Amplitude Level Evolution

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Particle Physics — University of Vienna

at the
"Taming the Accuracy of Parton Showers"
CERN/Remote | 2 July 2020

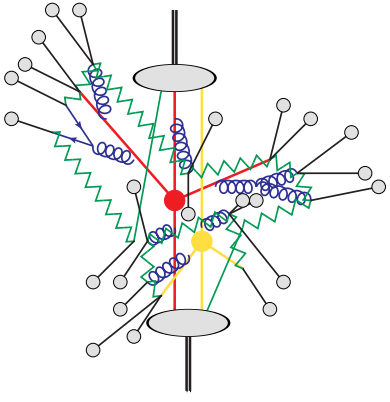
QCD, Event Generators & Phenomenology

QCD description of collider reactions:
Complexity challenges precision.

Hard partonic scattering:
NLO QCD routinely

Jet evolution — parton showers:
NLL sometimes, mostly unclear

Multi-parton interactions
Hadronization



$$\mathrm{d}\sigma \sim \mathrm{d}\sigma_{\mathrm{hard}}(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Bottlenecks

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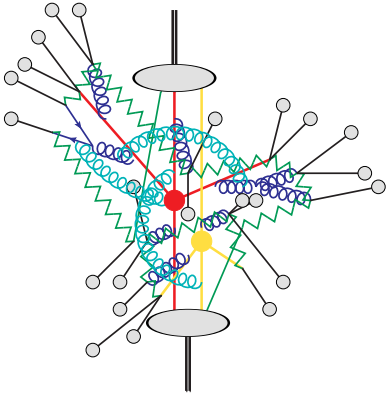
Parton shower algorithms

Lack a systematic expansion, obstruct fully differential NNLO for the hard process, open questions regarding mass effects and unstable particles.

Hadronization models

Lack constraints from perturbative evolution: Hiding perturbative corrections? Genuine uncertainties/constraints?

Rethink foundations of parton showers.



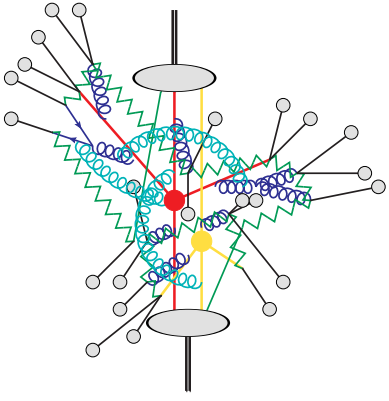
Outline

Loads of material, will focus on:

- Colour space and large-N expansions
- Collinear subtractions and structure of evolution
- Algorithmic aspects & numerical results
- Relation to colour reconnection

Will skip/discuss offline:

- Spin
- Ordering variable
- Coloumb/Glauber issues



Parton Branching at Amplitude Level

[Nagy, Soper — 2014 …]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

$$\sigma = \sum_n \int \underbrace{\mathrm{Tr}\left[\mathbf{A}_n(\mu)\right]}_{\text{density operator}} \underbrace{u(p_1, \ldots, p_n)}_{\text{observable}} \underbrace{\mathrm{d}\phi_n}_{\text{phase space integration}}$$

$A_0 = |\mu\rangle\langle\mu|$

$$\mathbf{A}_n(q_\perp; \{p\}_n) = \int \mathrm{d}R_n \mathbf{V}_{q_\perp, q_{n\perp}} \mathbf{D}_n \mathbf{A}_{n-1}(q_{n\perp}; \{p\}_{n-1}) \mathbf{D}_n^\dagger \mathbf{V}^\dagger_{q_\perp, q_{n\perp}} \Theta(q_\perp \leq q_{n\perp})$$

Evolution equation

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp; \{p\}_n)}{\partial q_\perp} = -\mathbf{\Gamma}_n(q_\perp)\, \mathbf{A}_n(q_\perp; \{p\}_n) - \mathbf{A}_n(q_\perp; \{p\}_n)\, \mathbf{\Gamma}_n^\dagger(q_\perp)$$
$$+ \int \mathrm{d}R_n\, \mathbf{D}_n(q_{n\perp})\, \mathbf{A}_{n-1}(q_{n\perp}; \{p\}_{n-1})\, \mathbf{D}_n^\dagger(q_{n\perp})\, q_\perp\, \delta(q_\perp - q_{n\perp})$$

Parton Branching at Amplitude Level

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
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$$\sigma = \sum_n \int \mathrm{Tr}\left[\mathbf{A}_n(\mu)\right]\, u(p_1, \ldots, p_n)\, \mathrm{d}\phi_n$$

density operator — observable — phase space integration

$$\mathbf{A}_n(q_\perp; \{p\}_n) = \int \mathrm{d}R_n \mathbf{V}_{q_\perp, q_{n\perp}} \mathbf{D}_n \mathbf{A}_{n-1}(q_{n\perp}; \{p\}_{n-1}) \mathbf{D}_n^\dagger \mathbf{V}^\dagger_{q_\perp, q_{n\perp}} \Theta(q_\perp \leq q_{n\perp})$$

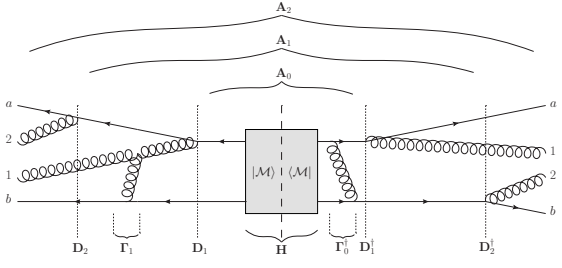
$$\mathbf{V}_{a,b} = \mathrm{Pexp}\left(-\int_a^b \frac{\mathrm{d}q_\perp}{q_\perp} \mathbf{\Gamma}_n(q_\perp)\right)$$

$$\mathbf{D}_n(q_{n\perp}; q_n \cup \{\tilde{p}\}_{n-1})\, \mathbf{O}\, \mathbf{D}_n^\dagger(q_{n\perp}; q_n \cup \{\tilde{p}\}_{n-1}) =$$

$$\sum_{i_n, j_n} \int \delta q_{n\perp}^{(i_n, j_n)}(q_{n\perp})\, \mathbf{S}_n^{i_n}\, \mathbf{O}\, \mathbf{S}_n^{j_n\,\dagger} + \sum_{i_n} \int \delta q_{n\perp}^{(i_n, \vec{n})}(q_{n\perp})\, \mathbf{C}_n^{i_n}\, \mathbf{O}\, \mathbf{C}_n^{i_n\,\dagger}$$

soft contributions

collinear contributions



The Colour Flow Basis

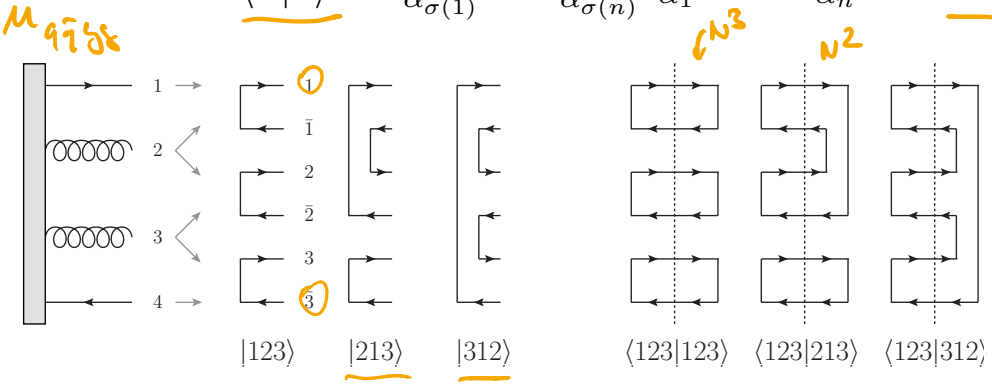
Not truly a basis, but handy from a computational point of view.

[Plätzer – EPJ C 74 (2014) 2907]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[see also Prestel & Isaacson, Hoeche & Reichelt]

$$|\mathcal{M}\rangle = \sum_{\sigma} \mathcal{M}_\sigma |\sigma\rangle \qquad \mathbf{A} = \sum_{\tau,\sigma} \mathcal{A}_{\tau\sigma} |\tau\rangle\langle\sigma| \qquad |\sigma\rangle = \left| \begin{matrix} 1 & \cdots & n \\ \sigma(1) & \cdots & \sigma(n) \end{matrix} \right\rangle = \delta^{\alpha_1}_{\bar{\alpha}_{\sigma(1)}} \cdots \delta^{\alpha_n}_{\bar{\alpha}_{\sigma(n)}}$$

$$\langle\sigma|\tau\rangle = \delta^{\alpha_1}_{\bar{\alpha}_{\sigma(1)}} \cdots \delta^{\alpha_n}_{\bar{\alpha}_{\sigma(n)}} \delta^{\bar{\alpha}_{\tau(1)}}_{\alpha_1} \cdots \delta^{\bar{\alpha}_{\tau(n)}}_{\alpha_n} = N^{n-\#\mathrm{transpositions}(\sigma,\tau)}$$

$\mathcal{M}_{q\bar{q}gg}$



Additional 1/N enhancement/suppression when squaring the evolved amplitude.

