

Interpretation of charged Higgs effects in low energy flavour physics

THIRD INTERNATIONAL WORKSHOP

cH[±]arged 2010



**Prospects for Charged Higgs
Discovery at Colliders**

Uppsala University, Sweden, 27–30 September 2010

Tobias Hurth



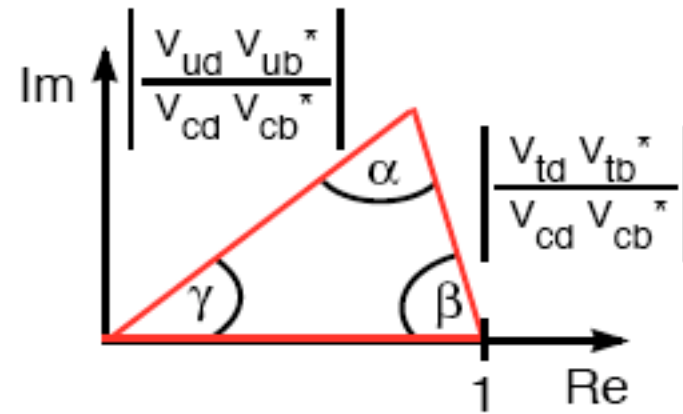
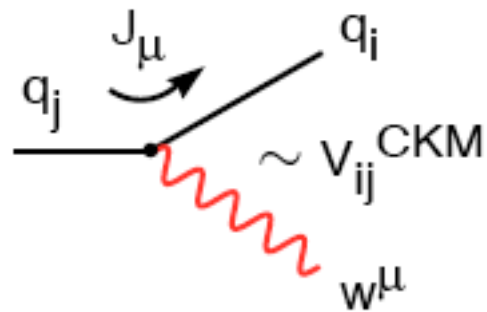
Plan of the talk

- Prologue: Flavour problem of SM
- Flavour problem of NP
- Natural flavour violation (NFV) in 2HDMs
- Parameter bounds in 2HDMs
- Minimal flavour violation (MFV)
- NFV and MFV beyond tree level

Prologue

Flavour in the SM

CKM mechanism of flavour mixing and CP violation: V_{CKM} , J_{CKM}



$$\text{Im}[V_{ij} V_{kl} V_{il}^* V_{kj}^*] = J_{CKM} \sum_{m,n=1}^3 \epsilon_{ikm} \epsilon_{jln}$$

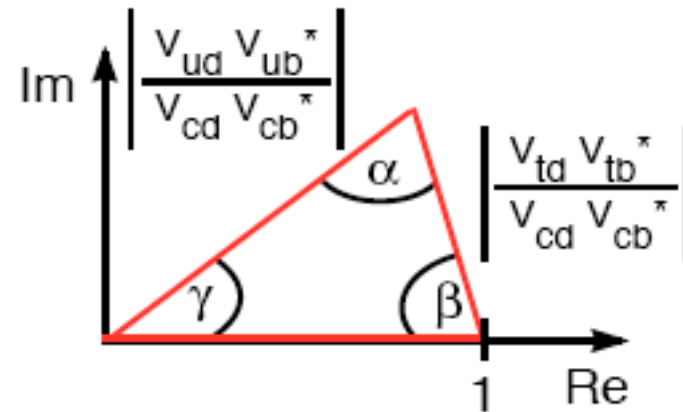
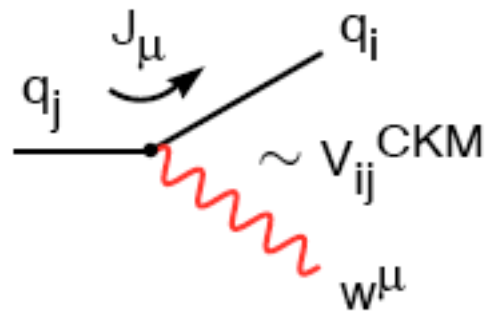
$$J_{CKM} \sim \mathcal{O}(10^{-5})$$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Prologue

Flavour in the SM

CKM mechanism of flavour mixing and CP violation: V_{CKM} , J_{CKM}



$$\text{Im}[V_{ij} V_{kl} V_{il}^* V_{kj}^*] = J_{\text{CKM}} \sum_{m,n=1}^3 \epsilon_{ikm} \epsilon_{jln} \quad J_{\text{CKM}} \sim \mathcal{O}(10^{-5})$$

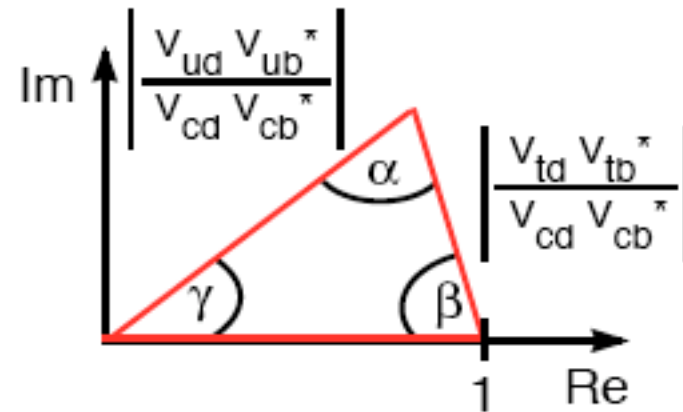
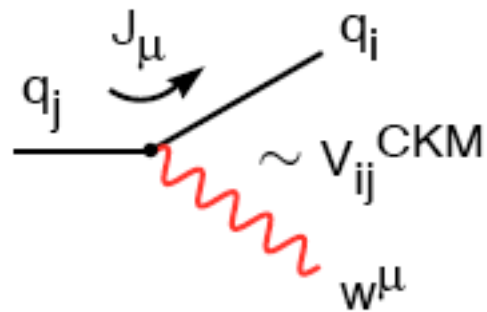
All present measurements (BaBar, Belle, CLEO, CDF, D0,....) of rare decays ($\Delta F = 1$), of mixing phenomena ($\Delta F = 2$) and of all CP violating observables at tree and loop level are consistent with the CKM theory.

Impressing success of SM and CKM theory !!

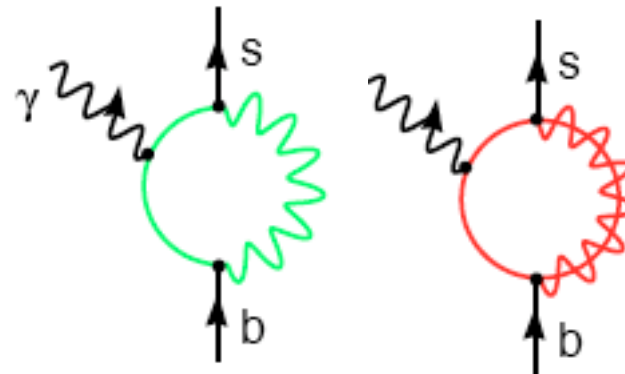
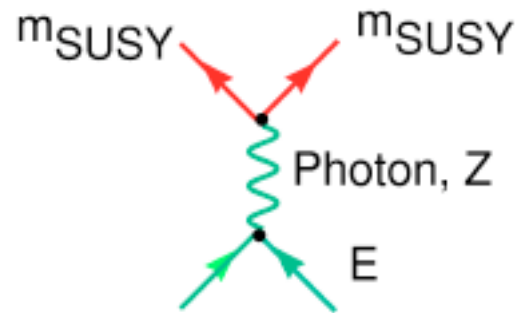
Prologue

Flavour in the SM

CKM mechanism of flavour mixing and CP violation: V_{CKM} , J_{CKM}



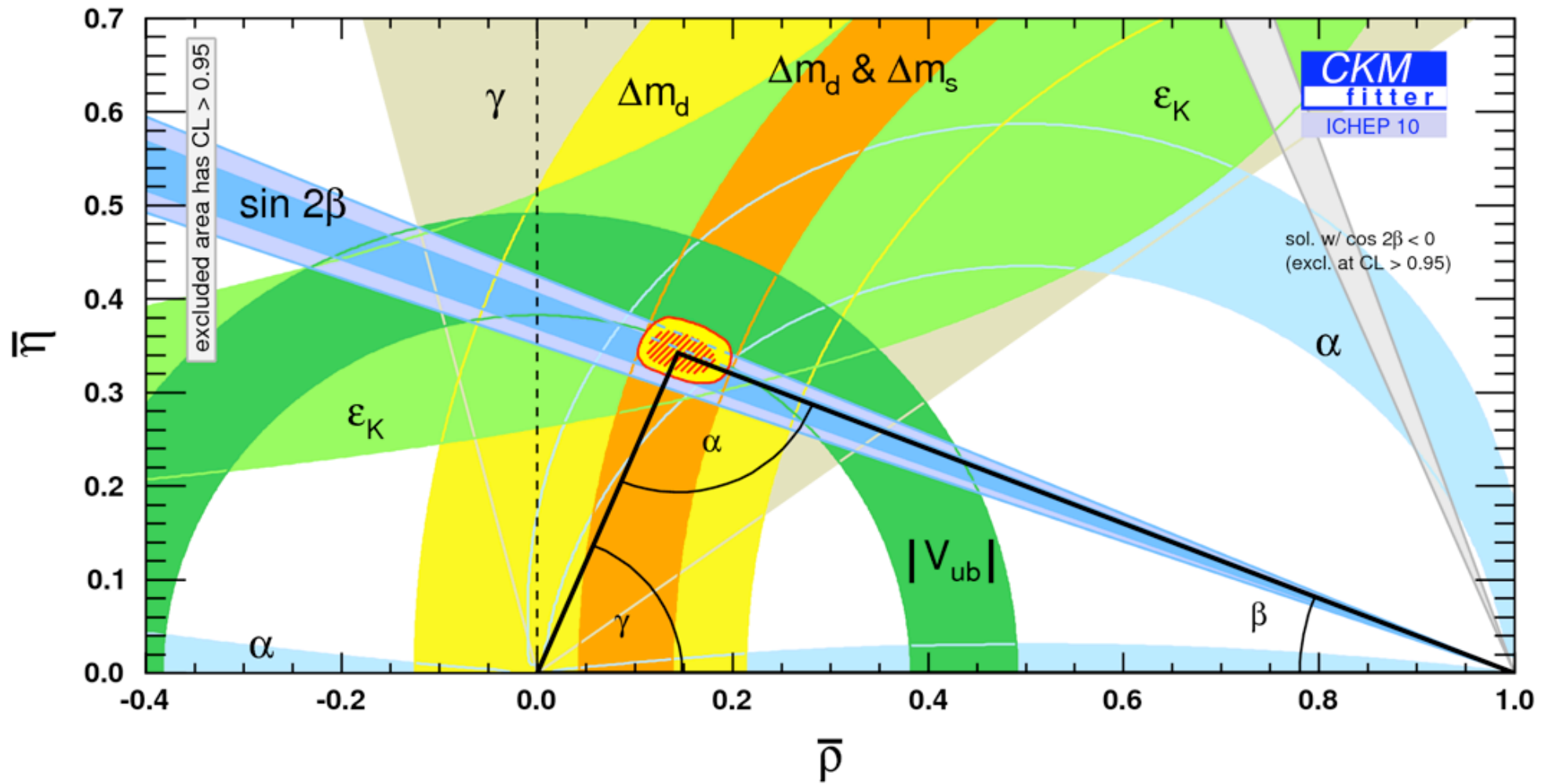
This success is somehow unexpected !!



Flavour-changing-neutral-currents as loop-induced processes are highly-sensitive probes for possible new degrees of freedom

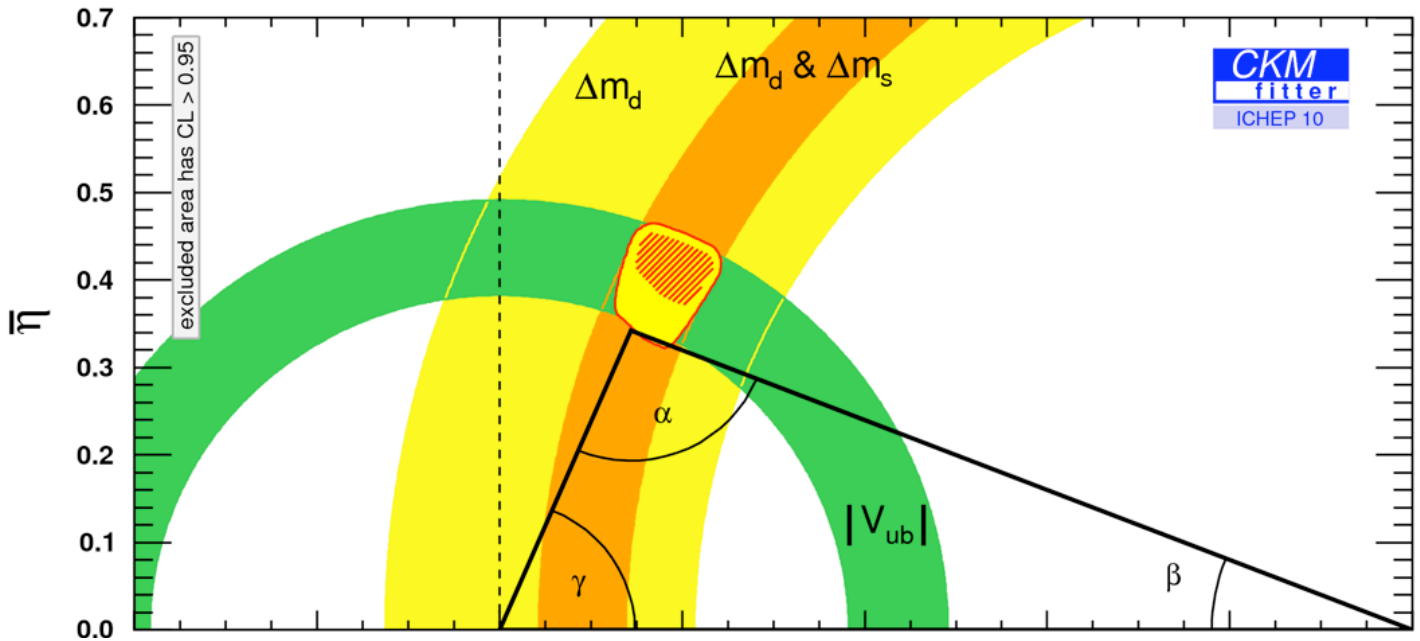
Impressing success of SM and CKM theory !!

Global fit, consistency check of the CKM theory.

