\( \alpha_s \rightarrow 0: \) easier said than done

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Taming the accuracy of MC generators, CERN, July 3 2020
Structure of NLL:

$$\Sigma_{\text{NLL}}(\alpha_s L, \alpha_s) = e^{g_1(\alpha_s L)L + g_2(\alpha_s L) + ...}$$

NLL accuracy test:

$$\frac{\Sigma_{\text{MC}}}{\Sigma_{\text{NLL}}} \xrightarrow{\alpha_s \to 0} f(\lambda = \alpha_s L) \approx 1$$

Here: discuss challenges associated with $\alpha_s \to 0$
Say we take $\lambda = \alpha_s L = 0.5$

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$\alpha_s \ll 1 \Rightarrow L \gg 1$
The challenge: numbers

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- $\alpha_s \ll 1 \Rightarrow L \gg 1$
- Extra room to resolve emissions:
  - $\Rightarrow$ Shower cut should be smaller
  - $\Rightarrow$ extra $\sqrt{\epsilon}$ ($\delta L = \log(1/\epsilon)$)
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**Challenge 1:**

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The challenge: numbers

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Challenge 1:
Deal with numbers over large numerical range \( \Rightarrow \) precision impaired

Challenge 2:

\[
g_1(\alpha_s L) L \gg 1 \Rightarrow \sum(\lambda, \alpha_s) \ll 1
\]

\( \Rightarrow \) no events with standard “unweighted” techniques

(e.g. \( \lambda = 0.5, \alpha_s = 0.005 \Rightarrow \sum_{y23}(L) \sim 10^{-29} \))
Sudakov suppression: $\beta_{\text{shower}} = \beta_{\text{obs}}$ (easy case)

Estimate contribution $e^{L_{\text{approx}}}$ to obs

PanScales: \[
\begin{align*}
L_{\text{approx}} &= \ln k_t - \beta_{\text{obs}}|\eta|, \\
\eta &= \bar{\eta} + \frac{1}{\beta} \ln \rho
\end{align*}
\]

consider bins in $L_{\text{approx}}$

focus on a single bin
Sudakov suppression: $\beta_{\text{shower}} = \beta_{\text{obs}}$ (easy case)

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Element 1: weights

- Generate first emission in given bin
- Weight corresponding to Sudakov above the considered bin
Sudakov suppression: $\beta_{\text{shower}} = \beta_{\text{obs}}$ (easy case)

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\[\text{chosen } L_{\text{approx}} \text{ bin}
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Estimate contribution $e^{L_{\text{approx}}}$ to obs

PanScales:

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\begin{cases}
L_{\text{approx}} = \ln k_t - \beta_{\text{obs}} |\eta|,
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Element 1: weights

- Generate first emission in given bin
- Weight corresponding to Sudakov above the considered bin

Element 2: dynamic cut-off

- Cut shower at factor $e^{-\delta L}$ below largest contribution

Gregory Soyez (PanScales)
Sudakov suppression: $\beta_{\text{shower}} \neq \beta_{\text{obs}}$ (harder case)

$$\beta_{\text{shower}} = 1/2$$
$$\beta_{\text{obs}} = 0$$

Estimate contribution $e^{L_{\text{approx}}}$ to obs

PanScales:

$$L_{\text{approx}} = \ln k_t - \beta_{\text{obs}} |\eta|,$$
$$\eta = \bar{\eta} + \frac{1}{\beta} \ln \rho$$

Element 1: weights + event veto
- Generate first emission in given bin
- Weight corresponding to Sudakov above the considered bin
- Veto event if $L_{\text{approx}} > L_{\text{approx}}^{(\text{min})}$

Element 2: emission veto
- Veto emissions at factor $e^{-\delta L}$ below largest contribution
For most shapes, $\sqrt{\epsilon} = \sqrt{\epsilon_{\text{double}}} \approx 10^{-8}$ ($\delta L \approx 18$) does the job (covered range between $\sqrt{\epsilon}L$ and $L$)
Numerical range

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- For $k_t$ subjet multiplicity, need full clustering
  \[ \Rightarrow \] need full kinematic range
  \[ \Rightarrow \] rely on higher-precision types

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(Note that multiplicity is a series in $\sqrt{\alpha_s}L$)
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For $E_t$ in a slice, naive $\delta L$ does not work; use angular cut-off instead.
Other considerations: extrapolation to 0

- From \( n \) values of \( \alpha_S \) (typically \( n = 3 \)), determine a polynomial of degree \( n - 1 \)
- The constant term is the extrapolation up to \( O(\alpha_S^n) \)
- Statistical uncertainties easily propagated
- Can try to get an estimate of “systematic” uncertainties using different sets of \( \alpha_S \) values
  (mostly helpful for multiplicity where convergence was slower)
Computation of angles

- computing “1 − cos θ” has precision $\sqrt{\epsilon}$ in collinear limit.
- switch to a cross product at small angles to bring down to $\epsilon$
- alignment along the $z$ axis $\Rightarrow$ angles down to $\epsilon \theta_{\text{max}}$
- benefit from precision at various stages, e.g. precise determination of the thrust axis

Multiplicities can get large

- $N \propto \exp(\sqrt{\alpha_s L^2}) \propto \exp(\sqrt{\lambda L})$ blows up if $L \gg 1$ at fixed $\lambda$
- currently limited by previous prescriptions but speed useful nonetheless
- adopted a “Roulette Wheel” strategy for dipole choice
- fast observable calculations (e.g. clustering)