Pion and kaon LFWFs and QCD structure functions in the forward limit

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In collaboration with:

L. Chang, C.D. Roberts, J. Rodriguez-Quintero + ...

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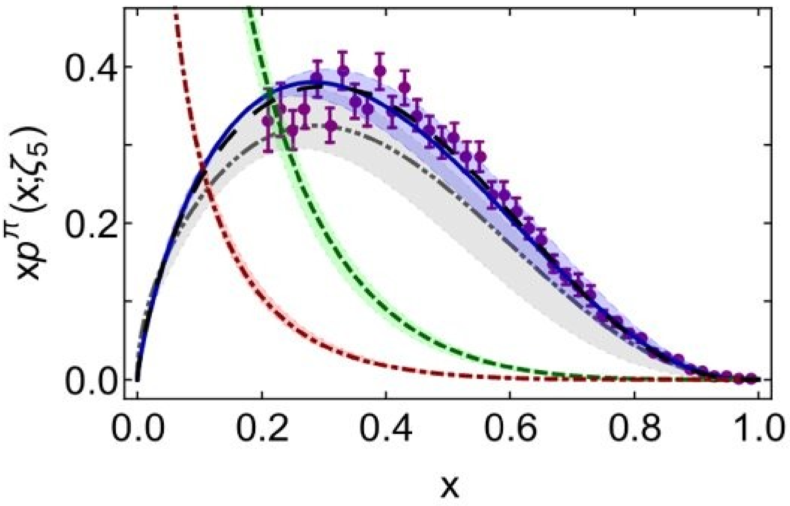
Pion PDFs: Recap

M. Ding, K. Raya, D. Binosi, L. Chang, C.D. Roberts, S.M. Schmidt

Chin.Phys. 44 (2020) no.3, 031002
“Drawing insights from pion parton distributions”

Phys.Rev. D101 (2020) no.5, 054014
“Symmetry, symmetry breaking, and pion parton distributions”

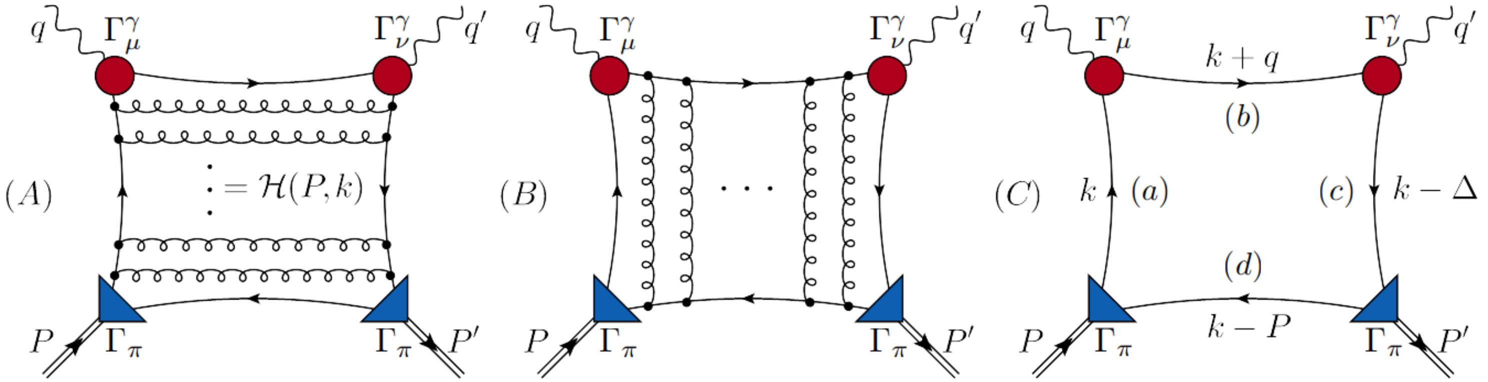


Pion PDFs

- In the RL truncation, the collection of **symmetry preserving diagrams** is:

(M. Ding's, C. Mezrag's talks)