The Colour Flow Basis

[Plätzer – EPJ C 74 (2014) 2907]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Colour charge operators decompose into colour line operators

$$\left(\mathbf{T}_i = \lambda_i \, \mathbf{t}_{c_i} - \bar{\lambda}_i \, \bar{\mathbf{t}}_{\bar{c}_i} - \frac{1}{N}(\lambda_i - \bar{\lambda}_i) \, \mathbf{s} \right)$$

$\lambda_i = \bar{\lambda}_i = 1$ g

$\lambda_i = 1 \quad \bar{\lambda}_i = 0$ q

$$\mathbf{t}_\alpha|\sigma\rangle = \mathbf{t}_\alpha \left| \begin{matrix} 1 & \cdots & \alpha & \cdots & n \\ \sigma(1) & \cdots & \sigma(\alpha) & \cdots & \sigma(n) \end{matrix} \right\rangle = \left| \begin{matrix} 1 & \cdots & \alpha & \cdots & n & n+1 \\ \sigma(1) & \cdots & n+1 & \cdots & \sigma(n) & \sigma(\alpha) \end{matrix} \right\rangle$$

$$\mathbf{s}|\sigma\rangle = \mathbf{s} \left| \begin{matrix} 1 & \cdots & \cdots & n \\ \sigma(1) & \cdots & \cdots & \sigma(n) \end{matrix} \right\rangle = \left| \begin{matrix} 1 & \cdots & \cdots & n & n+1 \\ \sigma(1) & \cdots & \cdots & \sigma(n) & n+1 \end{matrix} \right\rangle$$

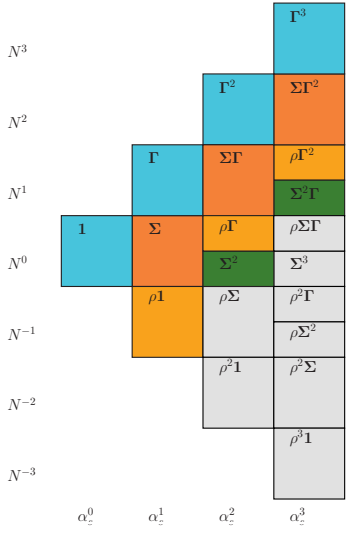
$T_i \cdot T_j$

swap $(\alpha, n+1)$

$$\mathbf{t}_\alpha|\sigma\rangle = \mathbf{s}_{\alpha,n+1} \, \mathbf{s} \, |\sigma\rangle$$

Expanding around Large-N

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]



virtuals	reals
(0 flips) $\times\, 1 \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[\ldots]\mathbf{t}\vert_{2\text{ flips}} \times 1$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[\ldots]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{s}[\ldots]\mathbf{s}\vert_{0\text{ flips}} \times N^{-2}$
(1 flip) $\times\, \alpha_s \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r}$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[\ldots]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$
(0 flips) $\times\, \alpha_s N^{-1} \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r}$
(0 flips) $\times\, \alpha_s^2 \times (\alpha_s N)^n$ (2 flips) $\times\, \alpha_s^2 \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r}$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[\ldots]\mathbf{t}\vert_{2\text{ flips}}$

$$\alpha_s N \sim 1$$

$\alpha_s^n N^n$

$$[\tau|\mathbf{\Gamma}|\sigma\rangle = N\delta_{\tau\sigma}\Gamma_\sigma + \Sigma_{\tau\sigma} + \frac{1}{N}\delta_{\tau\sigma}\rho$$

[Plätzer – EPJ C 74 (2014) 2907]

$\Sigma_{\tau\tau} = 0$

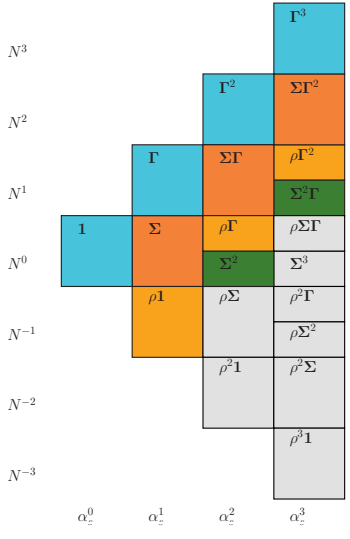
$$\alpha_s \Gamma = (\alpha_s N)\left(\Gamma + \frac{1}{N}\Sigma + \frac{1}{N^2}\rho\right)$$

exp(..) pert. pert.

$C_F \overset{?}{\leftrightarrow} \frac{C_A}{2}$

Expanding around Large-N

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]



virtuals	reals
(0 flips) $\times 1 \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\,\mathbf{t}[\ldots]\mathbf{t}\vert_{2\text{ flips}} \times 1$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\,\mathbf{t}[\ldots]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\,\mathbf{s}[\ldots]\mathbf{s}\vert_{0\text{ flips}} \times N^{-2}$
(1 flip) $\times\, \alpha_s \times (\alpha_s N)^n$	$(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r}$ $(\mathbf{t}[\ldots]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\,\mathbf{t}[\ldots]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$
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$$[\tau|\mathbf{\Gamma}|\sigma\rangle = N\delta_{\tau\sigma}\Gamma_\sigma + \Sigma_{\tau\sigma} + \frac{1}{N}\delta_{\tau\sigma}\rho$$

[Plätzer – EPJ C 74 (2014) 2907]

Systematically sum colour enhanced terms

$$\alpha_s N \sim 1$$

$$\mathbf{V}_n^{\text{LC+NLC}}|\sigma\rangle = V_\sigma^{(n)}|\sigma\rangle - \frac{1}{N}\sum_\tau \delta_{\#\text{transpositions}(\tau,\sigma),1}\Sigma_{\sigma\tau}^{(n)}|\tau\rangle$$

$$\Sigma_{\sigma\tau}^{(n)} = N\frac{e^{-W_\sigma^{(n)}} - e^{-W_\tau^{(n)}}}{W_\sigma^{(n)} - W_\tau^{(n)}} \times \sum_{\substack{i,k \text{ c.c. in } \sigma \\ j,l \text{ c.c. in } \sigma}} \left(\lambda_i\lambda_j W_{ij}^{(n)} + \bar\lambda_k\bar\lambda_l W_{kl}^{(n)} - \lambda_i\bar\lambda_l W_{il}^{(n)} - \bar\lambda_k\lambda_j W_{kj}^{(n)}\right)\delta_{\substack{i,l \text{ c.c. in } \tau \\ k,j \text{ c.c. in } \tau}}$$

dipole flips at next-to-leading colour

Expanding around Large-N

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Re-derive BMS equation: Prototype of constructing a dipole shower

$e^+e^- \to q\bar{q}$

$$\text{Leading}^{(l)}_{\tau\sigma}\left[\mathbf{A}\right] = \sum_{k=0}^{l} \mathcal{A}_{\tau\sigma}\Big|_{1/N^k} \delta_{\#\text{transpositions}(\tau,\sigma),l-k}$$

$$\text{Leading}^{(0)}_{\tau\sigma}\left[\mathbf{V}_n\mathbf{A}_n\mathbf{V}_n^\dagger\right] = \delta_{\tau\sigma}\left|V_\sigma^{(n)}\right|^2 \text{Leading}^{(0)}_{\tau\sigma}\left[\mathbf{A}_n\right]$$

$$\text{Leading}^{(0)}_{\tau\sigma}\left[\mathbf{D}_n\mathbf{A}_{n-1}\mathbf{D}_n^\dagger\right] = \delta_{\tau\sigma}\sum_{i,j \text{ c.c. in } \sigma\backslash n} \lambda_i\bar{\lambda}_j R_{ij}^{(n)} \text{Leading}^{(0)}_{\tau\backslash n,\sigma\backslash n}\left[\mathbf{A}_{n-1}\right]$$

$$V_\sigma^{(n)} = \exp\left(-N\sum_{i,j \text{ c.c. in } \sigma} \lambda_i\bar{\lambda}_j W_{ij}^{(n)}\right)$$

colour connected dipoles

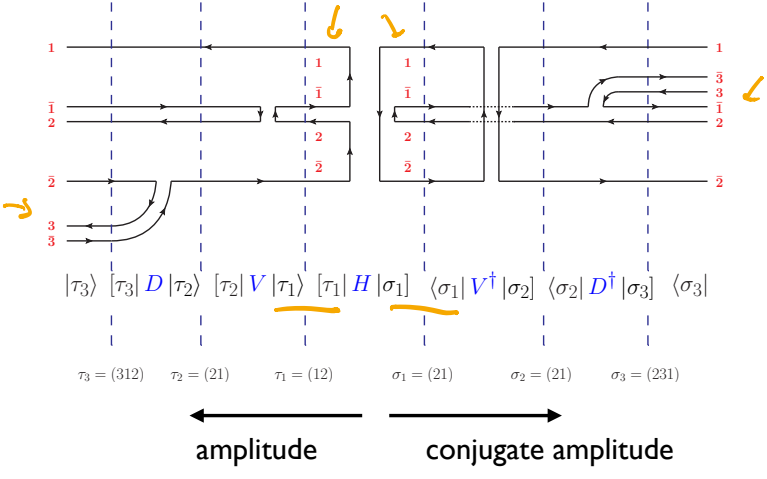
Algorithmic Framework

[De Angelis, Forshaw, Plätzer — 2018 …]

CVolver library implements numerical evolution in colour space

origins in [Plätzer – EPJ C 74 (2014) 2907]

Full evolution needs to sample colour flows differently in amplitude and conjugate amplitude, and account for differences in virtual evolution at the amplitude level.



Weights are complex during evolution.

