Sudakov Veto Algorithms & Resampling

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at the
"Taming the Accuracy of Parton Showers"
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Work with Malin Sjödahl and Jimmy Olsson

Sudakov Veto Algorithms

Sudakov-type densities central to Showers

[Olsson, Plätzer, Sjödahl — arXiv:1912.02436]
[Plätzer, Sjödahl — Eur.Phys.J.Plus 127 (2012) 26]
See also Verheyen — Lönnblad — Hoeche, Schumann, Siegert

no emission

$$\frac{\mathrm{d}S_P(q|Q,z,x)}{\mathrm{d}q\ \mathrm{d}z} = \Delta_P(Q_0|Q,x)\delta(q-Q_0)\delta(z-z_0) + \Delta_P(q|Q,x)P(q,z,x)\theta(Q-q)\theta(q-Q_0)$$

emission

$$\Delta_P(q|Q,x) = \exp\left(-\int_q^Q \mathrm{d}k \int \mathrm{d}z\ P(k,z,x)\right)$$

Integrates to one regardless of form of P and cutoff!

Positive P with known overestimate allows to use standard veto algorithm, including e.g. adaptive methods in Herwig with the ExSample library.

[Plätzer — Eur.Phys.J.C 72 (2012) 1929]

$Q' \leftarrow Q$
loop
  $r \leftarrow$**rnd**
  if $r \leq \Delta_R(Q_0|Q',x)$ **then**
    return Q_0
  else
    solve $r = \Delta_R(q|Q',x)$ for q
    select z in proportion to $R(q,z,x)$
    return q with probability $P(q,z,x)/R(q,z,x)$
  end if
  $Q' \leftarrow q$
end loop

Colour Matrix Element Corrections

Correct real emission by exact colour correlations,
obtain iterative corrections by amplitude evolution.

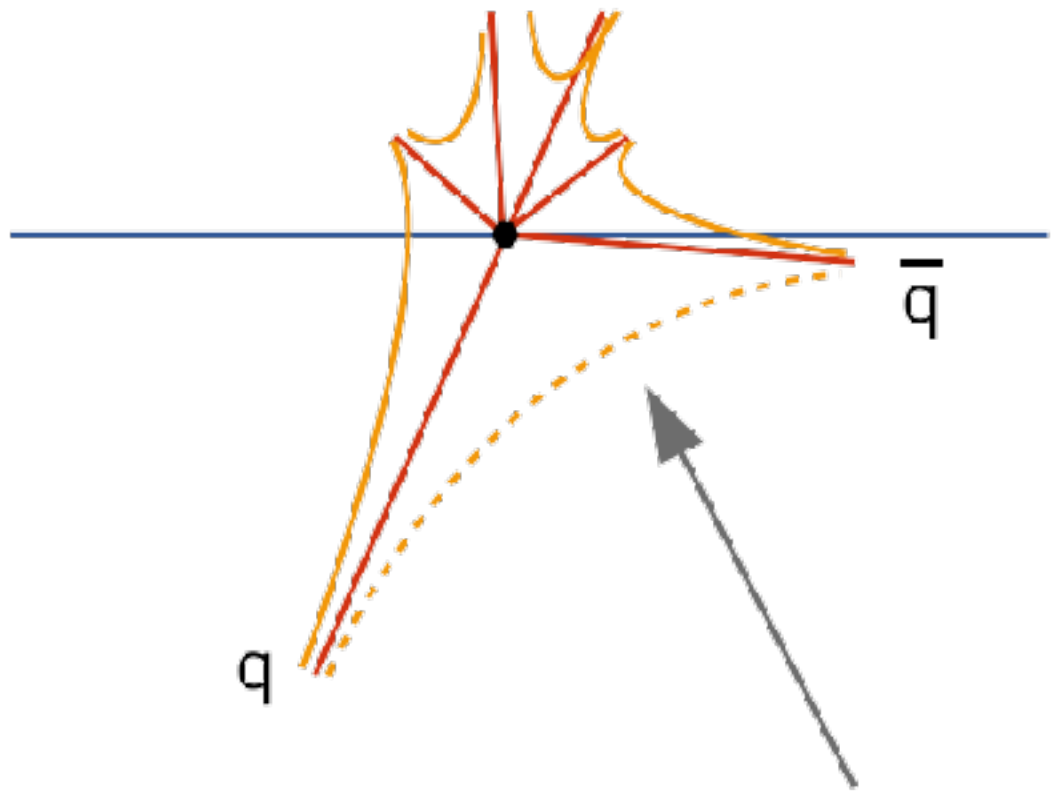
[Plätzer, Sjödahl – JHEP 1207 (2012) 042]
[Plätzer, Sjödahl, Thoren – JHEP 11 (2018) 009]

$$\mathcal{P}_{ij,k}(p_\perp^2, z; p_{\tilde{ij}}, p_{\tilde{k}}) = \mathcal{J}(p_\perp^2, z; p_{\tilde{ij}}, p_{\tilde{k}}) V_{ij,k}(p_\perp^2, z; p_{\tilde{ij}}, p_{\tilde{k}}) \times \frac{-1}{\mathbf{T}_{\tilde{ij}}^2} \frac{\langle \mathcal{M}_n | \mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_k | \mathcal{M}_n \rangle}{|\mathcal{M}_n|^2}$$