L. Chang et al., Phys.Lett. B737 (2014) 23-29.
M. Ding et al., Phys.Rev. D101 (2020) no.5, 054014.

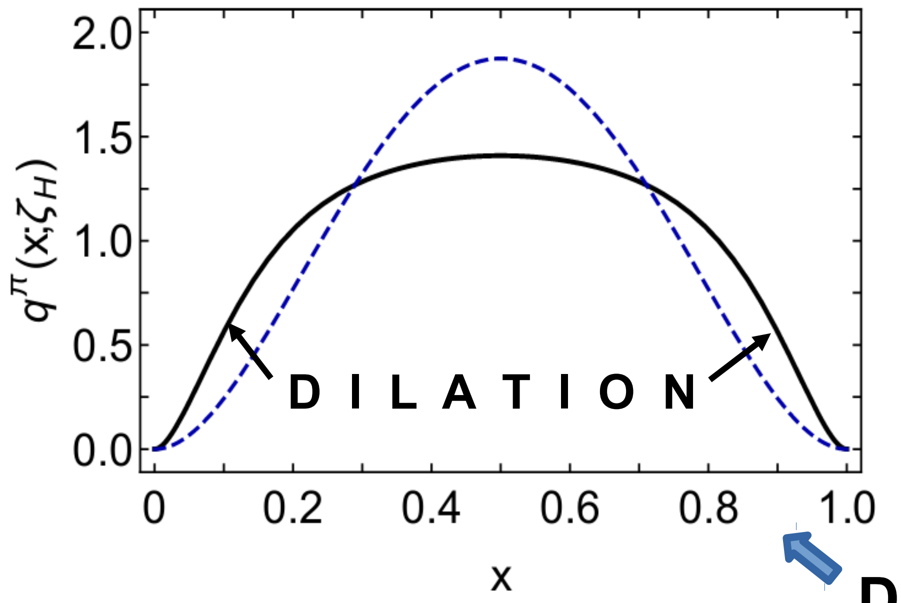


- We compute the moments of the distribution, at **hadronic scale** ζ_H, for the valence-quark distribution function.

Pion PDFs: hadronic scale

* M. Ding et al.,
Chin.Phys. 44 (2020) no.3, 031002
Phys.Rev. D101 (2020) no.5, 054014

- Valence-quark PDF at ζ_H is reconstructed from its Mellin moments.



$[\text{Blue}] : q_\pi(x) = 30x^2(1-x)^2$

Parton-like *model.* **Phys.Lett. B737 (2014) 23-29**

$[\textbf{Black}]$: DSE **Prediction**

$$q_\pi(x;\zeta_H) = 213.32\, x^2(1-x)^2$$
$$\times[1 - 2.9342\sqrt{x(1-x)} + 2.2911x(1-x)]$$

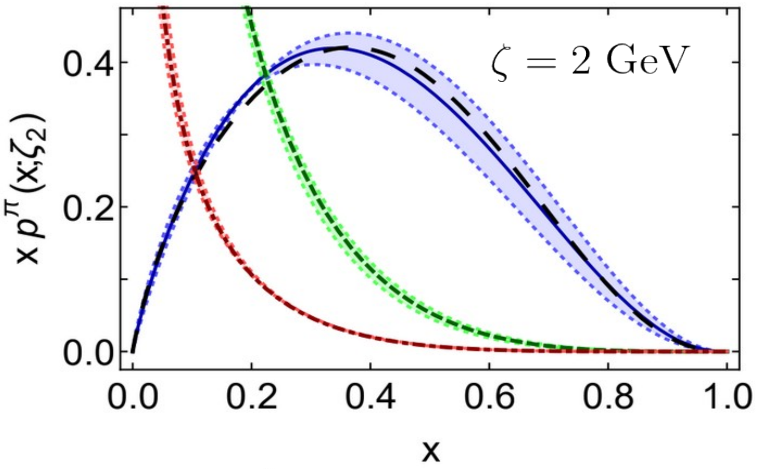
Fully non-perturbative computation*.

Dilation owing to **DCSB** (as in the PDA).

- **DCSB** manifests in wave functions, form factors, distribution amplitudes and parton distribution functions.

Pion PDFs: lattice

- Excellent agreement with **lattice** and **experimental** results.



ζ_2	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
Ref. [34]	0.24(2)	0.09(3)	0.053(15)
Ref. [35]	0.27(1)	0.13(1)	0.074(10)
Ref. [36]	0.21(1)	0.16(3)	
Herein	0.24(2)	0.098(10)	0.049(07)

Lattice moments:
Detmold et al., Brommel et al., Oehm *et al.*

- **Valence** content is **roughly 50%**.
- Gluon and sea contributions:

$$<x_{\text{gluon}}> = 0.41(2)\ ,\ <x_{\text{sea}}> = 0.11(2)$$

- It **confirms** the large gluon momentum fraction of:

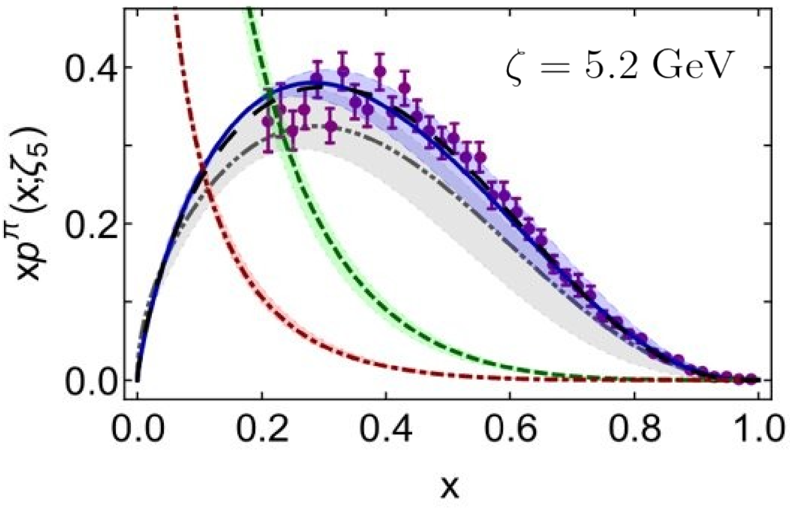
C. Chen et al., Phys.Rev. D93 (2016) no.7, 074021.
M.B. Hecht et al., Phys.Rev. C63, 025213 (2001).