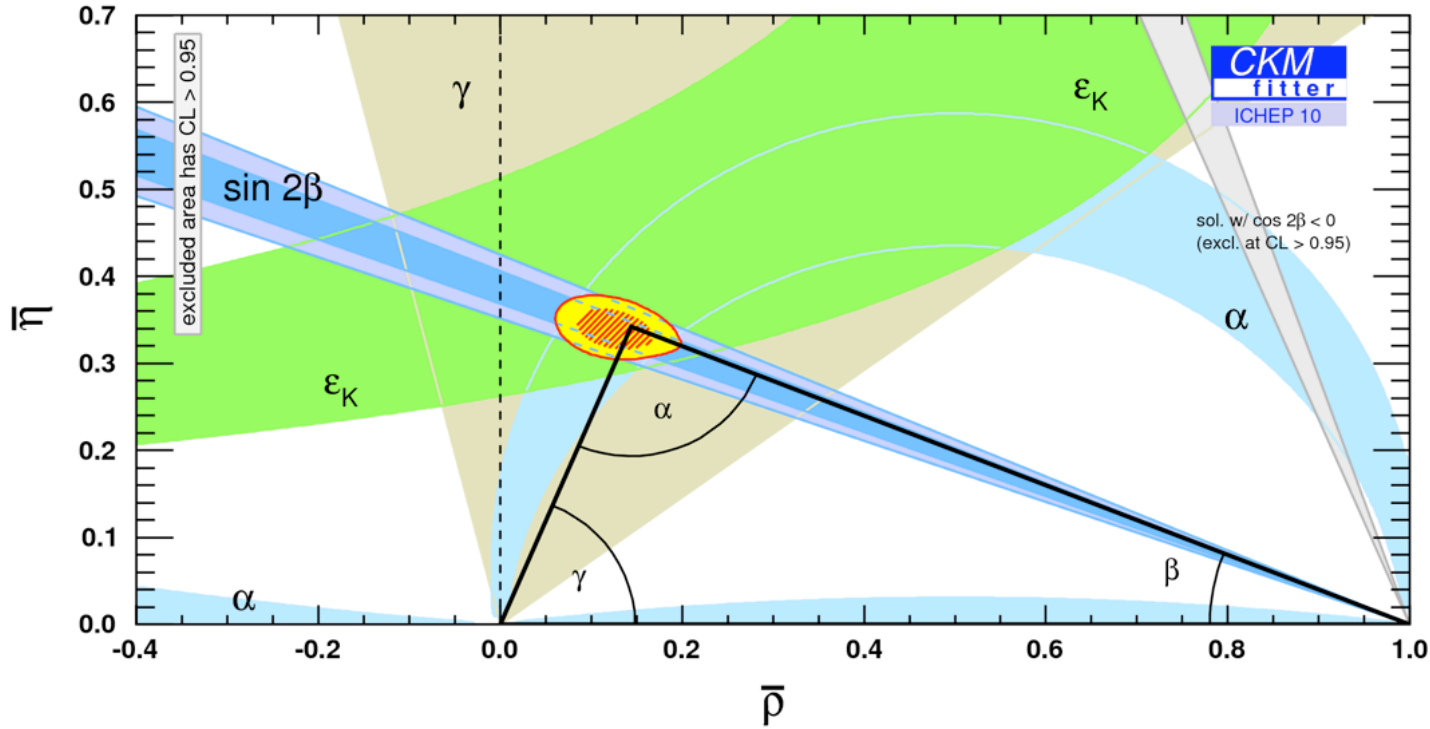


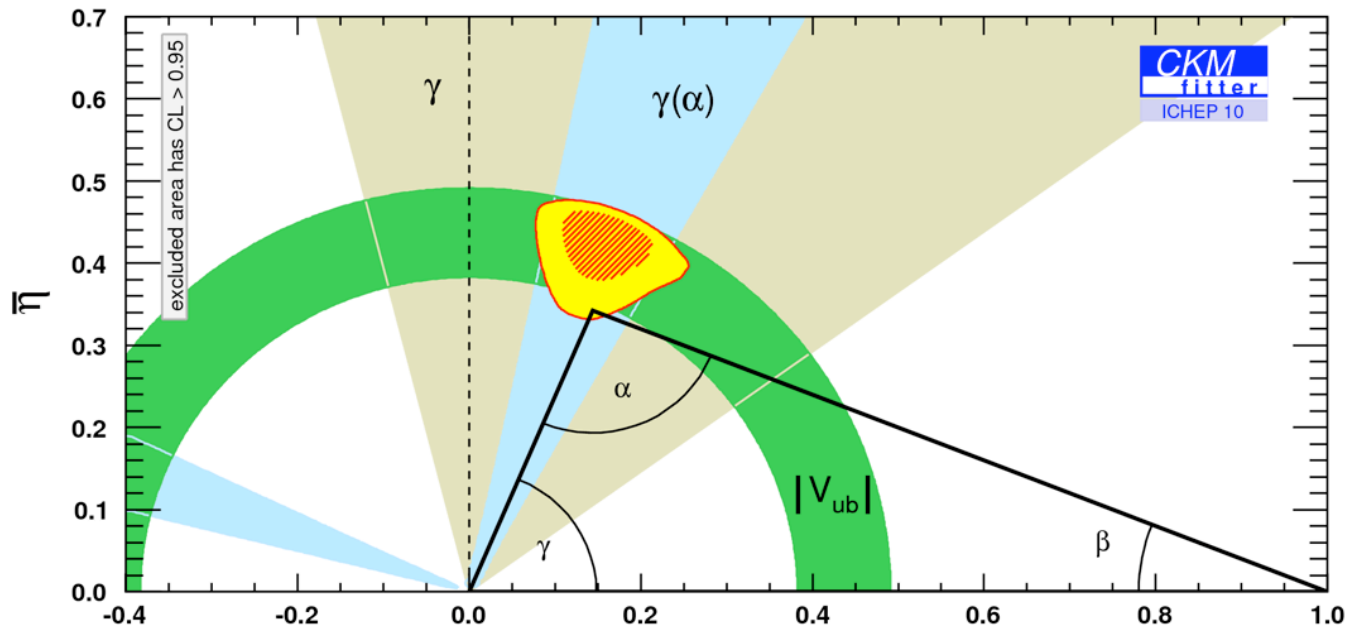
Closer Look:

CP conserving

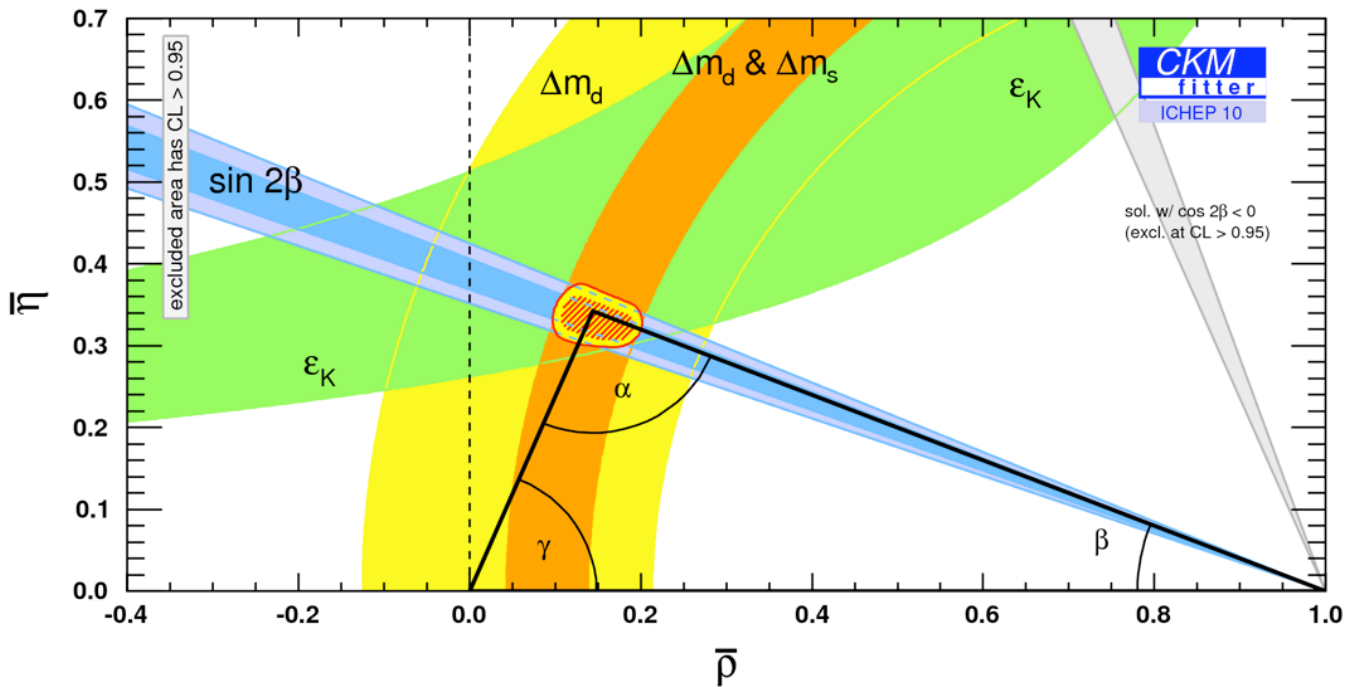


CP violating observables



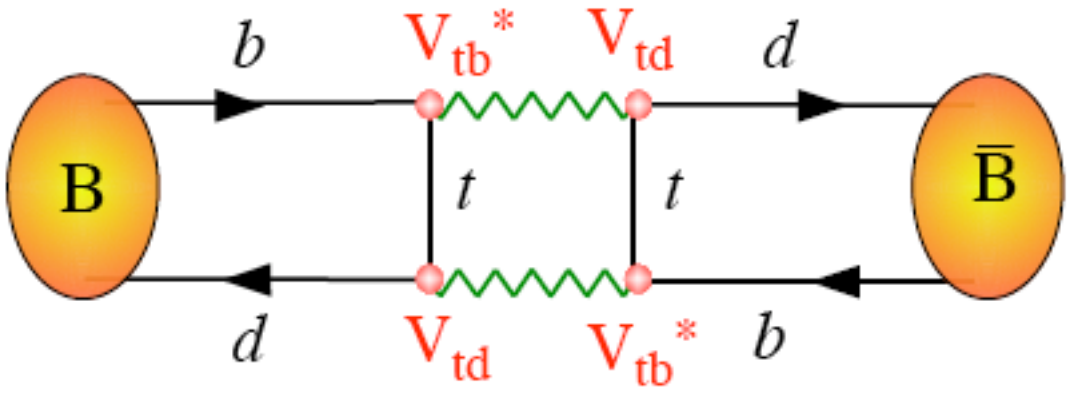
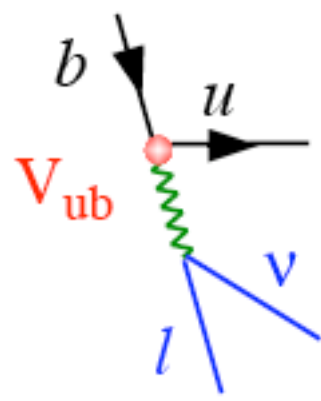


Tree processes



Loop processes

Most surprising is the consistency between the tree-level and loop-induced observables



Semileptonic tree-decays

versus

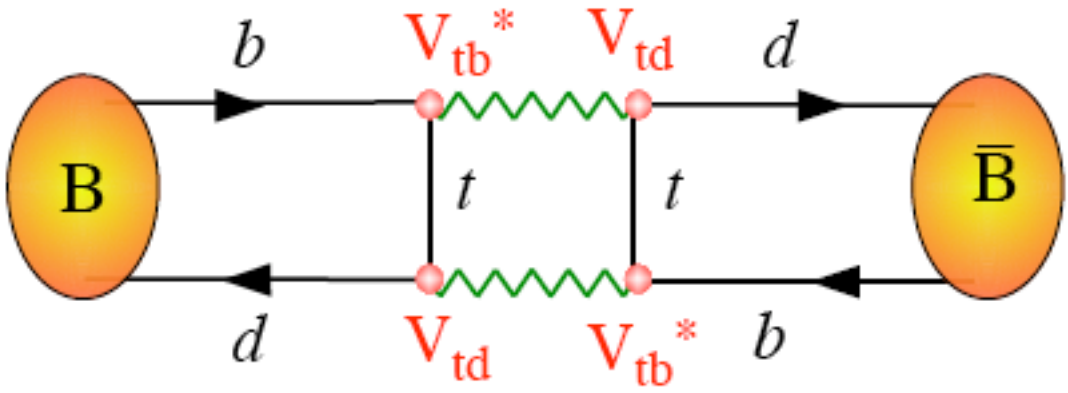
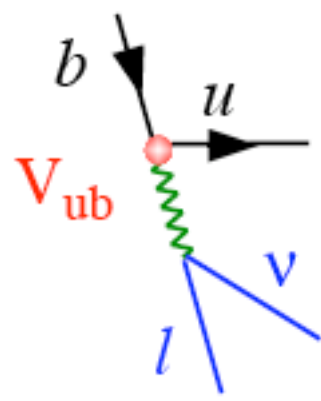
Neutral-meson mixing

$\Delta F = 2$

SM-dominated

Potentially more sensitive
to New Physics

Most surprising is the consistency between the tree-level and loop-induced observables



Semileptonic tree-decays versus Neutral-meson mixing $\Delta F = 2$

SM-dominated

Potentially more sensitive to New Physics

There is much more data not shown in the unitarity fits which confirms the SM predictions of flavour mixing like rare decays ($\Delta F = 1$)

Nobel Prize 2008



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***CP*-Violation in the Renormalizable Theory of Weak Interaction**

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KIKUYASHI and Toshihide MARIKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quark sector without introducing any other new fields. Some possible models of CP-violation are also discussed.

When we apply the renormalizable theory of weak interaction^{1) to the hadron system, we have some limitations on the hadron model. It is well known that there exist, in the case of the triplet model, a difficulty of the strangeness changing neutral current and that the quartet model is free from this difficulty. Furthermore, Maki and one of the present authors (TM) have shown^{2) that, in the latter case, the strong interaction must be chiral SU(4) × SU(4) invariant as precisely as the conservation of the third component of the isospin I₃. In addition to these arguments, for the theory to be realistic, CP-violating interactions should be incorporated in a gauge invariant way. This requirement will impose further limitations on the hadron model and the CP-violating interactions itself. The purpose of the present paper is to investigate this problem. In the following it will be shown that in the case of the above-mentioned quartet model, we cannot make a CP-violating interaction without introducing any other new fields when we require the following conditions: a) The case of the fourth member of the quartet, which we will call ζ, is sufficiently large, b) the model should be consistent with our well-established knowledge of the semi-leptonic processes. After that some possible ways of bringing CP-violation into the theory will be discussed.}}

We consider the quartet model with a charge assignment of Q, Q-1, Q-1 and Q for p, n, l and ζ, respectively, and we take the same underlying gauge group SU_{weak}(2) × SU(3) and the scalar doublet field φ as those of Weinberg's original model.^{3) Thus, hadronic parts of the Lagrangian can be divided in the following way:}

$$L_{had} = L_{kin} + L_{mass} + L_{strong} + L,$$

where L_{kin} is the gauge-invariant kinetic part of the quartet field φ, so that it contains interactions with the gauge fields. L_{mass} is a generalized mass term of φ, which includes Yukawa couplings to ψ where they contribute to the mass of ψ through the spontaneous breaking of gauge symmetry. L_{strong} is a strong-inter-

action part which conserves I₃ and therefore chiral SU(4) × SU(4) invariant.^{5) We assume C and P invariance of L_{strong}. The last term denotes residual interaction parts if they exist. Since L_{mass} includes couplings with φ, it has possibilities of violating CP-conservation. As is known as Higgs phenomenon,^{6) these massless components of φ can be absorbed into the massive gauge fields and eliminated from the Lagrangian. Even after this has been done, both scalar and pseudoscalar parts remain in L_{mass}. For the mass term, however, we can eliminate such pseudoscalar parts by applying an appropriate constant gauge transformation on φ, which does not affect on L_{strong} due to gauge invariance.}}

Now we consider possible ways of assigning the quartet field to representations of the SU_{weak}(2). Since this group is commutative with the Lorentz transformation, the left and right components of the quartet field, which are respectively defined as φ_L = 1/2(1+i)φ and φ_R = 1/2(1-i)φ, do not mix each other under the gauge transformation. Then, each component has three possibilities:

- A) 4 = 2 + 2,
- B) 4 = 2 + 1 + 1,
- C) 4 = 1 + 1 + 1 + 1,

where the n, k, s, n' denotes an n-dimensional representation of SU(2). The present scheme of charge assignment of the quartet does not permit representations of n ≥ 3. As a result, we have nine possibilities which we will denote by (A, A), (A, B), ..., where the former (latter) in the parentheses indicates the transformation properties of the left (right) component. Since all members of the quartet should take part in the weak interaction, and size of the strangeness changing neutral current is bounded experimentally to a very small value, the cases of (B, C), (C, B) and (C, C) should be abandoned. The models of (B, A) and (C, A) are equivalent to those of (A, B) and (A, C), respectively, except relative signs between vector and axial vector parts of the weak current. Since p₁/p₂ ratios are measured only for composite states, this difference of the relative signs would be related to a dynamical problem of the composite system. So, we investigate in detail the cases of (A, A), (A, B), (A, C) and (B, B).

1) Case (A, C)

This is the most natural choice in the quartet model. Let us denote two (SU_{weak}(2)) doublets and four singlets by L₁, L₂, R₁⁺, R₁⁰, R₁⁻ and R₂⁺, where superscript p(s) indicates p-like (s-like) charge states. In this case, L_{mass} takes, in general, the following form:

$$L_{mass} = \sum_{i,j=1}^4 [M_{ij}^{++} L_{i1} R_{j1}^{++} + M_{ij}^{+0} L_{i1} R_{j1}^{+0}] + h.c.,$$

$$p^{\pm} m \begin{pmatrix} p^{\pm} \\ p^{\pm} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (1)$$

2) Case (A, A)

In a similar way, we can show that no CP-violation occurs in this case as far as L⁺ = 0. Furthermore this model would reduce to an exactly U(6) symmetric one.

Summarizing the above results, we have no realistic models in the quark sector as far as L⁺ = 0. Now we consider some examples of CP-violation through L⁺. Hereafter we will consider only the case of (A, C). The first one is to introduce another scalar doublet φ'. Then, we may consider an interaction with this new field

$$L' = \phi \phi G \frac{1-i}{2} \phi_2 + h.c., \quad (1')$$

$$\phi = \begin{pmatrix} \bar{p}^+ & \bar{p}^0 & 0 & 0 \\ -\bar{p}^0 & \bar{p}^+ & 0 & 0 \\ 0 & 0 & \bar{n}^+ & \bar{p}^+ \\ 0 & 0 & -\bar{p}^+ & \bar{n}^+ \end{pmatrix}, \quad G = \begin{pmatrix} c_1 & c_2 & c_3 & 0 \\ 0 & d_1 & 0 & d_2 \\ c_3 & c_2 & c_1 & 0 \\ 0 & d_2 & 0 & d_1 \end{pmatrix}$$

where c_i and d_i are arbitrary complex numbers. Since we have already made use of the gauge transformation to get rid of the CP odd part from the quark mass term, there remains no such arbitrariness. Furthermore, we note that an arbitrariness of the phase of φ cannot absorb all the phases of c_i and d_i. So, this interaction can cause a CP-violation.

