Color Evolution in Parton Showers

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Perturbative Cross Section

The main focus of this workshop is to calculate the pQCD cross sections as precise as possible, thus we have a pretty integral

$$\sigma[O_J] = \sum_m \frac{1}{m!} \sum_{\{a,b,f_1,\ldots,f_m\}} \int_0^1 d\eta_{\rm a} \overbrace{\int_{\eta_{\rm a}}^1 \frac{dz}{z}\, \Gamma^{-1}_{aa'}(z,\mu^2)\, f_{a'/A}(\eta_{\rm a}/z,\mu^2)}^{\textit{Bare PDF}}$$

$$\times \int_0^1 d\eta_{\rm b} \int_{\eta_{\rm b}}^1 \frac{d\bar{z}}{\bar{z}}\, \Gamma^{-1}_{bb'}(\bar{z},\mu^2)\, f_{b'/A}(\eta_{\rm b}/\bar{z},\mu^2)$$

$$\times \int d\phi(\eta_{\rm a}\eta_{\rm b}s,\{p,f\}_m)\, \big\langle M(\{p,f\}_m)\big|\, \underbrace{O_J(\{p,f\}_m)}_{\textit{IR safe measurement operator}}\, \overbrace{\big|M(\{p,f\}_m)\big\rangle}^{\textit{Partonic matrix element}}$$

$$+ \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{\mu_J^2}\right)$$

Error of the factorization
(Cannot be beaten by calculating higher and higher order.)

and here the MSbar parton in parton renormalised PDF is

$$\Gamma_{aa'}(z,\mu^2) = \delta(1-z)\delta_{aa'} - \frac{\alpha_{\rm s}(\mu^2)}{2\pi}\frac{1}{\epsilon}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} P_{aa'}(z) + \cdots \quad .$$

Statistical Space

Introducing the statistical space we can represent the QCD density operator as a vector

$$\sigma[O_J] = \underbrace{\big(1\big|}_{\text{All the initial and final state sums and integrals}} \mathcal{O}_J \overbrace{\big[\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\big]}^{\text{Bare PDFs for both incoming hadrons}} \underbrace{\big|\rho(\mu^2)\big)}_{|M\rangle\langle M|}$$

QCD density operator

Describes the fully exclusive partonic final states.

The physical cross section is RG invariant as well as the QCD density operator and the bare PDF.

$$\mu^2 \frac{d}{d\mu^2} \big|\rho(\mu^2)\big) = \mu^2 \frac{d}{d\mu^2} \big[\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\big] = 0 + \mathcal{O}(\alpha_{\mathrm{s}}^{k+1})$$

Perturbative expansion of the density operator

$$\big|\rho(\mu^2)\big) = \sum_{n=0}^{k} \left[\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right]^n \sum_{\substack{n_{\mathrm{R}}=0 \\ n_{\mathrm{R}}+n_{\mathrm{V}}=n}}^{n} \sum_{n_{\mathrm{V}}=0}^{n} \big|\rho^{(n_{\mathrm{R}},n_{\mathrm{V}})}(\mu^2)\big)$$

Number of real radiations

Number of loops

Statistical Space

A vector in the statistical space can be translated as

$$\left(\{p,f,c,s,c',s'\}_m\middle|\rho\right) \Longleftrightarrow \left\langle\{c,s\}_m\middle|M(\{p,f\}_m)\right\rangle\left\langle M(\{p,f\}_m)\middle|\{c',s'\}_m\right\rangle$$

An operator in the statistical space corresponds to a direct products of the corresponding quantum operators:

$$\mathcal{A}(\mu^2) \Longleftrightarrow A^L(\mu^2)\otimes A^R(\mu^2)^\dagger$$

When operators act on a state we have

$$\cdots\mathcal{A}_3(\mu_3^2)\mathcal{A}_2(\mu_2^2)\mathcal{A}_1(\mu_1^2)\big|\rho\big) \Longleftrightarrow \cdots A_3^L(\mu_3^2)A_2^L(\mu_2^2)A_1^L(\mu_1^2)\big|M\big\rangle\big\langle M\big|A_1^R(\mu_1^2)^\dagger A_2^R(\mu_2^2)^\dagger A_3^R(\mu_3^2)^\dagger\cdots$$

pQCD Cross Sections

ZN, D. Soper, **Phys.Rev. D98** (2018) no.1, 014034

Subtractions

Singularities cancel each other here

$$\sigma[O_J] = \overbrace{\big(1\big|\,\big[\mathcal{F}(\mu^2)\circ\mathcal{Z}_F(\mu^2)\big]\,\mathcal{D}(\mu^2)}\,\underbrace{\mathcal{D}^{-1}(\mu^2)\,\mathcal{O}_J\,\big|\rho(\mu^2)\big)}_{=|\rho_{\rm H}(\mu^2))}$$

Hard part, finite in d=4 dimension

$$+\mathcal{O}(\alpha_{\rm s}^{k+1}L^{2k+2})$$

$$+\mathcal{O}(\Lambda^2_{QCD}/\mu_J^2)$$

By solving the corresponding renormalization group equation

Unitary shower

Hopefully n << 2k+1

$$\sigma[O_J] = \big(1\big|\mathcal{O}_J\,\overbrace{\mathcal{U}(\mu_{\rm f}^2,\mu^2)}\,\underbrace{\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu^2)}\,\mathcal{F}(\mu^2)\,\big|\rho_{\rm H}(\mu^2)\big) + \mathcal{O}(\overbrace{\alpha_{\rm s}^{k+1}L^n}) + \mathcal{O}(\mu_{\rm f}^2/\mu_J^2)$$

Resummation of threshold effects

Unitary Shower Operator

Here we focus on the the unitary shower

$$\mathcal{U}(\mu_{\rm f}^2,\mu_{\rm H}^2)=\mathbb{T}\exp\left(\int_{\mu_{\rm f}^2}^{\mu_{\rm H}^2}\frac{d\mu^2}{\mu^2}\,\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\left[\mathcal{S}^{(1)}(\mu^2)+\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\mathcal{S}^{(2)}(\mu^2)+\cdots\right]\right)$$

Here we are interested only at first order level:

Real emissions — **Imaginary part of the 1-loop contributions**

$$\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\mathcal{S}^{(1)}(\mu^2)=\overbrace{\mathcal{H}(\mu^2)}-\underbrace{\mathcal{V}_{\rm Re}(\mu^2)}-\overbrace{{\rm i}\pi\mathcal{V}_{{\rm i}\pi}(\mu^2)}$$

Inclusive splitting operator

Unitary condition tells us:

$$\big(1\big|\mathcal{S}^{(1)}(\mu^2)=\big(1\big|\big[\mathcal{H}(\mu^2)-\mathcal{V}_{\rm Re}(\mu^2)\big]=\big(1\big|\mathcal{V}_{{\rm i}\pi}(\mu^2)=0$$

Evolution Equation

The shower operator obeys the following integral equation:

$$\mathcal{U}(\mu_2^2,\mu_1^2) = \mathcal{N}(\mu_2^2,\mu_1^2) + \int_{\mu_2^2}^{\mu_1^2} \frac{d\mu^2}{\mu^2}\,\mathcal{U}(\mu_2^2,\mu^2)\mathcal{H}(\mu^2)\mathcal{N}(\mu^2,\mu_1^2)$$

where the no-splitting (**Sudakov**) operator is

$$\mathcal{N}(\mu_2^2,\mu_1^2) = \mathbb{T}\exp\left\{-\int_{\mu_2^2}^{\mu_1^2}\frac{d\mu^2}{\mu^2}\underbrace{\left[\mathcal{V}_{\mathrm{Re}}(\mu^2)+\mathrm{i}\pi\mathcal{V}_{\mathrm{i}\pi}(\mu^2)\right]}\right\}$$

This is not a diagonal operator and it is impossible to diagonalise when we have ~O(10) partons.