$\mathrm{Re}[\langle 0|z\rangle M_\sigma M_z^\dagger]$ … $R \ni |M|^2 = \mathrm{tr}[A_{\sigma z}\langle z|\sigma\rangle]$

$= W_{event}$

Algorithmic Framework

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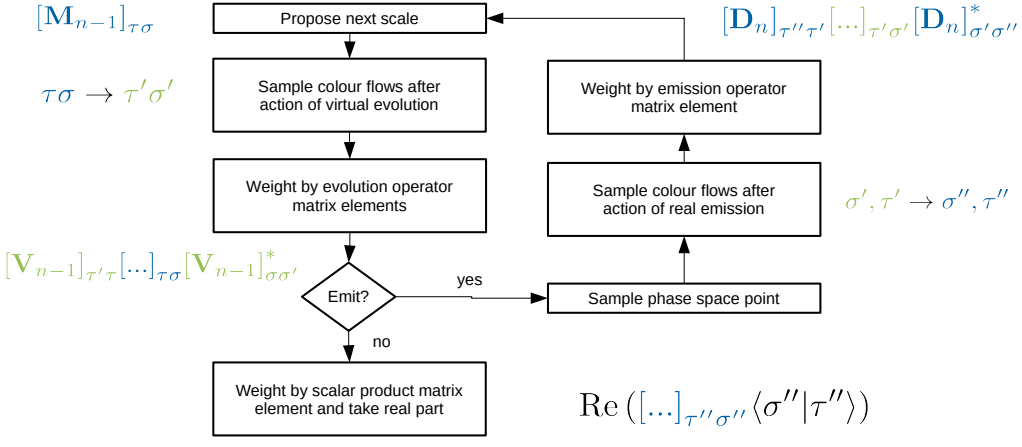
Weights are complex during evolution.

$$\mathbf{A}_n(E) = \mathbf{V}(E, E_n)\mathbf{D}_n\mathbf{A}_{n-1}(E_n)\mathbf{D}_n^\dagger\mathbf{V}^\dagger(E, E_n)\theta(E - E_n)$$



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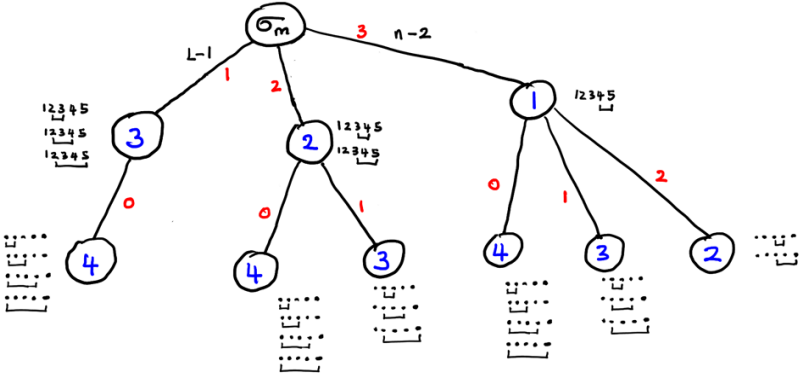
Full evolution needs to sample colour flows differently in amplitude and conjugate amplitude, and account for differences in virtual evolution at the amplitude level.

Weights are complex during evolution.

Importance sampling in colour space rules: $\#(\tau,\tau') \sim 1/N^{\#(\tau,\tau')}$

Enumerate and address permutations with fixed cycle length:



Collinear Subtractions

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[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

Identify and subtract collinear singularities in soft evolution

softness

ordering for soft evolution

$$\ln \mathbf{W}_{ab} = \frac{\alpha_s}{2\pi} \sum_{i<j} \mathbb{T}_i^g \cdot \mathbb{T}_j^g \int_{a^2}^{b^2} \frac{\mathrm{d}q^2}{q^2} \int \frac{\mathrm{d}^3 k}{2E} \frac{1}{\pi\,(S\cdot k)^2}$$

$$\left(K^2(p_i,p_j;k) \frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j} \delta(q^2 - K^2(p_i,p_j;k))\, \theta_{ij}(k) \right.$$

$$\left. - \frac{K^2(p_i;k)}{n_i\cdot n} \delta(q^2 - K^2(p_i;k))\theta_i(k) - \frac{K^2(p_j;k)}{n_j\cdot n} \delta(q^2 - K^2(p_j;k))\theta_j(k) \right)$$

ordering for collinear evolution

Energy ordering

$$\ln \mathbf{W}_{ab}\Big|_{\text{energy}} = \frac{\alpha_s}{\pi} \sum_{i<j} \mathbb{T}_i^g \cdot \mathbb{T}_j^g \int_a^b \frac{\mathrm{d}E}{E} \int \frac{\mathrm{d}\Omega}{4\pi} \left(\frac{n_i\cdot n_j - n_i\cdot n - n_j\cdot n}{n_i\cdot n\; n\cdot n_j} \right)$$

$$= \frac{\alpha_s}{\pi} \sum_{i<j} \mathbb{T}_i^g \cdot \mathbb{T}_j^g \int_a^b \frac{\mathrm{d}E}{E} \ln \frac{n_i\cdot n_j}{2}$$

5th FF

$$\ln \mathbf{K}_{ab}\Big|_{\text{energy}} = \frac{\alpha_s}{\pi} \sum_i (\mathbb{T}_i^g)^2 \int_a^b \frac{\mathrm{d}E}{E} \int \frac{\mathrm{d}\Omega}{4\pi} \frac{2}{n_i\cdot n}$$

reg.

(Dipole) pt ordering

$$\ln \mathbf{W}_{ab}\Big|_{k_T} = \frac{\alpha_s}{\pi} \sum_{i<j} \mathbb{T}_i^g \cdot \mathbb{T}_j^g \int_a^b \frac{\mathrm{d}k_\perp}{k_\perp} \int \frac{\mathrm{d}y\,\mathrm{d}\phi}{2\pi} \left(\theta_{ij}(k) - \theta_i(k)\right)$$

$$\ln \mathbf{K}_{ab}\Big|_{k_T} = \frac{\alpha_s}{2\pi} \sum_i (\mathbb{T}_j^g)^2 \int_a^b \frac{\mathrm{d}k_\perp}{k_\perp} \int_\alpha^1 \frac{\mathrm{d}z}{1-z+\alpha} \int \frac{\mathrm{d}\phi}{2\pi}$$

$$= \frac{\alpha_s}{2\pi} \sum_i (\mathbb{T}_j^g)^2 \int_a^b \frac{\mathrm{d}k_\perp}{k_\perp} \int_0^{1-\alpha} \frac{\mathrm{d}z}{1-z} \int \frac{\mathrm{d}\phi}{2\pi}$$

Collinear Subtractions

[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

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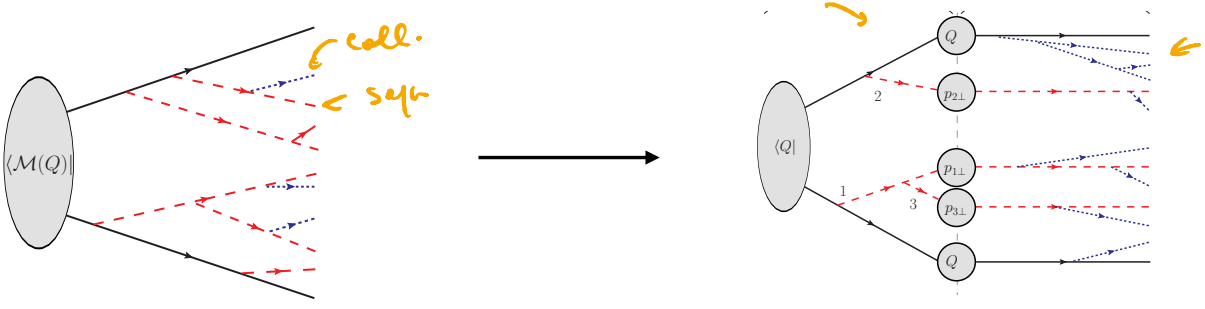
$$\left(K^2(p_i,p_j;k) \frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j} \delta(q^2 - K^2(p_i,p_j;k))\,\theta_{ij}(k) \right.$$

$$\left. - \frac{K^2(p_i;k)}{n_i\cdot n} \delta(q^2 - K^2(p_i;k))\theta_i(k) - \frac{K^2(p_j;k)}{n_j\cdot n} \delta(q^2 - K^2(p_j;k))\theta_j(k) \right)$$



The Fifth Form Factor (reloaded)

[De Angelis, Forshaw, Plätzer — 2018 …]

$$\frac{k_i \cdot k_j}{k_i \cdot k \; k \cdot k_j} - \frac{1}{k \cdot k_i} - \frac{1}{k \cdot k_j}$$

Vishnoto ✓ $\int d\Omega$

reals ↯

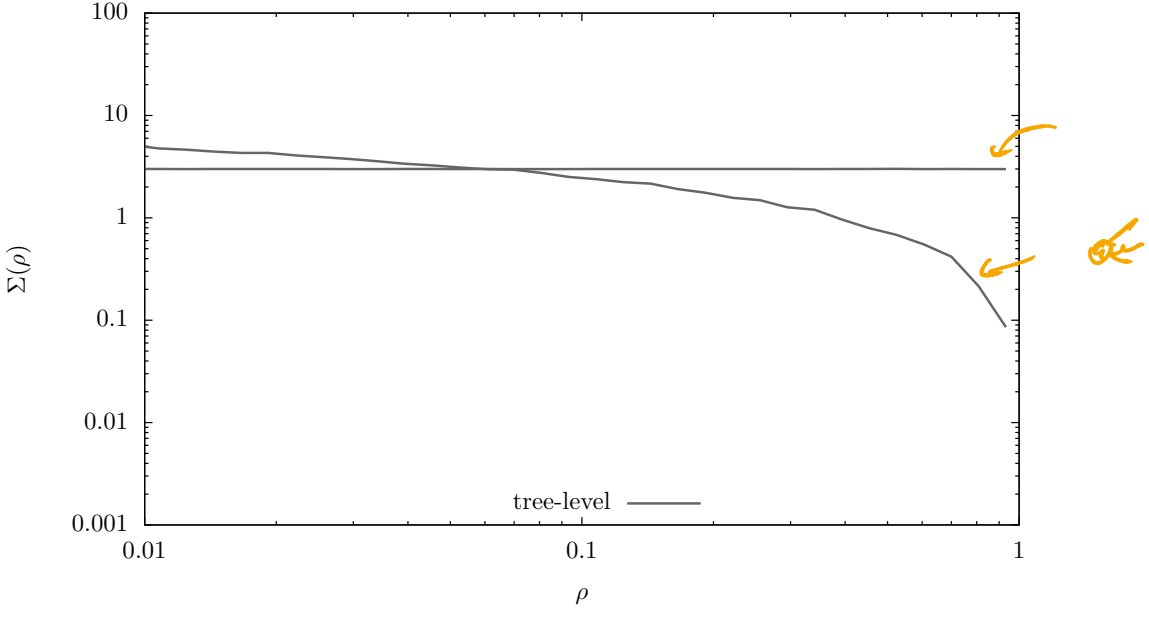
$$\frac{k_i \cdot k_j}{k_i \cdot k \; k \cdot k_j} - S_{ij} - S_{ji}$$

$$S_{ij} = \frac{1}{k_i \cdot k} - \frac{k_j \cdot k - k_i \cdot k_j}{k_i \cdot k_j \; k_i \cdot k}$$