Another one is a possibility associated with the strong interaction. Let us consider a scalar (pseudoscalar) field S which mediates the strong interaction. For the interaction to be renormalizable and SU_{weak}(2) invariant, it must belong to a (4, 4*) + (4*, 4) representation of chiral SU(4) × SU(4) and interact with φ through scalar and pseudoscalar couplings. It also interacts with φ and possible renormalizable forms are given as follows:

$$\begin{aligned} & \text{tr}(G_1 S^{\dagger} \phi^{\dagger} + h.c.), \\ & \text{tr}(G_2 S^{\dagger} \phi^{\dagger} G_3 \phi S) + h.c., \\ & \text{tr}(G_4 S^{\dagger} \phi^{\dagger} G_5 G_6 S \phi^{\dagger}) + h.c., \end{aligned} \quad (1'')$$

with

$$G = \begin{pmatrix} \rho^+ & \rho^0 & 0 & 0 \\ -\rho^0 & \rho^+ & 0 & 0 \\ 0 & 0 & \rho^+ & \rho^+ \\ 0 & 0 & -\rho^+ & \rho^+ \end{pmatrix},$$

where G_i is a 4 × 4 complex matrix and we have used a 4 × 4 matrix representation for S. It is easy to see that these interaction terms can violate CP-conservation.

where M_{ij}⁺ and M_{ij}⁰ are arbitrary complex numbers. We can eliminate three Goldstone modes φ, by putting

$$\phi = e^{i\alpha} \begin{pmatrix} 0 \\ 1+i\epsilon \end{pmatrix} \quad (2)$$

where i is a vacuum expectation value of φ' and ε is a massive scalar field. Therefore, performing a diagonalization of the remaining mass term, we obtain

$$L_{mass} = \phi \eta \phi \left(1 + \frac{\epsilon}{2} \right),$$

$$\eta = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix}, \quad \eta^{-1} = \begin{pmatrix} \rho \\ \sigma \\ \tau \\ \lambda \end{pmatrix} \quad (3)$$

Then, the interaction with the gauge field in L_{had} is expressed as

$$\frac{1}{2} A_{ij}^{\dagger} i \partial_{\mu} A_{\mu} \epsilon_{ij} \frac{1+i}{2} \phi. \quad (4)$$

Here, A_{ij} is the representation matrix of SU_{weak}(2) for this case and explicitly given by

$$A_{ij} = \frac{A_i + iA_j}{2} = K \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix} K^{-1}, \quad A_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

where U is a 2 × 2 unitary matrix. Here and hereafter we neglect the gauge field corresponding to U(1) which is irrelevant to our discussion. With an appropriate phase convention of the quartet field we can take U as

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (6)$$

Therefore, if L⁺ = 0, no CP-violations occur in this case. It should be noted, however, that this argument does not hold when we introduce one more fermion doublet with the same charge assignment. This is because all phases of elements of a 2 × 2 unitary matrix cannot be absorbed into the phase convention of six fields. This problem of CP-violation will be discussed later on.

3) Case (A, B)

This is a rather delicate case. We denote two left doublets, one right doublet and two singlets by L₁, L₂, R₁, R₂⁺ and R₂⁰, respectively. The general form

Next we consider a 6plet model, another interesting model of CP-violation. Suppose that 6plet with charges (2, 2, 0, 0, Q-1, Q-1, Q-1) is decomposed into SU_{weak}(2) multiplets as 2 + 2 + 2 and 1 + 1 + 1 + 1 + 1 + 1 for left and right components, respectively. Just as the case of (A, C), we have a similar expression for the charged weak current with a 3 × 3 instead of 2 × 2 unitary matrix in Eq. (5). As we pointed out in this case we cannot absorb all phases of matrix elements into the phase convention and one takes, for example, the following expression:

$$\begin{pmatrix} \cos \theta & -\sin \theta \cos \theta & & & -\sin \theta & \sin \theta \\ \sin \theta \cos \theta & \cos \theta \cos \theta \cos \theta & -\sin \theta \sin \theta \sin \theta & \cos \theta & \cos \theta \sin \theta & \sin \theta \sin \theta \\ \sin \theta \sin \theta & \cos \theta \sin \theta \cos \theta & +\cos \theta \sin \theta \sin \theta & \cos \theta \sin \theta \sin \theta & \cos \theta \sin \theta \sin \theta & -\cos \theta \sin \theta \sin \theta \end{pmatrix} \quad (1'')$$

Then, we have CP-violating effects through the interference among these different current components. An interesting feature of this model is that the CP-violating effects of lowest order appear only in ββ⁰ non-leptonic processes and in the semi-leptonic decay of neutral strange mesons (we are not concerned with higher order with the new quantum number) and not in the other semi-leptonic, ΔS=0 non-leptonic and pure-leptonic processes.

So far we have considered only the straightforward extensions of the original Weinberg's model. However, other schemes of underlying gauge groups and/or scalar fields are possible. Georgi and Glashow's model^{7) is one of them. We can easily see that CP-violation is incorporated into their model without introducing any other fields than (many) new fields which they have introduced already.}

References

- 1) S. Weinberg, Phys. Rev. Letters 18 (1967), 1544, 1545, 1548.
- 2) E. Maki and T. Marikawa, KIPP-146 (unpubl.), April 1972.
- 3) P. W. Higgs, Phys. Letters 12 (1964), 132, 133 (1964), 306.
- 4) S. Glashow, C. N. Hagen and T. W. Kibble, Phys. Rev. Letters 19 (1967), 816.
- 5) H. Georgi and S. L. Glashow, Phys. Rev. Letters 22 (1972), 1546.

Equations

Equation (13) should read as

$$\begin{pmatrix} \cos \theta & -\sin \theta \cos \theta & & & -\sin \theta \sin \theta \\ \sin \theta \cos \theta & \cos \theta \cos \theta \cos \theta & -\sin \theta \sin \theta \sin \theta & \cos \theta & \cos \theta \sin \theta \\ \sin \theta \sin \theta & \cos \theta \sin \theta \cos \theta & +\cos \theta \sin \theta \sin \theta & \cos \theta \sin \theta \sin \theta & \cos \theta \sin \theta \sin \theta \\ \cos \theta \cos \theta \cos \theta & -\sin \theta \sin \theta \sin \theta & \cos \theta \sin \theta \sin \theta & \cos \theta \sin \theta \sin \theta & \cos \theta \sin \theta \sin \theta \end{pmatrix} \quad (13)$$

of L_{mass} is given by

$$L_{mass} = \sum_{i,j=1}^4 [\mu_{ij} L_{i1} R_{j1} + M_{ij}^{++} L_{i1} R_{j1}^{++} + M_{ij}^{+0} L_{i1} R_{j1}^{+0}] + h.c.,$$

where μ_{ij}, M_{ij}⁺ and M_{ij}⁰ are arbitrary complex numbers. After diagonalization of mass terms (in this case, the CP odd part of coupling with φ does not disappear in general) each multiplet can be expressed as follows:

$$\begin{aligned} L_{11} &= \frac{1-i}{2} \begin{pmatrix} \cos \theta \cos \theta \phi \\ \cos \theta \sin \theta \phi \\ \sin \theta \cos \theta \phi \\ \sin \theta \sin \theta \phi \end{pmatrix}, \quad L_{21} = \frac{1-i}{2} \begin{pmatrix} \sin \theta \phi \\ -\sin \theta \cos \theta \phi \\ \cos \theta \sin \theta \phi \\ \cos \theta \phi \end{pmatrix}, \\ R_{11} &= \frac{1-i}{2} \begin{pmatrix} \sin \theta \cos \theta \phi \\ \sin \theta \phi \\ \cos \theta \cos \theta \phi \\ \cos \theta \sin \theta \phi \end{pmatrix}, \quad R_{21}^{++} = \frac{1-i}{2} \begin{pmatrix} \cos \theta \phi \\ \cos \theta \sin \theta \phi \\ \sin \theta \cos \theta \phi \\ \sin \theta \phi \end{pmatrix}, \\ R_{21}^{+0} &= \frac{1-i}{2} \begin{pmatrix} \cos \theta \sin \theta \phi \\ \cos \theta \phi \\ \sin \theta \cos \theta \phi \\ \sin \theta \phi \end{pmatrix}, \end{aligned} \quad (7)$$

where phase factors α, β and γ satisfy two relations with the masses of the quartet:

$$\begin{aligned} e^{i\alpha} \cos \theta \sin \theta \cos \theta \cos \theta &= m_1 \cos \theta \sin \theta e^{-i\beta} m_2 \sin \theta, \\ e^{i\gamma} \cos \theta \cos \theta \cos \theta &= -m_3 \sin \theta \cos \theta e^{-i\delta} m_4 \cos \theta. \end{aligned} \quad (8)$$

Owing to the presence of phase factors, there exists a possibility of CP-violation also through the weak current. However, the strangeness changing neutral current is proportional to sin θ cos θ and its experimental upper bound is roughly

$$\sin \theta \cos \theta < 10^{-3}. \quad (9)$$

Thus, making an approximation of sin θ = 0 (the other choice cos θ = 0 is less critical) we obtain from Eq. (8)

$$\begin{aligned} m_1/m_2 &= \cos \theta \cos \theta, \\ m_3/m_4 &= \sin \theta \sin \theta. \end{aligned} \quad (10)$$

We have no lowlying particle with a quantum number corresponding to ζ, so that m_ζ, which is a measure of chiral SU(4) × SU(4) breaking, should be sufficiently large compared to the masses of the other members. However, the present experimental results on the p₁/p₂ ratios of the octet baryon β-decay would not permit sin θ > 0.16. Thus, it seems difficult to reconcile the hierarchy of chiral symmetry breaking with the experimental knowledge of the semi-leptonic process.

3) Case (B, B)

As a previous case, in this case also, occurrence of CP-violation is possible, but in order to suppress |ΔS|=1 neutral currents, coefficients of the axial-vector part of ΔS=0 and |ΔS|=1 weak currents must take signs opposite to each other. This contradicts again the experiments on the baryon β-decay.

Next we consider a 6-plet model, another interesting model of *CP*-violation. Suppose that 6-plet with charges $(Q, Q, Q, Q-1, Q-1, Q-1)$ is decomposed into $SU_{\text{weak}}(2)$ multiplets as $2+2+2$ and $1+1+1+1+1+1$ for left and right components, respectively. Just as the case of (A, C) , we have a similar expression for the charged weak current with a 3×3 instead of 2×2 unitary matrix in Eq. (5). As was pointed out, in this case we cannot absorb all phases of matrix elements into the phase convention and can take, for example, the following expression:

$$\begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \cos \theta_2 & -\sin \theta_1 \sin \theta_2 \\ \sin \theta_1 \cos \theta_2 & \cos \theta_1 \cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3 e^{i\delta} & \cos \theta_1 \cos \theta_2 \sin \theta_3 + \sin \theta_2 \cos \theta_3 e^{i\delta} \\ \sin \theta_1 \sin \theta_2 & \cos \theta_1 \sin \theta_2 \cos \theta_3 + \cos \theta_2 \sin \theta_3 e^{i\delta} & \cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \sin \theta_3 e^{i\delta} \end{pmatrix}. \quad (13)$$

Then, we have *CP*-violating effects through the interference among these different current components. An interesting feature of this model is that the *CP*-violating effects of lowest order appear only in $\Delta S \neq 0$ non-leptonic processes and in the semi-leptonic decay of neutral strange mesons (we are not concerned with higher states with the new quantum number) and not in the other semi-leptonic, $\Delta S = 0$ non-leptonic and pure-leptonic processes.

So far we have considered only the straightforward extensions of the original Weinberg's model. However, other schemes of underlying gauge groups and/or scalar fields are possible. Georgi and Glashow's model⁶⁾ is one of them. We can easily see that *CP*-violation is incorporated into their model without introducing any other fields than (many) new fields which they have introduced already.

References

- 1) S. Weinberg, Phys. Rev. Letters **19** (1967), 1264; **27** (1971), 1688.
- 2) Z. Maki and T. Maskawa, RIFP-146 (preprint), April 1972.
- 3) P. W. Higgs, Phys. Letters **12** (1964), 132; **13** (1964), 508.
G. S. Guralnik, C. R. Hagen and T. W. Kibble, Phys. Rev. Letters **13** (1964), 585.
- 4) H. Georgi and S. L. Glashow, Phys. Rev. Letters **28** (1972), 1494.

Errata:

Equation (13) should read as

$$\begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \cos \theta_2 & -\sin \theta_1 \sin \theta_2 \\ \sin \theta_1 \cos \theta_2 & \cos \theta_1 \cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3 e^{i\delta} & \cos \theta_1 \cos \theta_2 \sin \theta_3 + \sin \theta_2 \cos \theta_3 e^{i\delta} \\ \sin \theta_1 \sin \theta_2 & \cos \theta_1 \sin \theta_2 \cos \theta_3 + \cos \theta_2 \sin \theta_3 e^{i\delta} & \cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \cos \theta_3 e^{i\delta} \end{pmatrix}. \quad (13)$$

However,...

- CKM mechanism is **the dominating effect** for CP violation and flavour mixing in the quark sector;

but there is still room for **sizable new effects and new flavour structures** (the flavour sector has only be tested at the 10% level in many cases).
- The SM does **not** describe the flavour phenomena in **the lepton sector**.

Flavour problem of SM

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge}(A_i, \psi_i) + \mathcal{L}_{Higgs}(\Phi, \psi_i, v)$$

- Gauge principle governs the gauge sector of the SM.

Flavour problem of SM

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge}(A_i, \psi_i) + \mathcal{L}_{Higgs}(\Phi, \psi_i, v)$$

- Gauge principle governs the gauge sector of the SM.

- No guiding principle in the flavour sector:

CKM mechanism (3 Yukawa SM couplings) provides a phenomenological description of quark flavour processes, but leaves significant hierarchy of quark masses and mixing parameters unexplained.

Flavour problem of SM

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge}(A_i, \psi_i) + \mathcal{L}_{Higgs}(\Phi, \psi_i, v)$$

- Gauge principle governs the gauge sector of the SM.

- No guiding principle in the flavour sector:

CKM mechanism (3 Yukawa SM couplings) provides a phenomenological description of quark flavour processes, but leaves significant hierarchy of quark masses and mixing parameters unexplained.

Compare for example:

$$|V_{us}| \approx 0.2, |V_{cb}| \approx 0.04, |V_{ub}| \approx 0.004 \text{ versus } g_s \approx 1, g \approx 0.6, g' \approx 0.3$$

Many open fundamental questions of particle physics are related to flavour :

- How many families of fundamental fermions are there ?
- How are neutrino and quark masses and mixing angles are generated ?
- Do there exist new sources of flavour and CP violation ?
- Is there CP violation in the QCD gauge sector ?
- Relations between the flavour structure in the lepton and quark sector ?

