The PDF4LHC benchmarks on LHC observables

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PDF4LHC BENCHMARK SETTINGS

The PDF4LHC benchmark study: NNPDF

- The NNPDF results are based in the NNPDF2.0 analysis (arxiv:1002.4407): *"A first unbiased global NLO determination of PDFs and their uncertainties"*
- NNPDF2.0 uses default value $\alpha_s\left(M_Z^2\right) = 0.119$. PDF sets in the range $0.114 \le \alpha_s\left(M_Z^2\right) \le 0.124$ are available in the NNPDF webpage

  `http://sophia.ecm.ub.es/nnpdf`

  and will also be available in LHAPDF
- We present both the 7 TeV and the 14 TeV results
- We provide results for PDF+α_s combined uncertainties obtained with exact error propagation, with the choice

$$\alpha_s\left(M_Z^2\right) = 0.119 \pm 0.002\,(68\%\mathrm{C.L.})$$

  Latest PDG average: $\alpha_s\left(M_Z^2\right) = 0.1184 \pm 0.0007 \longrightarrow$ If we were to assume this value of δ_{α_s}, conclusions for combination of PDF+α_s unchanged

PDF AND α_s UNCERTAINTIES IN NNPDF

On the combination of PDF and α_s uncertainties

- NNPDF provides PDF sets for any value of α_s $\to$ The choice of central value $\alpha_s^{(0)}$ and its uncertainty δ_{α_s} is up to the user
- PDF+α_s uncertainties in a cross section $\sigma\left(\mathrm{PDF},\alpha_s\right)$ can be combined
  1. Addition in quadrature using PDF sets with varying α_s:

$$(\delta\sigma)^{\pm}_{\alpha_s} = \sigma\left(\mathrm{PDF}^{(\pm)}, \alpha_s^{(0)} \pm \delta_{\alpha_s}\right) - \sigma\left(\mathrm{PDF}^{(0)}, \alpha_s^{(0)}\right) \ ,$$

$$(\delta\sigma)^{\pm}_{\mathrm{PDF}+\alpha_s} = \sqrt{\left[(\delta\sigma)^{\pm}_{\alpha_s}\right]^2 + \left[(\delta\sigma)^{\pm}_{\mathrm{PDF}}\right]^2} \ .$$

  2. Full error correlation:

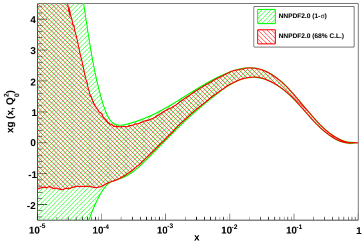
$$\langle\sigma\rangle_{\mathrm{rep}} = \frac{1}{N_{\mathrm{rep}}} \sum_{j=1}^{N_\alpha} \sum_{k_j=1}^{N_{\mathrm{rep}}^{\alpha_s^{(j)}}} \sigma\left(\mathrm{PDF}^{(k_j,j)}, \alpha_s^{(j)}\right) \ ,$$

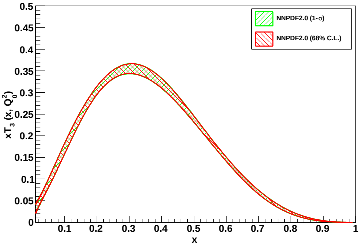
$$(\delta\sigma)^{\pm}_{\mathrm{PDF}+\alpha_s} = \sqrt{\langle\sigma^2\rangle_{\mathrm{rep}} - \langle\sigma\rangle^2_{\mathrm{rep}}} \ , \qquad N_{\mathrm{rep}}^{\alpha_s^{(j)}} \propto \exp\left(-\frac{\left(\alpha_s^{(j)} - \alpha_s^{(0)}\right)^2}{2\delta^2_{\alpha_s}}\right) \ .$$

- In these benchmarks we compute always uncertainties using both methods

Uncertainty bands: 1–sigma vs. 68% Confidence Levels

- The 1–sigma uncertainty bands only coincide with 68% Confidence Levels when underlying distributions are gaussian
- The two methods typically coincide when PDF errors are small (data region, like in T_3 at large-x) - but yield very different results when errors are larger (extrapolation regions, like small-x gluon) → Sizable deviations from gaussianity
- In these benchmarks we compute always uncertainties using both methods