Collinear Subtractions in Real Life

[DeAngelis, Forshaw, Plätzer – unpublished]

$\Sigma(\rho)$



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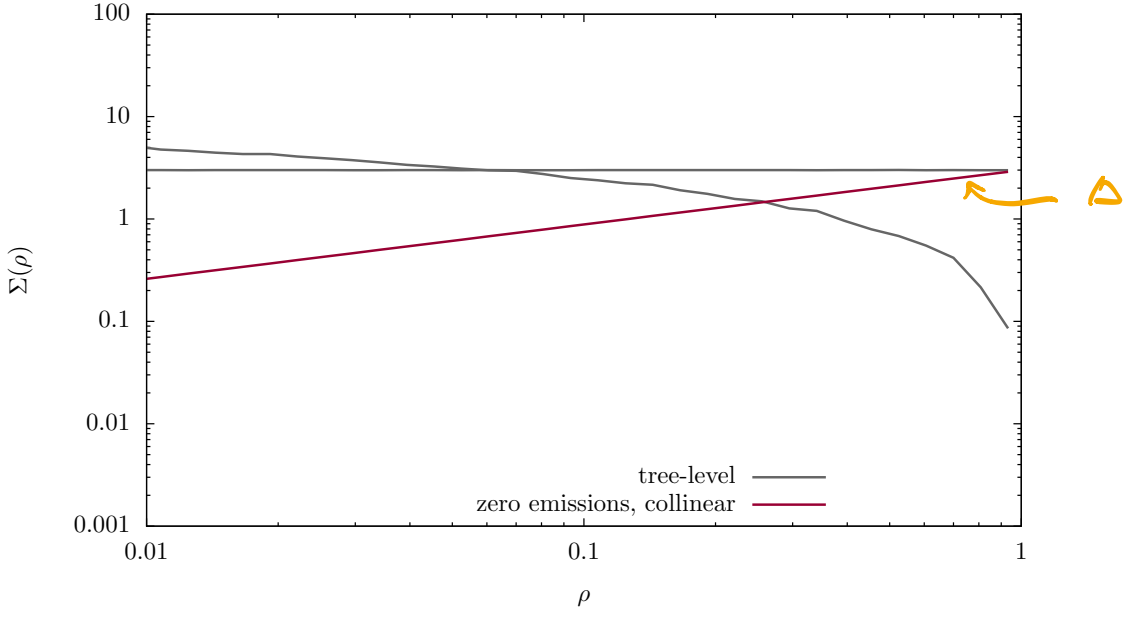

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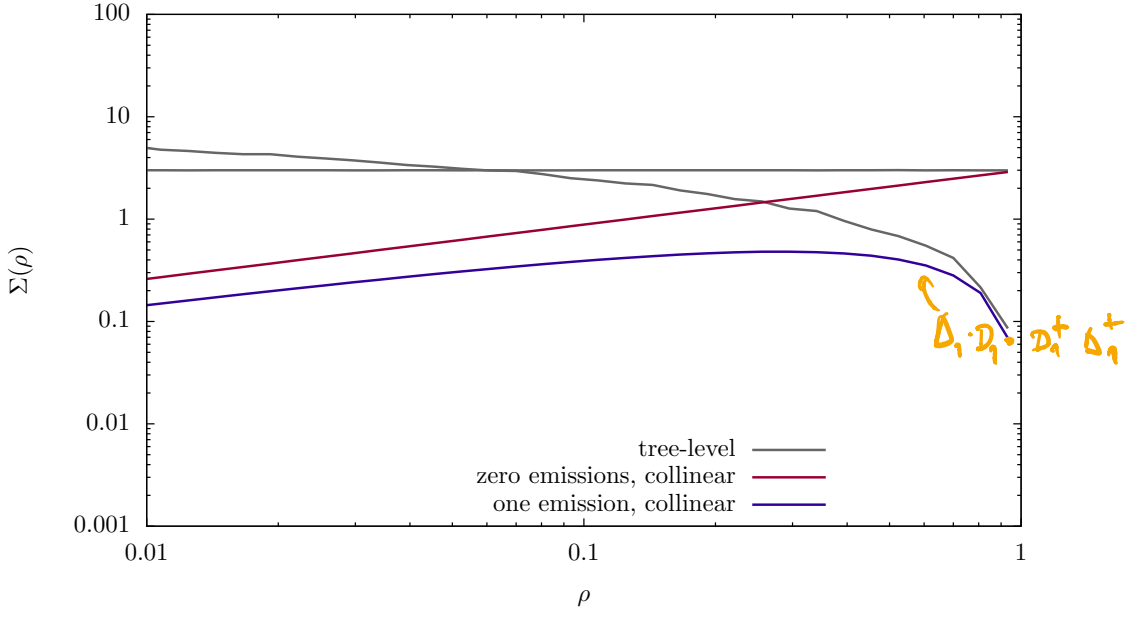

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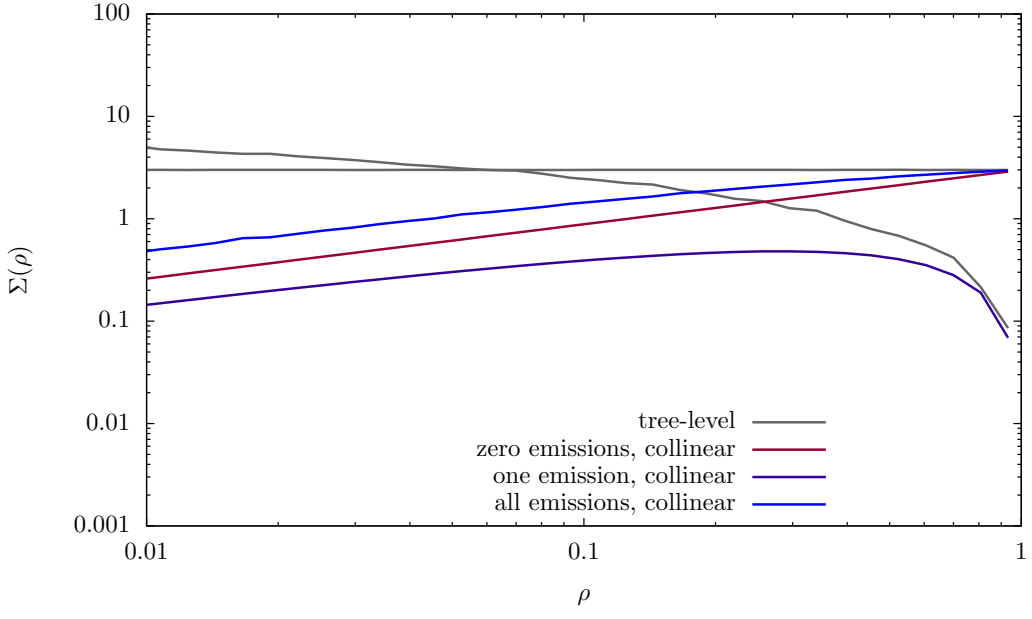
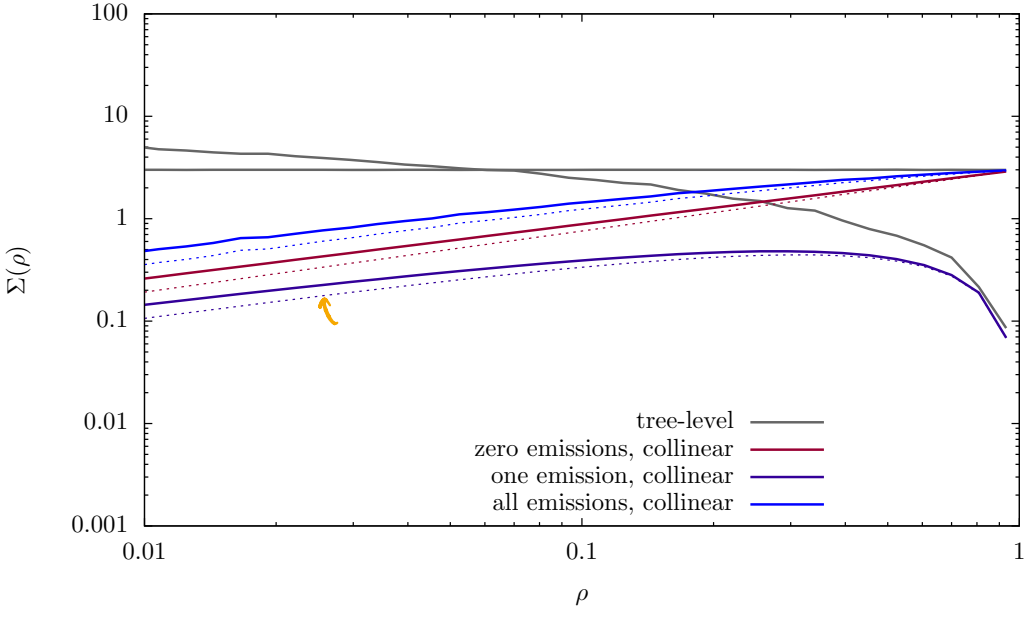