- And the **trend** observed in:

P.C. Barry et al., Phys. Rev. Lett.121, 152001 (2018).

Pion PDFs: experiment

- Excellent agreement with **lattice** and **experimental** results.



ζ_5	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
Ref. [31]	0.17(1)	0.060(9)	0.028(7)
Herein	0.21(2)	0.076(9)	0.036(5)

Lattice results:
Sufian *et al.*, Phys. Rev. D99, 074507 (2019)

- **Valence** content is **roughly 42%**.
- Gluon and sea contributions:

$$<x_{\text{gluon}}> = 0.45(1)\ ,\ <x_{\text{sea}}> = 0.14(2)$$

- Pointwise form of the **lattice** result **agrees** with the **DSE** prediction.
- **Disparate** treatments arrive at the *same* prediction.

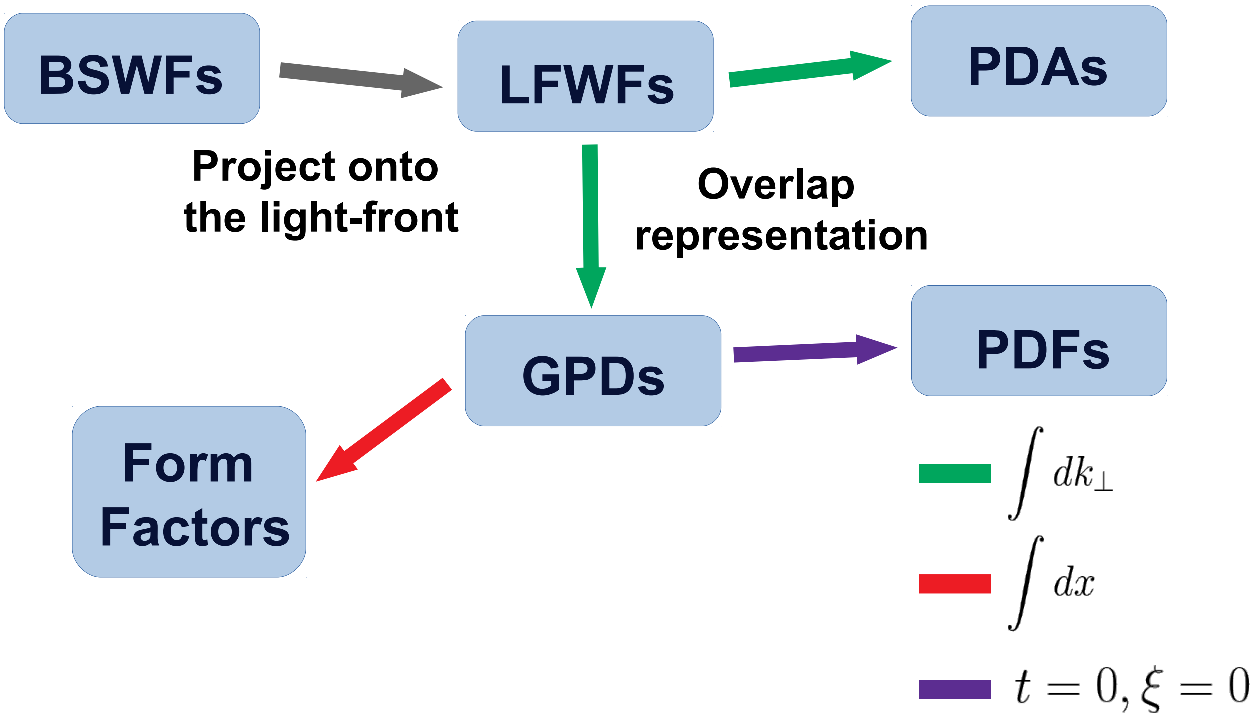
→ **Urgent need for new data!!!**

Experimental data:
J. S. Conway *et al.*, Phys. Rev. D39, 92 (1989)
M. Aicher *et al.*, Phys. Rev.Lett.105, 252003 (2010)

Light-front wave functions (LFWFs) and so on...

Light-front wave function approach

➢ **Goal:** get a **broad picture** of the pion/kaon structure.