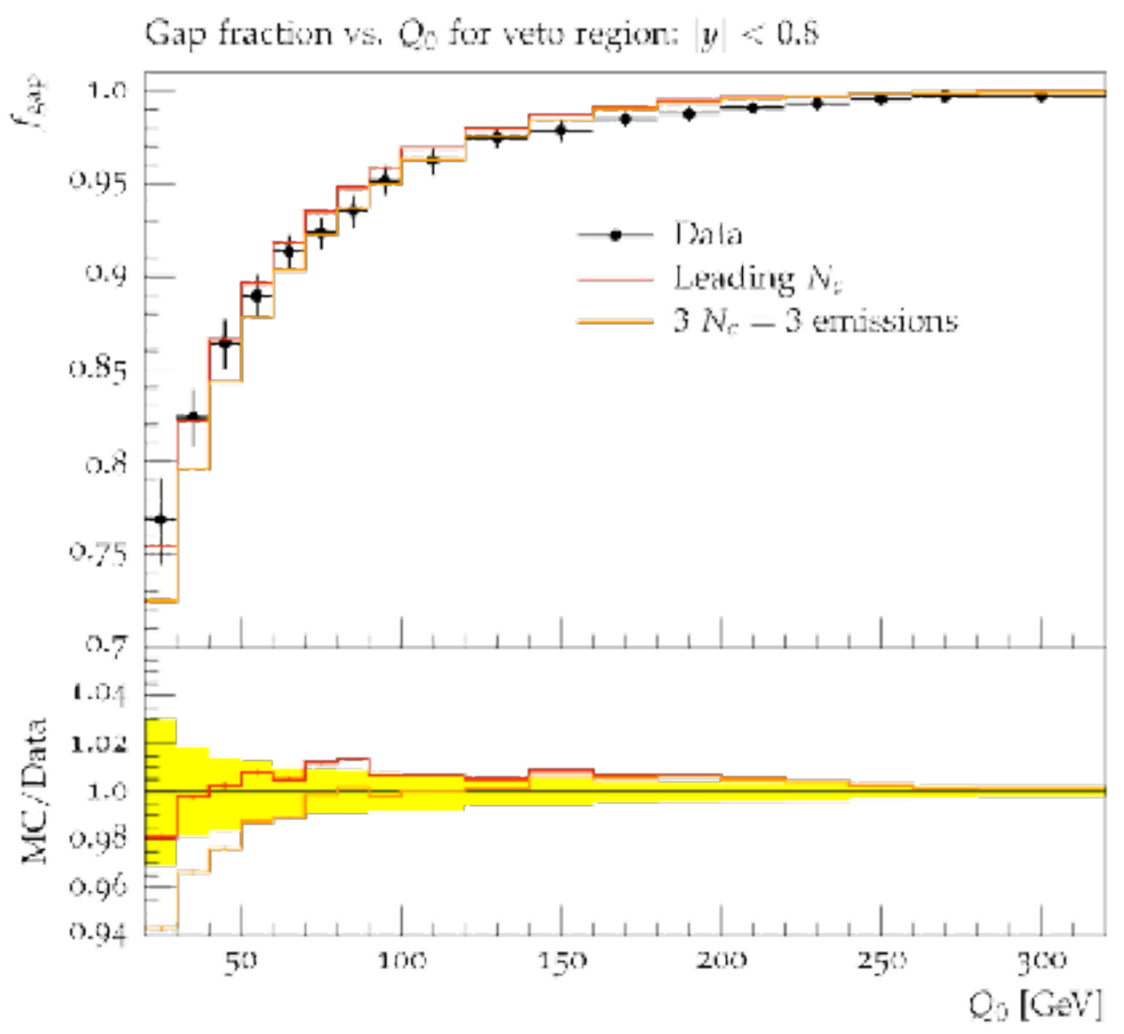
$$M_{n+1} = -\sum_{i \neq j} \sum_{k \neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j} \frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}_{\tilde{ij}}^2} T_{\tilde{k},n} M_n T_{\tilde{ij},n}^\dagger$$

Dipole branching algorithms can be supplemented by correction factors for real emission, but still lack virtual contributions beyond leading-N.

Different from amplitude level evolution.



Some subleading-N corrections can be restored.



Available in Herwig 7.2, including collinear contributions.

