Phi_{\mathrm{FJ}})\right\}$$

. The second radiation is generated by the usual POWHEG mechanism.

$$\mathrm{d}\sigma = \bar{B}(\Phi_{\mathrm{FJ}})\,\mathrm{d}\Phi_{\mathrm{FJ}}\left\{\Delta_{\mathrm{pwg}}(\Lambda_{\mathrm{pwg}}) + \mathrm{d}\Phi_{\mathrm{rad}}\Delta_{\mathrm{pwg}}(p_{\mathrm{T,rad}})\frac{R(\Phi_{\mathrm{FJ}},\Phi_{\mathrm{rad}})}{B(\Phi_{\mathrm{FJ}})}\right\}$$

. if emissions are strongly ordered, same emission probabilities as in k_t-ordered shower
$\rightarrow$ LL shower accuracy preserved

MiNNLO$_{\text{PS}}$: implementation II

Final master formula:

$$\frac{d\bar{B}(\Phi_{FJ})}{d\Phi_{FJ}} = \exp[-\tilde{S}(p_T)]\left\{ \frac{\alpha_S(p_T)}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{FJ}}\right]^{(1)}\left(1+\frac{\alpha_S(p_T)}{2\pi}[\tilde{S}(p_T)]^{(1)}\right) + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{FJ}}\right]^{(2)} + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^3[D(p_T)]^{(3)}F_\ell^{\text{corr}}(\Phi_{FJ})\right\}$$

. The second radiation is generated by the usual `POWHEG` mechanism.

Subleading terms:

- 1908.06987: D truncated to $\alpha_S^3(p_T)$.
  Formally correct, but many terms that were present in the starting formula (total derivative) are lost
- 2006.04133: avoiding the truncation minimizes the contamination from higher-order terms:

$$\left(\frac{\alpha_S(p_T)}{2\pi}\right)^3[D(p_T)]^{(3)} \to \underbrace{-\frac{d\tilde{S}(p_T)}{dp_T}\mathcal{L}(p_T)+\frac{d\mathcal{L}(p_T)}{dp_T}}_{D} - \frac{\alpha_S(p_T)}{2\pi}[D(p_T)]^{(1)} - \left(\frac{\alpha_S(p_T)}{2\pi}\right)^2[D(p_T)]^{(2)}$$

- by keeping the full expression for D:
  . inclusive observables agree better with fixed-order
  . scale variation band more reliable (truncation → throw away subleading, but log-enhanced, terms)

MiNNLO$_{\rm PS}$: implementation III

$$\frac{d\bar{B}_{\rm MiNNLO_{PS}}(\Phi_{\rm FJ})}{d\Phi_{\rm FJ}} = \exp[-\tilde{S}(p_{\rm T})]\Bigg\{\frac{\alpha_{\rm S}(p_{\rm T})}{2\pi}\left[\frac{d\sigma_{\rm FJ}}{d\Phi_{\rm FJ}}\right]^{(1)}\left(1+\frac{\alpha_{\rm S}(p_{\rm T})}{2\pi}[\tilde{S}(p_{\rm T})]^{(1)}\right) + \left(\frac{\alpha_{\rm S}(p_{\rm T})}{2\pi}\right)^2\left[\frac{d\sigma_{\rm FJ}}{d\Phi_{\rm FJ}}\right]^{(2)} + \left(\frac{\alpha_{\rm S}(p_{\rm T})}{2\pi}\right)^3[D(p_{\rm T})]^{(3)}F_\ell^{\rm corr}(\Phi_{\rm FJ})\Bigg\}$$

PDFs evolution and scale setting:

- no cutoff on physical phase space $\Phi_{\rm FJ}$: p_T can get arbitrarily small in phase space and Sudakov integral
- We always include μ_R scale variation in Sudakov integrand $\rightarrow$ extra uncertainty wrt nominal fixed-order