Let's define a systematic approximation!

LC+ Approximation

ZN, D. Soper, **JHEP06** (2012) 044

The **real splittings** are described by

$$\mathcal{H}(\mu^2)\Big|\{p,f,c',c\}_m\Big) \propto \sum_{l,k} H_{lk}(\mu^2)\Big|\{p,f\}_m\Big) \left\{ t_l^\dagger \Big|\{c\}_m\Big\rangle\Big\langle\{c'\}_m\Big| t_k + t_k^\dagger \Big|\{c\}_m\Big\rangle\Big\langle\{c'\}_m\Big| t_l \right\}$$

The index l always represents the emitter parton and the emitted parton can be collinear only with l.

The **inclusive splitting operator** is

$$\mathcal{V}_{\mathrm{Re}}(\mu^2)\Big|\{p,f,c',c\}_m\Big) \propto \sum_{l,k} V_{lk}(\mu^2)\Big|\{p,f\}_m\Big) \left\{ \Big|\{c\}_m\Big\rangle\Big\langle\{c'\}_m\Big| [t_k \cdot t_l^\dagger] + [t_l \cdot t_k^\dagger] \Big|\{c\}_m\Big\rangle\Big\langle\{c'\}_m\Big| \right\}$$

We need an approximation (only in the color space) that

- **can handle color interference** contributions
- is as minimal approximation as possible
- is **exact in the collinear and soft-collinear** regions
- makes some **harm only in the wide angle soft** region
- preserves unitary

LC+ Approximation

We insert a projection **only on the spectator side**

$$t_k^\dagger \big| \{c\}_m \big\rangle \longrightarrow C(l, m+1) t_k^\dagger \big| \{c\}_m \big\rangle$$
$$\big\langle \{c'\}_m \big| t_k \longrightarrow \big\langle \{c'\}_m \big| t_k C(l, m+1)^\dagger$$

The **operator** $C(l, m+1)$ is defined by its action on the basis states:

$$C(l, m+1) \big| \{\hat{c}\}_{m+1} \big\rangle = \begin{cases} \big| \{\hat{c}\}_{m+1} \big\rangle & \text{if } l \text{ and } m+1 \text{ are color connected in } \{\hat{c}\}_{m+1} \\ 0 & \text{otherwise} \end{cases}$$

(In string basis l and $m+1$ are color connected when they are next to each other along the fermion line.)

In the inclusive splitting operator, the color simplifies a lot:

$$[t_l \cdot t_k^\dagger] \big| \{c\}_m \big\rangle \longrightarrow [t_l \cdot C(l, m+1) t_k^\dagger] \big| \{c\}_m \big\rangle = \big| \{c\}_m \big\rangle \frac{t_l^2}{1 + \delta_{\mathrm{g} f_l}}$$

$$\big\langle \{c'\}_m \big| [t_k \cdot t_l^\dagger] \longrightarrow \big\langle \{c'\}_m \big| [t_k C(l, m+1)^\dagger \cdot t_l^\dagger] = \frac{t_l^2}{1 + \delta_{\mathrm{g} f_l}} \big\langle \{c'\}_m \big|$$

LC+ Approximation

In LC+ approximation every basis state is eigenstate of the inclusive splitting operator

$$\mathcal{V}^{\mathrm{LC+}}(\mu^2)\big|\{p,f,c',c\}_m\big) = \lambda(\{p,f,c',c\}_m,\mu^2)\big|\{p,f,c',c\}_m\big)$$

and we have **Sudakov factor** instead of Sudakov operator

$$\mathcal{N}^{\mathrm{LC+}}(\mu_2^2,\mu_1^2)\big|\{p,f,c',c\}_m\big) = \exp\left\{-\int_{\mu_2^2}^{\mu_1^2}\frac{d\mu^2}{\mu^2}\lambda(\{p,f,c',c\}_m,\mu^2)\right\}\big|\{p,f,c',c\}_m\big)$$

Based on this we can define the **LC+ parton shower** and its evolution equation is

$$\mathcal{U}^{\mathrm{LC+}}(\mu_2^2,\mu_1^2) = \mathcal{N}^{\mathrm{LC+}}(\mu_2^2,\mu_1^2) + \int_{\mu_2^2}^{\mu_1^2}\frac{d\mu^2}{\mu^2}\,\mathcal{U}^{\mathrm{LC+}}(\mu_2^2,\mu^2)\mathcal{H}^{\mathrm{LC+}}(\mu^2)\mathcal{N}^{\mathrm{LC+}}(\mu^2,\mu_1^2)$$

Beyond LC+

ZN, D. Soper, **Phys.Rev. D99** (2019) no.5, 054009

Now we can define the operators of the soft wide angle emissions

$$\mathcal{H}(\mu^2) = \mathcal{H}^{\mathrm{LC+}}(\mu^2) + \Delta\mathcal{H}(\mu^2)$$
$$\mathcal{V}(\mu^2) = \mathcal{V}_{\mathrm{Re}}(\mu^2) + \mathrm{i}\pi\mathcal{V}_{\mathrm{i}\pi}(\mu^2) = \mathcal{V}^{\mathrm{LC+}}(\mu^2) + \Delta\mathcal{V}(\mu^2)$$

With these the full shower is

$$\mathcal{U}(\mu_2^2,\mu_1^2) = \mathcal{U}^{\mathrm{LC+}}(\mu_2^2,\mu_1^2) + \int_{\mu_2^2}^{\mu_1^2}\frac{d\mu^2}{\mu^2}\,\mathcal{U}(\mu_2^2,\mu^2)\left[\Delta\mathcal{H}(\mu^2) - \Delta\mathcal{V}(\mu^2)\right]\mathcal{U}^{\mathrm{LC+}}(\mu^2,\mu_1^2)$$

One can expand this in terms of the soft wide angle operators at a given order. In principle this is what we want, but this form is not efficient for implementation.
Let's **try something else**:

$$\mathcal{N}(\mu_2^2,\mu_1^2) = \underbrace{\mathcal{X}(\mu_2^2,\mu_1^2)}\,\mathcal{N}^{\mathrm{LC+}}(\mu_2^2,\mu_1^2)$$

Hopefully it is **simple enough** to deal with it perturbatively.