Sudakov Veto Algorithms

universität wien

Sudakov-type densities central to Showers

[Olsson, Plätzer, Sjödahl — arXiv:1912.02436]
[Plätzer, Sjödahl — Eur.Phys.J.Plus 127 (2012) 26]
Also cf. shower variations e.g.
[Bellm, Plätzer, et al. — Phys.Rev.D 94 (2016) 3, 034028]

no emission

$$\frac{\mathrm{d}S_P(q|Q,z,x)}{\mathrm{d}q\ \mathrm{d}z} = \Delta_P(Q_0|Q,x)\delta(q-Q_0)\delta(z-z_0) + \Delta_P(q|Q,x)P(q,z,x)\theta(Q-q)\theta(q-Q_0)$$

emission

$Q' \leftarrow Q, w \leftarrow w_0$
loop
A trial splitting scale and variables, q, z, are generated according to $S_R(q|Q', z, x)$, for example using Alg. 1.
if $q = Q_0$ **then**
There is no emission and the cut-off scale Q_0 is returned while the event weight is kept at w.
else
if rnd$\leq \epsilon$ **then**
The trial splitting variables q, z are accepted, and

$$w \leftarrow w \times \frac{1}{\epsilon} \times \frac{P(Q',z,x)}{R(Q',z,x)}. \qquad (3)$$

else
The emission is rejected, and the algorithm continues with

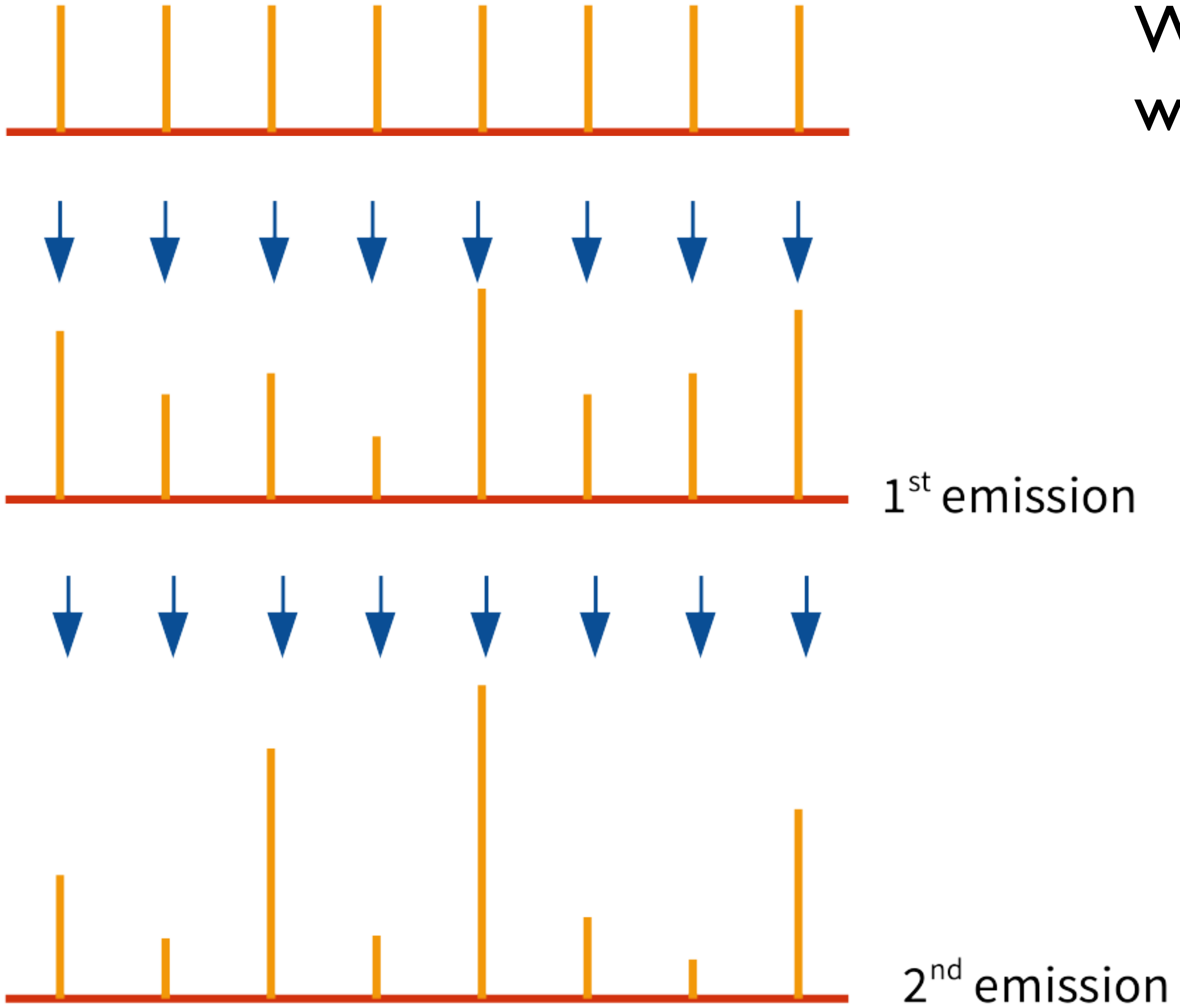
$$w \leftarrow w \times \frac{1}{1-\epsilon} \times \left(1 - \frac{P(q,z,x)}{R(q,z,x)}\right)$$
$$Q' \leftarrow q. \qquad (4)$$

end if
end if
end loop

Negative P or unknown overestimate requires weighted veto algorithm, with in principle arbitrary proposal kernel and veto probability.