The uncertainty on the uncertainty

- The PDF uncertainties on physical observables are stochastic variables themselves, with the associated fluctuations
- In order to check if differences between PDF uncertainties are statistically significant, one needs to compute the variance on the variance:
  1. Sample estimator:

$$\sigma[\sigma^2] = \left(\frac{1}{N_{\rm rep}} \left[m_4[q] - \frac{N_{\rm rep}-3}{N_{\rm rep}-1} \left(\bar{\sigma}^2\right)^2 \right] \right)^{1/2} , \qquad (1)$$

  2. Jackknife method:

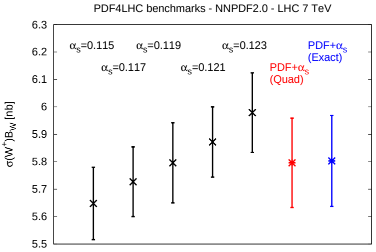
$$\sigma_i^2(x) = \frac{1}{N_{\rm rep}-2} \sum_{j=1, j\neq i}^{N_{\rm rep}} (x_j - \mu_i(x))^2; \quad i = 1, \ldots, N_{\rm rep} , \quad \overline{\sigma_i^2} = \frac{1}{N_{\rm rep}} \sigma_i^2 ,$$

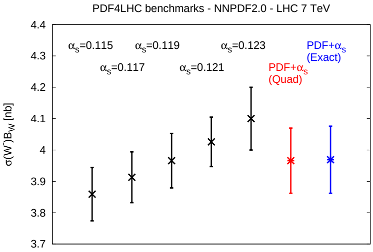
$$\sigma[\sigma^2] = \left(\sum_{i=1}^{N_{\rm rep}} \left[(\sigma_i^2)^2 - \overline{\sigma_i^2}^2 \right] \right)^{1/2} ,$$

- Two different determinations of PDF uncertainties, σ_1^2 and σ_2^2, which differ by $|\sigma_1^2 - \sigma_1^2| \leq \sigma[\sigma^2]$ are statistically equivalent

PDF4LHC BENCHMARK RESULTS
LHC 7 TeV

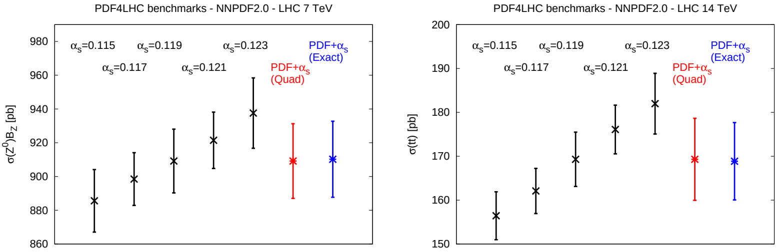
Benchmark observables - W^+ and W^- 7 TeV





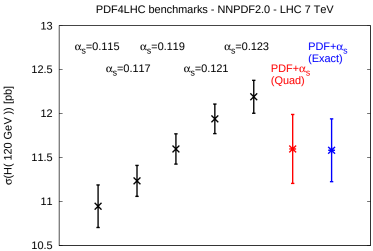
	$\sigma(W^+)\mathrm{Br}\left(W^+ \to l^+\nu_l\right)$	$\sigma(W^-)\mathrm{Br}\left(W^- \to l^+\nu_l\right)$
α_s=0.115	5.65 $\pm$ 0.13 nb	3.86 $\pm$ 0.09 nb
α_s=0.117	5.73 $\pm$ 0.13 nb	3.91 $\pm$ 0.08 nb
α_s=0.119	5.80 $\pm$ 0.15 nb	3.97 $\pm$ 0.09 nb
α_s=0.121	5.87 $\pm$ 0.13 nb	4.03 $\pm$ 0.08 nb
α_s=0.123	5.98 $\pm$ 0.14 nb	4.10 $\pm$ 0.10 nb
PDF+α_s - quad	5.80 $\pm$ 0.16 nb	3.97 $\pm$ 0.10 nb
PDF+α_s exact (68% CL)	5.80 $\pm$ 0.17($\pm$0.14) nb	3.97 $\pm$ 0.11($\pm$0.10) nb