Collinear Subtractions in Real Life

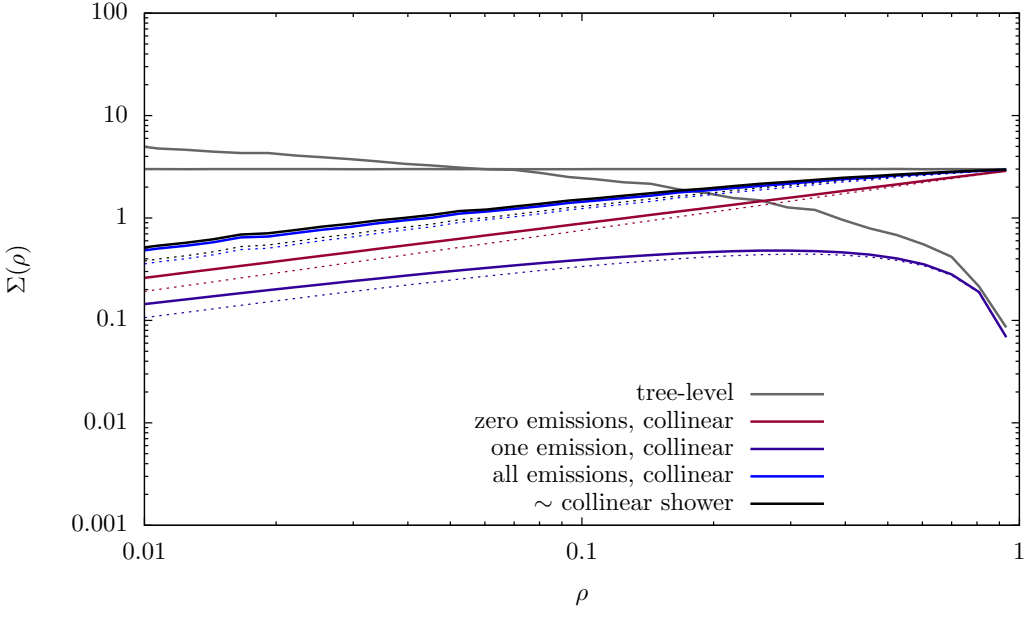
[DeAngelis, Forshaw, Plätzer – unpublished]



"$C_F \leftrightarrow \frac{C_A}{2}$"

Collinear Subtractions in Real Life

[DeAngelis, Forshaw, Plätzer – unpublished]



Collinear Subtractions in Real Life

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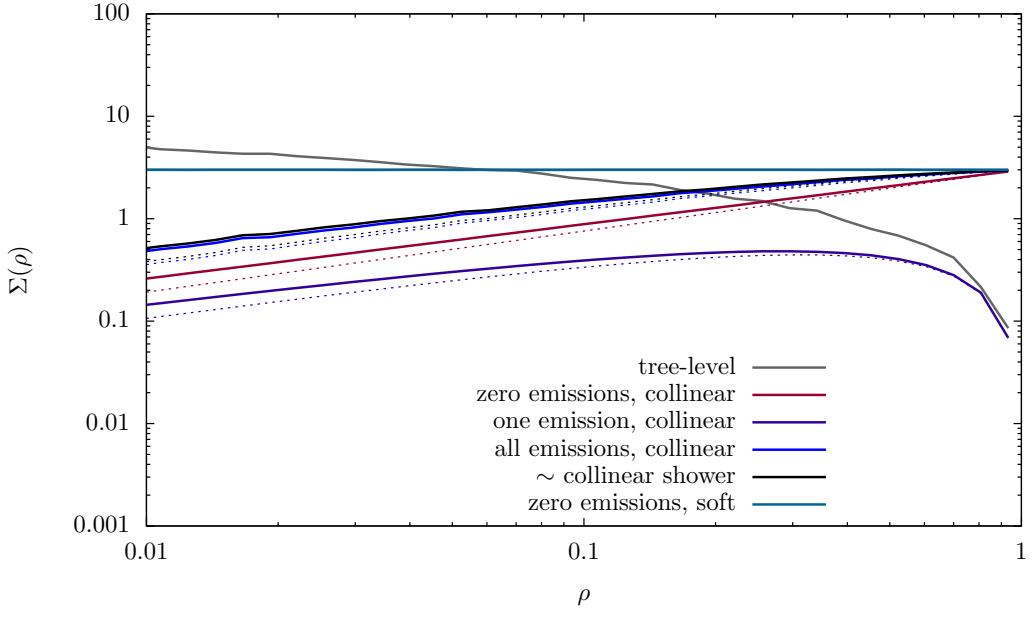
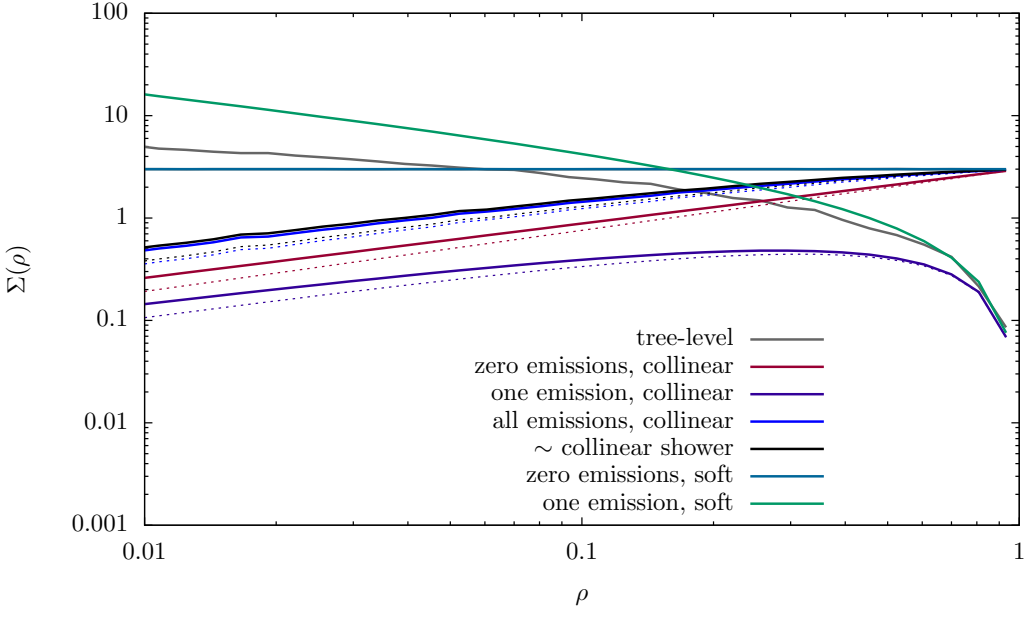


$\ln \frac{A_i \cdot A_j}{2}$

Collinear Subtractions in Real Life

[DeAngelis, Forshaw, Plätzer – unpublished]



Collinear Subtractions in Real Life

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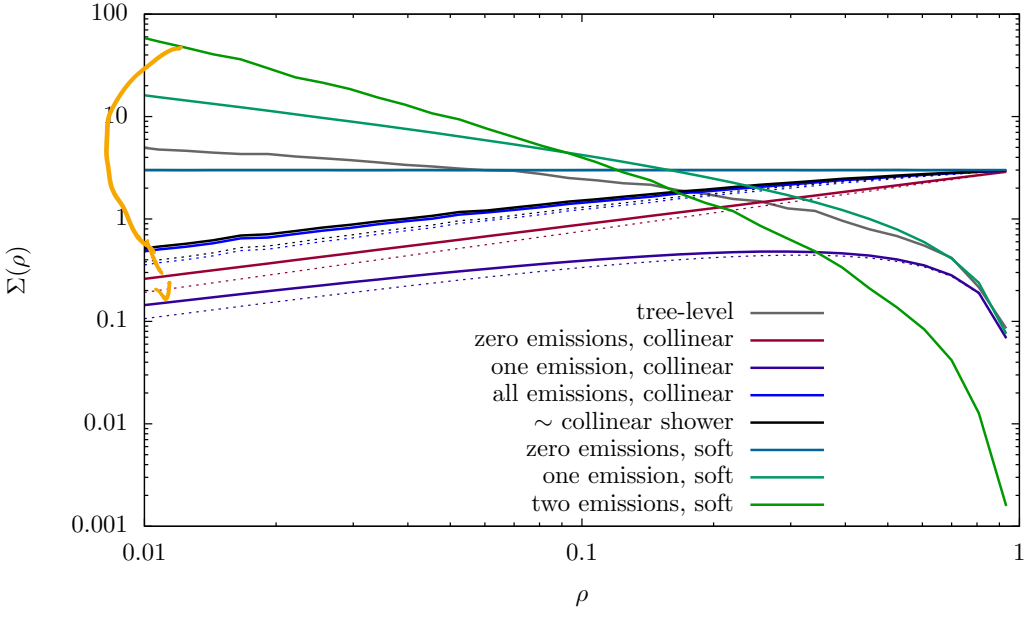
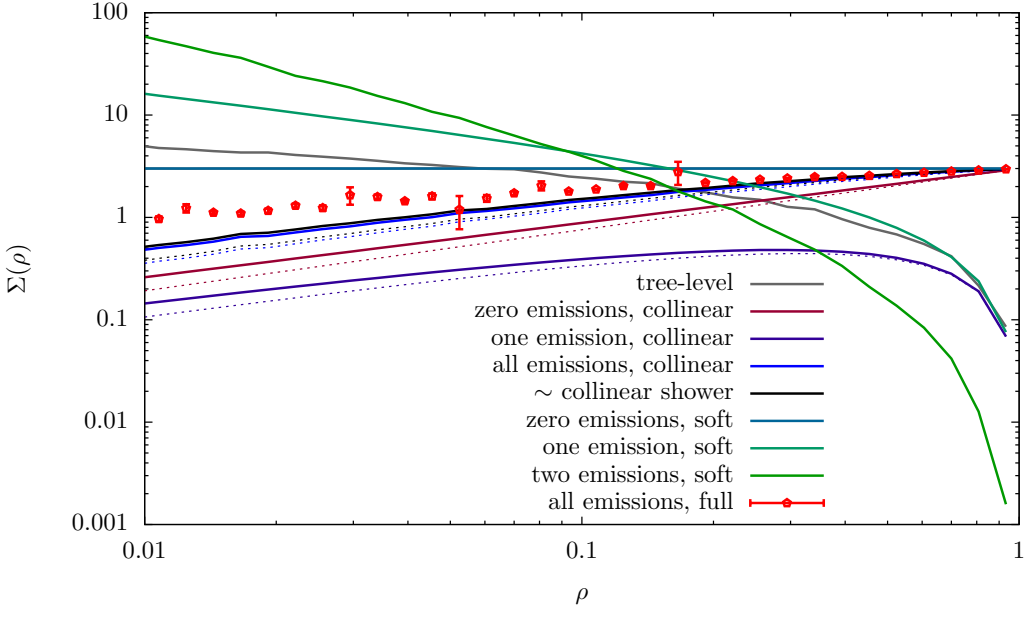


Collinear Subtractions in Real Life

[DeAngelis, Forshaw, Plätzer – unpublished]



Colour Matrix Element Corrections

Correct real emission by exact colour correlations, obtain iterative corrections by amplitude evolution.