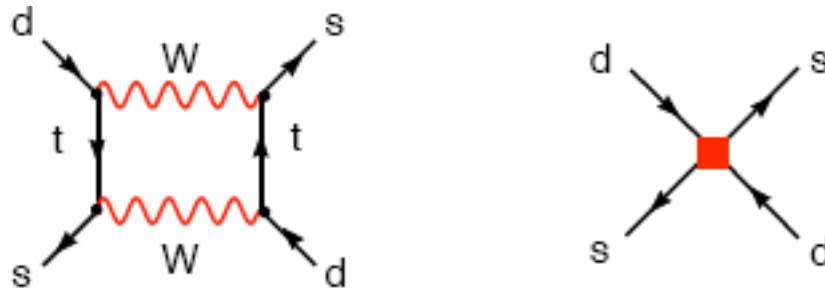
Flavour problem of New Physics or how do FCNCs hide

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs} + \sum_i \frac{c_i^{New}}{\Lambda_{NP}} \mathcal{O}_i^{(5)} + \dots$$

- SM as effective theory valid up to cut-off scale Λ_{NP}

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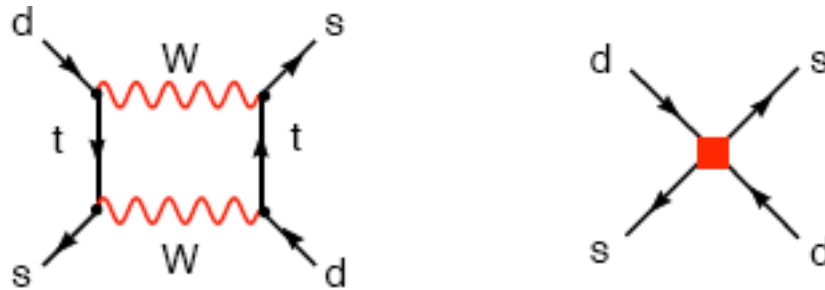


$$c^{SM}/M_W^2 \times (\bar{s}d)^2 + c^{New}/\Lambda_{NP}^2 \times (\bar{s}d)^2 \quad \Rightarrow \quad \Lambda_{NP} > 10^4 \text{ TeV}$$

(tree-level, generic new physics)

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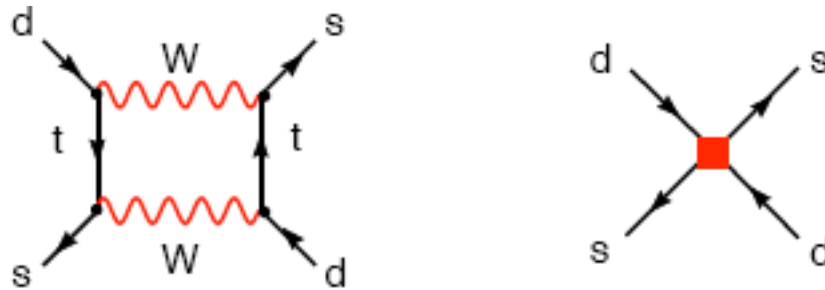
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- Natural stabilisation of Higgs boson mass (hierarchy problem)
(i.e. supersymmetry, little Higgs, extra dimensions) $\Rightarrow \Lambda_{NP} \leq 1 \text{ TeV}$
- EW precision data \leftrightarrow little hierarchy problem $\Rightarrow \Lambda_{NP} \sim 3 - 10 \text{ TeV}$

Possible New Physics at the TeV scale has to have a very non-generic flavour structure

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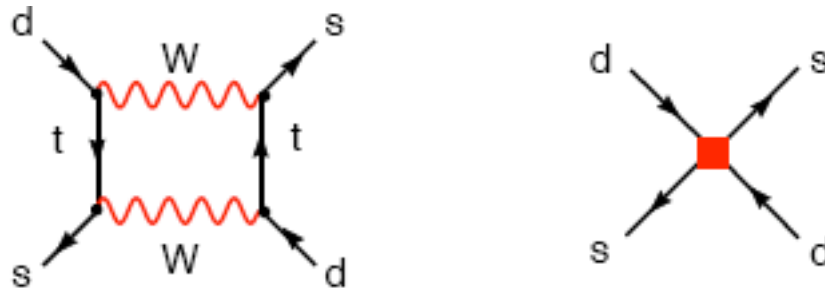
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Ambiguity of new physics scale from flavour data

$$(C_{SM}^i/M_W + C_{NP}^i/\Lambda_{NP}) \times \mathcal{O}_i$$

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The indirect information will be most valuable when the general nature of new physics will be identified in the direct search (LHC), especially when the mass scale of the new physics will be fixed.

Example: Supersymmetry

- In the general MSSM too many contributions to flavour violation
 - CKM-induced contributions from H^+ , χ^+ exchanges (quark mixing)
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- Dynamics of flavour \leftrightarrow mechanism of SUSY breaking ($BR(b \rightarrow s\gamma) = 0$ in exact supersymmetry)

Two-Higgs-doublet models (THDMs)

The two-Higgs-doublet model (THDM) constitutes one of the simplest extensions of the SM.

Many new-physics scenarios, including supersymmetry, can lead to a low-energy spectrum containing the SM fields plus
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I follow here recent concise analysis of [Buras, Carlucci, Gori, Isidori, arXiv:1005.5310](#) which summarizes, but also clarifies the recent discussion.

In the most general version, the fermionic couplings of the neutral scalars are **nondiagonal in flavour** leading to **FCNC on the tree level**:

$$\mathcal{L}_Y^{\text{gen}} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u1} U_R H_1^c + \bar{Q}_L X_{d2} D_R H_2^c + \bar{Q}_L X_{u2} U_R H_2 + \text{h.c.}$$

$$H_{1(2)}^c = -i\tau_2 H_{1(2)}^* \quad X_i \text{ are } 3 \times 3 \text{ matrices with a generic}$$

$$M_d = \frac{1}{\sqrt{2}} (v_1 X_{d1} + v_2 X_{d2}) \quad M_u = \frac{1}{\sqrt{2}} (v_1 X_{u1} + v_2 X_{u2})$$

$$\langle H_{1(2)}^\dagger H_{1(2)} \rangle = v_{1(2)}^2 / 2 \quad \text{with } v^2 = v_1^2 + v_2^2 \approx (246 \text{ GeV})^2$$

Classical solution: Natural Flavour Conservation

Glashow, Weinberg, Phys.Rev.D15,1958(1977); Paschos, Phys.Rev.D15,1966(1977)

- Assumption that only one Higgs field can couple to a given quark species

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Implementation: requiring the invariance of $\mathcal{L}_Y^{\text{gen}}$ under $U(1)_{PQ}$

(simple definition: D_R and H_1 have opposite charge, all others neutral)

or use a discrete subgroup of $U(1)_{PQ}$: the Z_2 symmetry under which

$$H_1 \rightarrow -H_1, \quad D_R \rightarrow -D_R \quad \text{and all other fields unchanged}$$

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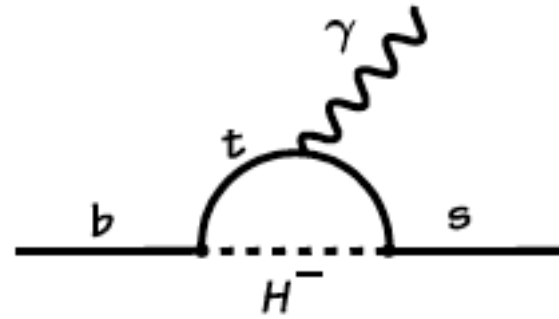
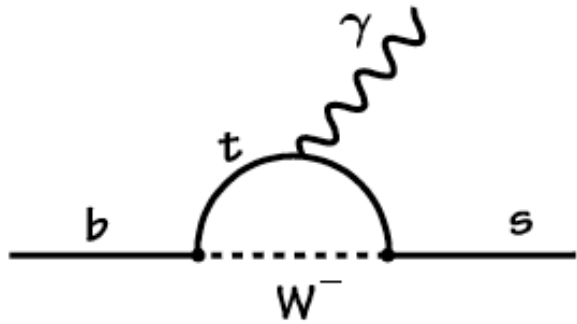
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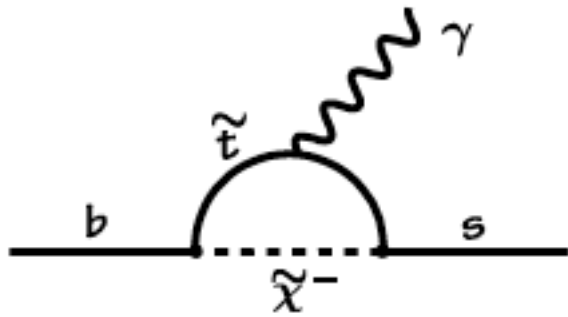
Caveat: Tree-level implementations are not stable under quantum corrections

Parameter bounds in THDMs

Two key observables: $\bar{B} \rightarrow X_s \gamma$



Charged Higgs contribution always adds to the SM one !



Within supersymmetry possible cancellation with chargino contribution.

Note: There are generically new contributions via squark mixing !

There is much more data not shown in the unitarity fits which confirms the SM predictions of flavour mixing like rare decays

Status of the inclusive mode $\bar{B} \rightarrow X_s \gamma$

HFAG: $\mathcal{B}(B \rightarrow X_s \gamma) = (3.57 \pm 0.24) \times 10^{-4}$ (for $E_\gamma > 1.6$ GeV)

VS

SM: $\mathcal{B}(B \rightarrow X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$ (for $E_\gamma > 1.6$ GeV) [PRL98,022003\(2007\)](#)
NNLO calculation by M.Misiak et al.

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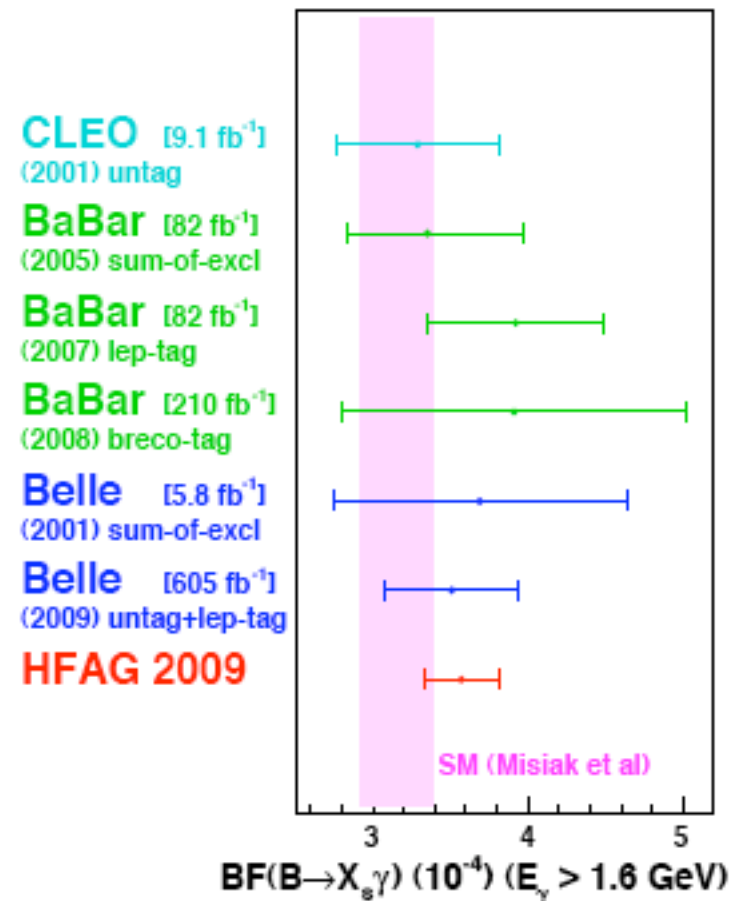
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Parameter bounds from flavour physics are model-dependent

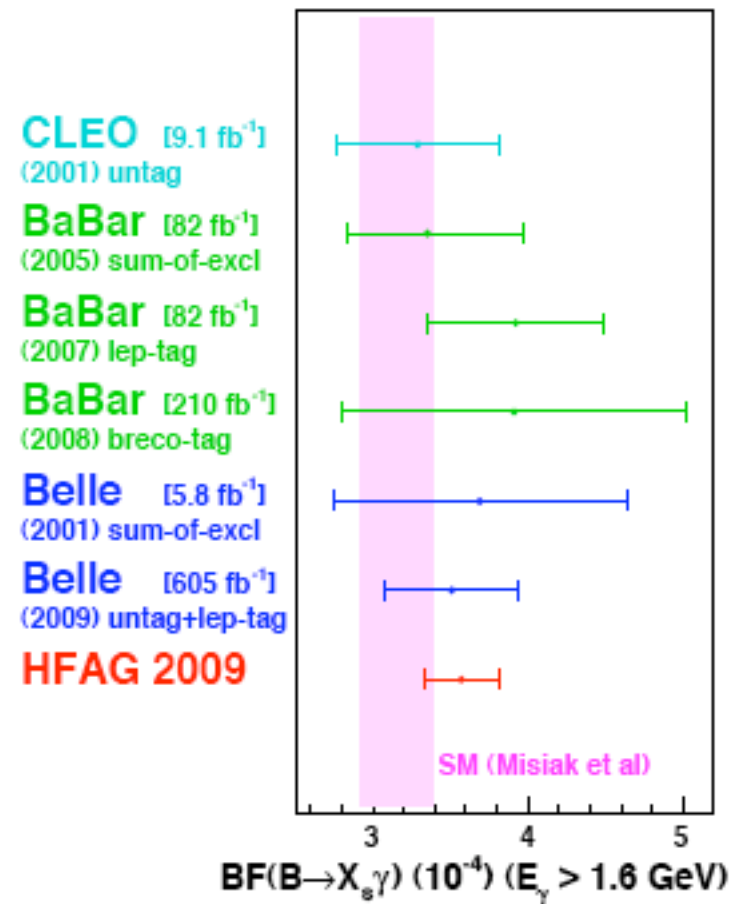
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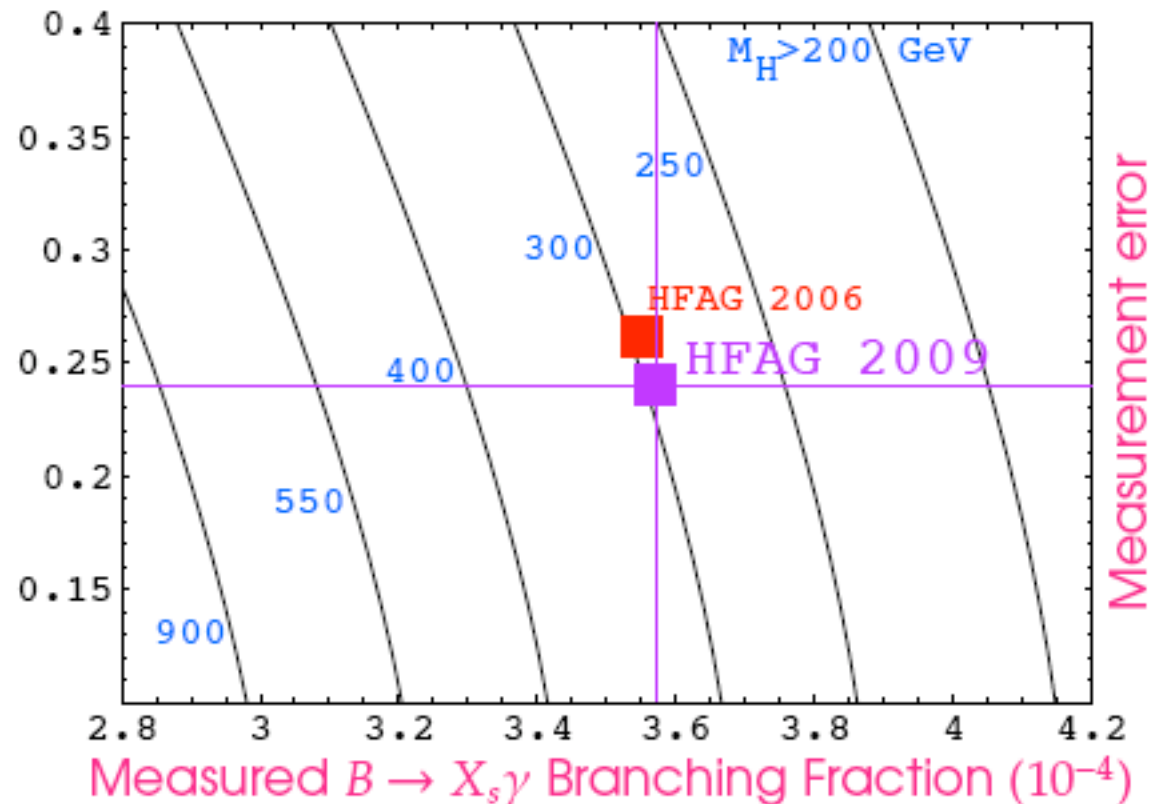
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Charged Higgs bound (2HDM)