The idea:
Compute ***everything*** from the **LFWF.**

The inputs:
Solutions from quark **DSE** and meson **BSE**.

The alternative inputs:
Model **BSWF** from realistic DS-BS ***predictions***.

N. Chouika et al., Phys.Lett. B780 (2018) 287-293.
C. Mezrag et al., Few Body Syst. 57 (2016) no.9, 729-772

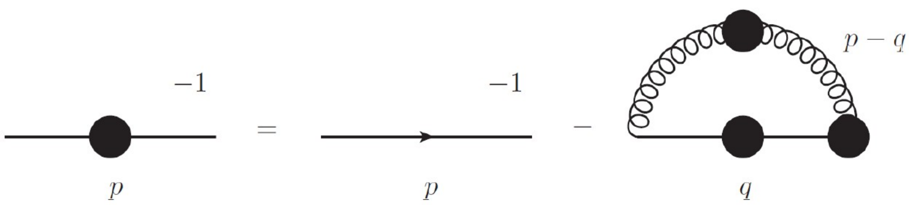
Light-front wave functions

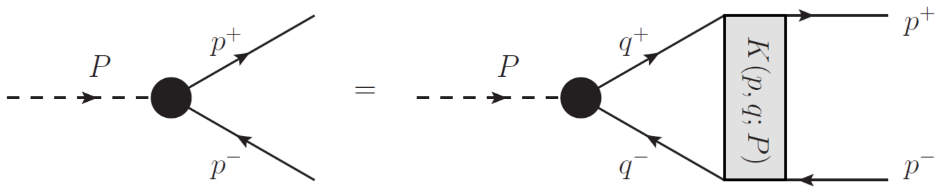
- The BSWF is the sandwich of the Bethe-Salpeter amplitude and quark propagators:

$$\chi_H(k_-^H; P_H) = S_q(k)\Gamma_H(k_-^H; P_H)S_{\bar{q}}(k - P_H)\ ,\ k_-^H = k - P_H/2\ .$$

$P^2 = -m_H^2$ **: meson's mass**; Γ_H **BS amplitude**; $S_{q(\bar{q})}$ **quark (antiquark) propagator**

- Quark propagator and BSA should come from solutions of:





Quark DSE **Meson BSE**

- **Alternative** first step: **construct** an *educated* **guess**.

LFWF: model

- Starting with the Kaon as a example, we employ a Nakanishi-like representation:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \underbrace{\mathcal{M}(k; P_K)}_{1} \int_{-1}^{1} d\omega \, \underbrace{\rho_K(\omega)}_{2} \underbrace{\mathcal{D}(k; P_K)}_{3} \,,$$

1: Matrix structure (leading BSA):

$$\mathcal{M}(k; P_K) = -\gamma_5[\gamma \cdot P_K M_u + \gamma \cdot k(M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}] \,,$$

2: Sprectral weight: To be described later.

3: Denominators: $\mathcal{D}(k; P_K) = \Delta(k^2, M_u^2)\Delta((k - P_K)^2, M_s^2)\hat{\Delta}(k_{\omega-1}^2, \Lambda_K^2) \,,$

where: $\Delta(s,t) = [s+t]^{-1}$, $\hat{\Delta}(s,t) = t\Delta(s,t)$.

S-S Xu et al., Phys.Rev. D97 (2018) no.9, 094014.

LFWF: model

- **Algebraic** manipulation yields:

$$\chi_K^{(2)}(k_-^K; P_K) = \mathcal{M}(k; P_K) \int_0^1 d\alpha \, 2\chi_K(\alpha; \sigma^3(\alpha)) \; , \; \sigma = (k - \alpha P_K)^2 + \Omega_K^2 \; ,$$

Depends on Feynman and model **parameters**.

Scalar function, **reads** as:

$$\chi_K(\alpha; \sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv \right] \frac{\rho_K(\omega)}{n_K} \frac{\Lambda_K^2}{\sigma^3} \; .$$

- **$\rho_K(\omega)$** will play a **crucial role** in determining the meson's observables.
- Realisitc DSE predictions will help us determine it.

Pion PDF as benchmark!

Chin.Phys. 44 (2020) no.3, 031002
Phys.Rev. D101 (2020) no.5, 054014

Light-front wavefunction

- The **pseudoscalar LFWF** can be written:

$$f_K \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \mathrm{tr}_{CD} \int_{dk_{||}} \delta(n \cdot k - x n \cdot P_K) \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K) \,.$$

- The **moments** of the distribution:

$$< x^m >_{\psi_K^{\uparrow\downarrow}} = \int_0^1 dx x^m \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{1}{f_K n \cdot P} \int_{dk_{||}} \left[\frac{n \cdot k}{n \cdot P}\right]^m \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K)$$

$$\int_0^1 d\alpha \alpha^m \left[\frac{12}{f_K} \mathcal{Y}_K(\alpha; \sigma^2)\right] \;, \quad \mathcal{Y}_K(\alpha; \sigma^2) = [M_u(1-\alpha) + M_s \alpha] \mathcal{X}(\alpha; \sigma_\perp^2) \,.$$

Uniqueness of Mellin moments →

$$\boxed{\psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K} \mathcal{Y}_K(x; \sigma_\perp^2)}$$

S-S Xu et al.,
Phys.Rev. D97 (2018) no.9, 094014.