- 1908.06987: freezing of $\mu_{R/F} = K_{R/F}\, p_{\rm T}$ in PDFs, $\alpha_{\rm S}(\mu_R)$ and $d\sigma_{\rm FJ}$.
  - stay above PDF cutoff ($\Lambda_{\rm PDF}$) AND make sure cross-section at freezing point is already very suppressed by Sudakov.
  - for $K_{R/F} = 1$, freezing at 1.25 GeV (1 GeV) for Higgs (DY)
- 2006.04133: no freezing
  - below $\Lambda_{\rm PDF}$ we continue DGLAP evolution through HOPPET
  - exactly the same running of $\alpha_{\rm S}$ everywhere (including in the Sudakov, that we evaluate numerically, improving its numerical stability)

MiNNLO$_{\text{PS}}$: implementation IV

$$\frac{d\bar{B}_{\mathrm{MiNNLO_{PS}}}(\Phi_{\mathrm{FJ}})}{d\Phi_{\mathrm{FJ}}} = \exp[-\tilde{S}(p_{\mathrm{T}})]\left\{ \frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{\mathrm{FJ}}}{d\Phi_{\mathrm{FJ}}}\right]^{(1)}\left(1+\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}[\tilde{S}(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{\mathrm{FJ}}}{d\Phi_{\mathrm{FJ}}}\right]^{(2)} + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)}F_{\ell}^{\mathrm{corr}}(\Phi_{\mathrm{FJ}})\right\}$$

Scale setting & logs far from Sudakov region:

- large p_{T}: $d\sigma \to d\sigma_{\mathrm{FJ}}$
  - `MiNLO'` original prescription: $d\sigma_{\mathrm{FJ}}$ computed with $\mu_{R/F} = p_{\mathrm{T}}$
  - depending on physics goal, legitimate also to set $\mu_{R/F} \sim Q$ in $d\sigma_{\mathrm{FJ}}$, when p_{T} is large
- large p_{T}: switch off logarithms in Sudakov, $[\tilde{S}(p_{\mathrm{T}})]^{(1)}$ and $[D(p_{\mathrm{T}})]^{(3)}$
  - original `MiNLO'`: . Θ function
  - 1908.06987: . modified logs in Sudakov, $[\tilde{S}(p_{\mathrm{T}})]^{(1)}$ and $[D(p_{\mathrm{T}})]^{(3)}$

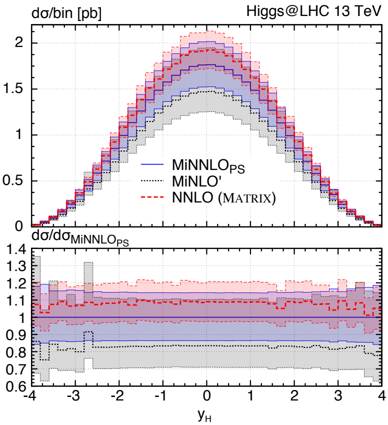
$$\ln\frac{Q}{p_{\mathrm{T}}} \to L \equiv \frac{1}{p}\ln\left(1+\left(\frac{Q}{p_{\mathrm{T}}}\right)^p\right) \qquad \mu_{R/F} = K_{R/F}Qe^{-L} \qquad p = 6$$

  - 2006.04133: . modified logs + option to use profiled scales

$$\mu_{R/F} = K_{R/F}\left(Q\,e^{-L} + Q_0\,g(p_{\mathrm{T}})\right) \qquad Q_0 \lesssim 2\ \text{GeV}$$