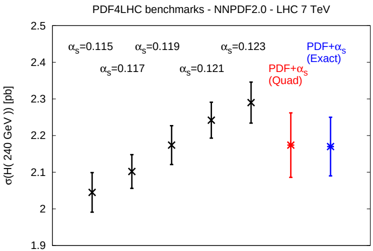
Benchmark observables - Z^0 and $t\bar{t}$ 7 TeV



	$\sigma(Z^0)\mathrm{Br}\left(Z^+ \to l^+ l^-\right)$	$\sigma(t\bar{t})$
α_s=0.115	886 $\pm$ 18 pb	156 $\pm$ 5 pb
α_s=0.117	898 $\pm$ 16 pb	162 $\pm$ 5 pb
α_s=0.119	909 $\pm$ 19 pb	169 $\pm$ 6 pb
α_s=0.121	921 $\pm$ 17 pb	176 $\pm$ 6 pb
α_s=0.123	937 $\pm$ 21 pb	182 $\pm$ 7 pb
PDF+α_s (quad)	909 $\pm$ 22 pb	169 $\pm$ 9 pb
PDF+α_s (exact)	910 $\pm$ 22($\pm$19) pb	169 $\pm$ 9($\pm$9) pb

Benchmark observables - Higgs 7 TeV

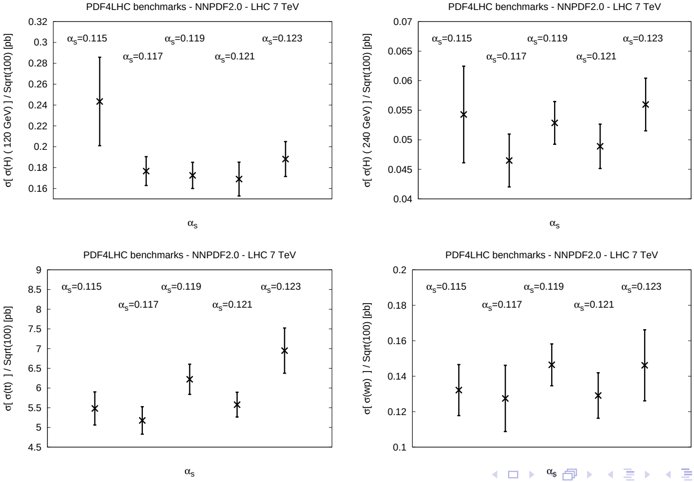




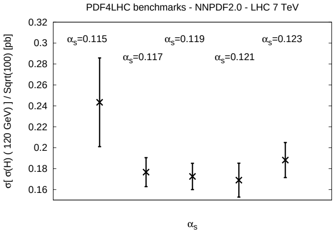
	$\sigma(H)$(120 GeV)	$\sigma(H)$(180 GeV)	$\sigma(H)$(240 GeV)
α_s=0.115	10.95 ± 0.24 pb	4.23 ± 0.10 pb	2.05 ± 0.05 pb
α_s=0.117	11.23 ± 0.18 pb	4.39 ± 0.08 pb	2.10 ± 0.05 pb
α_s=0.119	11.60 ± 0.17 pb	4.54 ± 0.09 pb	2.17 ± 0.05 pb
α_s=0.121	11.94 ± 0.17 pb	4.68 ± 0.08 pb	2.24 ± 0.05 pb
α_s=0.123	12.19 ± 0.19 pb	4.77 ± 0.09 pb	2.29 ± 0.06 pb
PDF+α_s (quad)	11.60 ± 0.39 pb	4.54 ± 0.17 pb	2.17 ± 0.09 pb
PDF+α_s (exact)	11.58 ± 0.36(±0.38) pb	4.53 ± 0.15(±0.16) pb	2.17 ± 0.08(±0.08) pb

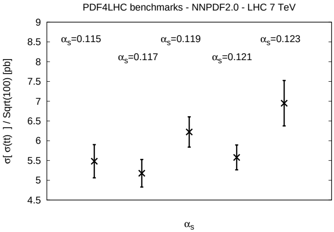
The uncertainty on the PDF uncertainty

Statistical equivalence $\rightarrow$ Compare $\sigma\,[\sigma]\,/\sqrt{N_{rep}}$ for benchmark processes