Beyond LC+

The evolution equation is

$$\mathcal{U}(\mu_2^2,\mu_1^2)=\mathcal{X}(\mu_2^2,\mu_1^2)\mathcal{N}^{\rm LC+}(\mu_2^2,\mu_1^2)+\int_{\mu_2^2}^{\mu_1^2}\frac{d\mu^2}{\mu^2}\,\mathcal{U}(\mu_2^2,\mu^2)\left[\mathcal{H}^{\rm LC+}(\mu^2)+\Delta\mathcal{H}(\mu^2)\right]\mathcal{X}(\mu^2,\mu_1^2)\mathcal{N}^{\rm LC+}(\mu^2,\mu_1^2)$$

When we iterate this equation we can **control the number of ΔH** operator **insertions**.

The X operator obeys its evolution equation:

$$\mathcal{X}(\mu_2^2,\mu_1^2)=1-\int_{\mu_2^2}^{\mu_1^2}\frac{d\mu^2}{\mu^2}\;\mathcal{X}(\mu_2^2,\mu^2)\,\mathcal{N}^{\rm LC+}(\mu_2^2,\mu^2)\Delta\mathcal{V}(\mu^2)\,\mathcal{N}^{\rm LC+}(\mu_2^2,\mu^2)^{-1}$$

It is not immediately obvious but this operator depends **only pure soft** contributions

$$\mathcal{X}(\mu_2^2,\mu_1^2)=1-\int_{\mu_2^2}^{\mu_1^2}\frac{d\mu^2}{\mu^2}\;\mathcal{X}(\mu_2^2,\mu^2)\,\mathcal{N}^{\rm LC+}_{\rm soft}(\mu_2^2,\mu^2)\Delta\mathcal{V}(\mu^2)\,\mathcal{N}^{\rm LC+}_{\rm soft}(\mu_2^2,\mu^2)^{-1}$$

and the soft **Sudakov factor** is

$$\mathcal{N}^{\rm LC+}_{\rm soft}(\mu_2^2,\mu_1^2)\big|\{p,f,c',c\}_m\big)=\exp\left\{-\int_{\mu_2^2}^{\mu_1^2}\frac{d\mu^2}{\mu^2}\,\underbrace{\lambda_{\rm soft}(\{p,f,c',c\}_m,\mu^2)}\right\}\big|\{p,f,c',c\}_m\big)$$

It is rather **simple** and can be computed "***quasi-analytically***".

Beyond LC+

Expanding the shower operator in terms of ΔH and ΔV operators, we can write

$$\mathcal{U}(\mu_2^2,\mu_1^2) = \mathcal{U}^{\mathrm{LC+}}(\mu_2^2,\mu_1^2) + \underbrace{\mathcal{U}^{(1)}(\mu_2^2,\mu_1^2)}_{\sim\mathcal{O}([\Delta\mathcal{H}(\mu^2)-\Delta\mathcal{V}(\mu^2)])} + \overbrace{\mathcal{U}^{(2)}(\mu_2^2,\mu_1^2)}^{\sim\mathcal{O}([\Delta\mathcal{H}(\mu^2)-\Delta\mathcal{V}(\mu^2)]^2)} + \ldots$$

This expansion is systematic and the **unitary condition** is satisfied term by term,

$$(1|\mathcal{U}^{\mathrm{LC+}}(\mu_2^2,\mu_1^2) = (1|$$

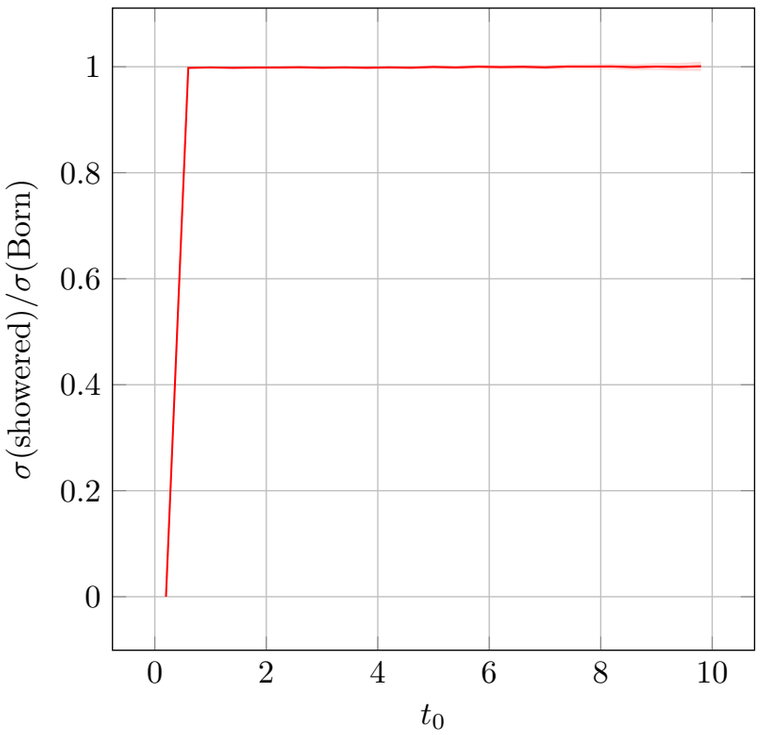
and for the corrections

$$(1|\mathcal{U}^{(k)}(\mu_2^2,\mu_1^2) = 0 \qquad \text{for } k = 1,2,3,\ldots$$

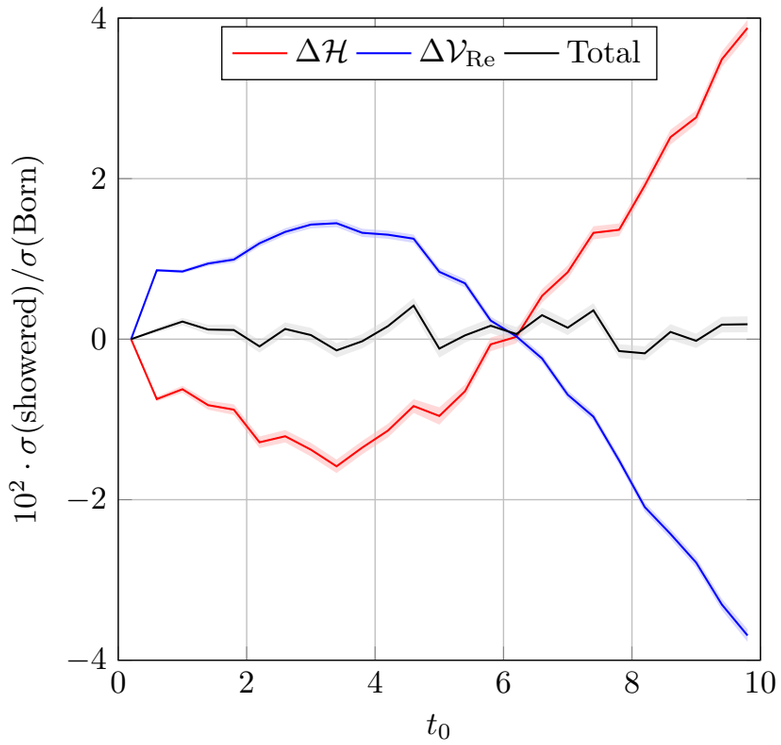
We can test this numerically!