$m_{H^+} > 300$ GeV



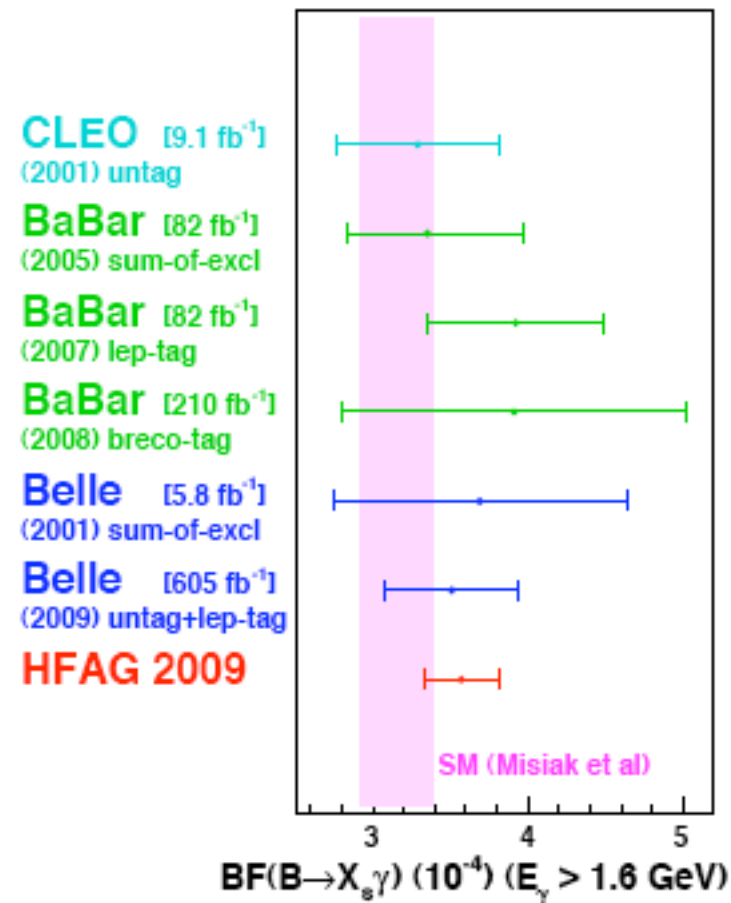
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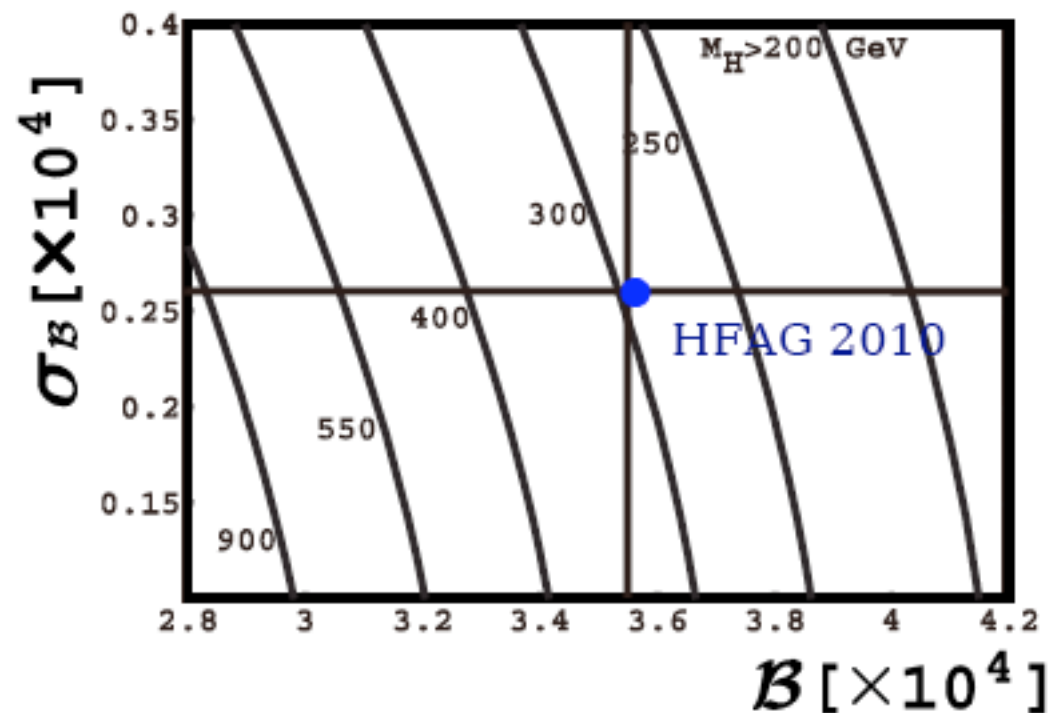
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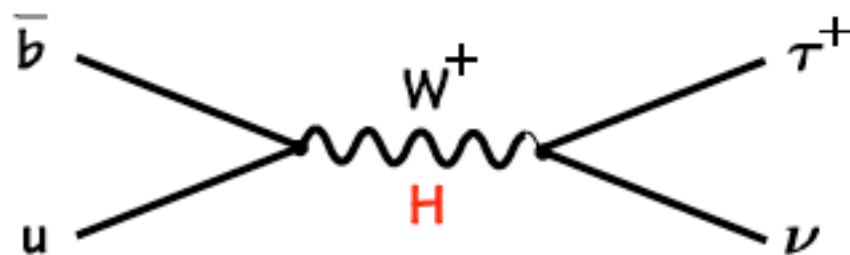
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Charged Higgs bound (2HDM TypeII)
 $M_{H^+} > 300$ GeV



$B^+ \rightarrow \tau^+ \nu$



$$B_{\text{SM}}(B^+ \rightarrow \tau^+ \nu) = \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right) f_B^2 |V_{ub}|^2 \tau_B$$

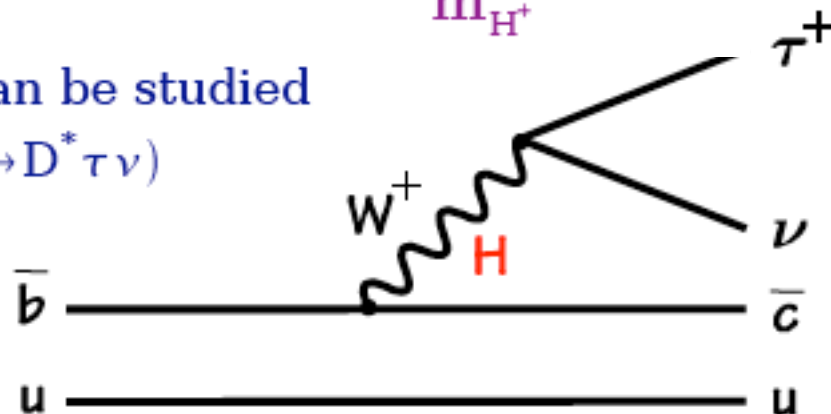
$$2\text{HDM (type II): } B(B^+ \rightarrow \tau^+ \nu) = B_{\text{SM}} \times \left(1 - \frac{m_B^2}{m_{H^+}^2} \tan^2 \beta\right)^2$$

Also $B \rightarrow D^* \tau \nu$

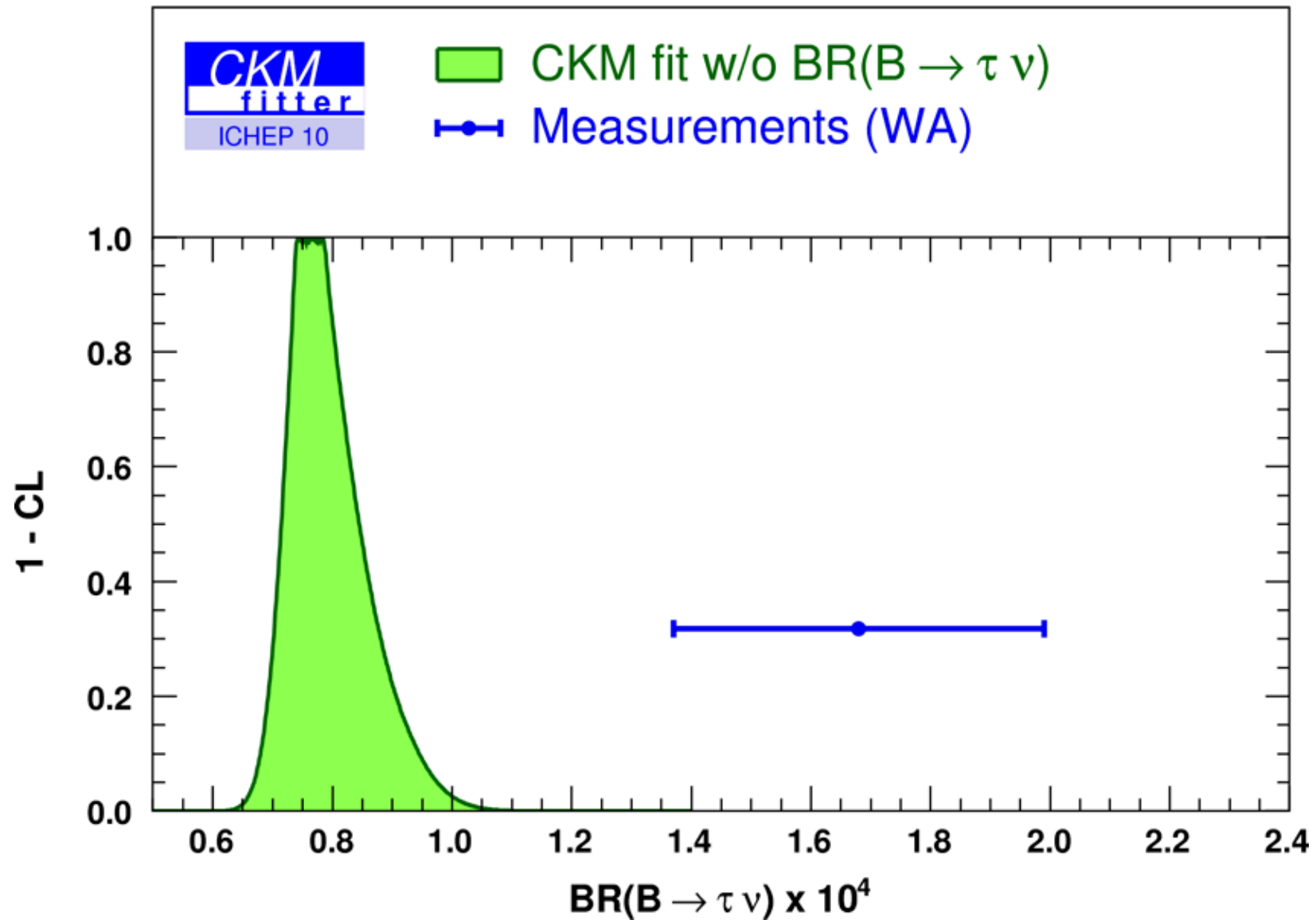
$$2\text{HDM (type II): } B(B \rightarrow D^* \tau^+ \nu) = G_F^2 \tau_B |V_{cb}|^2 f(F_V, F_S, \frac{m_B^2}{m_{H^+}^2} \tan^2 \beta)$$

uncertainties from form factors F_V and F_S can be studied

with $B \rightarrow D l \nu$ (more form factors in $B \rightarrow D^* \tau \nu$)

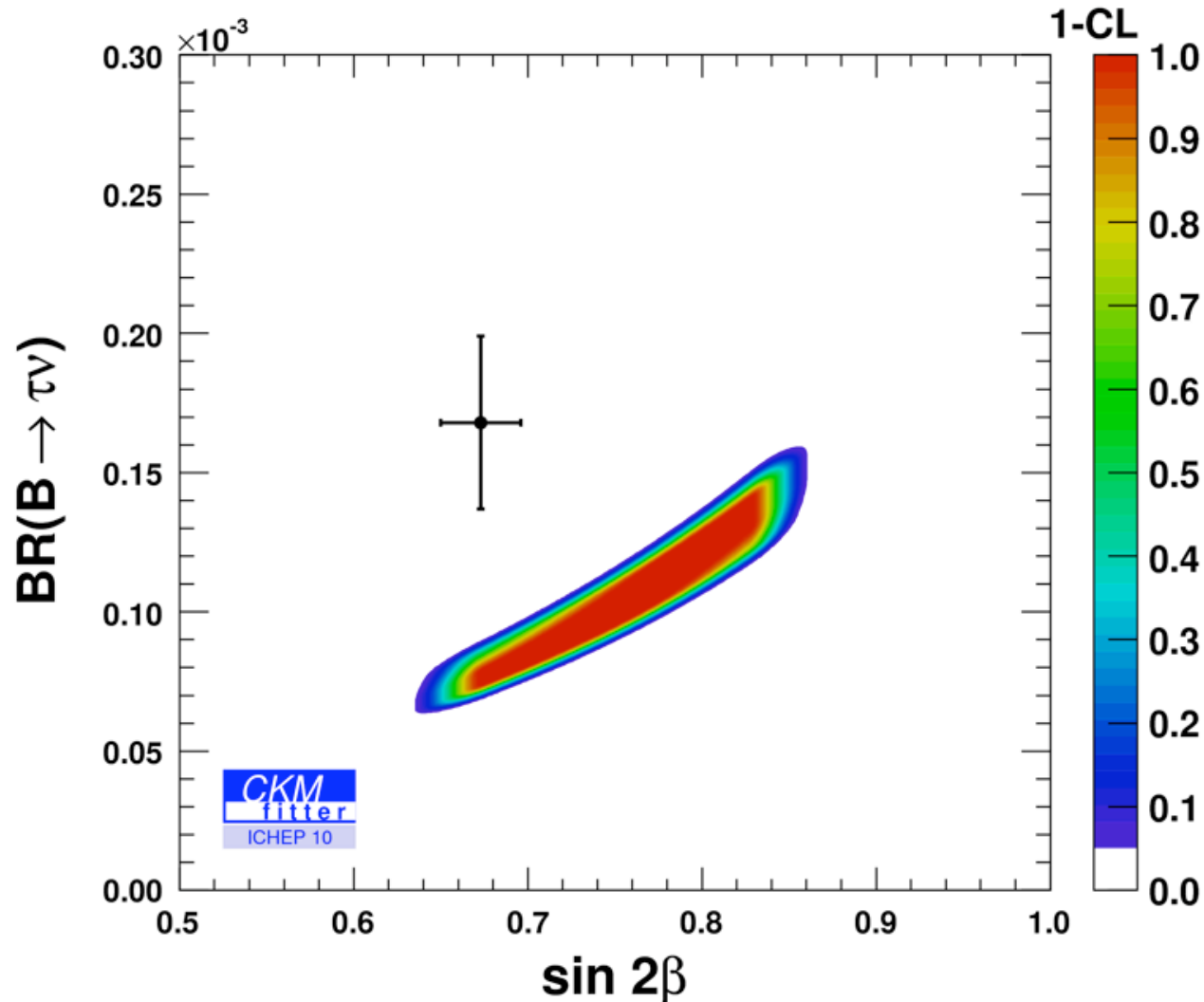


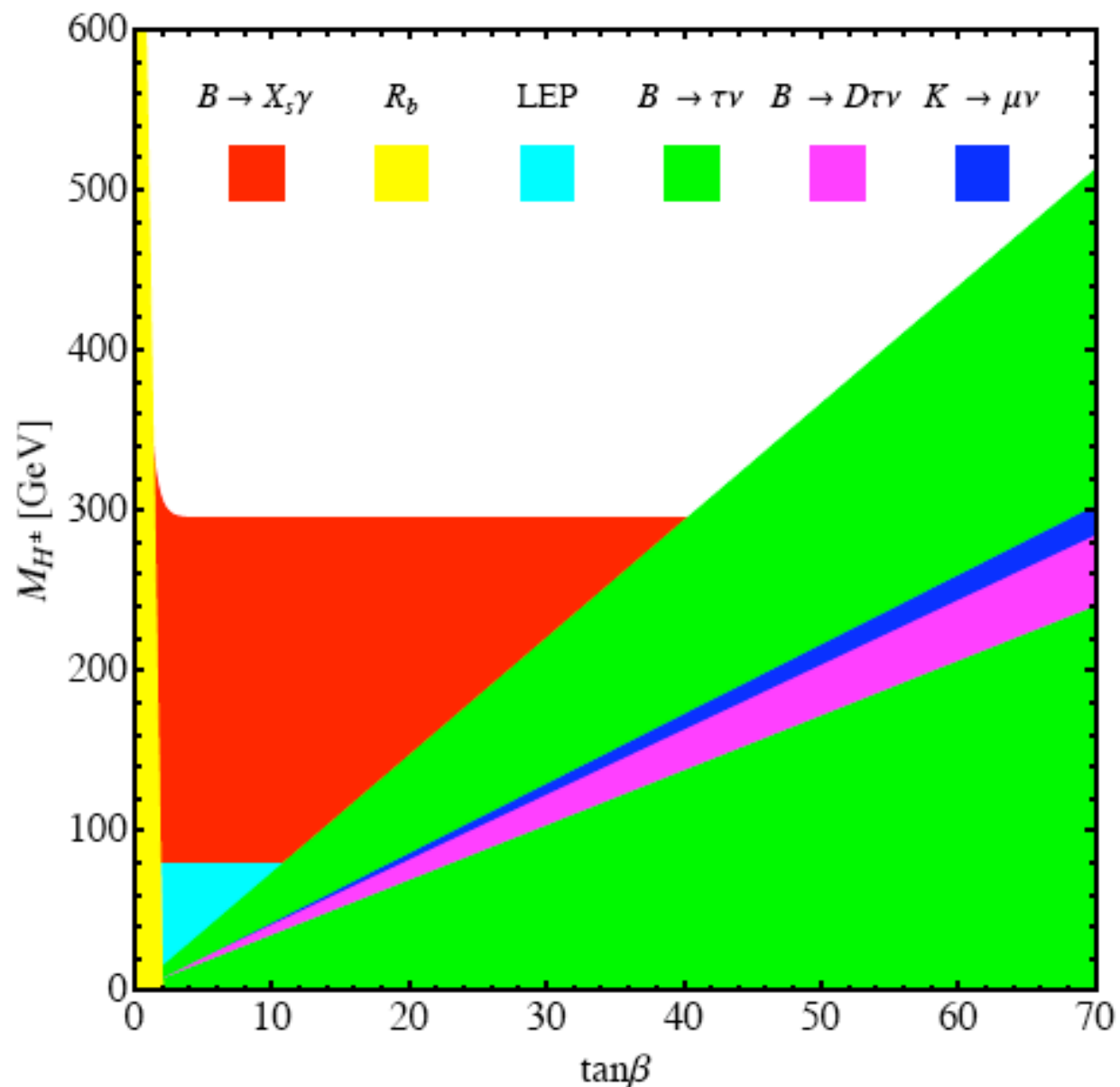
“Tension” between direct measurement and indirect fit prediction
(2.8σ)