The uncertainty on the PDF uncertainty

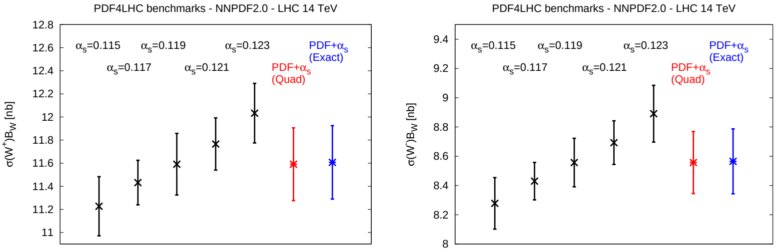




- For all LHC benchmark observables for a reasonable α_s range, $0.117 \le \alpha_s \le 0.121$, PDF uncertainties on observables are statistically equivalent.
- When comparing PDF uncertainties between different sets or different values of α_s, one should account for the uncertainty on the uncertainty

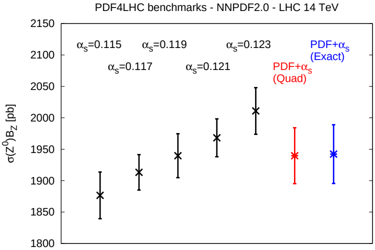
PDF4LHC BENCHMARK RESULTS
LHC 14 TeV

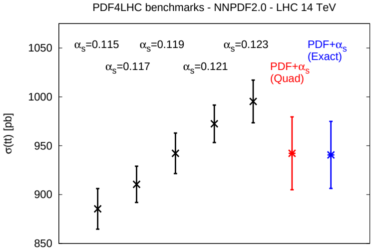
Benchmark observables - W^+ and W^- 14 TeV



	$\sigma(W^+)\mathrm{Br}\left(W^+ \to l^+\nu_l\right)$	$\sigma(W^-)\mathrm{Br}\left(W^- \to l^+\nu_l\right)$
α_s=0.115	11.23 $\pm$ 0.26 nb	8.28 $\pm$ 0.18 nb
α_s=0.117	11.43 $\pm$ 0.19 nb	8.43 $\pm$ 0.13 nb
α_s=0.119	11.59 $\pm$ 0.27 nb	8.56 $\pm$ 0.17 nb
α_s=0.121	11.77 $\pm$ 0.23 nb	8.69 $\pm$ 0.15 nb
α_s=0.123	12.03 $\pm$ 0.26 nb	8.89 $\pm$ 0.19 nb
PDF+α_s quad	11.59 $\pm$ 0.31 nb	8.56 $\pm$ 0.21 nb
PDF+α_s exact (68% CL)	11.61 $\pm$ 0.32 ($\pm$0.28) nb	8.56 $\pm$ 0.22 ($\pm$0.21) nb

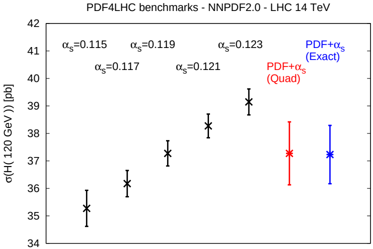
Benchmark observables - Z^0 and $t\bar{t}$ 14 TeV

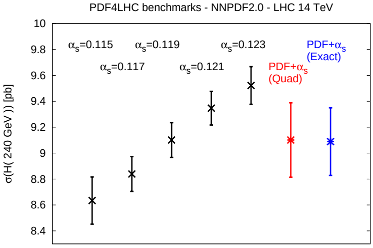




	$\sigma(Z^0)\mathrm{Br}\left(Z^+ \to l^+ l^-\right)$	$\sigma(t\bar{t})$
α_s=0.115	1876 $\pm$ 37 pb	885 $\pm$ 20 pb
α_s=0.117	1913 $\pm$ 28 pb	910 $\pm$ 19 pb
α_s=0.119	1939 $\pm$ 35 pb	942 $\pm$ 21 pb
α_s=0.121	1968 $\pm$ 30 pb	972 $\pm$ 19 pb
α_s=0.123	2010 $\pm$ 37 pb	995 $\pm$ 22
PDF+α_s (quad)	1939 $\pm$ 44 pb	942 $\pm$ 37 pb
PDF+α_s (exact)	1942 $\pm$ 47($\pm$42) pb	940 $\pm$ 34($\pm$35) pb