Unitary Test

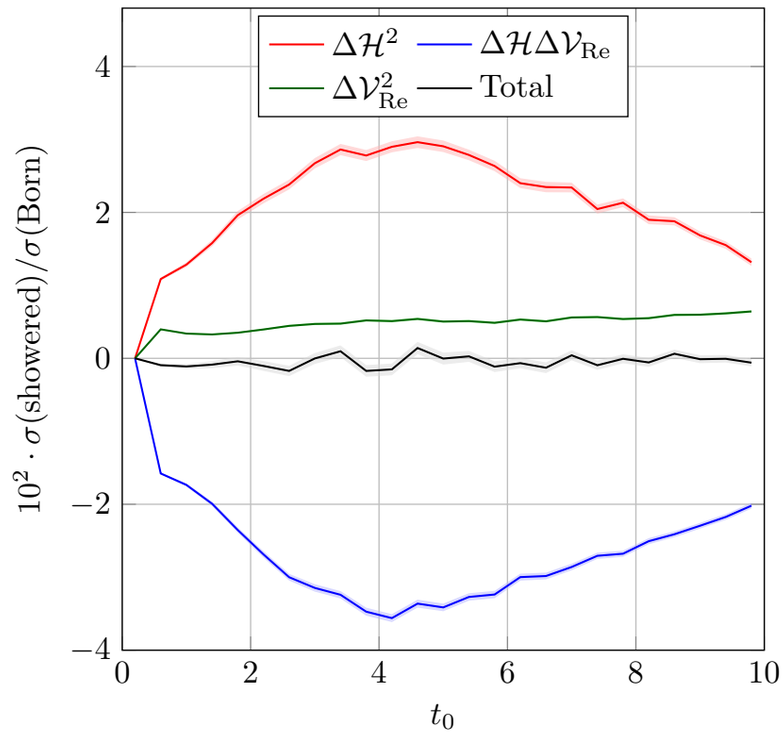
LC+ contribution



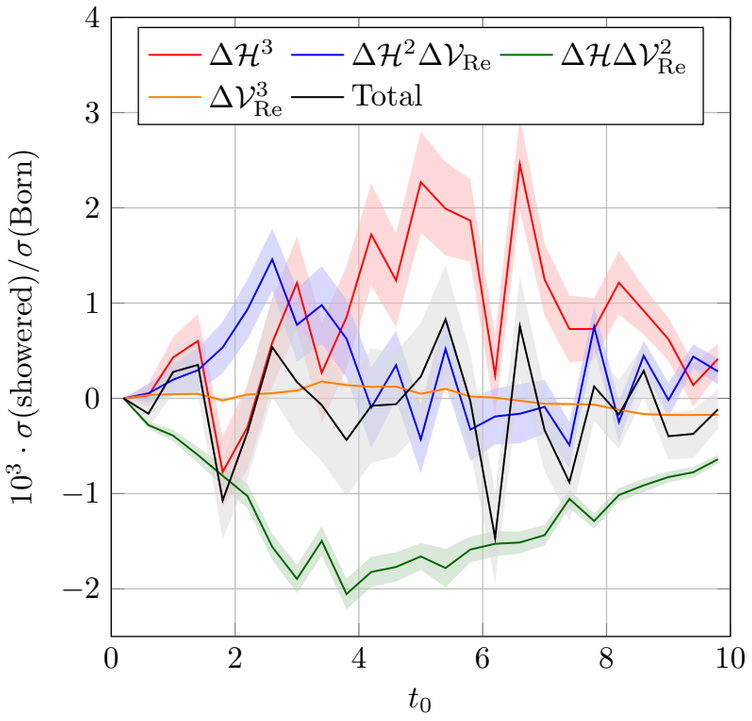
$A + B = 1$ contributions



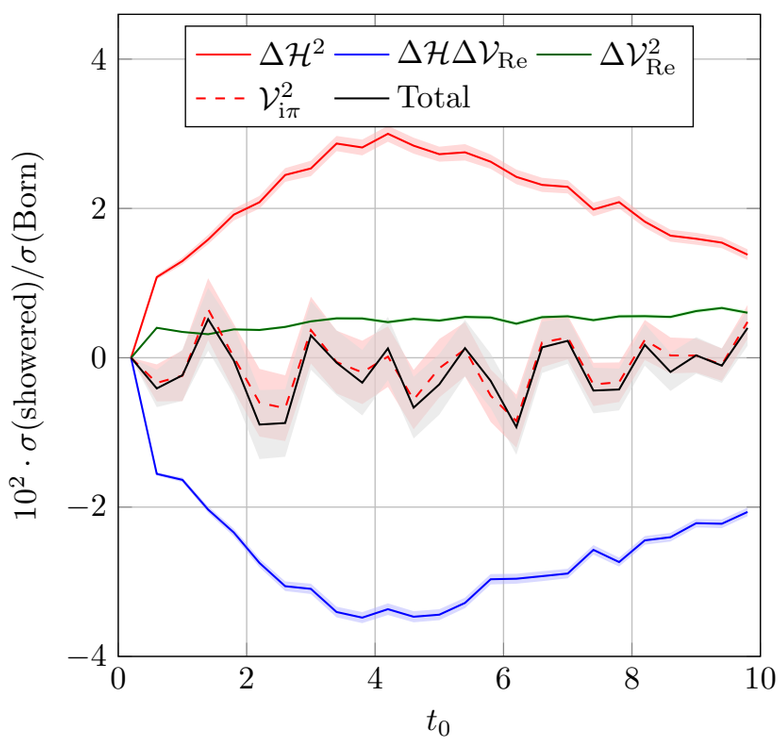
$A + B = 2$ contributions



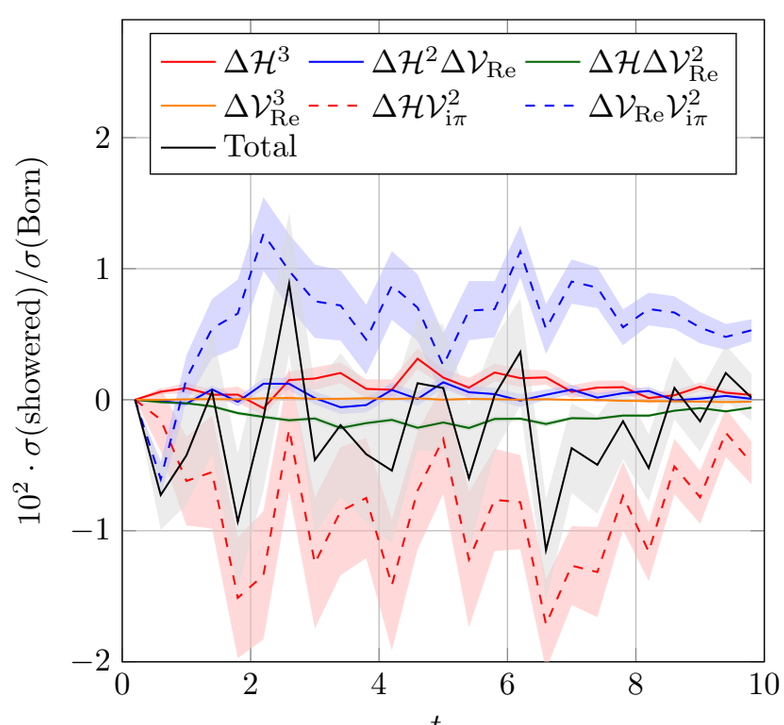
$A + B = 3$ contributions



$A + B + C = 2$ contributions

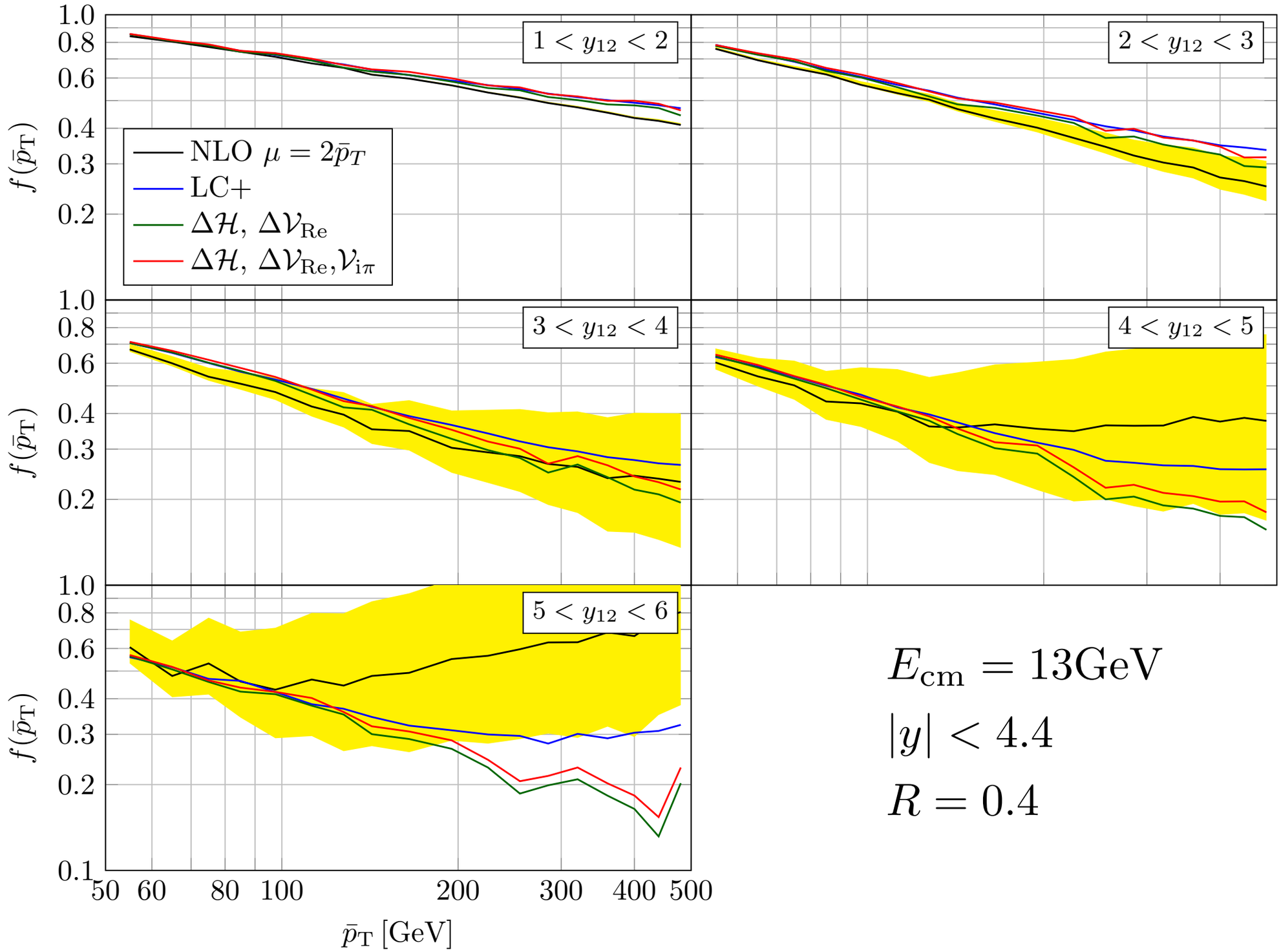


$A + B + C = 3$ contributions



Rapidity Gap Survival

ZN, D. Soper, **arXiv:1905.07176**

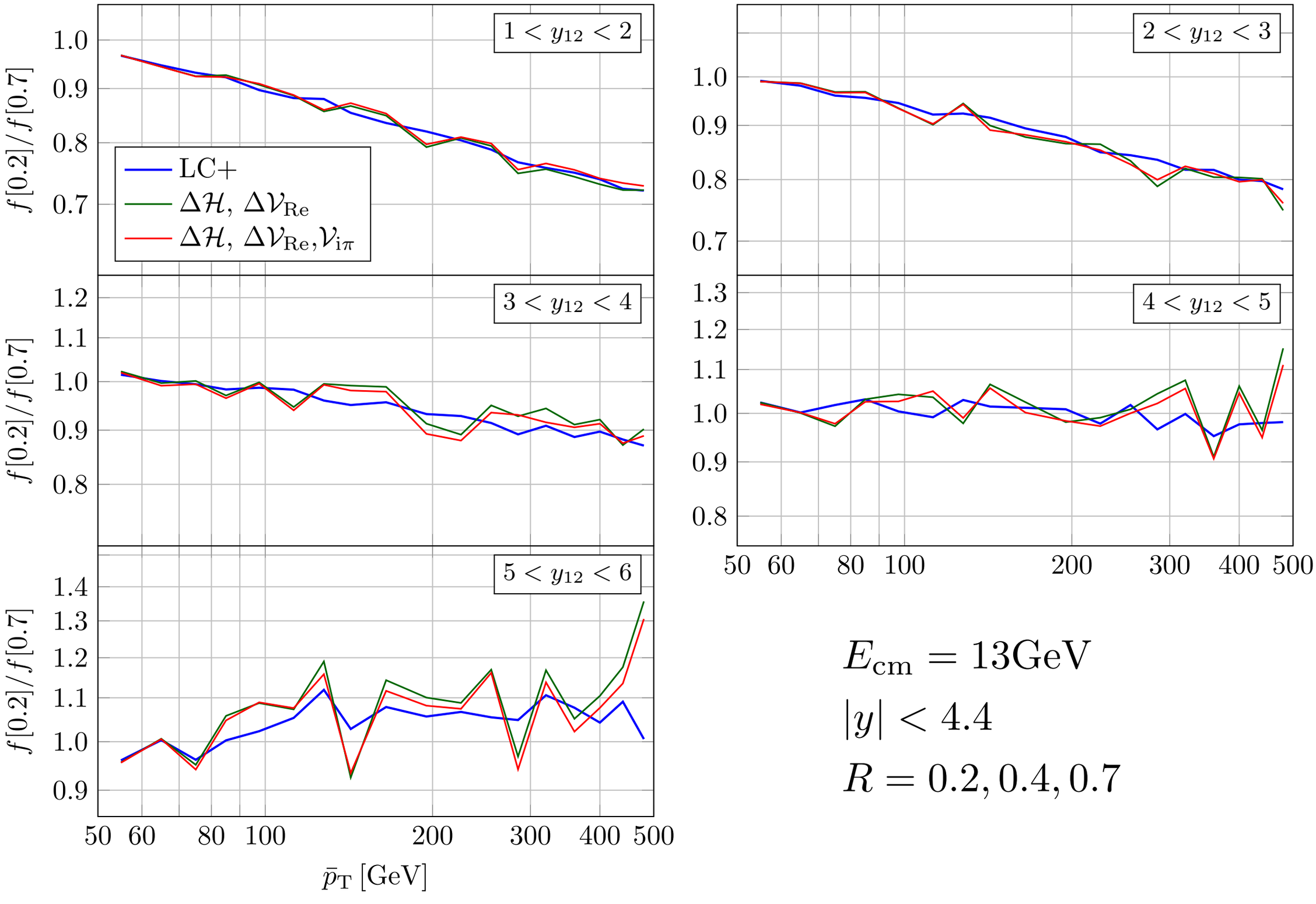


$E_{\rm cm} = 13{\rm GeV}$

$|y| < 4.4$

$R = 0.4$

Rapidity Gap Survival



$$E_{\mathrm{cm}} = 13\mathrm{GeV}$$

$$|y| < 4.4$$

$$R = 0.2, 0.4, 0.7$$

Exponentiating the Phase Terms

Evolution Equation

The evolution equation is