Specific correlation between $\sin\beta$ and $B(B \rightarrow \tau\nu)$ in the global fit

$$\frac{\text{BR}(B \rightarrow \tau\nu)}{\Delta\text{md}} = \frac{3}{4} \frac{\pi}{m_W^2} \frac{m_\tau^2}{S(\alpha_t)} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \tau B^+ \frac{1}{E_{\text{Ed}}} \frac{1}{|V_{ud}|^2} \left(\frac{\sin\beta}{\sin\gamma}\right)^2$$



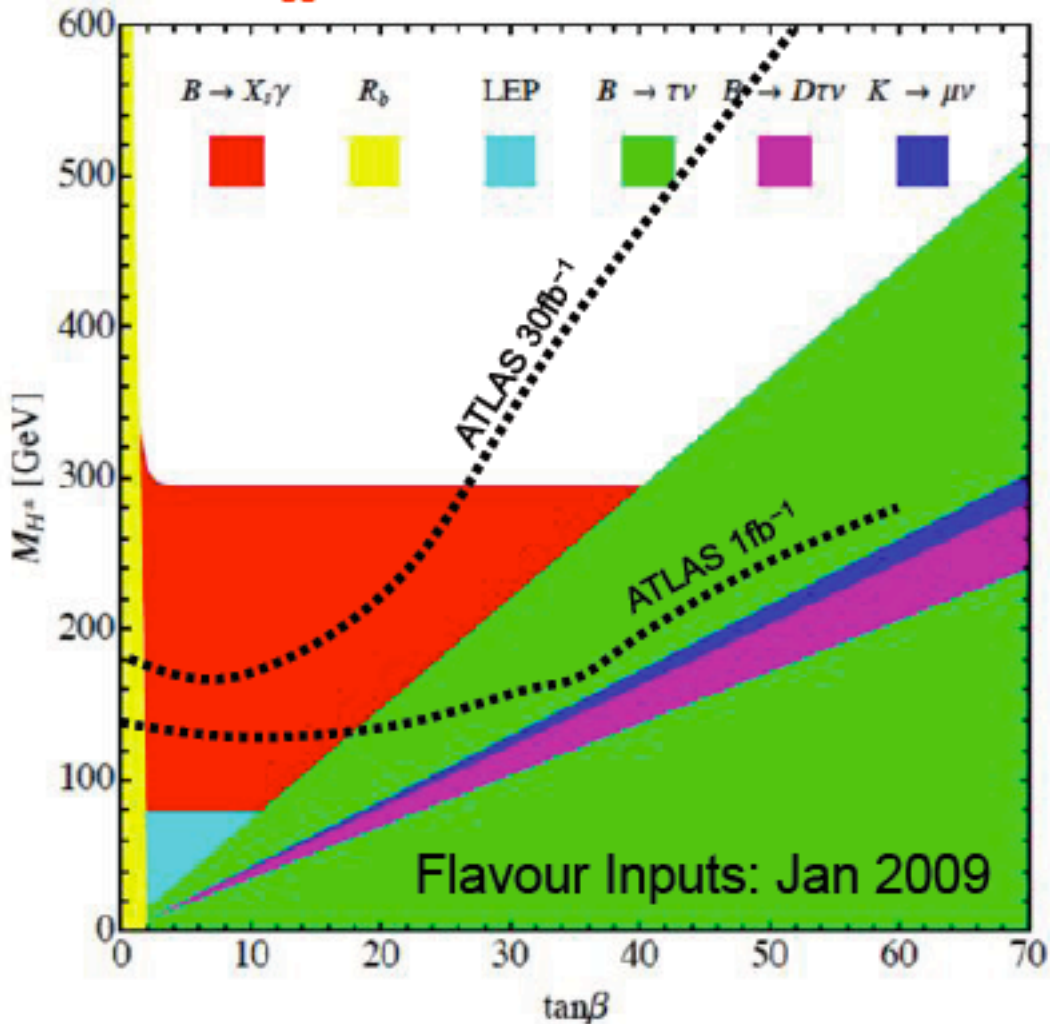


See also Deschamps et al.(CKMfitter), arXiv:0907.5135. Mahmoudi, Stal, arXiv:0907.1791.
Erikson, Mahmoudi, Stal, arXiv:0808.3551.

see talks by Mahmoudi and Kolda for more details

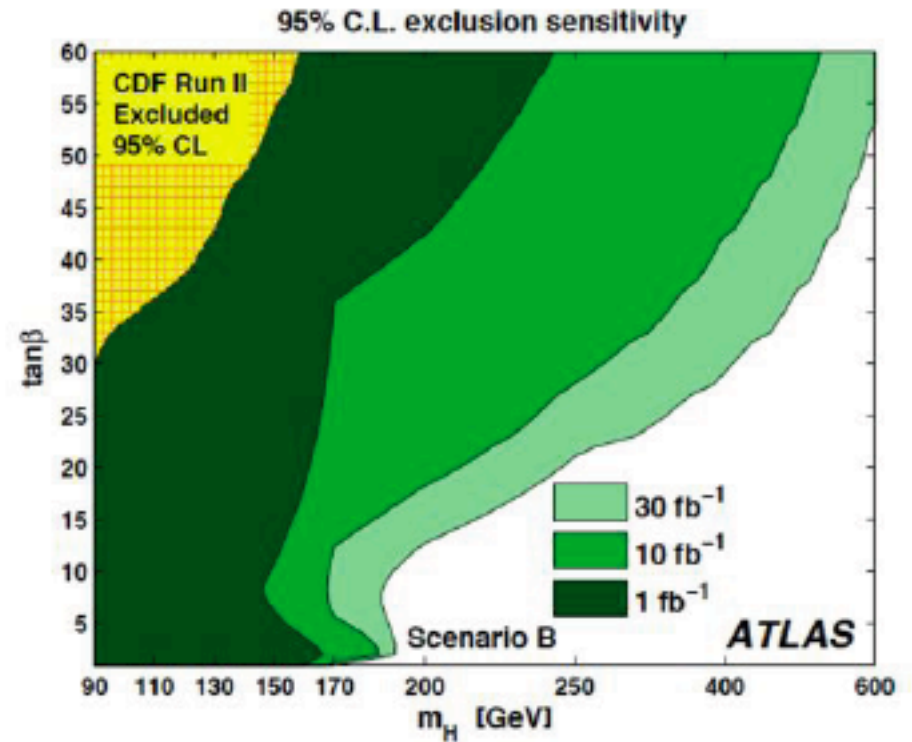
LHC versus Flavour constraints

Combined Higgs search constraint from ATLAS: arXiv:0901.1502



U. Haisch 0805.2141
2HDM

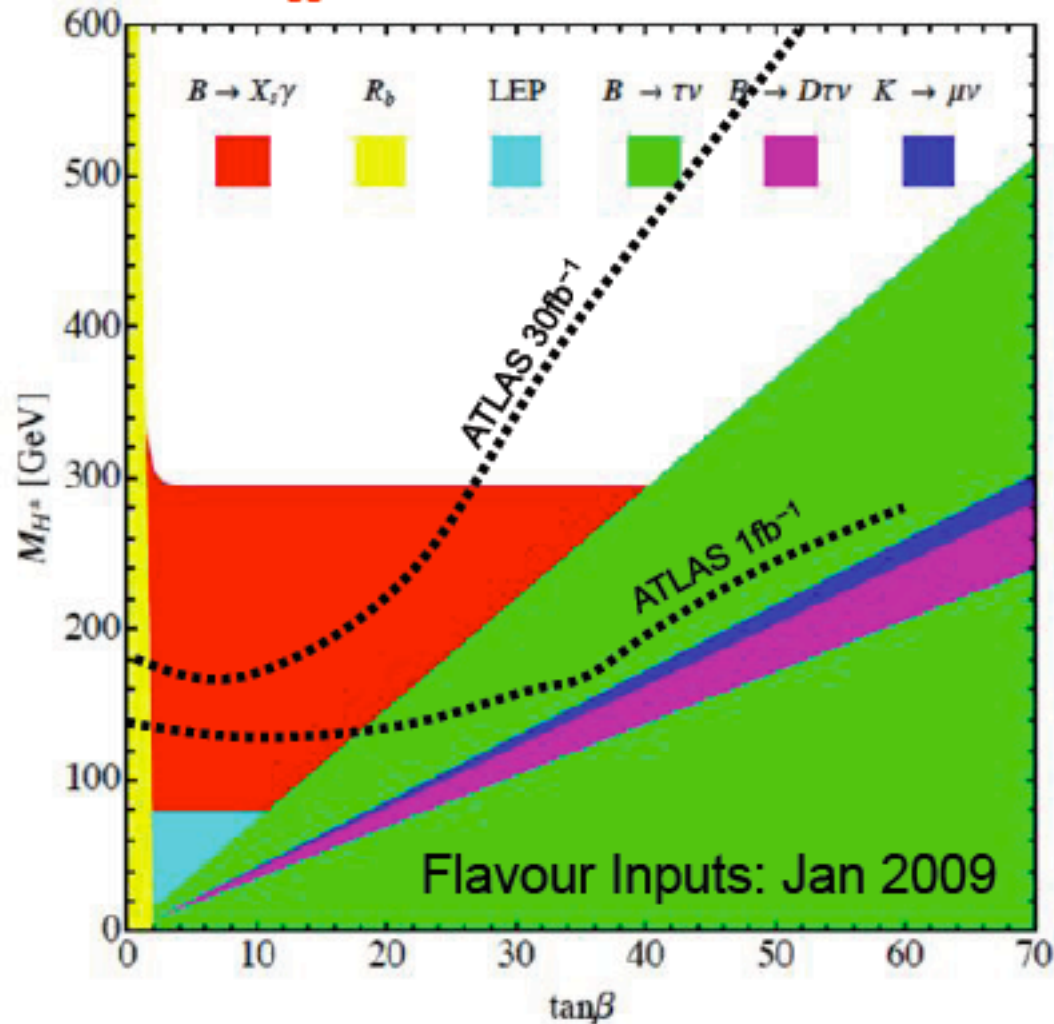
Converted constraints expected from ATLAS onto the plot by hand.



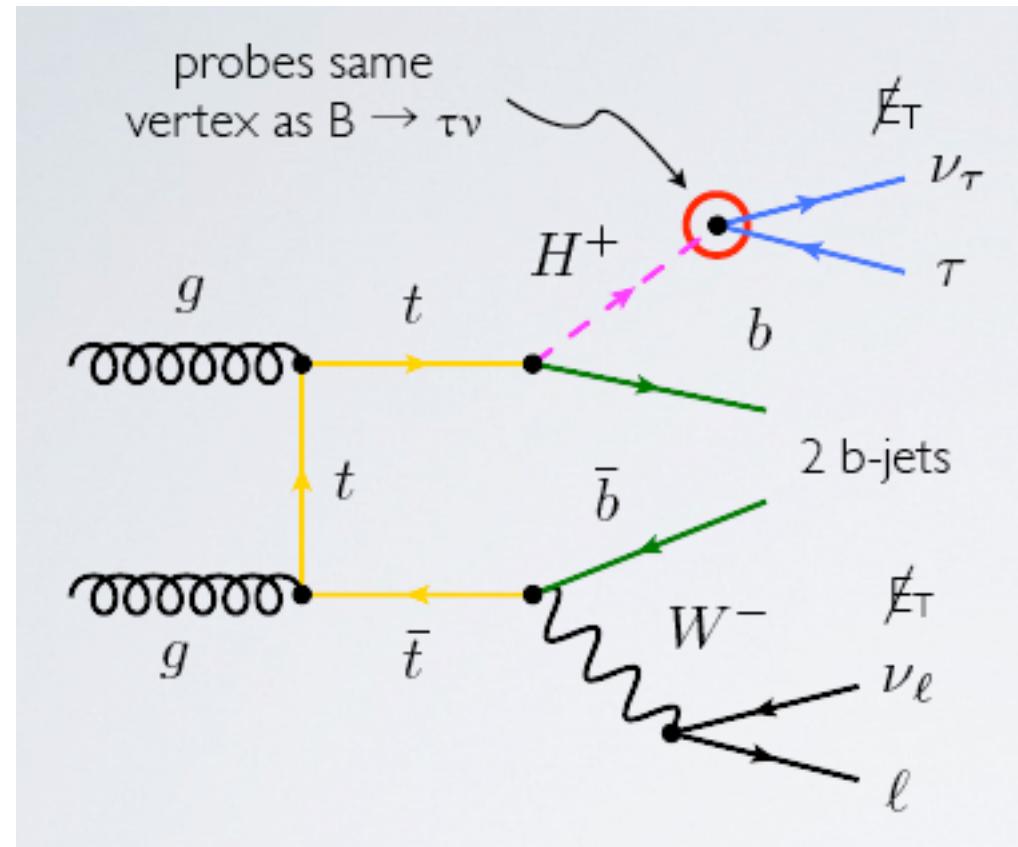
Courtesy of Adrian Bevan

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Courtesy of Uli Haisch

Minimal flavour violation hypothesis

- SM gauge interactions are universal in quark flavour space:
flavour symmetry $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$
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Complete flavour symmetry of SM gauge Lagrangian:

$$U(3)^3 = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \times U(1)_B \times U(1)_Y \times U(1)_{PQ}$$

$U(1)$ symmetries related to baryon number, hypercharge and the Peccei-Quinn symmetry

Minimal flavour violation hypothesis

- SM gauge interactions are universal in quark flavour space:
flavour symmetry $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$
- Symmetry is only broken by the Yukawa couplings Y_U and Y_D responsible for the quark masses

- Any new physics model in which all flavour- and CP-violating interactions can be linked to the known Yukawa couplings is MFV
- RG-invariant definition based on the flavour symmetry:
Yukawa couplings are introduced as background values of fields (spurions) transforming under the flavour group

d'Ambrosio, Giudice, Isidori, Strumia, hep-ph/0207036

Chivukula, Georgi, Phys.Lett.B188(1987)99

Hall, Randall, Phys.Rev.Lett.65(1990)2939

MFV at work

The flavour symmetry $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$

is broken by the Yukawa couplings only as in the SM $Y_D (3, 1, \bar{3}); Y_U (3, \bar{3}, 1)$

$$-\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} = Y_{ij}^d \overline{Q}_{Li}^I \phi D_{Rj}^I + Y_{ij}^u \overline{Q}_{Li}^I \tilde{\phi} U_{Rj}^I + \text{h.c.}$$

$$\overline{Q}_L(\bar{3}, 1, 1), D_R(1, 1, 3), U_R(1, 3, 1) \Rightarrow \mathcal{L}(1, 1, 1)$$

MFV: All effective field operators with higher dimension
also have to be invariant

Specific basis: $Y_D = \text{diag}(y_d, y_s, y_b)$, $Y_U = V_{CKM}^+ \times \text{diag}(y_u, y_c, y_t)$

Typical FCNC-operator with external d-type quarks: $\overline{Q}_{LL}^i (Y_U Y_U^+)_{ij} Q_{LL}^j \times L_L L_L$

$$\begin{aligned} \lambda_{FCij} &= (Y_U Y_U^+)_{ij} = (V_{CKM}^+ \times \text{diag}(y_u^2, y_c^2, y_t^2) \times V_{CKM})_{ij} \approx \\ &\approx (V_{CKM}^+ \times \text{diag}(0, 0, y_t^2) \times V_{CKM})_{ij} = y_t^2 \times V_{3,i}^* V_{3,j} \end{aligned}$$

Coupling λ_{FC} is the effective coupling ruling all FCNCs with external d-type quarks.