Benchmark observables - Higgs 14 TeV

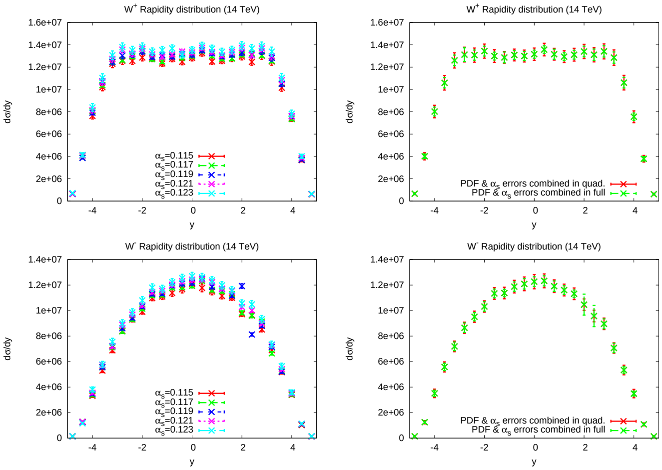




	$\sigma(H)(120\,\mathrm{GeV})$	$\sigma(H)(180\,\mathrm{GeV})$	$\sigma(H)(240\,\mathrm{GeV})$
α_s=0.115	35.27 $\pm$ 0.66 pb	15.95 $\pm$ 0.32 pb	8.63 $\pm$ 0.18 pb
α_s=0.117	36.17 $\pm$ 0.48 pb	16.34 $\pm$ 0.23 pb	8.83 $\pm$ 0.13 pb
α_s=0.119	37.27 $\pm$ 0.46 pb	16.83 $\pm$ 0.22 pb	9.10 $\pm$ 0.13 pb
α_s=0.121	38.27 $\pm$ 0.43 pb	17.28 $\pm$ 0.21 pb	0.35 $\pm$ 0.13 pb
α_s=0.123	39.15 $\pm$ 0.47 pb	17.65 $\pm$ 0.23 pb	9.52 $\pm$ 0.14 pb
PDF+α_s - quad	37.23 $\pm$ 1.06 pb	16.83 $\pm$ 0.51 pb	9.10 $\pm$ 0.26 pb
PDF+α_s - exact (68% CL)	37.23 $\pm$ 1.14 ($\pm$1.06) pb	16.81 $\pm$ 0.47($\pm$0.50) pb	9.09 $\pm$ 0.26($\pm$0.27) pb

Rapidity distributions - W^+ and W^- LHC 14 TeV

Results for rapidity distributions available in the NNPDF PDF4LHC benchmark doc



The uncertainty on the combined PDF+α_s uncertainty

Compare the uncertainty on the uncertainty $\sigma(\sigma)$ for all benchmark processes with either the sample estimator (I) or with the jackknife method (II)

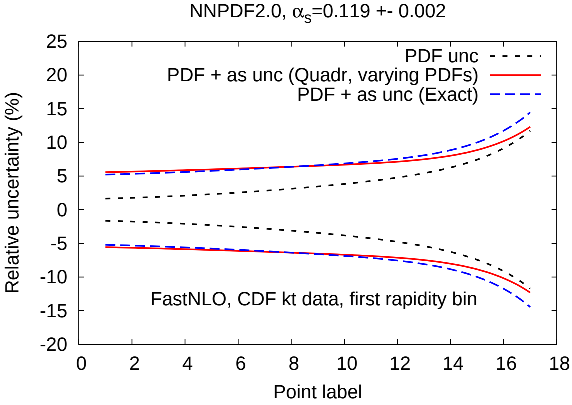
Process	σ	$\sigma(\sigma)$ (%) (I)	$\sigma(\bar{\sigma}) = \sigma(\sigma)/\sqrt{N_{\rm rep}}$ (%) (I)	$\sigma(\sigma)$ (%) (II)	$\sigma(\bar{\sigma})$ (%) (II)
W^+	0.31 nb	78	7.8	72	7.2
W^-	0.22 nb	77	7.7	70	7.0
Z^0	47 pb	78	7.8	71	7.1
h(120)	1.06 pb	61	6.1	50	5.0
h(180)	0.47 pb	61	6.1	50	5.0
h(240)	0.26 pb	62	6.2	51	5.1
$t\bar{t}$	34.4 pb	65	6.5	56	5.6