$$\mathcal{U}(\mu_2^2,\mu_1^2) = \mathcal{X}(\mu_2^2,\mu_1^2)\mathcal{N}^{\mathrm{LC+}}(\mu_2^2,\mu_1^2) + \int_{\mu_2^2}^{\mu_1^2} \frac{d\mu^2}{\mu^2}\,\mathcal{U}(\mu_2^2,\mu^2)\left[\mathcal{H}^{\mathrm{LC+}}(\mu^2) + \Delta\mathcal{H}(\mu^2)\right]\mathcal{X}(\mu^2,\mu_1^2)\mathcal{N}^{\mathrm{LC+}}(\mu^2,\mu_1^2)$$

The X operator obeys its evolution equation:

$$\mathcal{X}(\mu_2^2,\mu_1^2) = 1 - \int_{\mu_2^2}^{\mu_1^2} \frac{d\mu^2}{\mu^2}\;\mathcal{X}(\mu_2^2,\mu^2)\,\mathcal{N}^{\mathrm{LC+}}(\mu_2^2,\mu^2)[\Delta\mathcal{V}_{\mathrm{Re}}(\mu^2) + \underbrace{\mathrm{i}\pi\mathcal{V}_{\mathrm{i}\pi}(\mu^2)}]\,\mathcal{N}^{\mathrm{LC+}}(\mu_2^2,\mu^2)^{-1}$$

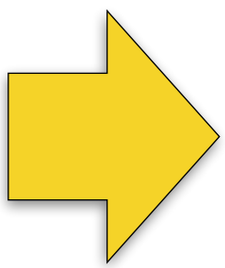
Can we exponentiate this term?

Here