✓ Compactness of this result is a merit of the algebraic model.

Light-front wavefunction

$$\psi_K^{\uparrow\downarrow}(x,k_\perp^2) = \frac{12}{f_K}\mathcal{Y}_K(x;\sigma_\perp^2)$$

- Thus, the **LFWF** is heavily influenced by **$\rho_K(\omega)$.**

$$\mathcal{Y}_K(\alpha;\sigma^2) = [M_u(1-\alpha)+M_s\alpha]\mathcal{X}_K(\alpha;\sigma_\perp^2)\ ,\quad \chi_K(\alpha;\sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv\right]\frac{\rho_K(\omega)}{n_K}\frac{\Lambda_K^2}{\sigma^3}\ .$$

- The explicit form of **$\rho_K(\omega)$** **determines** the form of **PDAs, GPDs, PDFs**, etc.

$$\Rightarrow \psi_K^{\uparrow\downarrow}(x,k_\perp^2) \sim \int d\omega \cdots \rho_K(\omega) \cdots$$

- <u>For example:</u>

$$\rho_\pi(\omega) \sim (1-\omega^2)$$

Asymptotic model

$$\phi(x) \sim x(1-x)$$ *Asymptotic* profile

$$q(x) \sim [x(1-x)]^2$$ *Parton-like* profile

C. Mezrag et al., Phys.Lett. B741 (2015) 190-196.
C. Mezrag et al., Few Body Syst. 57 (2016) no.9, 729-772

Light-front wavefunction

$$\psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K}\mathcal{Y}_K(x;\sigma_\perp^2)$$

- Our choice is given by **experience** and careful **analyses**.

$$u_G\rho_G(\omega) = \frac{1}{2b_0^G}\left[\text{sech}^2\left(\frac{\omega-\omega_0^G}{2b_0^G}\right) + \text{sech}^2\left(\frac{\omega+\omega_0^G}{2b_0^G}\right)\right]\left[1+\omega\, v_G\right],$$

Parameters

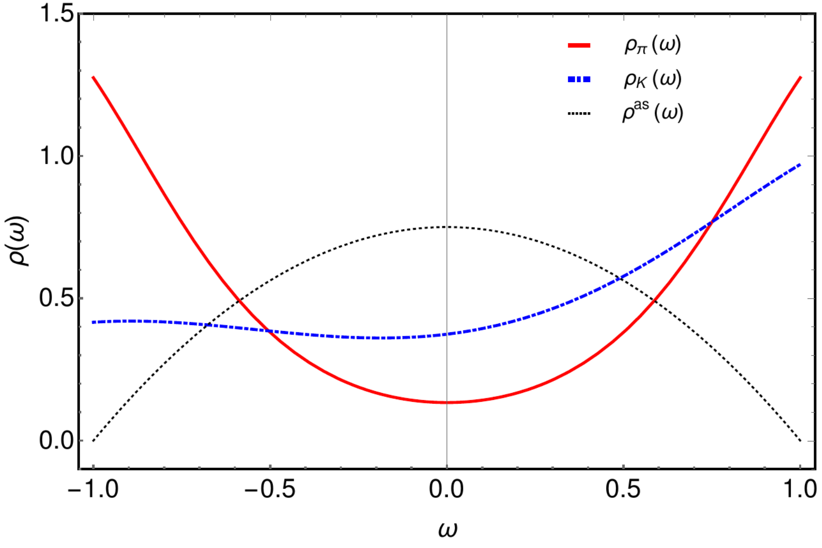
Λ_π	b_0^π	ω_0^π	ν_π	Λ_K	b_0^K	ω_0^K	ν_K
M_u	0.275	1.23	0	$2M_s$	0.5	1.3	0.4

$$m_\pi = 0.140 \text{ GeV}, m_K = 0.49 \text{ GeV}$$

$$M_u = 0.31 \text{ GeV}, M_s = 1.2\ M_u$$

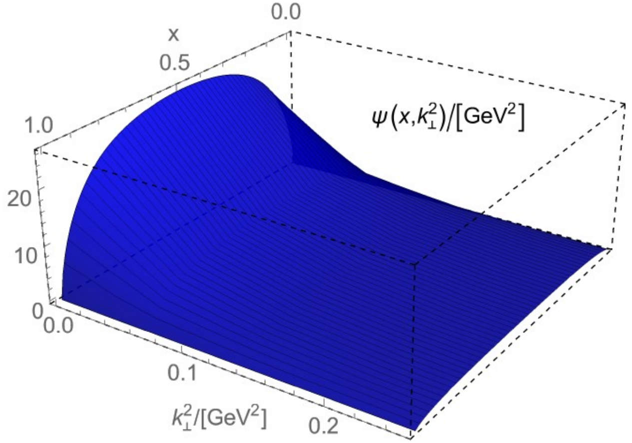
Typical values of **constituent** quark masses, from **realistic** DSEs **solutions**.

