Phase Evolution of Universe to the present Inert stage

(How dark matter can appear during cooling down of the Universe in the non-minimal Higgs model)

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Dark matter. Candidates

About 25 % of the Universe is made from Dark Matter (DM).
Different candidates for particles of DM:

Most probable: Neutral, mass $< 60 - 80$ GeV

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Fermion. Lightest superparticle — LSP.

Scalar. Higgs-like field — Inert Dark Model, IDM

G. Despande, L. Ma. *Phys. Rev.* **D 18** (1978) 2574 + many now

Idea:

SM with standard Higgs field ϕ_S is supplemented by Higgs field ϕ_D, having no interaction with matter fields and v.e.v. $= 0$).
G. Despande, L. Ma *Phys.Rev.* **D18** (1978) 2574; many papers now

Lagrangian: $\mathcal{L} = \mathcal{L}_{gf}^{SM} + \mathcal{L}_H + \mathcal{L}_Y\,; \quad \mathcal{L}_H = T - V\,.$

$\mathcal{L}_{gf}^{SM}$: $SU(2) \times U(1)$ SM interaction of gauge bosons and fermions;

$\mathcal{L}_H$: Higgs scalar Lagrangian= kinetic term T and the potential V;

$\mathcal{L}_Y$: Yukawa interaction of fermions with Higgs scalars.

General renormalizable Higgs potential, forbidding (ϕ_S, ϕ_D) mixing:

$$V = -\frac{1}{2}\left(m_{11}^2(\phi_S^\dagger\phi_S) + m_{22}^2(\phi_D^\dagger\phi_D)\right) + \\ +\frac{1}{2}\left(\lambda_1(\phi_S^\dagger\phi_S)^2 + \lambda_2(\phi_D^\dagger\phi_D)^2\right) + \lambda_3(\phi_S^\dagger\phi_S)(\phi_D^\dagger\phi_D) + \\ +\lambda_4(\phi_S^\dagger\phi_D)(\phi_D^\dagger\phi_S) + \frac{\lambda_5}{2}\left((\phi_S^\dagger\phi_D)^2 + (\phi_D^\dagger\phi_S)^2\right)$$

We fix $\lambda_5 < 0$, real without loss of generality.

$\mathcal{L}_Y$ contains only field ϕ_S, it does not create (ϕ_S, ϕ_D) mixing.

Useful abbreviation: $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad R = \frac{\lambda_{345}}{\sqrt{\lambda_1\lambda_2}}\,.$

We introduce D-parity and S-parity as invariance under transformations

$$S: \quad \phi_S \xrightarrow{S} -\phi_S, \quad \phi_D \xrightarrow{S} \phi_D, \quad SM \xrightarrow{S} SM,$$
$$D: \quad \phi_S \xrightarrow{D} \phi_S, \quad \phi_D \xrightarrow{D} -\phi_D, \quad SM \xrightarrow{D} SM,$$

where SM denote the SM fermions and gauge bosons.
Our potential preserves both D-parity and S-parity (Z_2 symmetry), S-parity conservation is violated by the Yukawa interaction.

But vacuum can violate these conservations.

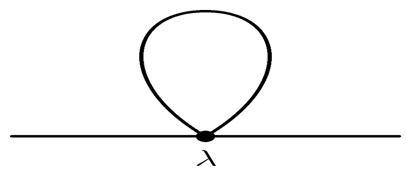
To have a *stable vacuum*, the potential must be positive at large quasi–classical values of fields (*positivity constraints*). In particular,

$$\lambda_1 > 0\,, \quad \lambda_2 > 0, \quad R + 1 > 0.$$

Temperature dependence

The Hubble constant is small $\Rightarrow$ statistical equilibrium. At the finite temperature the ground state of system is given by minimum of the Gibbs potential $V_G = Tr\left(V e^{-\widehat{H}/T}\right)/Tr\left(e^{-\widehat{H}/T}\right)$. The first correction to the V is given by diagrams of type (Matsubara technic):