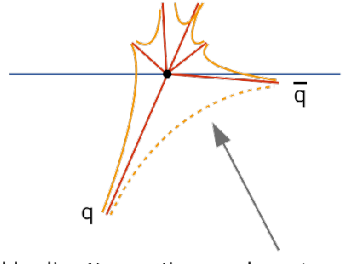
[Plätzer, Sjödahl – JHEP 1207 (2012) 042]
[Plätzer, Sjödahl, Thoren – JHEP 11 (2018) 009]

$$\mathcal{P}_{ij,k}(p_\perp^2, z; p_{\tilde{ij}}, p_{\tilde{k}}) = \mathcal{J}(p_\perp^2, z; p_{\tilde{ij}}, p_{\tilde{k}}) V_{ij,k}(p_\perp^2, z; p_{\tilde{ij}}, p_{\tilde{k}}) \times \frac{-1}{\mathbf{T}^2_{\tilde{ij}}} \frac{\langle \mathcal{M}_n | \mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_k | \mathcal{M}_n \rangle}{|\mathcal{M}_n|^2}$$

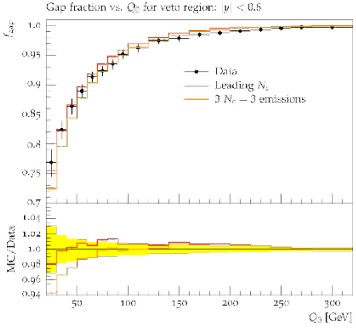
$$M_{n+1} = -\sum_{i \neq j} \sum_{k \neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j} \frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}^2_{\tilde{ij}}} T_{\tilde{k},n} M_n T^\dagger_{\tilde{ij},n}$$

Dipole branching algorithms can be supplemented by correction factors for real emission, but still lack virtual contributions beyond leading-N.

Different from amplitude level evolution.



Some subleading-N corrections can be restored.



Available in Herwig 7.2, including collinear contributions.

Colour Matrix Element Corrections vs Full Colour

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Colour matrix element corrections reconsidered.

[Holguin, Forshaw, Plätzer – arXiv:2003:06399]

[Hoeche, Reichelt – arXiv:2001.11492v1]

$$\frac{\mathrm{d}\sigma_{n+k+1}}{\sigma_{n+k}} = \mathrm{d}\Phi_{+1} 8\pi\alpha_\mathrm{s} \frac{\langle m_{n+k} | \mathbf{\Gamma}_{n+k}(\mathbf{1}) | m_{n+k} \rangle}{\langle m_{n+k} | m_{n+k} \rangle}$$

$$\mathbf{\Gamma}_n(\mathbf{\Gamma}) = -\sum_{\substack{i,j=1 \\ i\neq j}}^{n} \mathbf{T}_i \, \mathbf{\Gamma} \, \mathbf{T}_j \, \omega_{ij}$$

Cross-section unitarity is not sufficient to produce correct subleading-N virtual evolution.

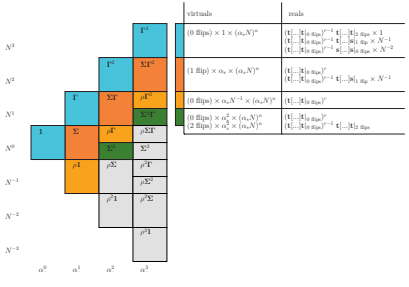
$$\mathrm{Tr}_\mathrm{norm}\left(e^{\mathbf{V}}\right) = e^{\mathrm{Tr}_\mathrm{norm}(\mathbf{V})} + \sum_{n\geq 2} \mathcal{O}\left(\alpha_\mathrm{s}^n N_\mathrm{c}^{n-2} (\mathrm{Tr}_\mathrm{norm}\delta\mathbf{V}^2 - (\mathrm{Tr}_\mathrm{norm}\delta\mathbf{V})^2)\right)$$

Compare to "exponent counting"

$$\Sigma(L) = \sum_{n=0}^{\infty} (N_\mathrm{c}\alpha_\mathrm{s})^n \sum_{m=0}^{n+1} C_{n,m}(L)$$

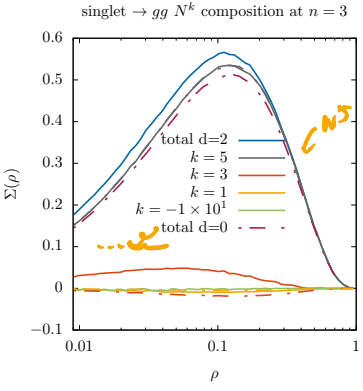
breaks for weak-counting NNLC effects

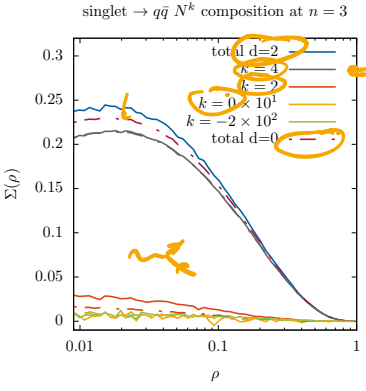
$$C_{n,m} = \underbrace{C_{n,m}^{(0)}}_{\mathrm{LC}_\Sigma} + \underbrace{\frac{1}{N_\mathrm{c}} C_{n,m}^{(1)}}_{\mathrm{NLC}_\Sigma} + \underbrace{\frac{1}{N_\mathrm{c}^2} C_{n,m}^{(2)}}_{\mathrm{NNLC}_\Sigma} + \ldots$$



Numerics with CVolver

[De Angelis, Forshaw, Plätzer — in progress]

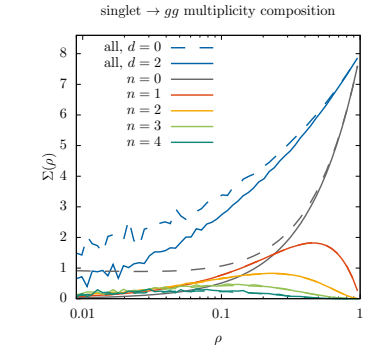


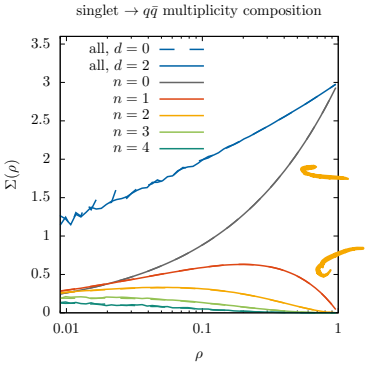


$d=2$ $1, \Sigma, \Sigma\Sigma$

$\mathrm{Tr}[A] \cdot n \cdot d\phi_n$

$= \sum_k N^k \cdot \Sigma_{k,n}$



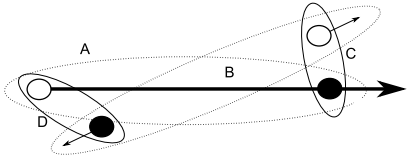


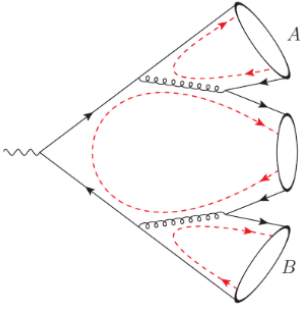
$$\Sigma(\rho) = \sum_{n=0}^{\infty} \Sigma_n(\rho)$$

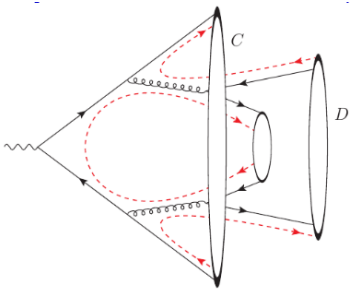
preliminary

Geometric & Baryonic Reconnection

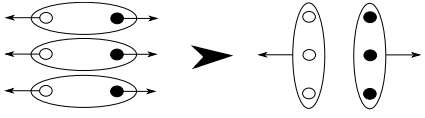
New model uses geometric measure instead of 'string length' and introduces baryonic degrees of freedom

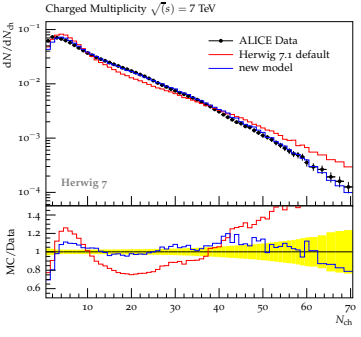


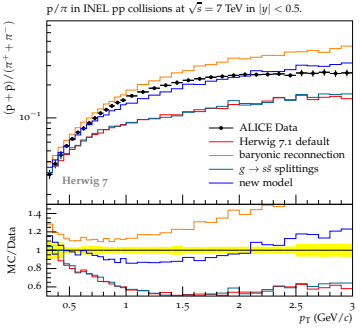




$$R_{q,qq} + R_{\bar{q},\bar{q}\bar{q}} < R_{q,\bar{q}} + R_{qq,\bar{q}\bar{q}}$$



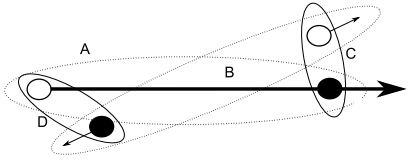




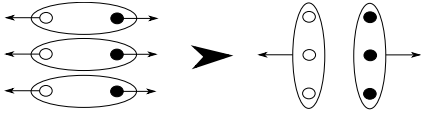
[Gieseke, Kirchgaesser, Plätzer – EPJ C 78 (2018) 99]