Weight distributions

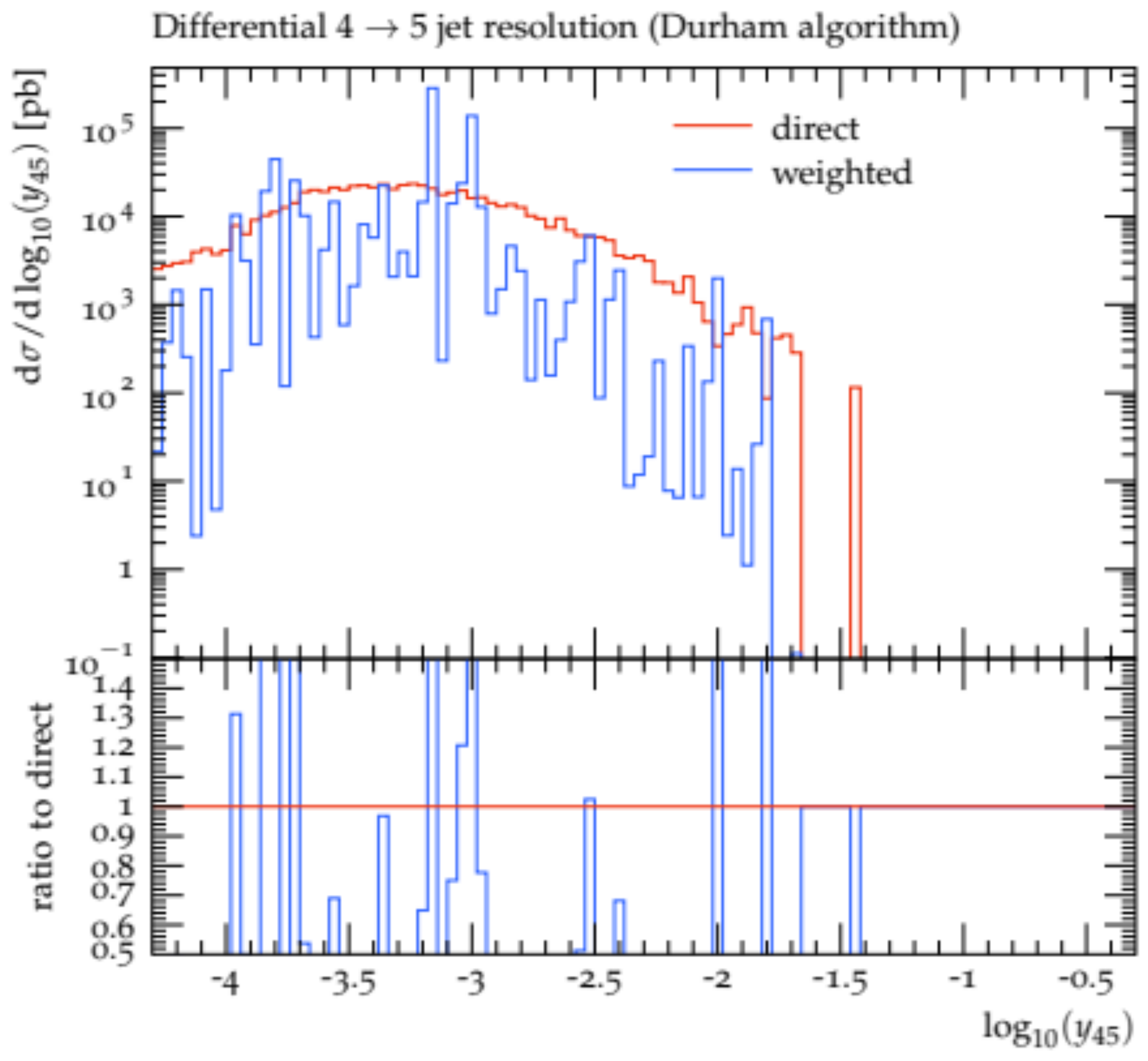
[Olsson, Plätzer, Sjödahl — arXiv:1912.02436]



Weighted branching algorithms exhibit prohibitive weight distributions & convergence issues.

$$w_{\text{veto}} = \frac{1 - \frac{P}{R}}{1 - \epsilon} \qquad w_{\text{accept}} = \frac{P}{\epsilon \, R}$$

[Python shower by Hoeche & in-house sandbox code]