At high enough temperature forms of potentials V_G and V coincide. Namely, the coefficients λ_i coincide, but mass terms evolve with temperature



$$m_{11}^2(T) = m_{11}^2(0) - c_1 T^2\,, \quad m_{22}^2(T) = m_{22}^2(0) - c_2 T^2\,,$$
$$c_1 = (3\lambda_1 + 2\lambda_3 + \lambda_4)/12 + (3g^2 + g'^2)/32 + (g_t^2 + g_b^2)A,$$
$$c_2 = (3\lambda_2 + 2\lambda_3 + \lambda_4)/12 + (3g^2 + g'^2)/32\,.$$

g and g' are coupling constants of gauge EW interaction. The Yukawa coupling constants of SM for t and b quarks are $g_t \approx 1$ and $g_b \approx 0.03$.

Simple analysis shows that in the case of neutral DM particle

$$At \;\; R > 0 \quad c_1 > 0,\; c_2 > 0\,,$$
$$At \;\; R < 0 \quad both \;\; signs \;\; of \;\; c_1 > 0,\; c_2 \;\; are \;\; possible, \;\; but \;\; c_1 + c_2 > 0\,.$$

Extrema of potential

The extrema of the potential define the values $\langle\phi_{S,D}\rangle$ of the fields $\phi_{S,D}$ via equations:

$$\partial V/\partial\phi_i|_{\phi_i=\langle\phi_i\rangle}=0\,,\qquad \partial V/\partial\phi_i^\dagger|_{\phi_i=\langle\phi_i\rangle}=0\,.$$

The extremum with the lowest value of energy (the global minimum of potential) realizes vacuum state of the model.
For each extremum with $\langle\phi_S\rangle\neq 0$ we choose the z axis in the weak isospin space so that $\langle\phi_S\rangle=\begin{pmatrix}0\\ v_S\end{pmatrix}$ with real $v_S>0$ ("neutral direction"). After this choice the most general form extremum is $\langle\phi_S\rangle=\dfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_S\end{pmatrix}$, $\quad\langle\phi_D\rangle=\dfrac{1}{\sqrt{2}}\begin{pmatrix}u\\ v_D\end{pmatrix}$. The vacuum with $u\neq 0$ is excluded by condition that DM is neutral particle.

At $u=0$ above extremum conditions have form of two equations:

$$v_S(-m_{11}^2+\lambda_1 v_S^2+\lambda_{345}v_D^2)=0\,, \quad v_D(-m_{22}^2+\lambda_2 v_D^2+\lambda_{345}v_S^2)=0\,.$$

with four solutions

name	v_D	v_S	D-parity conservation	S-parity conservation	state
EWs	0	0	$+$	$+$ (scalar sect.)	EW symmetric state
I_1	0	$\neq 0$	$+$	$-$	Inert state
I_2	$\neq 0$	0	$-$	$+$ (scalar sect.)	Inert-like state
M	$\neq 0$	$\neq 0$	$-$	$-$	Mixed state

EWs (Electroweak symmetry conserving phase): $v_S = v_D = 0$ preserve electroweak symmetry, D and S-parity (latter – in scalar sector only). It is vacuum at $m_{11}^2, m_{22}^2 < 0$.

I_1 (inert phase): $v_D = 0, \quad v_S^2 = v^2 = \dfrac{m_{11}^2}{\lambda_1}, \quad \mathcal{E}_{I_1}^{ext} = -\dfrac{m_{11}^4}{8\lambda_1}.$

S-parity is violated by choice of vacuum. ϕ_S is similar to Higgs field in SM. Its 4 components are splitted for 3 Goldstone modes + observable Higgs boson h_S with mass $M_{h_S}^2 = \lambda_1 v^2$ and with well-known couplings to fermions and gauge bosons.

This extremum preserves D-parity. Field ϕ_D is decomposed for 4 physical fields $D^\pm$, D_H, D_A, non-interacting with fermions and having only interactions like $D^+D^-W_\mu^+W_\mu^-$, $D^+D_HW^+$, D_HD_AZ and $D_HD_Hh_S$ with gauge bosons and ordinary Higgs boson. Mass relations:

$$M_{D_H}^2 = M_{D^\pm}^2 + (\lambda_4 + \lambda_5)v^2/2, \; M_{D_A}^2 = M_{D^\pm}^2 + (\lambda_4 - \lambda_5)v^2/2.$$

If this state is vacuum, D_H is DM particle.