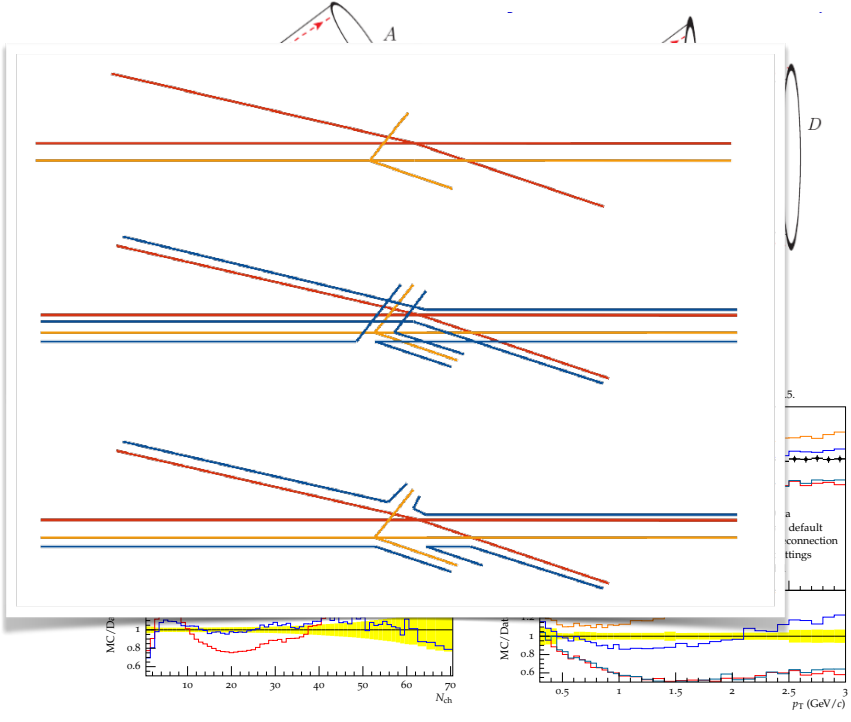
Geometric & Baryonic Reconnection

New model uses geometric measure instead of 'string length' and introduces baryonic degrees of freedom



$$R_{q,qq} + R_{\bar{q},\bar{q}\bar{q}} < R_{q,\bar{q}} + R_{qq,\bar{q}\bar{q}}$$

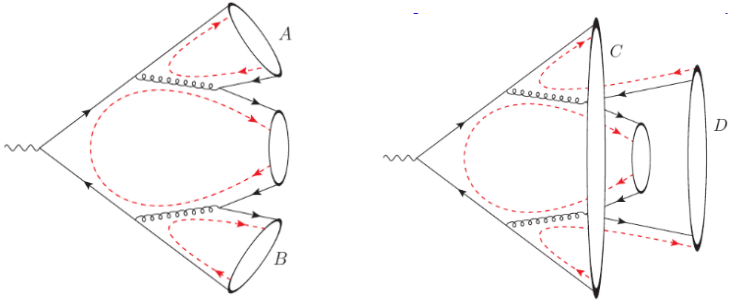


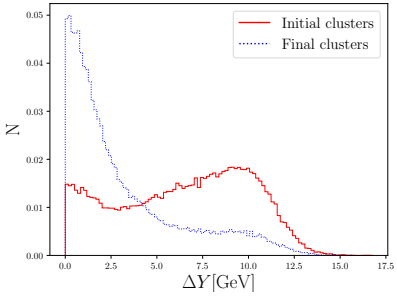


[Gieseke, Kirchgaesser, Plätzer – EPJ C 78 (2018) 99]

A New View on Colour Reconnection

[Gieseke, Kirchgaesser, Plätzer, Siodmok — JHEP 11 (2018) 149]
[Kirchgaesser, Plätzer — work in progress]

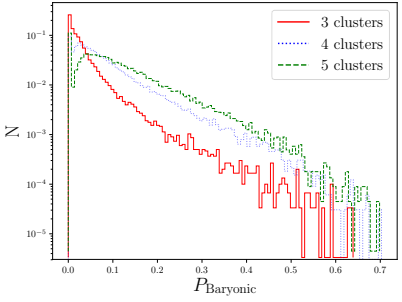




$$\mathcal{A}_{\tau\to\sigma} = \langle\sigma|\mathbf{U}\left(\{p\},\mu^2,\{M_{ij}^2\}\right)|\tau\rangle$$

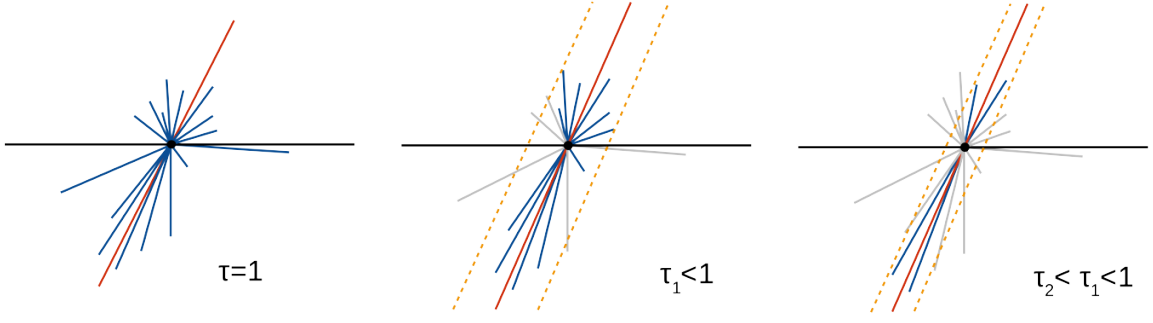
$$\mathcal{A}_{\tau\to B_{ijk}\otimes\tilde{\sigma}_{ijk}} = \langle B_{ijk}|\otimes\langle\tilde{\sigma}_{ijk}|\mathbf{U}\left(\{p\},\mu^2,\{M_{ij}^2\}\right)|\tau\rangle$$

$$|B_{ijk}\rangle = \frac{1}{N_B}\epsilon^{ijk}\epsilon_{\bar{i}\bar{j}\bar{k}}$$



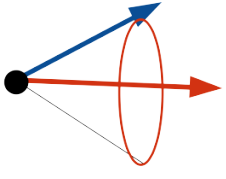
Thank you!

QCD Coherence

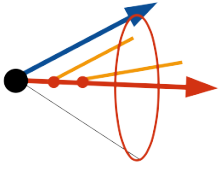


Resummation of observables which globally measure deviations from n-jet limit.

Use for analytic resummation and basis of parton shower algorithms.

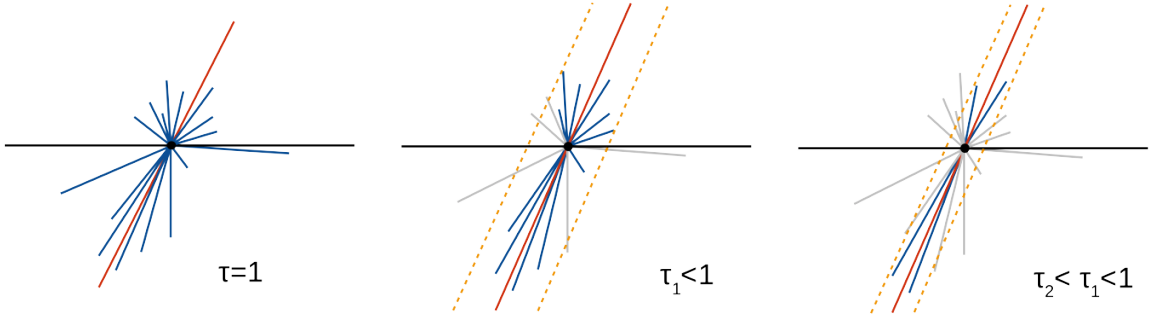


constructive interference in each collinear region



branchings order in ~ angle

QCD Coherence



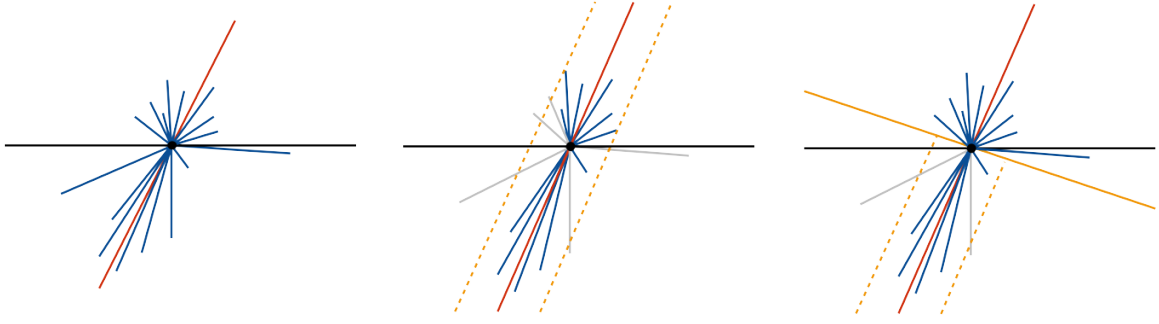
Parton branching algorithm:

Formulates iterative structure of contributions to cross sections. Could mean iterative structure of amplitudes or 'density operators'.

Parton shower:

Numerical implementation of a parton branching algorithm. Used to solve evolution equations stemming from a parton branching algorithm.

Non-global Observables



No global measure of deviation from jet configuration:
Coherent branching fails, full complexity of amplitudes strikes back.

Even with a dipole approach 1/N effects possibly become comparable to subleading logarithms, and intrinsically 10% effects. Cannot ignore in the quest for higher precision.