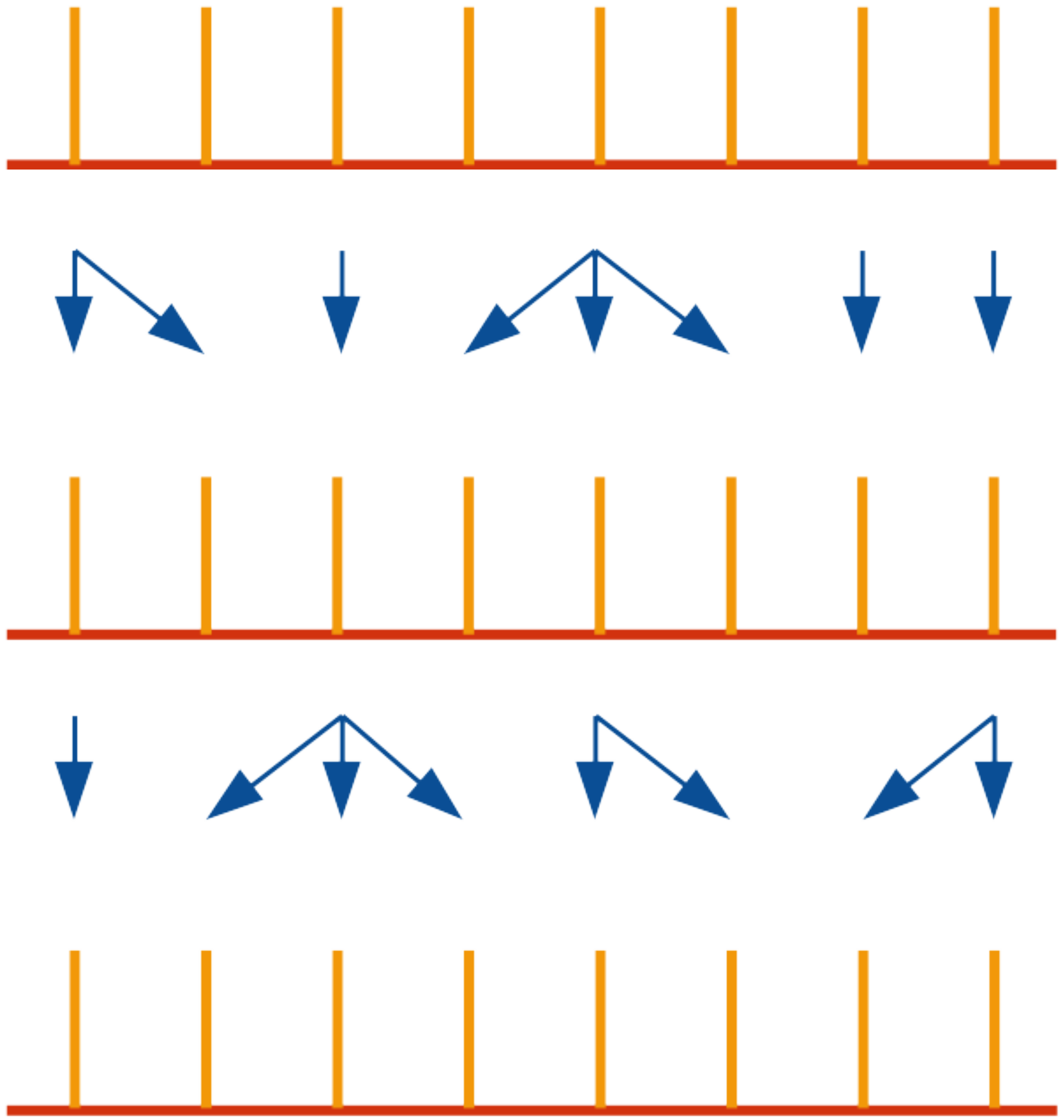


Resampling

[Olsson, Plätzer, Sjödahl — arXiv:1912.02436]