- The uncertainty on a PDF cannot be determined with an accuracy greater than the uncertainty on the uncertainty itself

PDF4LHC BENCHMARK RESULTS TEVATRON

Inclusive jets at Tevatron

CDF k_t inclusive jets, `fnt2004` FastNLO scenario
Excellent agreement for combination PDF+α_s uncertainties in quadrature and with exact error correlation.

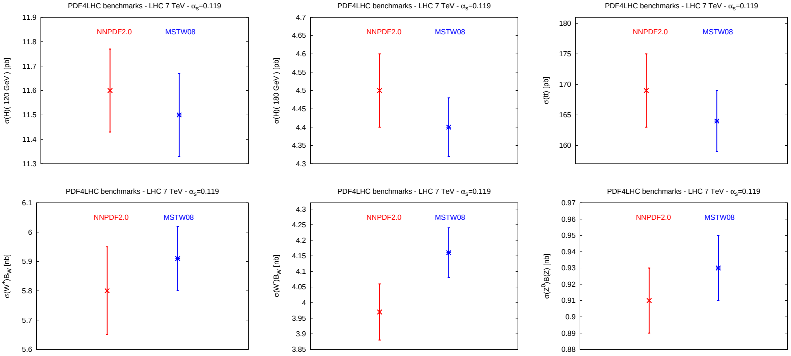


COMPARISON OF BENCHMARK RESULTS

Benchmark comparison - NNPDF vs. MSTW

- Compare benchmark results from NNPDF2.0 and MSTW08 for $\alpha_s = 0.119$ at LHC 7 TeV and 14 TeV - PDF uncertainties only, 1–sigma errors
- For MSTW08, take as PDF errors those for the $\alpha_s = 0.120$ set - differences cannot be statistically relevant
- We compare also the respective uncertainties, taking into account the NNPDF determination of the uncertainty on the uncertainty
- Update once results from all groups available

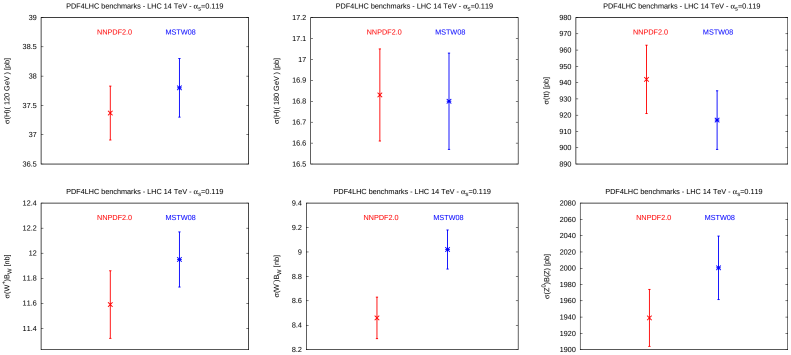
Benchmark comparison - NNPDF vs. MSTW 7 TeV



Excellent agreement once common value of α_s adopted!

(only exception $\rightarrow$ W^-)

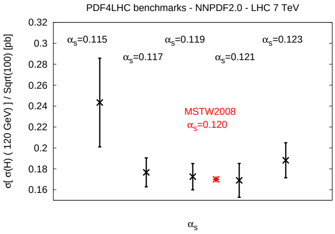
Benchmark comparison - NNPDF vs. MSTW 14 TeV

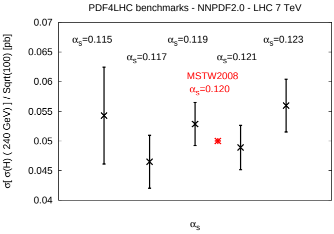


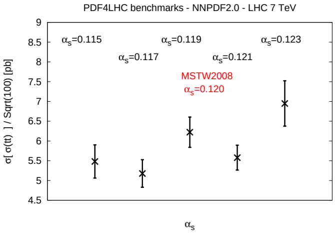
Excellent agreement once common value of α_s adopted!

