$\alpha_s \to 0$: easier said than done

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with M.Dasgupta, F.Dreyer, K.Hamilton, P.Monni and G.Salam (PanScales)

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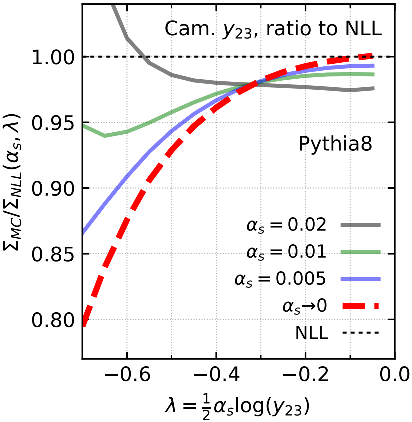
Taming the accuracy of MC generators, CERN, July 3 2020

The challenge: idea



Structure of NLL:

$$\Sigma_{\rm NLL}(\alpha_s L, \alpha_s) = e^{g_1(\alpha_s L)L + g_2(\alpha_s L)+\dots}$$

NLL accuracy test:

$$\frac{\Sigma_{\rm MC}}{\Sigma_{\rm NLL}} \overset{\alpha_s\to 0}{\longrightarrow} f(\lambda = \alpha_s L) \overset{?}{=} 1$$

Here: discuss challenges associated with $\alpha_s \to 0$

The challenge: numbers

Say we take $\lambda = \alpha_s L = 0.5$

α_s	L	
0.04	12.5	
0.02	25	
0.01	50	
0.005	100	

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  $\Rightarrow$ Shower cut should be smaller
  $\Rightarrow$ extra $\sqrt{\epsilon}$ $(\delta L = \log(1/\epsilon))$

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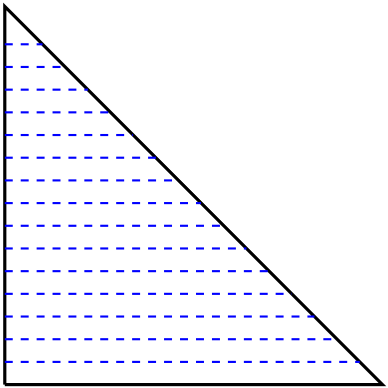
Challenge 2:

$$g_1(\alpha_s L)L \gg 1 \qquad \Rightarrow \qquad \Sigma(\lambda, \alpha_s) \ll 1$$

$\Rightarrow$ no events with standard "unweighted" techniques

(e.g. $\lambda = 0.5$, $\alpha_s = 0.005 \Rightarrow \Sigma_{y_{23}}(L) \sim 10^{-29}$)

Sudakov suppression: $\beta_{\text{shower}} = \beta_{\text{obs}}$ (easy case)



Estimate contribution $e^{L_{\text{approx}}}$ to obs

PanScales: $\begin{cases} L_{\text{approx}} = \ln k_t - \beta_{\text{obs}}|\eta|, \\ \eta = \bar{\eta} + \frac{1}{\beta} \ln \rho \end{cases}$

consider bins in L_{approx}

focus on a single bin

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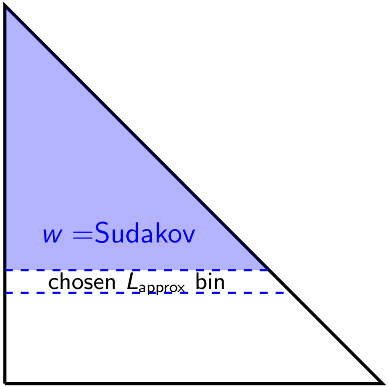

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- Generate first emission in given bin
- Weight corresponding to Sudakov above the considered bin

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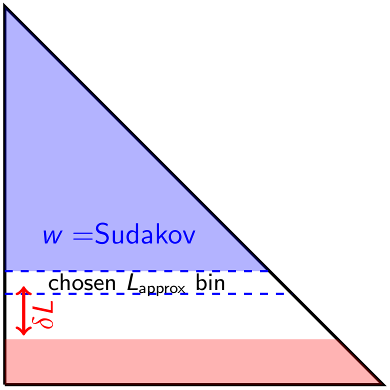

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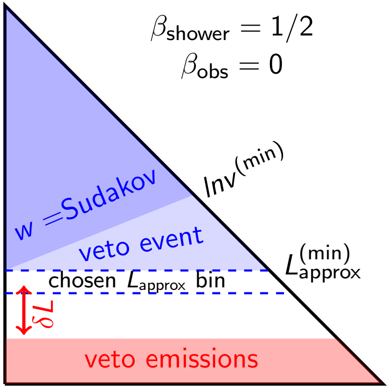
Element 2: dynamic cut-off

cut shower at factor $e^{-\delta L}$
below largest contribution



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Element 1: weights+event veto

- Generate first emission in given bin
- Weight corresponding to Sudakov above the considered bin
- Veto event if $L_{\text{approx}} > L^{(min)}_{\text{approx}}$

Element 2: emission veto

veto emissions at factor $e^{-\delta L}$
below largest contribution

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- For most shapes, $\sqrt{\epsilon} = \sqrt{\epsilon_{\text{double}}} \approx 10^{-8}$ ($\delta L \approx 18$) does the job (covered range between $\sqrt{\epsilon}L$ and L)

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- For k_t subjet multiplicity, need full clustering
  $\Rightarrow$ need full kinematic range
  $\Rightarrow$ rely on higher-precision types

type	precision		slow-down
`double`	$\epsilon \approx 10^{-16}$	$L \approx 37$	1
`ddreal`	$\epsilon \approx 10^{-31}$	$L \approx 71$	10
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- For E_t in a slice, naive δL does not work; use angular cut-off instead.

Other considerations: extrapolation to 0

- From n values of α_S (typically $n = 3$), determine a polynomial of degree $n - 1$
- The constant term is the extrapolation up to $\mathcal{O}(\alpha_s^n)$
- Statistical uncertainties easily propagated
- Can try to get an estimate of "systematic" uncertainties using different sets of α_s values

  (mostly helpful for multiplicity where convergence was slower)

Other considerations

Computation of angles

- computing "$1 - \cos\theta$" has precision $\sqrt{\epsilon}$ in collinear limit.
- switch to a cross product at small angles to bring down to ϵ
- alignment along the z axis $\Rightarrow$ angles down to $\epsilon\theta_{\text{max}}$
- benefit from precision at various stages, e.g. precise determination of the thrust axis

Multiplicities can get large

- $N \propto \exp(\sqrt{\alpha_s L^2}) \propto \exp(\sqrt{\lambda L})$ blows up if $L \gg 1$ at fixed λ
- currently limited by previous prescriptions but speed useful nonetheless
- adopted a "Roulette Wheel" strategy for dipole choice
- fast observable calculations (e.g. clustering)