$$\mathrm{i}\pi\mathcal{V}_{\mathrm{i}\pi}(t) = -4\mathrm{i}\pi\frac{\alpha_{\mathrm{s}}}{2\pi}\left([(T_{\mathrm{a}}\cdot T_{\mathrm{b}})\otimes 1] - [1\otimes(T_{\mathrm{a}}\cdot T_{\mathrm{b}})]\right)$$

and

$$\mathcal{N}^{\mathrm{LC+}}(t_2,t_1) = \exp\left[-\int_{t_1}^{t_2} d\tau\;\mathcal{V}^{\mathrm{LC+}}(\tau)\right]$$



$$\mathcal{N}^{\mathrm{E}}(t_2,t_1) = \exp\left[-\int_{t_1}^{t_2} d\tau\;(\mathcal{V}^{\mathrm{LC+}}(\tau) + \mathrm{i}\pi\mathcal{V}_{\mathrm{i}\pi}(\tau)\right]$$

No-splitting Operator

Statistical space → **Quantum space**

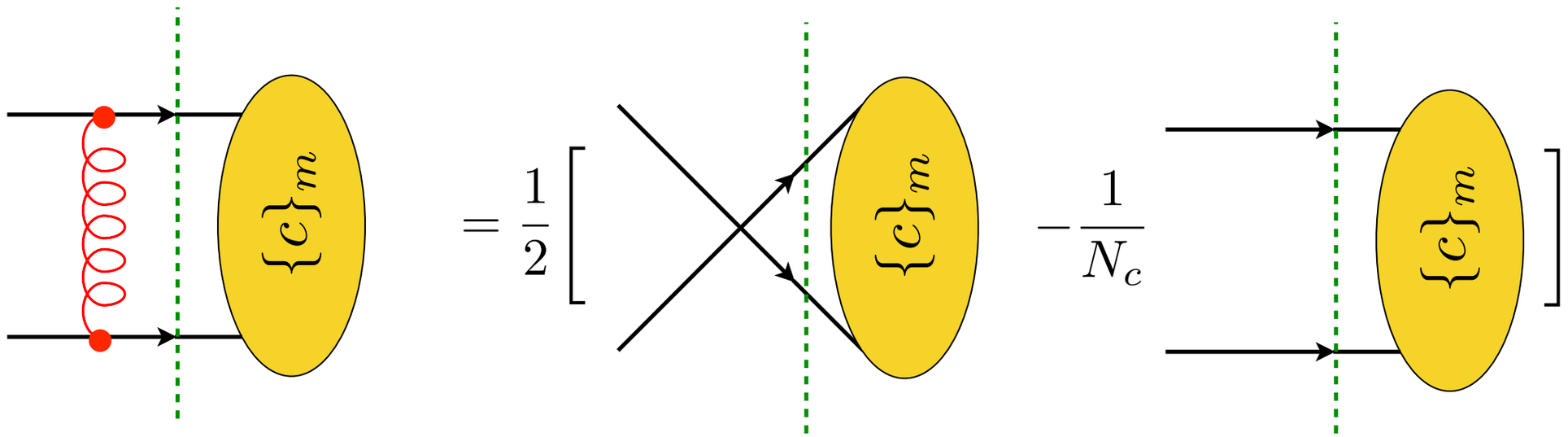
$$\mathcal{N}^{\mathrm{E}}(t_2,t_1)\Big|\{p,f,c'c\}_m\Big) = n(t_2,t_1,\{p,f\}_m)\Big|\{c\}_m\Big\rangle\Big\langle\{c'\}_m\Big|n(t_2,t_1,\{p,f\}_m)^\dagger$$