I_2 (inert-like phase): $v_D^2 = v^2 = \dfrac{m_{22}^2}{\lambda_2}\,, \quad \mathcal{E}_{I_2}^{ext} = -\dfrac{m_{22}^4}{8\lambda_2}$ looks similar to the inert phase, but – interaction with fermions!

D-parity is violated by choice of vacuum. ϕ_D **looks** similar to Higgs field in SM. Its 4 components are splitted for 3 Goldstone modes + observable Higgs boson D_H with mass $M_{D_H}^2 = \lambda_2 v^2$. It has standard 3-linear couplings to gauge bosons (decay $D_H \to W^+W^-$ is allowed) but have no coupling to fermions. Fermions are massless.

S-parity is violated by interaction with fermions. Field ϕ_S is decomposed for 4 physical fields $S^\pm$, S_H, S_A, interacting with fermions in standard manner. Mass relations and quartic couplings are similar to those for inert phase.

If this state is vacuum, we see no candidates for DM particle.

Certainly, at $m_{22}^2 < 0$ this I_2 does not exist.

M (mixed phase):

$$v_S^2 = \frac{m_{11}^2\lambda_2 - \lambda_{345}m_{22}^2}{\lambda_1\lambda_2 - \lambda_{345}^2}, \quad v_D^2 = \frac{m_{22}^2\lambda_1 - \lambda_{345}m_{11}^2}{\lambda_1\lambda_2 - \lambda_{345}^2}$$

$$\mathcal{E}_M^{ext} = -\frac{m_{11}^4\lambda_2 - 2\lambda_{345}m_{11}^2m_{22}^2 + m_{22}^4\lambda_1}{\lambda_1\lambda_2 - \lambda_{345}^2}, \qquad v^2 = v_S^2$$

This mixed phase is similar to that in 2HDM with Model I for Yukawa interaction. Both D- and S-parity are violated. Scalars h, H, A, $D^\pm$ + 3 Goldstones mix components of ϕ_D and ϕ_S.

The equations for this extremum have sense only if obtained values v_S^2 and v_D^2 are positive. This extremum can be minimum if only $R^2 < 1$.

Simple algebra allows to transform these conditions to the form

$$\text{At} \quad 1 > R > 0: \quad 0 < R\frac{m_{11}^2}{\sqrt{\lambda_1}} < \frac{m_{22}^2}{\sqrt{\lambda_2}} < \frac{m_{11}^2}{R\sqrt{\lambda_1}};$$

$$\text{At} \quad 0 > R > -1: \quad \frac{m_{22}^2}{\sqrt{\lambda_2}} > R\frac{m_{11}^2}{\sqrt{\lambda_1}}, \quad \frac{m_{22}^2}{\sqrt{\lambda_2}} > \frac{m_{11}^2}{R\sqrt{\lambda_1}}.$$

Thermal evolution of Universe

During cooling of the Universe mass terms in the scalar potential were changed $\Rightarrow$ phase states of system might be changed.
We show evolution in the plane $\mu_1(T) = m_{11}^2(T)/\sqrt{\lambda_1}, \quad \mu_2(T) = m_{22}^2(T)/\sqrt{\lambda_2}$. at different values of parameter $R = \lambda_{345}/\sqrt{\lambda_1\lambda_2}$.
The possible current states of Universe are represented in figures by small black dots $P\#$.