More precise:

- In MFV models with one Higgs doublet, all FCNC processes with external d -type quarks are governed by

$$(Y_U Y_U^\dagger)_{ij} \approx y_t^2 V_{3i}^* V_{3j} \quad \text{CKM hierarchy}$$

- If additional Higgs-doublets are added, then another spurion combination is numerically important: (terms suppressed by $m_{s,d}/m_b$ neglected)

$$(Y_D Y_D^\dagger)_{ij} \approx 2m_b^2 \tan^2 \beta / v^2 \Delta_{ij}, \quad \Delta = \text{diag}(0, 0, 1)$$

Thus, MFV allows for large- $\tan \beta$ effects in particular in helicity-suppressed observables $B \rightarrow \mu\mu$ and $B \rightarrow \tau\nu$.

$$B \rightarrow \mu\mu: \quad A_{\text{SM}} \sim m_\mu/m_b \Leftrightarrow A_{H^0, A^0} \sim \tan^3 \beta$$

Minimal flavour violation: formal solution of NP flavour problem

$$M(B_d - \bar{B}_d) \sim \frac{(V_{tb}^* V_{td})^2}{16 \pi^2 M_w^2} + \left(c_{\text{NP}} \frac{1}{\Lambda^2} \right)$$

← contribution of the new heavy degrees of freedom

c_{NP}	↙	~ 1	tree/strong + generic flavour	→	$\Lambda \gtrsim 2 \times 10^4 \text{ TeV [K]}$
	↘	$\sim 1/(16 \pi^2)$	loop + generic flavour	→	$\Lambda \gtrsim 2 \times 10^3 \text{ TeV [K]}$
	↘	$\sim (V_{ti}^* V_{tj})^2$	tree/strong + MFV	→	$\Lambda \gtrsim 5 \text{ TeV [K \& B}_d]$
	↘	$\sim (V_{ti}^* V_{tj})^2 / (16 \pi^2)$	loop + MFV	→	$\Lambda \gtrsim 0.5 \text{ TeV [K \& B}_d]$

Courtesy of Gino Isidori

- MFV implies **model-independent** relations between FCNC processes

$$\Delta F = 2 \quad \text{UTfit, arXiv:0707.0636} \quad \Delta F = 1 \quad \text{H., Isidori, Kamenik, Mescia, arXiv:0807.5039}$$

MFV predictions to be tested:

- usual CKM relations between $[b \rightarrow s] \leftrightarrow [b \rightarrow d] \leftrightarrow [s \rightarrow d]$ transitions:
 - we need high-precision $b \rightarrow s$, but also $s \rightarrow d$ measurements
 - $\mathcal{B}(\bar{B} \rightarrow X_d \gamma) \leftrightarrow \mathcal{B}(\bar{B} \rightarrow X_s \gamma)$, $\mathcal{B}(\bar{B} \rightarrow X_s \nu \bar{\nu}) \leftrightarrow \mathcal{B}(K \rightarrow \pi^+ \nu \bar{\nu})$
- CKM phase only source of CP violation:
 - phase measurements in $B \rightarrow \phi K_s$ or $\Delta M_{B_{(s/d)}}$ are not sensitive to new physics
- The usefulness of MFV-bounds/relations is obvious; **any measurement beyond those bounds indicate the existence of new flavour structures**
- **The MFV hypothesis is far from being verified**

New spurions allowed: Next-to-MFV

Agashe, Papucci, Perez, Pijol, hep-ph/0509117 Feldmann, Mannel, hep-ph/0611095

We still have to find explicit dynamical structures to realise MFV:

- Gauge-mediated supersymmetry
- $SO(10)$ GUT model with family symmetries
Dermisek,Raby,hep-ph/0507045 Straub et al.,arXiv:0707.3954
- Top-bottom- τ unification under attack of FCNC
Atmannsdorfer,Guadagnoli,Raby,Straub,arXiv:0801.4363
- Warped extra dimensions Weiler et al.,arXiv:0709.1714
- 5DMFV \Rightarrow 4DNMFV, Randall-Sundrum
Fitzpatrick,Perez,Randall,arXiv:0710.1869
- General formalism to describe specific sequence of flavour
symmetry breaking within MFV Feldmann,Mannel,arXiv:0801.1802

- MFV ansatz RG-invariant by construction

$$m_Q^2 = \alpha_1 \mathbb{1} + \beta_1 Y_u^\dagger Y_u + \beta_2 Y_d^\dagger Y_d + \beta_3 Y_d^\dagger Y_d Y_u^\dagger Y_u + \beta_3 Y_u^\dagger Y_u Y_d^\dagger Y_d ,$$

$$m_u^2 = \alpha_2 \mathbb{1} + \beta_5 Y_u Y_u^\dagger ,$$

$$m_d^2 = \alpha_3 \mathbb{1} + \beta_6 Y_d Y_d^\dagger ,$$

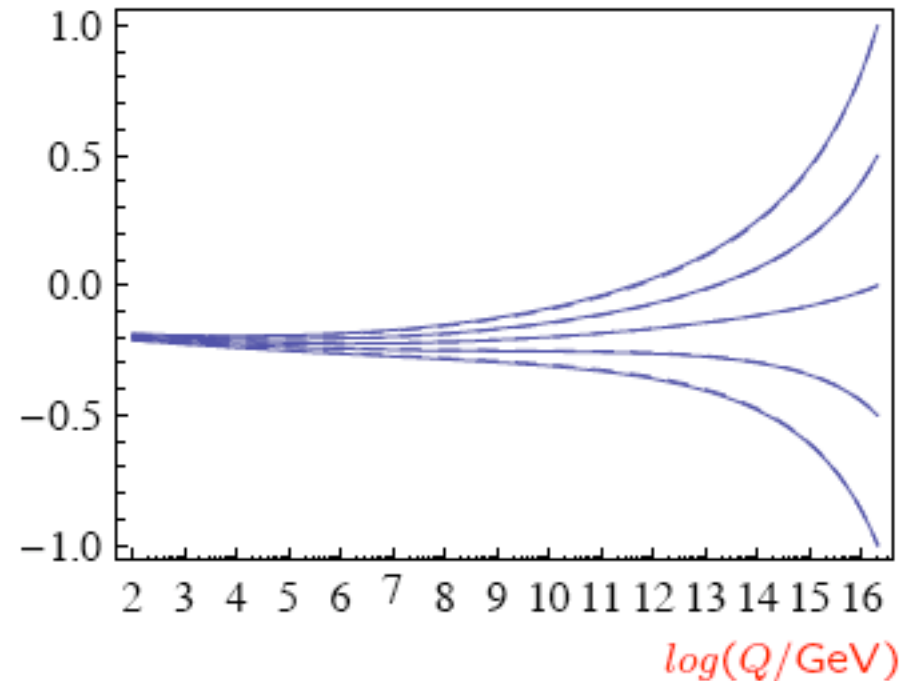
$$A_u = \alpha_4 Y_u + \beta_7 Y_u Y_d^\dagger Y_d ,$$

$$A_d = \alpha_5 Y_d + \beta_8 Y_d Y_u^\dagger Y_u ,$$

$$A_e = \alpha_e Y_e .$$

$$\frac{\beta_1}{\alpha_1}$$

'Spurion expansion' of soft terms



- MFV coefficients β_i at low energy insensitive to their GUT boundary conditions: (gluino contribution versus Yukawa effects)
- Result:** MFV-compatible change of boundary conditions at the high scale has barely any influence on the low scale spectrum. 'fixed points'

Back to THDMs

$$\mathcal{L}_Y^{\text{gen}} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u1} U_R H_1^c + \bar{Q}_L X_{d2} D_R H_2^c + \bar{Q}_L X_{u2} U_R H_2 + \text{h.c.}$$

$$U(3)^3 = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \times U(1)_B \times U(1)_Y \times U(1)_{PQ}$$

We assume that hypercharge is not explicitly broken and baryon number is conserved.

$SU(3)^3$ and $U(1)_{PQ}$ breaking mechanism allow to classify the structure of Yukawa interactions and the solutions to the flavour problem in this class of models:

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$SU(3)^3$ and $U(1)_{PQ}$ breaking mechanism allow to classify the structure of Yukawa interactions and the solutions to the flavour problem in this class of models:

- **Natural flavour violation hypothesis**

Require invariance of $\mathcal{L}_Y^{\text{gen}}$ under $U(1)_{PG}$ or discrete subgroup

- **Minimal flavour violation hypothesis**

Require that $SU(3)^3$ flavour symmetry is only broken by the two independent terms Y_D and Y_U

- Minimal flavour violation hypothesis

$$\mathcal{L}_Y^{\text{gen}} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u1} U_R H_1^c + \bar{Q}_L X_{d2} D_R H_2^c + \bar{Q}_L X_{u2} U_R H_2 + \text{h.c.}$$

Yukawa alignment

$$\begin{aligned} X_{d1} &= c_{d1} Y_d & X_{d2} &= c_{d2} Y_d \\ X_{u1} &= c_{u1} Y_u & X_{u2} &= c_{u2} Y_u \end{aligned} \quad \mathcal{O}(Y^1)$$

Quark mass terms and couplings to the neutral Higgs fields can be diagonalized simultaneously; no FCNC on tree level.

Jung, Pich, Tuzon, arXiv:0908.1554, 1001.0293.
see talk by Jung for more details

- Minimal flavour violation hypothesis

$$\mathcal{L}_Y^{\text{gen}} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u1} U_R H_1^c + \bar{Q}_L X_{d2} D_R H_2^c + \bar{Q}_L X_{u2} U_R H_2 + \text{h.c.}$$

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Open question for both options:

Stability beyond the tree level ?

Buras, Carlucci, Gori, Isidori, arXiv:1005.5310

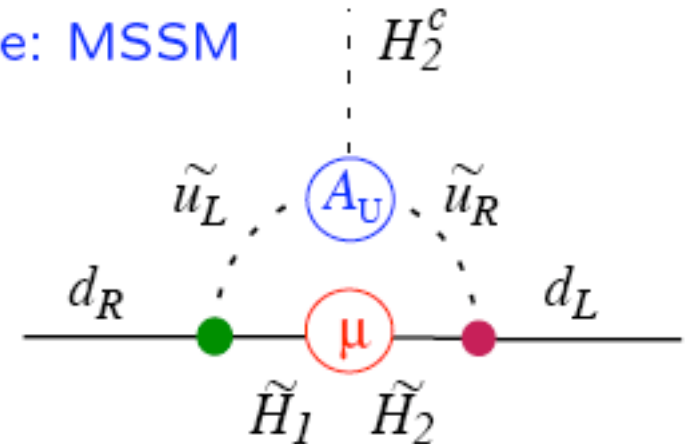
Natural flavour violation beyond tree level

- $U(1)_{PQ}$ must be explicitly broken in other sectors of the theory to avoid a massless pseudoscalar Higgs field (spontaneous breaking by the vev of H_2 implies a Goldstone boson)

Tree: $X_{d1} = Y_d$, $X_{d2} = 0$

Example: MSSM

Loop: $X_{d1} = Y_d + \dots$, $X_{d2} = \epsilon_d \Delta_d$



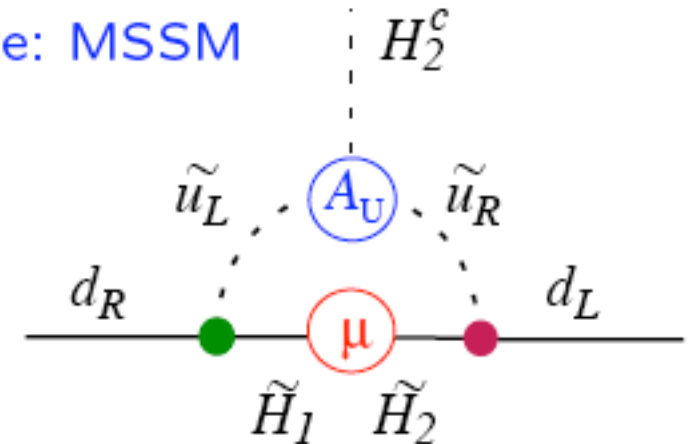
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Breaking of $U(1)_{PQ}$ only at loop level $\Rightarrow \epsilon_d \ll 1$

However: With $\epsilon_d \approx 10^{-2}$ one still induces too large FCNC unless Δ is very small or aligned with Y_d (MFV hypothesis)

Natural flavour violation beyond tree level

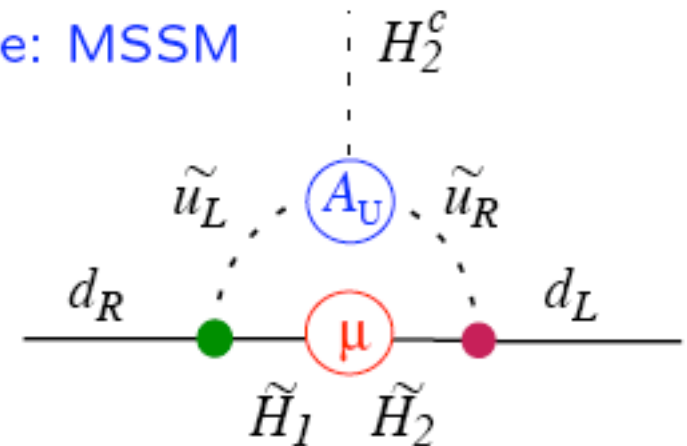
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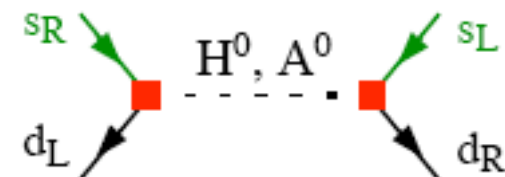
Tree: $X_{d1} = Y_d$, $X_{d2} = 0$ Example: MSSM H_2^c

Loop: $X_{d1} = Y_d + \dots$, $X_{d2} = \epsilon_d \Delta_d$



Fine-tuning necessary for CP violation in $K^0 - \bar{K}^0$ mixing

Integrating out the heavy Higgs fields leads to



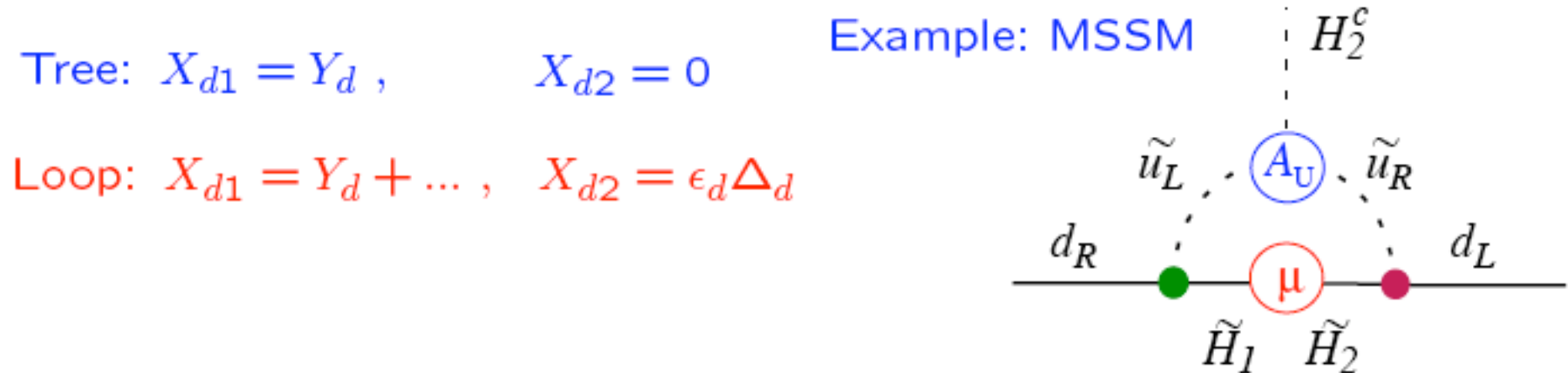
$$\mathcal{H}_\epsilon^{|\Delta S|=2} = -\frac{\epsilon_d^2}{c_\beta^2 M_H^2} (\tilde{\Delta}_d)_{21} (\tilde{\Delta}_d)_{12}^* (\bar{s}_L d_R) (\bar{s}_R d_L) + \text{h.c.}$$

$$|\epsilon_K^{\text{NP}}| < 0.2 |\epsilon_K^{\text{exp}}| \Rightarrow |\epsilon_d| \times \left| \text{Im}[(\tilde{\Delta}_d)_{21}^* (\tilde{\Delta}_d)_{12}] \right|^{1/2} \lesssim 3 \times 10^{-7} \times \frac{c_\beta M_H}{100 \text{ GeV}}$$

Natural flavour violation beyond tree level

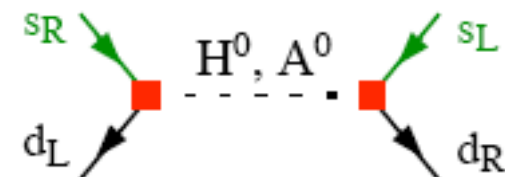
- $U(1)_{PQ}$ must be explicitly broken in other sectors of the theory to avoid a massless pseudoscalar Higgs field

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Flavour structures $\tilde{\Delta}_d$ need further protection \rightarrow MFV !