We have to compute the the no-splitting operator in the quantum space and that is

How messy is this?

$$n(t_2,t_1,\{p,f\}_m) = \mathbb{T}\exp\left[-\int_{t_1}^{t_2} dt\ \frac{\alpha_{\mathrm{s}}(t)}{2\pi}\left(\underbrace{a^{\mathrm{LC+}}(t,\{p,f\}_m)}-4\mathrm{i}\pi\ \overbrace{T_{\mathrm{a}}\cdot T_{\mathrm{b}}}\right)\right]$$

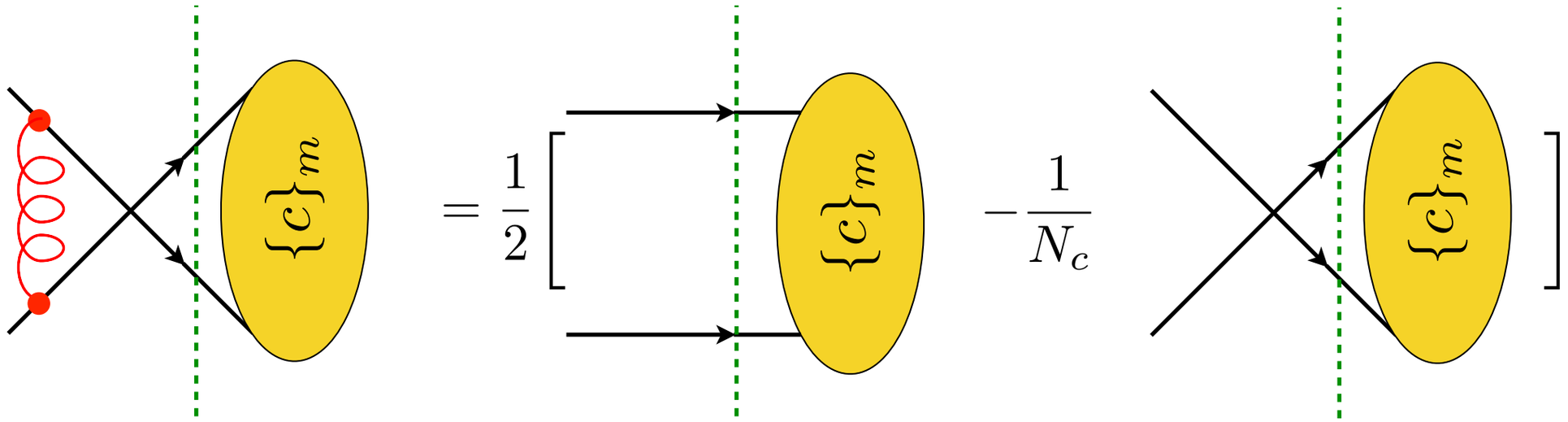
LC+ part
still diagonal in color space



A new state: $\Big|\{c_1\}_m\Big\rangle$ **Original state:** $\Big|\{c\}_m\Big\rangle$

No-splitting Operator

Now, what happens when phase operator acts on the new state?



$T_{\rm a} \cdot T_{\rm b} \left| \{c_1\}_m \right\rangle$ $\qquad$ $\left| \{c_1\}_m \right\rangle$ $\qquad$ $\left| \{c\}_m \right\rangle$

This is good! The phase operator rotates **only** in the **2 dimensional** subspace.

$$T_{\rm a} \cdot T_{\rm b} \left| \{c_n\}_m \right\rangle = \sum_{n'} M_{n'n} \left| \{c_{n'}\}_m \right\rangle$$

$$M = -\frac{1}{2} \begin{bmatrix} 1/N_{\rm c} & -1 \\ -1 & 1/N_{\rm c} \end{bmatrix}$$

Exponentiating a 2x2 matrix is easy!

No-splitting Operator

How about gluon incomings? For **quark-gluon** incoming partons we have a 4D subspace and the corresponding 4x4 matrix is

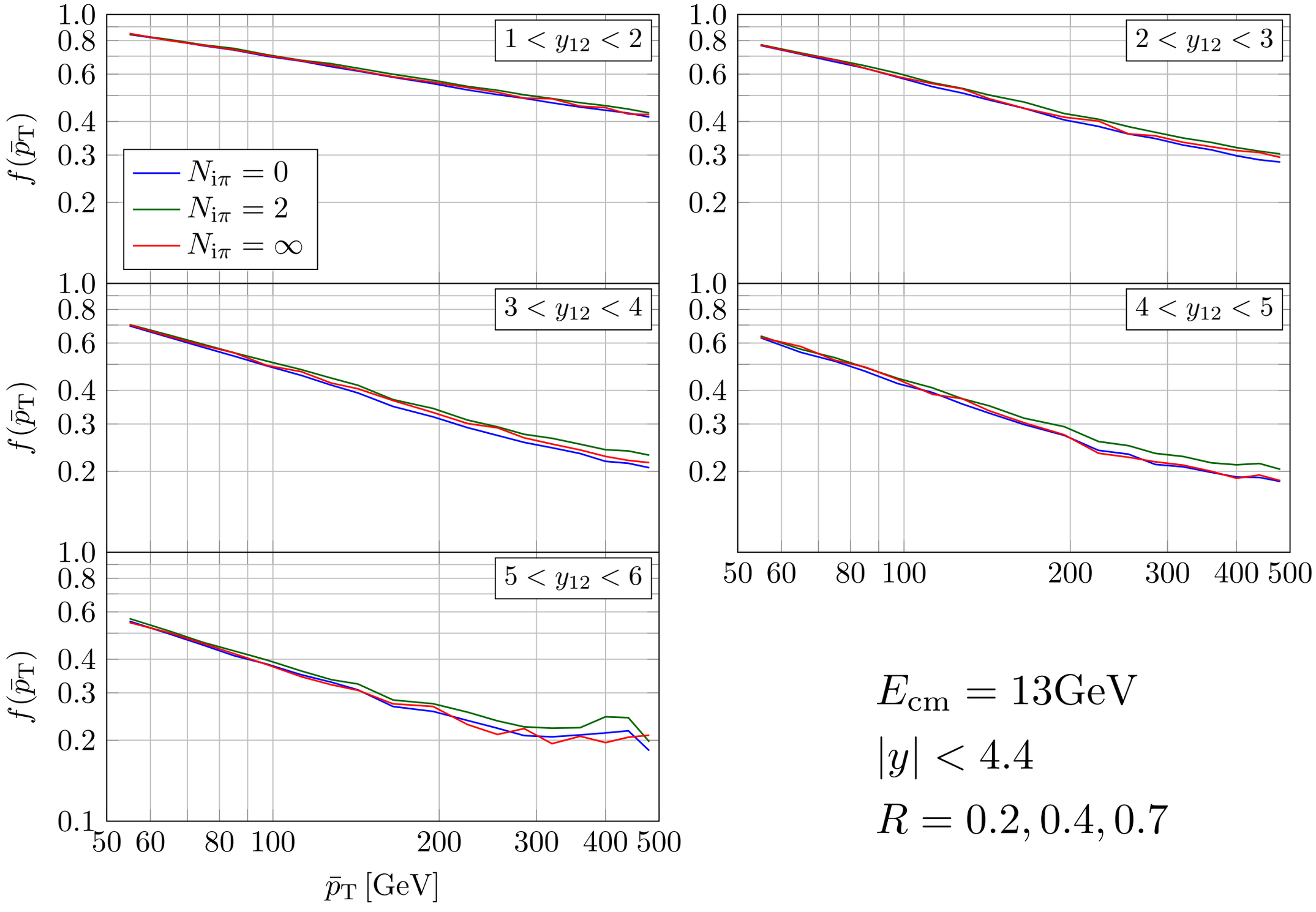
$$M = -\frac{1}{2}\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & N_c & 0 \\ 1 & 0 & 0 & N_c \end{bmatrix}$$