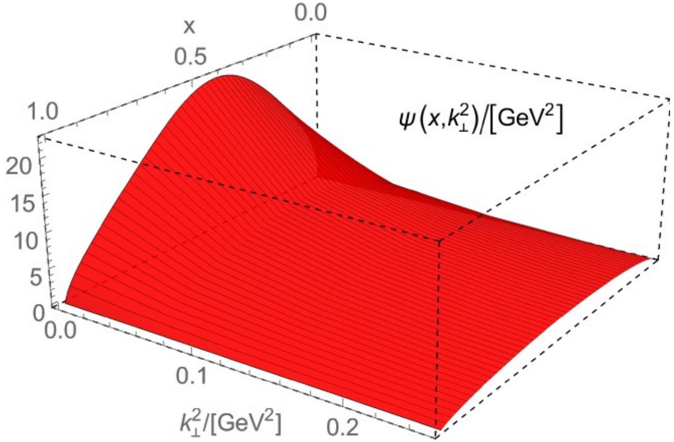
Profiles

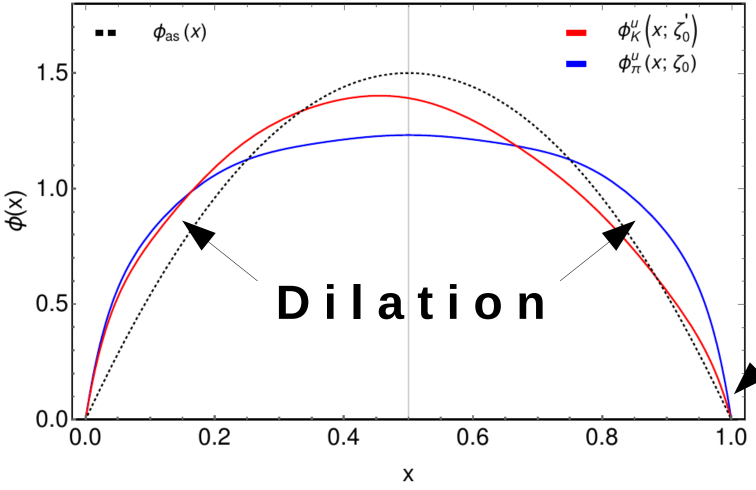


LFWFs and PDAs

$$\phi_M(x) = \frac{1}{16\pi^3}\int d^2\vec{k}_\perp \psi_M^{\uparrow\downarrow}(x,k_\perp^2)$$







Smooth fall $\sim (1-x)^{1+\gamma}$

~ 0.25[+]

$$\left\{ \begin{array}{l} <(2x-1)^2>^{\pi}_{\zeta_2} = 0.228 \\ <(2x-1)^2>^{K}_{\zeta_2} = 0.221 \end{array} \right\}$$