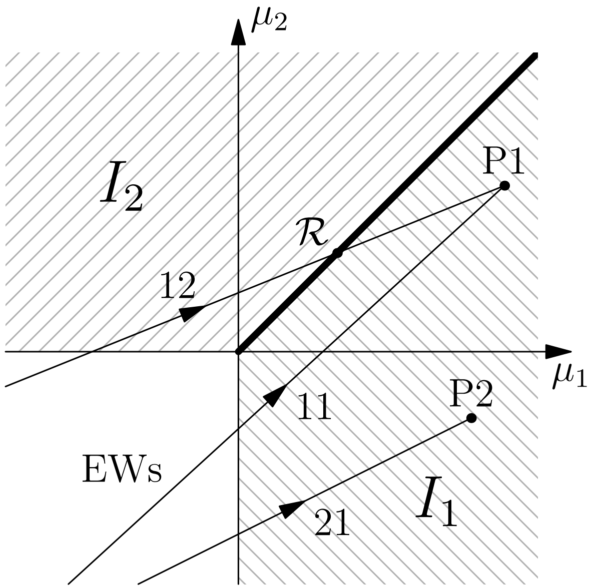
Basics:

present phase corresponds to the inert one, i.e. for modern values we have

$$\mu_1 > \mu_2, \ \mu_1 > 0 \qquad .$$

Evolution of physical states is given by rays, going to modern point $\boldsymbol{P\#}(\mu_1, \mu_2)$ and directed as vectors $\tilde{c}_1 = c_1/\sqrt{\lambda_1}$, $\tilde{c}_2 = c_2/\sqrt{\lambda_2}$ with $\tilde{c} = \tilde{c}_2/\tilde{c}_1$. Transition temperatures are easily calculated (crossing of straight lines).

The case $R > 1$



Phase diagram contains one quadrant with EWs phase and two sectors, describing the I_1 and I_2 phases. Phases I_1, I_2 are separated by *the phase transition line* $\mu_1 = \mu_2$. Two typical positions of today's state are represented by points $P1$ and $P2$. All possible phase evolutions are represented by rays 11 and 12 leading to the today's point $P1$ and by a ray 21 which leads to the today's point $P2$.

Ray 11: $\mathbf{0 < \mu_2 < \mu_1 \tilde{c}}$. $EWs \xrightarrow{II} I_1$

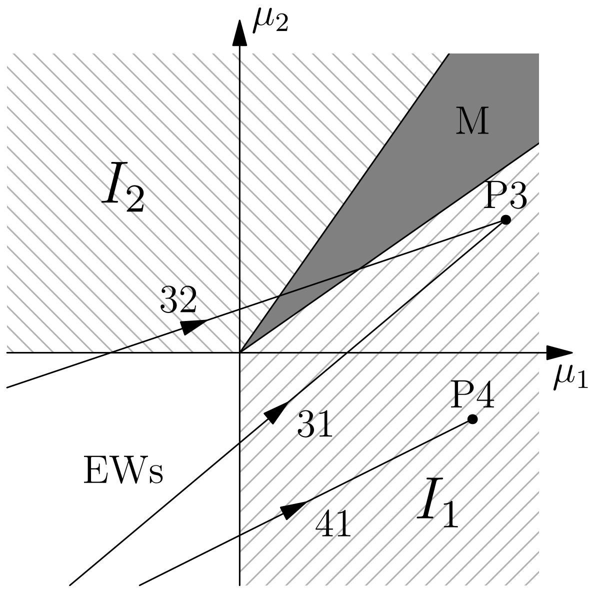
Ray 12: $\mathbf{\mu_2 > \mu_1 \tilde{c} > 0}$. $EWs \xrightarrow{II} I_2 \xrightarrow{I} I_1$

$I_2 \to I_1$ is the first-order phase transition with the latent heat

$$Q_{I_2 \to I_1} = T\frac{\partial \mathcal{E}_{I_2}}{\partial T} - T\frac{\partial \mathcal{E}_{I_1}}{\partial T}\bigg|_{\mu_2(T) \to \mu_1(T)} = (\mu_2 \tilde{c}_1 - \mu_1 \tilde{c}_2) T_{2,1}^2 / 4\,.$$