Integrating out the heavy Higgs fields leads to



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Natural flavour violation beyond tree level

- Even if natural flavour condition introduced via **exact** Z_2 symmetry

$$H_1 \rightarrow -H_1, D_R \rightarrow -D_R$$

NFV is not sufficient to protect FCNCs due to Z_2 invariant higher-dimensional operators of the type

$$\begin{aligned} \Delta\mathcal{L}_Y = & \frac{c_1}{\Lambda^2} \bar{Q}_L X_{u1}^{(6)} U_R H_2 |H_1|^2 + \frac{c_2}{\Lambda^2} \bar{Q}_L X_{u2}^{(6)} U_R H_2 |H_2|^2 + \\ & + \frac{c_3}{\Lambda^2} \bar{Q}_L X_{d1}^{(6)} D_R H_1 |H_1|^2 + \frac{c_4}{\Lambda^2} \bar{Q}_L X_{d2}^{(6)} D_R H_1 |H_2|^2 \end{aligned}$$

Only ϵ_d replaced by a parameter of order v^2/Λ^2

Flavour structures $X_i^{(6)}$ need further protection \rightarrow MFV !

Minimal flavour violation beyond tree level

$$\mathcal{L}_Y^{\text{gen}} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u1} U_R H_1^c + \bar{Q}_L X_{d2} D_R H_2^c + \bar{Q}_L X_{u2} U_R H_2 + \text{h.c.}$$

- Structure of Yukawa couplings within MFV

$$X_{d1} = Y_d, \text{ definition!}$$

$$X_{d2} = \epsilon_0 Y_d + \epsilon_1 Y_d Y_d^\dagger Y_d + \epsilon_2 Y_u Y_u^\dagger Y_d + \dots ,$$

$$X_{u1} = \epsilon'_0 Y_u + \epsilon'_1 Y_u Y_u^\dagger Y_u + \epsilon'_2 Y_d Y_d^\dagger Y_u + \dots ,$$

$$X_{u2} = Y_u, \text{ definition}$$

Note: we are free to redefine the two basic spurions Y_u and Y_d !

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- Quantum corrections can change the values of the ϵ_i at different energy scales, but they cannot modify this functional form.
- Choice $\epsilon_i = 0$ is not consistent, leads to heavy fine-tuning.

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Note: we are free to redefine the two basic spurions Y_u and Y_d !

- Quantum corrections can change the values of the ϵ_i at different energy scales, but they cannot modify this functional form.
- Choice $\epsilon_i = 0$ is not consistent, leads to heavy fine-tuning.
- Even when $\epsilon_i = \mathcal{O}(1)$ the expansion in terms of off-diagonal CKM matrix elements and small quark masses is rapidly convergent.

Similar stability argument as in the case of the soft-breaking terms in the MSSM.

- LHCb (5 years) $10fb^{-1}$: allows for wide range of analyses,
highlights: B_s mixing phase, angle γ , $B \rightarrow K^*\mu\mu$, $B_s \rightarrow \mu\mu$, $B_s \rightarrow \phi\phi$
then possibility for upgrade to $100fb^{-1}$
- Dedicated kaon experiments J-PARC E14 and CERN P-326/NA62:
rare kaon decays $K_L^0 \rightarrow \pi^0\nu\bar{\nu}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$
- Two proposals for a Super-B factory:
BELLE II at KEK and SuperB in Frascati ($75ab^{-1}$)
Super-B is a Super Flavour factory: besides precise B measurements,
CP violation in charm, lepton flavour violating modes $\tau \rightarrow \mu\gamma, \dots$

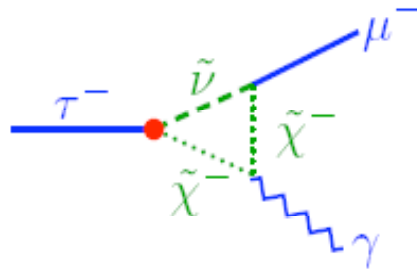
Opportunities at a Super Flavour Factory

see JHEP 0802 (2008) 110, arXiv:0710.3799

Measurement of lepton flavour violation

$\tau \rightarrow \mu \gamma$ and $\tau \rightarrow 3\mu$

$$\text{BR}(l_j^- \rightarrow l_i^- \gamma)|_{\text{SM}_R} \approx (m_\nu/M_W)^2 \sim \mathcal{O}(10^{-54})$$



Process	Expected 90%CL upper limited	4 σ Discovery Reach
$\mathcal{B}(\tau \rightarrow \mu \gamma)$	2×10^{-9}	5×10^{-9}
$\mathcal{B}(\tau \rightarrow \mu \mu \mu)$	2×10^{-10}	8.8×10^{-10}

Use modes to distinguish SUSY vs LHT

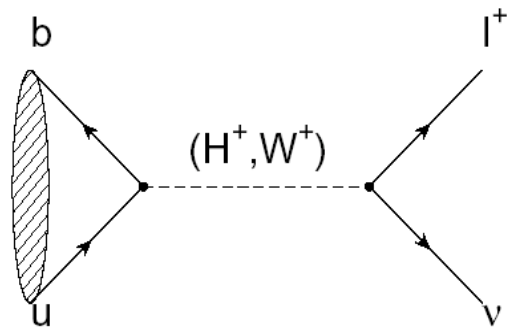
Blanke et al.

ratio	LHT	MSSM (dipole)	MSSM (Higgs)
$\frac{\mathcal{B}(\tau^- \rightarrow e^- e^+ e^-)}{\mathcal{B}(\tau^- \rightarrow e \gamma)}$	0.4...2.3	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$
$\frac{\mathcal{B}(\tau^- \rightarrow \mu^- \mu^+ \mu^-)}{\mathcal{B}(\tau^- \rightarrow \mu \gamma)}$	0.4...2.3	$\sim 2 \cdot 10^{-3}$	0.06...0.1
$\frac{\mathcal{B}(\tau^- \rightarrow e^- \mu^+ \mu^-)}{\mathcal{B}(\tau^- \rightarrow e \gamma)}$	0.3...1.6	$\sim 2 \cdot 10^{-3}$	0.02...0.04
$\frac{\mathcal{B}(\tau^- \rightarrow \mu^- e^+ e^-)}{\mathcal{B}(\tau^- \rightarrow \mu \gamma)}$	0.3...1.6	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$
$\frac{\mathcal{B}(\tau^- \rightarrow e^- e^+ e^-)}{\mathcal{B}(\tau^- \rightarrow e^- \mu^+ \mu^-)}$	1.3...1.7	~ 5	0.3...0.5
$\frac{\mathcal{B}(\tau^- \rightarrow \mu^- \mu^+ \mu^-)}{\mathcal{B}(\tau^- \rightarrow \mu^- e^+ e^-)}$	1.2...1.6	~ 0.2	5...10

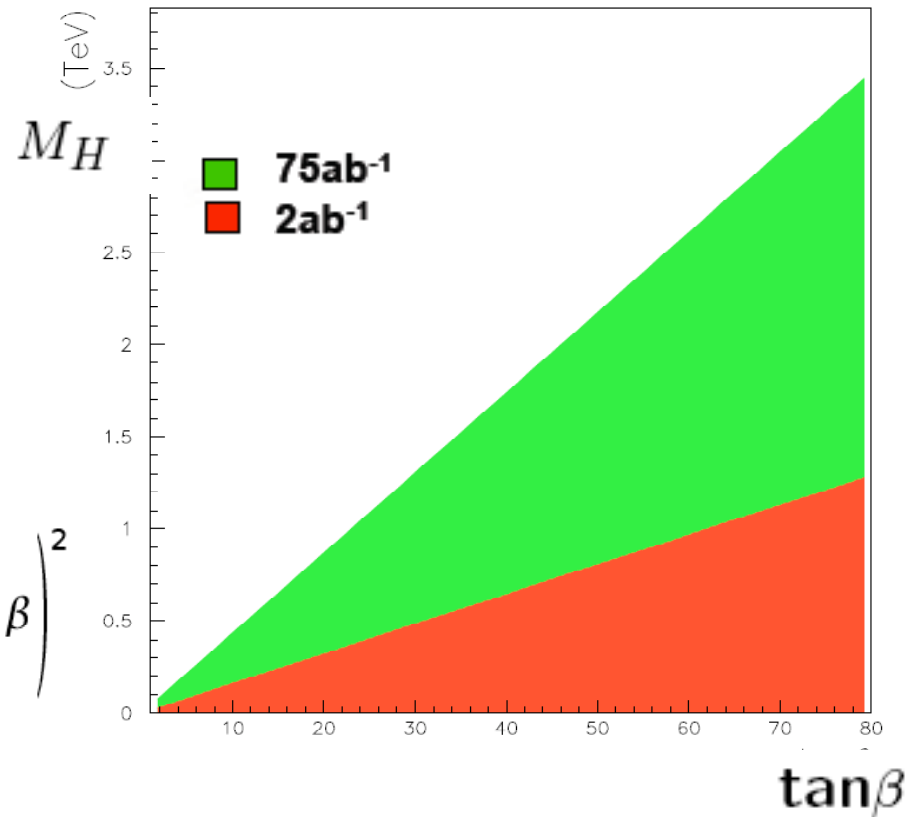
Superflavour factory: measurement of clean modes

$B \rightarrow \tau \nu$: **B factories 20%** **Super B factories 4%**

2HDM-II



$$\text{BR}(B \rightarrow \tau \nu) = \text{BR}_{\text{SM}}(B \rightarrow \tau \nu) \left(1 - \frac{m_B^2}{M_H^2} \tan^2 \beta \right)^2$$

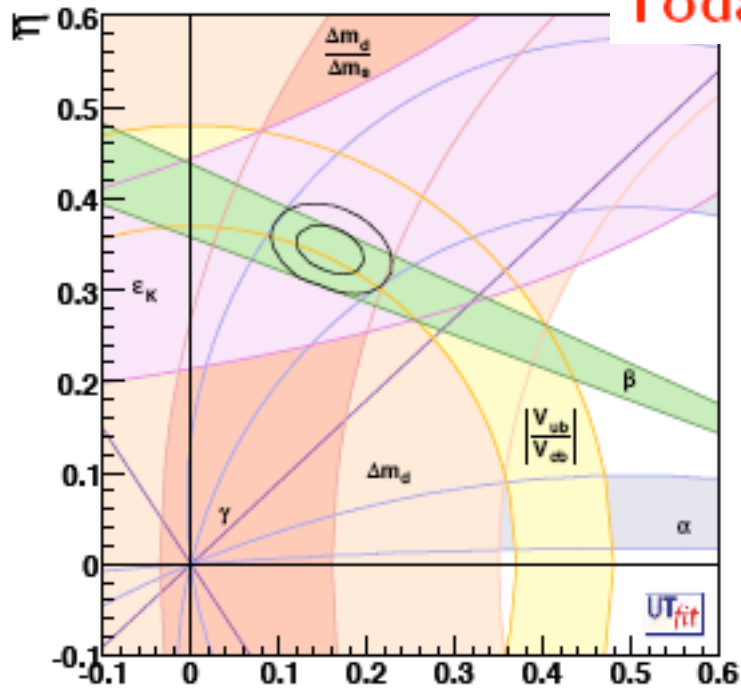


(Assuming SM branching fraction is measured)

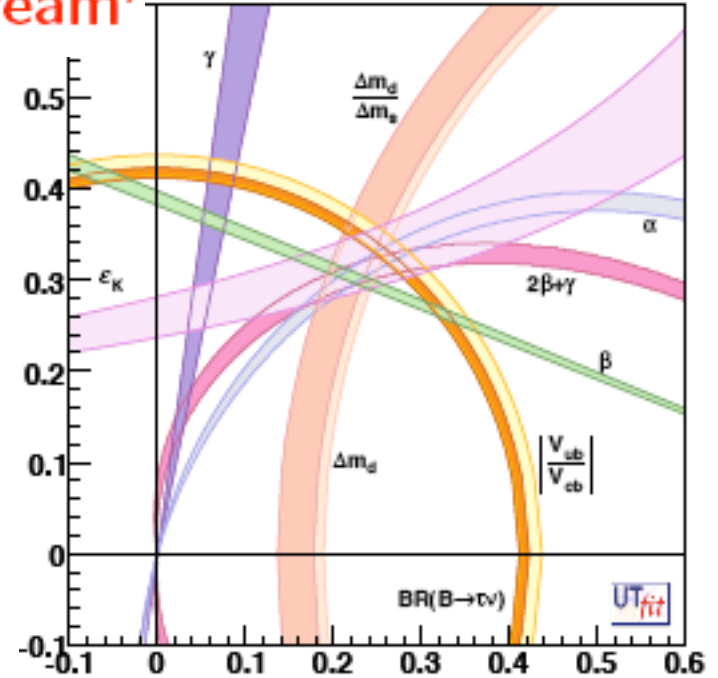
M_H (TeV)

Superflavour factory: CKM theory gets tested at 1%

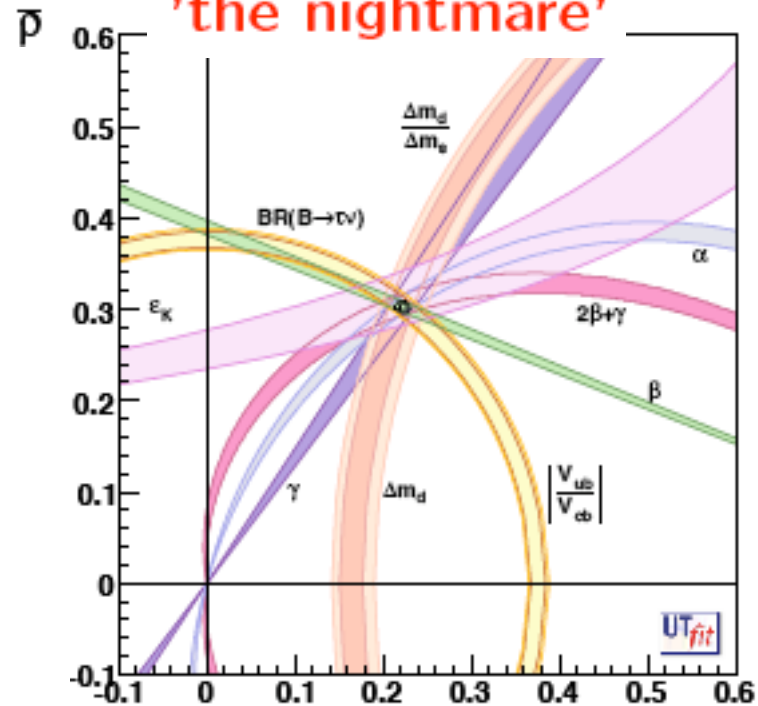
Today



'the dream'



'the nightmare'



Extra

Recall: SM basics

- Gauge group $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$

- Fermion representations

$$Q_{Li}^I(3, 2)_{+1/6}, \quad U_{Ri}^I(3, 1)_{+2/3}, \quad D_{Ri}^I(3, 1)_{-1/3}, \quad L_{Li}^I(1, 2)_{-1/2}, \quad E_{Ri}^I(1, 1)_{-1}.$$

Notation: left-handed quarks, Q_L^I : $SU(3)_C$, doublets of $SU(2)_L$ and carry hypercharge $Y = +1/6$

I interaction eigenstates

$i = 1, 2, 3$ flavor index

- Spontaneous symmetry breaking

$$\phi(1, 2)_{+1/2} \quad \langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad G_{\text{SM}} \rightarrow SU(3)_C \times U(1)_{\text{EM}}$$

$$\mathcal{L}_{\text{gauge}}(Q_L) = i \overline{Q_{Li}^I} \gamma_\mu \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{6} g' B^\mu \right) Q_{Li}^I$$

- $-\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} = Y_{ij}^d \overline{Q_{Li}^I} \phi D_{Rj}^I + Y_{ij}^u \overline{Q_{Li}^I} \tilde{\phi} U_{Rj}^I + \text{h.c.}$