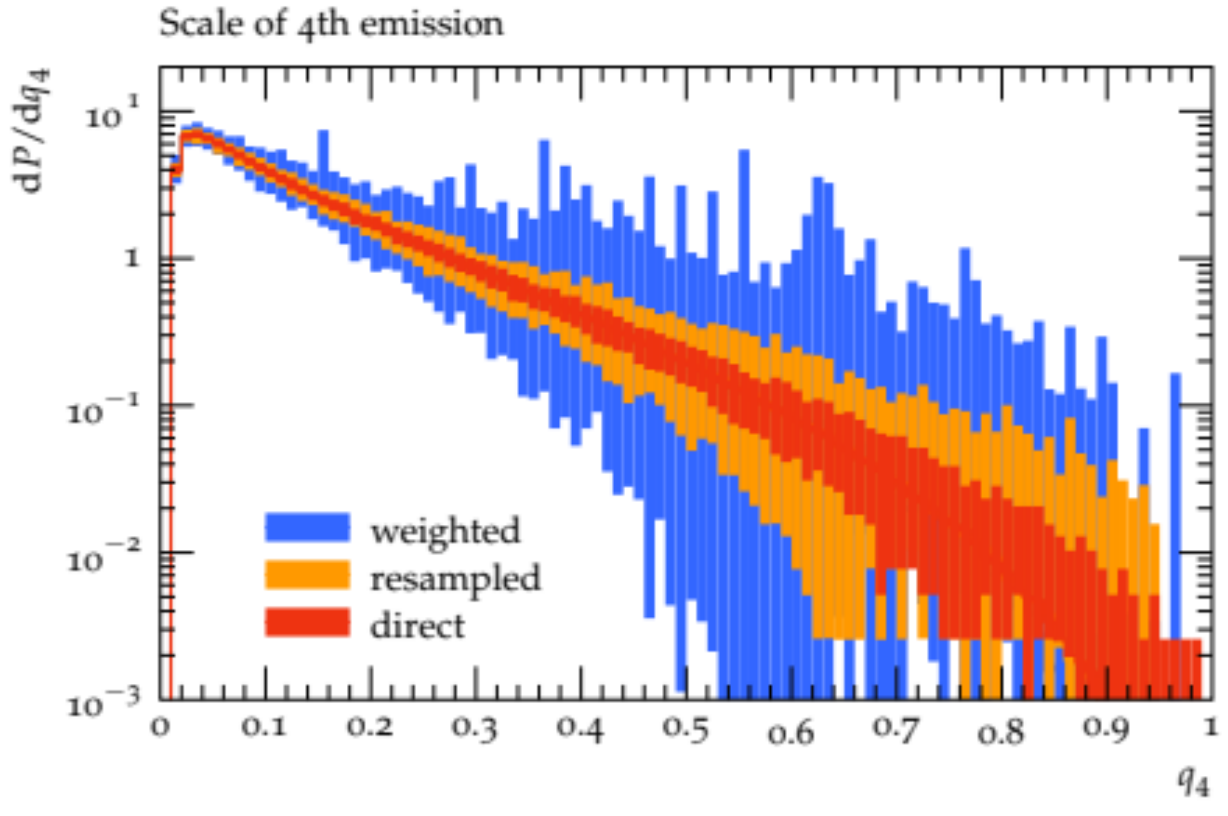
Resampling algorithms solve this issue, though strict event generator interpretation lost.

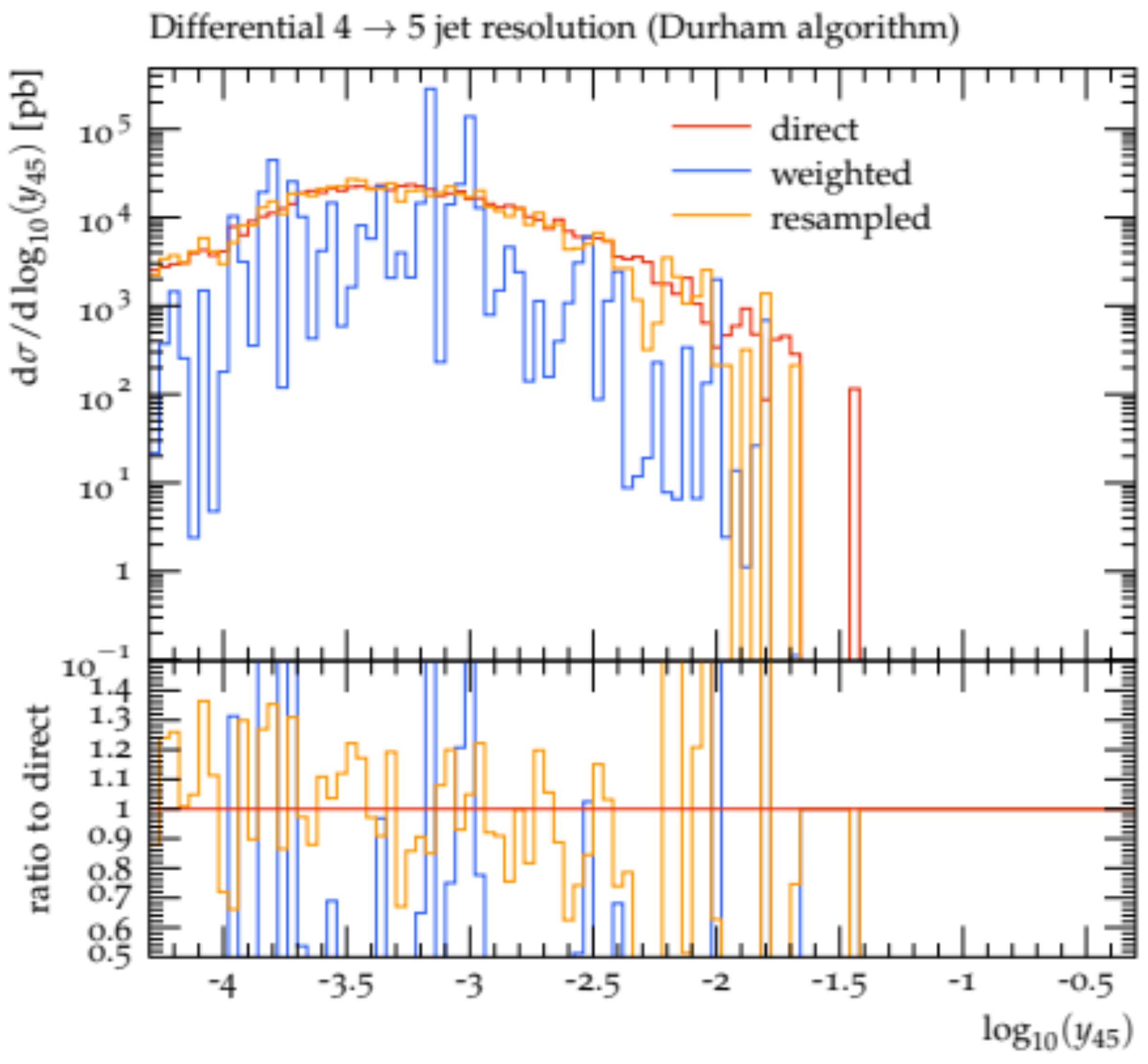
$$w_{\text{veto}} = \frac{1 - \frac{P}{R}}{1 - \epsilon} \qquad w_{\text{accept}} = \frac{P}{\epsilon\, R}$$