In the most complicated case, **gluon-gluon** initiated process, we have to deal with a 14D subspace and then we have

$$M = -\frac{1}{2}\begin{bmatrix} 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & N_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & N_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & N_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & N_c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & N_c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & N_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & N_c & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_c & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 2N_c & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 2N_c \end{bmatrix}$$

It is still easy to exponentiate numerically.

Rapidity Gap Survival



$E_{\mathrm{cm}} = 13\mathrm{GeV}$

$|y| < 4.4$

$R = 0.2, 0.4, 0.7$

Implementation

- **DEDUCTOR** is designed to do a better job with color, spin and resummation of large logarithms compared to other shower generators.
  - Lambda, kT and angular ordering
  - LC+ color treatment. It allows us to do color evolution at amplitude level
  - Threshold log resummation
  - Spin correlations are not yet computed
  - Fully exponentiated Glauber (Coulomb) gluon effects
  - Wide angle soft gluon effects perturbatively.
- Next version is available soon…
  - NLO matching at density operator level
- It is available from

`http://www.desy.de/~znagy/deductor`
`http://pages.uoregon.edu/soper/deductor`

Summary, Outlook

- **We have theory of parton shower algorithm.**
  - Defined at all order level and derived from fixed order perturbation theory (with the help of renormalization group).
  - It is implemented at LO level.
  - All the classical shower algorithm fit into this formalism.
- **But how good is it at summing up large logarithms?**
  - … stay tuned!

Shower Kernel

The generators of the unitary shower can be expanded in the coupling:

$$S(\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} S^{(1)}(\mu^2) + \left[\frac{\alpha_s(\mu^2)}{2\pi}\right]^2 S^{(2)}(\mu^2) + \cdots$$

and the first order term is rather simple

$$\frac{1}{\mu_S^2} S^{(1)}(\mu^2) = \frac{\partial}{\partial \mu_S^2} \left[\mathcal{F}(\mu^2)\mathcal{D}^{(1,0)}(\mu^2,\mu_S^2)\mathcal{F}^{-1}(\mu^2) + \mathcal{D}^{(0,1)}(\mu^2,\mu_S^2)\right]_{\mu_S^2=\mu^2} - \left.\frac{\partial \mathcal{V}^{(1)}(\mu^2,\mu_S^2)}{\partial \mu_S^2}\right|_{\mu_S^2=\mu^2}$$

$$= \left[\underbrace{\mathcal{F}(\mu^2)\frac{\partial \mathcal{D}^{(1,0)}(\mu^2,\mu_S^2)}{\partial \mu_S^2}\mathcal{F}^{-1}(\mu^2)}_{\text{Real operator}} - \underbrace{\frac{\partial \mathcal{F}(\mu^2) \circ \overline{\mathcal{D}}^{(1,0)}(\mu^2,\mu_S^2)}{\partial \mu_S^2}\mathcal{F}^{-1}(\mu^2)}_{\text{Integrated real operator}} + \underbrace{\mathrm{Im}\frac{\partial \mathcal{D}^{(0,1)}(\mu^2,\mu_S^2)}{\partial \mu_S^2}}_{\text{Glauber gluon}}\right]_{\mu_S^2=\mu^2}$$

Real operator
all the quantum numbers of the emitted parton is resolved

Integrated real operator
- all the quantum numbers of the emitted parton is integrated out
- it is not the contribution of the virtual graphs

Glauber gluon
imaginary part of the virtual graphs
$\sim i\pi$

Note, the first order kernel is **independent of** the real part of the virtual graphs.

Shower Kernel

At second order level we are not that lucky. The shower kernel is much more complicated:

$$\frac{1}{\mu_{\rm S}^2}S^{(2)}(\mu^2) = \mathcal{F}(\mu^2)\left(\frac{\partial\mathcal{D}^{(2)}(\mu^2,\mu_{\rm S}^2)}{\partial\mu_{\rm S}^2} - \mathcal{D}^{(1)}(\mu^2)\frac{\partial\mathcal{D}^{(1)}(\mu^2,\mu_{\rm S}^2)}{\partial\mu_{\rm S}^2}\right)_{\mu_{\rm S}^2=\mu^2}\mathcal{F}^{-1}(\mu^2)$$
$$-\left(\frac{\partial\mathcal{V}^{(2)}(\mu^2,\mu_{\rm S}^2)}{\partial\mu_{\rm S}^2} - \mathcal{V}^{(1)}(\mu^2)\frac{\partial\mathcal{V}^{(1)}(\mu^2,\mu_{\rm S}^2)}{\partial\mu_{\rm S}^2}\right)_{\mu_{\rm S}^2=\mu^2}$$
$$+\left[\mathcal{V}^{(1)}(\mu^2),\frac{1}{\mu^2}\mathcal{S}^{(1)}(\mu^2)\right]$$

This is highly non-trivial operator and cancelation of all the singularities in the first term is rather delicate.

$$\mathcal{D}^{(2)}(\mu^2,\mu_{\rm S}^2) = \overbrace{\mathcal{D}^{(2,0)}(\mu^2,\mu_{\rm S}^2)}^{\textbf{Double real}} + \underbrace{\mathcal{D}^{(1,1)}(\mu^2,\mu_{\rm S}^2)}_{\textbf{Real-virtual}} + \overbrace{\mathcal{D}^{(0,2)}(\mu^2,\mu_{\rm S}^2)}^{\textbf{Double virtual}}$$

and

$$\mathcal{D}^{(1)}(\mu^2,\mu_{\rm S}^2) = \overbrace{\mathcal{D}^{(1,0)}(\mu^2,\mu_{\rm S}^2)}^{\textbf{Single real}} + \underbrace{\mathcal{D}^{(0,1)}(\mu^2,\mu_{\rm S}^2)}_{\textbf{Single virtual}}$$