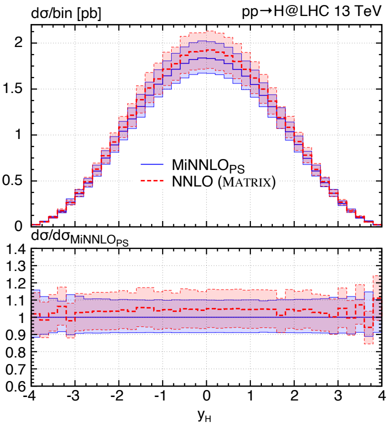
MiNNLO$_{PS}$: ggH

PS, no hadronization, no MPI



1908.06987

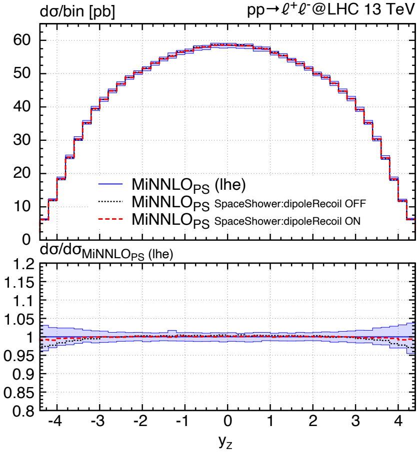
- $\sigma_{\text{MiNNLOPS}}/\sigma_{\text{NNLO}} = 0.92$
- larger scale uncertainy:
  . scale dependence in Sudakov FF
- flat ratios for y_H ✓



2006.04133

- different scale choice for finite terms + $D(p_T)$ not truncated
- $\sigma_{\text{MiNNLOPS}}/\sigma_{\text{NNLO}} = 0.96$ ✓

MiNNLO$_{PS}$: Drell-Yan

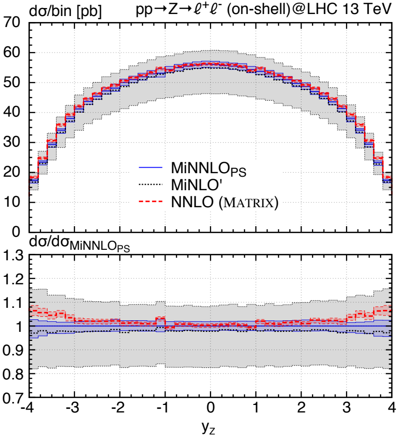


Impact of shower recoil scheme

- shower suppresses configuration at large $|y_Z|$
- due to the PYTHIA8 global recoil scheme
- effect is less pronounced if local recoil for emissions off initial-final colour dipoles
- these effects are formally subleading, but visible

MiNNLO$_{PS}$: Drell-Yan

PS, no hadronization, no MPI



1908.06987

- $\sigma_{\text{MiNNLOPS}}/\sigma_{\text{NNLO}} = 0.98$
- visible pattern for $|y_Z| > 3$
  . acceptable?
  . reduced, but present, also with local recoil
  . corner of phase-space, should we really expect exact agreement with NNLO?

MiNNLO$_{PS}$: Drell-Yan

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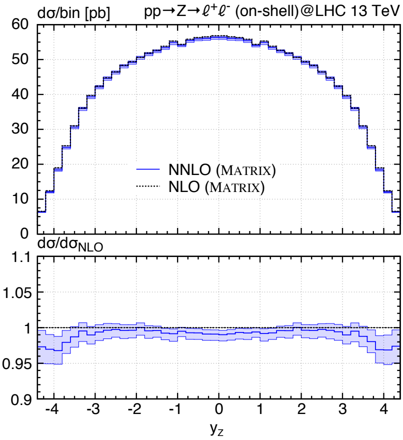

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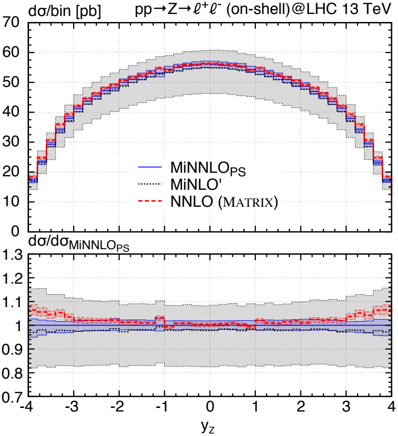
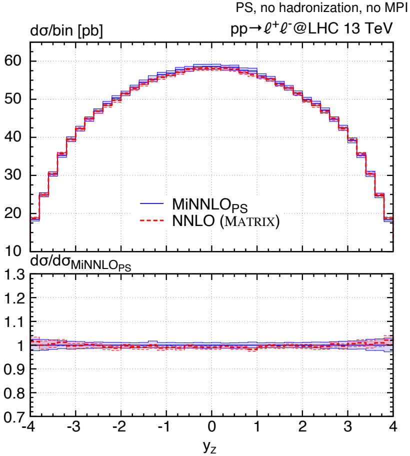