~ 0.23*

~ -0.03

$$<(2x-1)^1>^{K}_{\zeta_2} = -0.025$$

[+] J. Segovia et al., Phys.Lett. B731 (2014) 13-18

[+] L. Chang et al.,Phys.Rev.Lett. 110 (2013) no.13, 132001

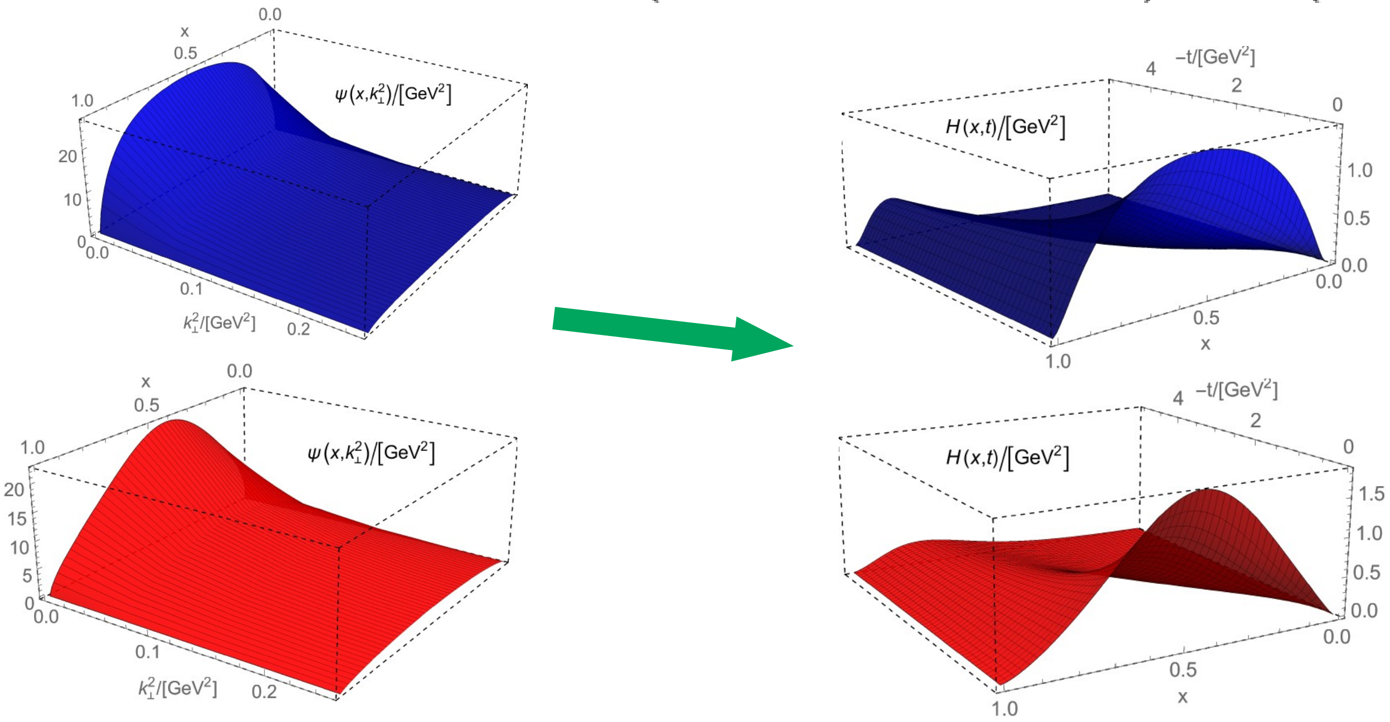
* G.S. Bali et al., JHEP 1908 (2019) 065

LFWFs and GPDs

LFWFs → **GPDs**

- In the **overlap representation**, the **GPD** reads as:

$$H_M^q(x,\xi,t) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\,\pi^3}\Psi^*_{u\bar{f}}\left(\frac{x-\xi}{1-\xi},\mathbf{k}_\perp + \frac{1-x}{1-\xi}\frac{\Delta_\perp}{2}\right)\Psi_{u\bar{f}}\left(\frac{x+\xi}{1+\xi},\mathbf{k}_\perp - \frac{1-x}{1+\xi}\frac{\Delta_\perp}{2}\right).$$



- ✓ Valence content only
- ✓ Valid in the **DGLAP** region
- ✓ ***Compatible*** with diagram approach

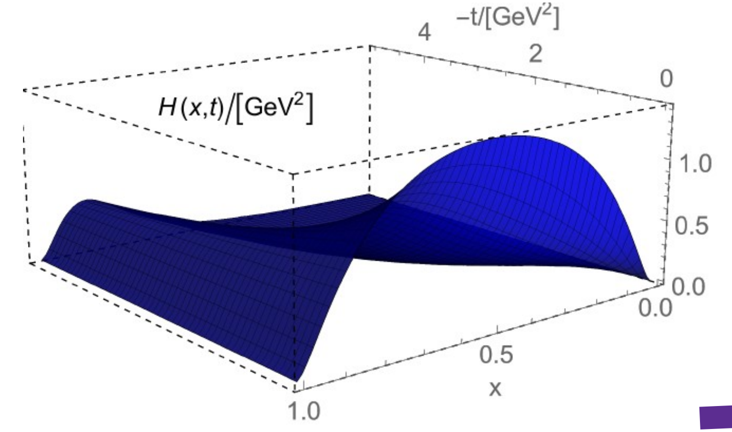
(J R-Q's, C. Mezrag's talks)