Parton Showers from Amplitude Evolution

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[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Start from amplitude evolution equations

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = -\,\mathbf{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\mathbf{\Gamma}_n^\dagger(q_\perp) + \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\perp})\,\mathbf{A}_{n-1}(q_{n\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\perp})\,q_\perp\,\delta(q_\perp - q_{n\perp})$$

$$\mathrm{d}\sigma_n(\mu) = \left(\prod_{i=1}^{n}\mathrm{d}\Pi_i\right)\mathrm{Tr}\,\mathbf{A}_n(\mu) \qquad \Sigma(\mu;\{p\}_0,\{v\}) = \int \sum_n \mathrm{d}\sigma_n(\mu)\,u(\{p\}_n,\{v\})$$

Combine insight from soft evolution, large-N expansions and collinear subtractions:

- Can we reproduce existing algorithms as well-defined limits of amplitude evolution?
- Can we use this to obtain an ideal combination of coherent and dipole branching?

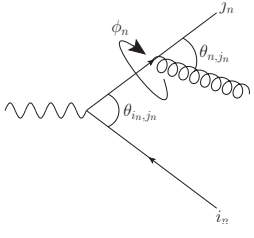
Collinear Subtractions & Angular Ordering

[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Collinear subtractions within a dipole?
Recall angular ordering and coherent branching:

$$\frac{n_{i_n}\cdot n_{j_n}}{n_{i_n}\cdot n\; n_{j_n}\cdot n} = P_{i_n j_n} + P_{j_n i_n}, \quad \text{where} \quad 2P_{i_n j_n} = \frac{n_{i_n}\cdot n_{j_n} - n_{i_n}\cdot n}{n_{i_n}\cdot n\; n_{j_n}\cdot n} + \frac{1}{n_{i_n}\cdot n}$$

Azimuthal average will result in angular ordering and simplify colour structures.



$$\left\langle |\mathcal{M}_n|^2\, u(\{p\}_n)\right\rangle_{1,\dots,n} = \left\langle |\mathcal{M}_n|^2\right\rangle_{1,\dots,n} \left\langle u(\{p\}_n)\right\rangle_{1,\dots,n}$$
$$+ \sum_{m=1}^{n} \sigma_m(\left\langle |\mathcal{M}_n|^2\right\rangle_{1,\dots,n})\, \sigma_m(\left\langle u(\{p\}_n)\right\rangle_{1,\dots,n})\, \mathrm{Cor}_m(\left\langle |\mathcal{M}_n|^2\right\rangle_{1,\dots,n}, \left\langle u(\{p\}_n)\right\rangle_{1,\dots,n})$$
$$+ \text{higher order correlations}$$

irrelevant for global observables at NLL

Collinear Subtractions & Angular Ordering

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[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

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Azimuthal average will result in angular ordering and simplify colour structures.

$$\zeta\frac{\partial\,\langle|\mathcal{M}_n(\zeta)|^2\rangle_{1,\ldots,n}}{\partial\zeta} \approx$$

$$-\sum_{j_{n+1}}\sum_{v}\frac{\alpha_s}{\pi}\int \mathrm{d}z\,\mathcal{P}_{vv_{j_{n+1}}}(z)\,\langle\Theta_{\text{on shell}}\rangle_{n+1}\,\langle|\mathcal{M}_n(\zeta)|^2\rangle_{1,\ldots,n} + \sum_{v}\frac{\alpha_s}{\pi}\mathcal{P}_{vv_{j_n}}(z_n)$$

$$\times\,\langle\Theta_{\text{on shell}}\rangle_n\int \mathrm{d}^4p_{j_n}\;\delta^4(p_{j_n} - z_n^{-1}\tilde{p}_{j_n})\;\langle|\mathcal{M}_{n-1}(\zeta_{n,j_n})|^2\rangle_{1,\ldots,n-1}\;\zeta_{n,j_n}\;\delta(\zeta-\zeta_{n,j_n})$$

includes momentum mapping and physical phase space boundaries for on-shell partons

Dipoles, Recoil & Partitioning

[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Colour in dipole shower evolution follows the BMS derivation.

$$
\begin{aligned}
q_\perp \mathrm{Leading}^{(0)}_{\tau\sigma}\left[\frac{\partial \hat{\mathbf{A}}_n(q_\perp)}{\partial q_\perp}\right] \approx -\frac{\alpha_\mathrm{s}}{\pi}\int \frac{\mathrm{d}S_2^{(q_{n+1})}}{4\pi} \sum_{i_{n+1},j_{n+1}\,\mathrm{c.c.\,in}\,\sigma} \\
\times 4\lambda_{i_{n+1}}\bar{\lambda}_{j_{n+1}} N_\mathrm{c} \int \delta q_{n+1\,\perp}^{(i_{n+1},j_{n+1})}(q_\perp)\,\Theta_{\mathrm{on\,shell}}\,\delta_{\tau\sigma}\,\mathrm{Leading}^{(0)}_{\tau\sigma}\left[\hat{\mathbf{A}}_n(q_\perp)\right] \\
+\int\left(\prod_{i_n}\mathrm{d}^4 p_{i_n}\right) \sum_{i_n,j_n\,\mathrm{c.c.\,in}\,\sigma} \lambda_i\bar{\lambda}_j N_\mathrm{c} \int \delta q_{n\,\perp}^{(i_n,j_n)}(q_{n\,\perp})\,\mathfrak{R}^{\mathrm{soft}}_{i_n j_n} \\
\times \delta_{\tau\sigma}\,\mathrm{Leading}^{(0)}_{\tau\backslash n\,\sigma\backslash n}\left[\hat{\mathbf{A}}_{n-1}(q_{n\,\perp})\right]\; q_\perp\,\delta(q_\perp - q_{n\,\perp}).
\end{aligned}
$$

Combination with collinear contributions: partition using coherent branching logic

$$
\frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\; p_{j_n}\cdot q_n} \longrightarrow \frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\; p_{j_n}\cdot q_n} - \frac{T\cdot p_{j_n}}{T\cdot q_n}\frac{1}{p_{j_n}\cdot q_n} + \frac{T\cdot p_{i_n}}{T\cdot q_n}\frac{1}{p_{i_n}\cdot q_n}
$$

Dipoles, Recoil & Partitioning

[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Evolution now per colour flow matrix element:

$$
\begin{aligned}
& q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp} \\
& \approx -\frac{\alpha_s}{\pi} \sum_{i_{n+1}^c} \int \mathrm{d}q_\perp^{(i_{n+1}^c, \overline{i^c}_{n+1})} \delta(q_\perp^{(i_{n+1}^c, \overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z\, \Theta_{\text{on shell}}\, P_{v_{i_n} v_{i_n}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2 \\
& + \frac{\alpha_s}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n} \right) \mathfrak{R}_{i_n^c}^{\text{dipole}}\, P_{v_{i_n} v_{i_n}}(z_n)\, q_\perp \delta(q_{n\perp}^{(i_n^c, \overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i_n^c, \overline{i^c}_n)})|^2
\end{aligned}
$$

New dipole shower evolution, reduces to coherent branching upon azimuthal average and BMS evolution for large-angle soft.

$$
\mathfrak{R}_{i_n^c}^{\text{dipole}} = \left(\frac{1}{2} + \mathrm{Asym}_{i_n^c \overline{i^c}_n}(q_n) \right) \mathfrak{R}_{i_n^c}
$$

$$
(q_{n\perp}^{(i_n^c, \overline{i^c}_n)})^2 = \frac{2(p_{i_n^c} \cdot q_n)(p_{\overline{i^c}_n} \cdot q_n)}{p_{i_n^c} \cdot p_{\overline{i^c}_n}}
$$

$$
\mathrm{Asym}_{i_n^c \overline{i^c}_n}(q_n) = \left[\frac{T \cdot p_{i_n^c}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i_n^c \overline{i^c}_n)})^2}{p_{i_n^c} \cdot q_n} - \frac{T \cdot p_{\overline{i^c}_n}}{4T \cdot q_n} \frac{(q_{n\perp}^{(i_n^c \overline{i^c}_n)})^2}{p_{\overline{i^c}_n} \cdot q_n} \right]
$$

Dipoles, Recoil & Partitioning

[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Recoil needs to be addressed separately.

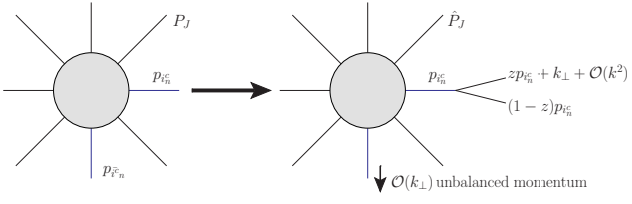
$$\hat{q}_n = (1 - z_n) p_{i_n^c} + k_\perp + \frac{(q_{n\perp}^{(i_n^c \overline{i^c}_n)})^2}{1 - z_n} \frac{p_{\overline{i^c}_n}}{2 p_{i_n^c} \cdot p_{\overline{i^c}_n}},$$

$$\hat{p}_{i_n^c} = z_n p_{i_n^c}, \qquad (q_{n\perp}^{(i_n^c \overline{i^c}_n)})^2 = -k_\perp^2, \qquad k_\perp \cdot p_{i_n^c} = k_\perp \cdot p_{\overline{i^c}_n} = 0$$

balance only
longitudinal fractions

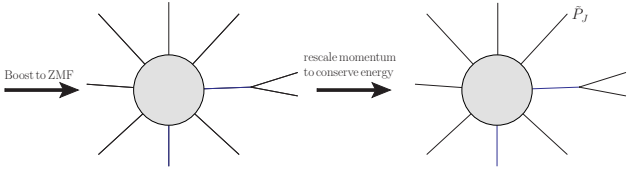
Redistribute recoil globally, but per emission, inspired by Herwig's kinematic reconstruction.

recent discussion in:
[Bewick, Ferrario, Richardson, Seymour — arXiv:1904:11866]



This scheme is free of the issues encountered for local dipole recoils.

[Dasgupta, Dreyer, Hamilton, Monni, Salam — JHEP 09 (2018) 033]



Parton Branching at Amplitude Level

universität wien

Formulate iterative structures quite generally, with the goal of systematically approximating the iteration, not "iterating an approximation".

Similar in spirit to Nagy & Soper

Theoretical control

Guiding principle to incremental improvements of existing algorithms.

Actual predictions

Explore new methods and paradigms in their own right.

Non-global observables and accuracy for global observables both set the level of complexity.