1908.06987



2006.04133

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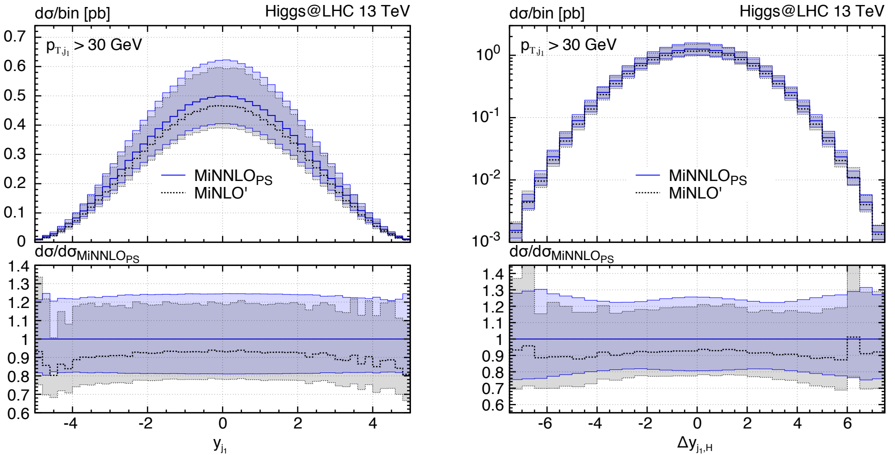
  . reduced, but present, also with local recoil
  . corner of phase-space, should we really expect exact agreement with NNLO?

- $\sigma_{\mathrm{MiNNLOPS}}/\sigma_{\mathrm{NNLO}} = 1.004$ ✓
- $D(p_{\mathrm{T}})$ not truncated + local recoil
  $\rightarrow$ no shape

MiNNLO$_{PS}$: ggH

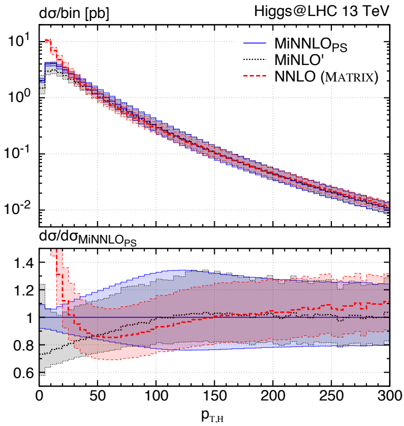
PS, no hadronization, no MPI



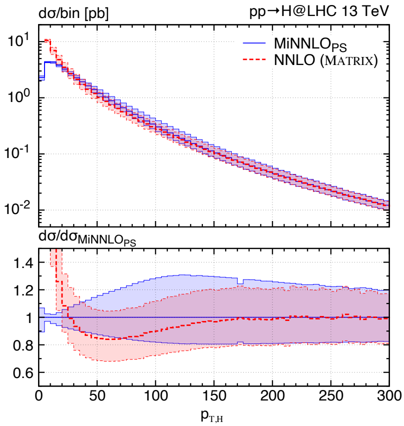
- plots from 1908.06987 [same pattern in 2006.04133]
- $p_{\mathrm{T,j}} > 30$ GeV, $R = 0.4$, anti-k_{T}
- $[D(p_{\mathrm{T}})]^{(3)} \leftrightarrow \Phi_{\mathrm{FJ}}$ ✓

MiNNLO$_{PS}$: ggH

PS, no hadronization, no MPI



1908.06987



2006.04133

- differences expected: different scale settings
- MiNLO′ original prescription: $\mu_{R/F} \sim p_T$

$\mu_{R/F} \sim Q$ when p_T is large

Summary

[Monni,Nason,ER,Wiesemann,Zanderighi '19] + [Monni,ER,Wiesemann '20]

- NNLO+PS without reweighting
- better analytical understanding
- efficient event generation (factor 2 slower than MiNLO′)
- no unphysical merging scale
- leading-log accuracy of (p_{T}-ordered) PS preserved
- subleading effects due to PS recoiling scheme
- understood origin of features initially observed at large y_Z
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Some points for discussion

- Interplay of perturbative and non-perturbative effects
- Details of the matching to PS (especially in non strongly-ordered regions)
- Regular terms in $d\sigma_{\mathrm{FJ}}$: which scale?