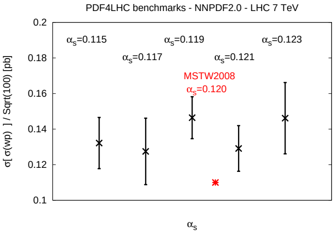
(only exception $\rightarrow$ W^-)

Comparison of PDF uncertainties









THE PDF4LHC BENCHMARKS: THE NNPDF RECOMMENDATION

PDF+α_s combined uncertainties

PDG 2009 average is $\alpha_s\left(M_Z^2\right)_{\rm PDG} = 0.1184 \pm 0.0007$ while PDF fits find:

	$\alpha_s(M_Z^2)$	
ABKM	0.1135 ± 0.0014	heavy quarks: FFN $N_f = 3$
ABKM	0.1129 ± 0.0014	heavy quarks: BMSN-approach
BBG [42]	$0.1134\ ^{+0.0019}_{-0.0021}$	valence analysis, NNLO
AMP06 [37]	0.1128 ± 0.0015	
JR [52]	0.1124 ± 0.0020	dynamical approach
MSTW [43]	0.1171 ± 0.0014	
BBG [42]	$0.1141\ ^{+0.0020}_{-0.0022}$	valence analysis, N^{3}LO

- Determinations of α_s from PDF analyses, by construction, cannot be more accurate than PDG global average
- Therefore, PDF determinations should be provided for a reasonable range of α_s values, centered at $\alpha_s\left(M_Z^2\right)_{\rm PDG}$, for their reference PDF sets, leaving α_s determinations to dedicated studies

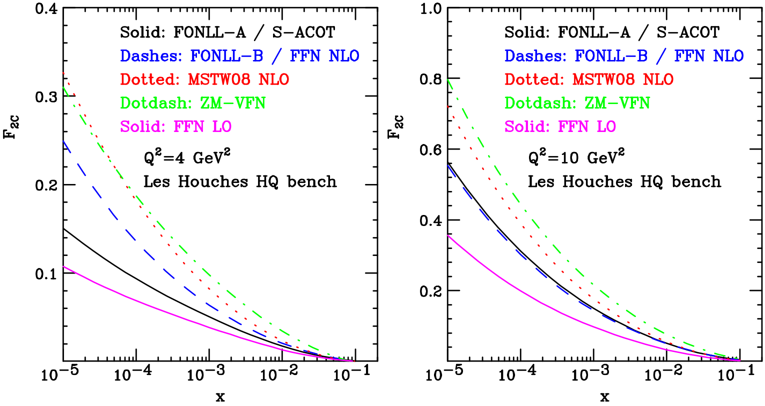
PDF+α_s combined uncertainties

- PDF sets should be provided for a series of values of α_s $\rightarrow$ Choice of $\alpha_s^{(0)}$, δ_{α_s} should be left to the PDF users
- Through all benchmark examples addition in quadrature of PDF and α_s uncertainties and exact error correlation in excellent agreement
- No empirical justification so far prevents to add in quadrature PDF and α_s uncertainties.
- PDF uncertainties are essentially unchanged for a reasonable range of α_s (within their own uncertainties).

All evidence suggest that all PDF groups could (and should) provide PDF sets for a reasonable common range of α_s

A caveat on HQ

Differences between GM-VFN HQ schemes are as large as ZM-VFN/FFN difference
Until theoretical uncertainties associated to GM HQ scheme choice throughly examined, differences between ZM and GM PDF sets provide a reasonable estimate of these uncertainties.



Determination of total PDF uncertainty

1. When comparing predictions from different PDF sets, it is mandatory to always adopt the same value of α_s (close to $\alpha_s\left(M_Z^2\right)_{\rm PDG}$)
2. If the above condition is met, **it is not necessary** to include PDF+α_s combined uncertainties in the total PDF uncertainty
3. If predictions from different sets differ by more than 1.7–sigma with PDF uncertainty only, compute 90% C.L. uncertainties and take the envelope (and divide by $C_{90} = 1.642$ to get 1–sigma errors)
4. Predictions from **all global PDF** sets (which can satisfy condition (1)) should be included in the envelope
5. More work (benchmark-like) required to further understand difference between PDF sets