1st emission

2nd emission

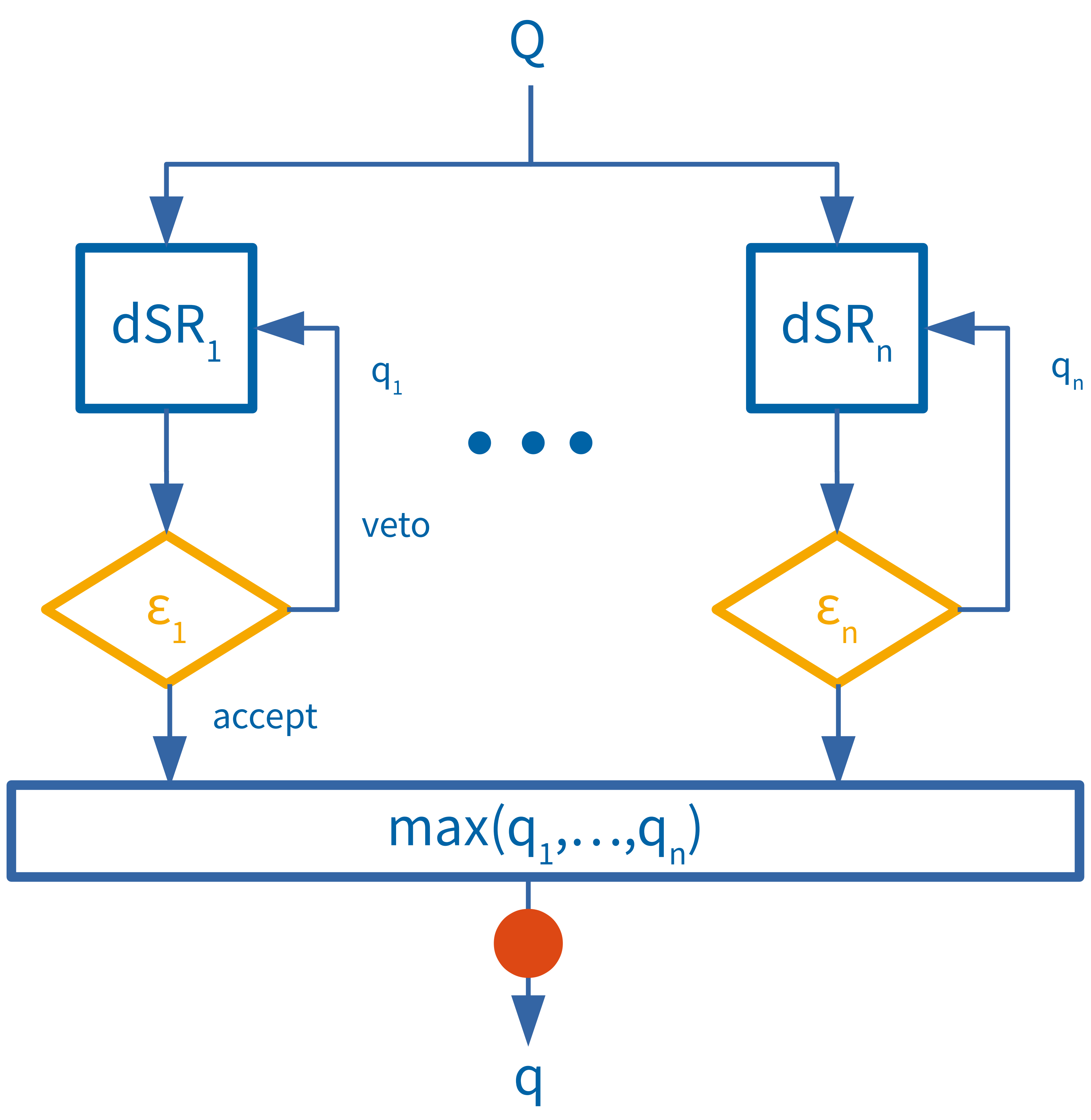


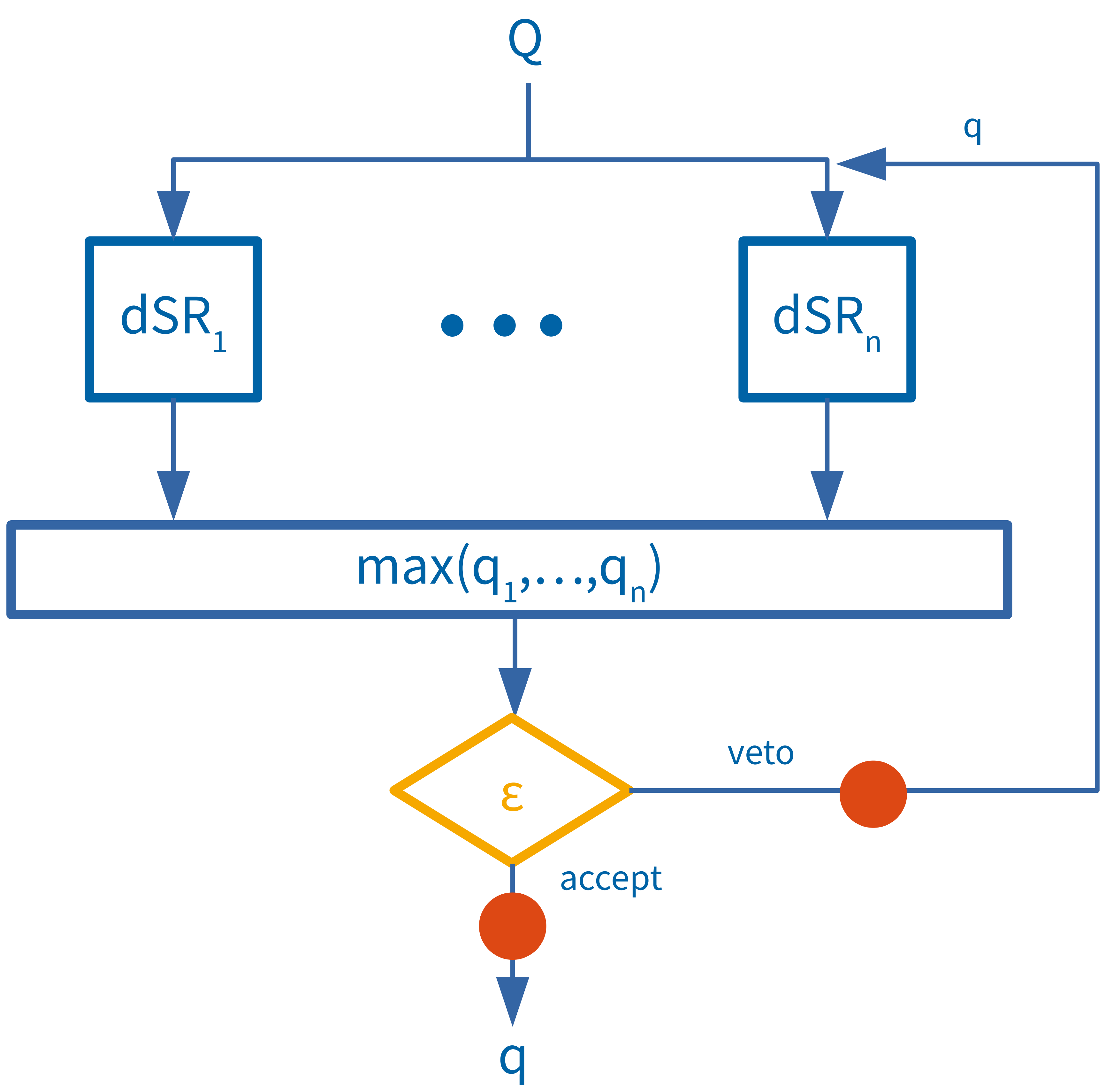


Resampling

[Olsson, Plätzer, Sjödahl — arXiv:1912.02436]

Resampling can intervene at various intermediate steps in veto algorithm setups





Points for discussion

Weighted (veto) algorithms seem to be inevitable in pushing showers to higher orders. Essentially caused by breakdown of unitarity and/or indefinite splitting kernels.

Resampling is a very powerful method to stabilise sequential Monte Carlos over long time horizons. It is a way out of the convergence problems we have been struggling with in several applications.

Active field of research in statistics/applied mathematics, uncertainties, parallelisation, etc. are known and can be explored.

Shift in paradigm however required:
No independent events anymore — can accept this for calculating distributions.

Thank you!