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Thank you for your attention!

Extra slides

NNLO+PS for color-singlet production

- starting from a MiNLO′ generator, it's possible to match a PS simulation to NNLO.
- FJ−MiNLO′ (+POWHEG) generator gives F-FJ @ NLOPS:

	F (inclusive)	F+j (inclusive)	F+$2j$ (inclusive)
✓ F-FJ @ NLOPS	NLO	NLO	LO
F @ NNLOPS	NNLO	NLO	LO

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  - by construction NNLO accuracy on inclusive observables; [✓]
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- possible to obtain $pp \to WW$ @ NNLOPS: so far, the most complex NNLOPS result

MiNNLO$_{\rm PS}$: further details

Luminosities and $D(p_{\rm T})$ term:

$$\mathcal{L}(\Phi_{\rm F}, p_{\rm T}) = B^{(\rm F)}_{cc'}(\Phi_{\rm F}) \Big\{ \Big[C_{ci} \otimes f_i\Big](p_{\rm T})\ H(p_{\rm T})\ \Big[C_{c'j} \otimes f_j\Big](p_{\rm T})\ +\ \Big[G_{ci} \otimes f_i\Big](p_{\rm T})\ H(p_{\rm T})\ \Big[G_{c'j} \otimes f_j\Big](p_{\rm T}) \Big\}$$

$$\begin{aligned}[D(p_{\rm T})]^{(3)} &= -\left[\frac{{\rm d}\tilde{S}(p_{\rm T})}{{\rm d}p_{\rm T}}\right]^{(1)} [\mathcal{L}(p_{\rm T})]^{(2)} - \left[\frac{{\rm d}\tilde{S}(p_{\rm T})}{{\rm d}p_{\rm T}}\right]^{(2)} [\mathcal{L}(p_{\rm T})]^{(1)} - \left[\frac{{\rm d}\tilde{S}(p_{\rm T})}{{\rm d}p_{\rm T}}\right]^{(3)} [\mathcal{L}(p_{\rm T})]^{(0)} + \left[\frac{{\rm d}\mathcal{L}(p_{\rm T})}{{\rm d}p_{\rm T}}\right]^{(3)} \\ &= \frac{2}{p_{\rm T}}\left(A^{(1)} \ln\frac{Q^2}{p_{\rm T}^2} + B^{(1)}\right)[\mathcal{L}(p_{\rm T})]^{(2)} + \frac{2}{p_{\rm T}}\left(A^{(2)} \ln\frac{Q^2}{p_{\rm T}^2} + \tilde{B}^{(2)}\right)[\mathcal{L}(p_{\rm T})]^{(1)} + \frac{2}{p_{\rm T}}\left(A^{(3)} \ln\frac{Q^2}{p_{\rm T}^2}\right)[\mathcal{L}(p_{\rm T})]^{(0)} + \left[\frac{{\rm d}\mathcal{L}(p_{\rm T})}{{\rm d}p_{\rm T}}\right]^{(3)}\end{aligned}$$

Profiled scales:

$$\mu_{R/F} = K_{R/F}\left(Q\, e^{-L} + Q_0\, g(p_{\rm T})\right) \qquad Q_0 \lesssim 2\ \text{GeV}$$

$$g(p_{\rm T}) = 1 \qquad \text{or} \qquad g(p_{\rm T}) = \frac{1}{1 + Q/Q_0 e^{-L}}$$

MiNNLO$_{PS}$: effects from truncation at LH level