Ray 21: $\mathbf{\mu_2 < 0}$. $EWs \xrightarrow{II} I_1$

The case $1 > R > 0$



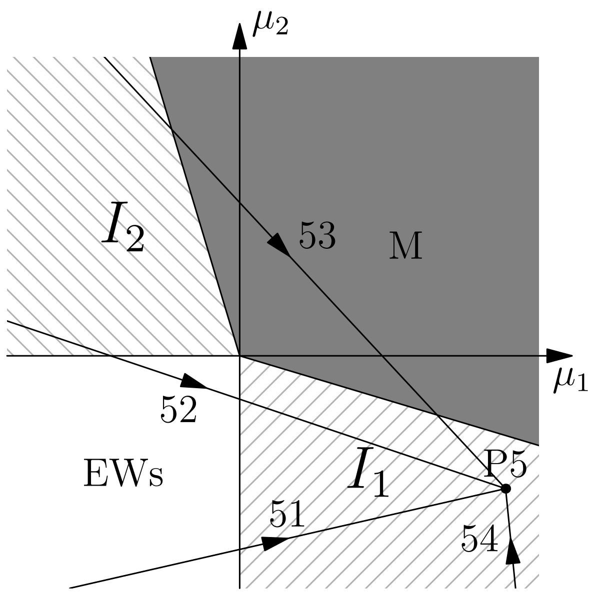
The phase diagram is modified in comparison with previous case. The new sector – the mixed phase M – occurs in the region $0 < R\mu_1 < \mu_2 < \mu_1/R$. As before, since $R > 0$ both $c_{1,2} > 0$ and consequently $\tilde{c} > 0$.

The possible today's states are given points $P3$ and $P4$, for which $R\mu_1 > \mu_2$.

For the rays 31 and 41 evolutions are as for the rays 11 and 21.

Ray 32: $\boldsymbol{\mu_2 > \mu_1 \tilde{c}}$. $EWs \xrightarrow{II} I_2 \xrightarrow{II} M \xrightarrow{II} I_1$

The case $0 > R > -1$



The mixed phase M region covers more than upper right quadrant of (μ_1, μ_2) plane $\mu_2 > \mu_1/R$, $\mu_2 > \mu_1 R$. Since currently we are in the inert vacuum, we have $\mu_2 < R\mu_1$ $(\mu_2 < 0)$. $\Rightarrow$ Only one type of today's point $P5$. New: $\tilde{c}_1$ and $\tilde{c}_2$ can be either positive or negative.

Ray 51 – similar evolution as rays 21, 41.

Ray 52: $\tilde{c}_1 > 0$, $\tilde{c}_2 < 0$, $\mu_2 < \mu_1\tilde{c}$. $I_2 \xrightarrow{II} EWs \xrightarrow{II} I_1$

Ray 53: $\tilde{c}_1 > 0$, $\tilde{c}_2 < 0$, $\mu_2^2 > \mu_1^2\tilde{c}$. $I_2 \xrightarrow{II} M \xrightarrow{II} I_1$

Ray 54: $\tilde{c}_1 < 0$, $\tilde{c}_2 > 0$. $I_1 \to I_1$

Summary

• The way from EW symmetric phase to modern inert phase can pass through the phase with no candidates for Dark Matter particles.

In these cases after EWSB transition Universe pass through inert-like phase with no candidates for DM, massless fermions, etc. After that system comes into the modern inert phase either via one first order phase transition or via two second order phase transitions.

• Modern inert state of Universe can be developed also from the high temperature state having no EW symmetry.

• When temperature tends to second order phase transition point, the mass of some particle tend to zero, after this transition the mass of some other particles grows from zero value.

• In contrast to standard picture, last phase transition to inert phase can take place at low enough temperature. It gives new starting point for calculation of modern abundance of component of Universe.

• We calculated thermal evolution in the very high temperature approximation, i.e. for $T^2 \gg |m_{ii}^2|$. The most interesting effects are expected at lower temperatures where more precise calculations, are necessary. Let us list simplest expected modifications of our description:

1. Cubic terms like $\phi^3 T$ will appear. They are important near phase transition point, as they can transform some second-order phase transitions into the first-order transitions.
2. The parameters become depend on temperature by more complex rule than that we used. $\Rightarrow$ the rays, depicted thermal evolutions in figures, can become non-straight. The bending of these rays can be different in different points of our plots and at different λ_i.

It can give possible phase evolution even reacher that discussed above.

However, we expect that the general picture will not change too much.

• Spread of results for more complex models is almost evident.
The simplest example gives the model with one D-field and pair of S-fields is evident. Such model allows to include CP violated Higgs sector for description of reality.