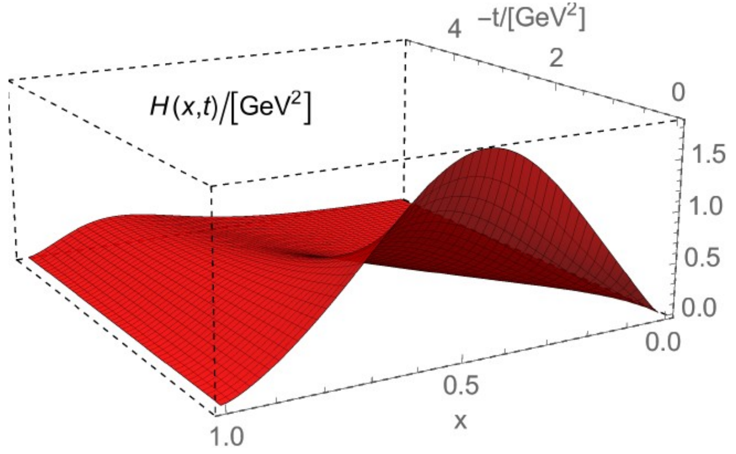
LFWFs and PDFs

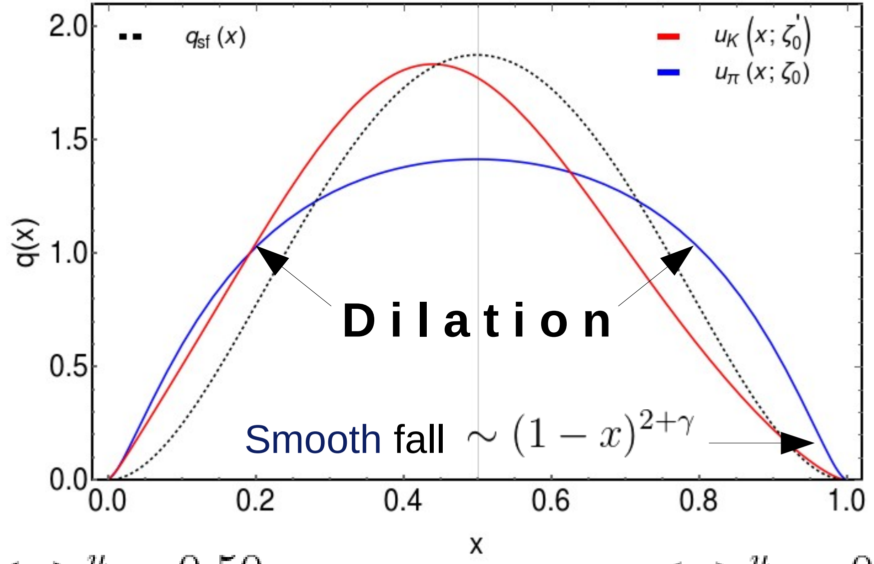
GPD → **PDF**

- The **PDF** is obtained from the **forward limit** of the **GPD**.

$$q(x) = H(x,0,0)$$





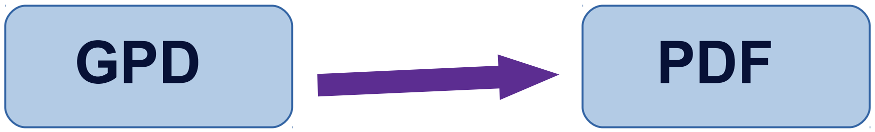


$$<x>^u_\pi = 0.50 \qquad <x>^u_K = 0.46$$

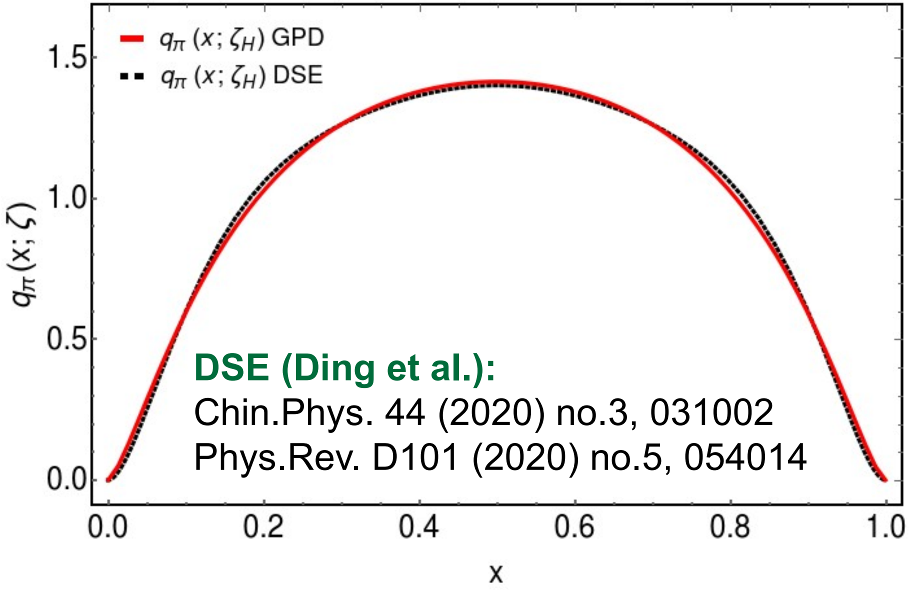
- ζ_H: **all** the momentum is carried by the valence-quarks.
- **Defined**, *unambiguously*, from the **PI** charge.

(J. Rodriquez-Quintero's talk)

Pion PDFs

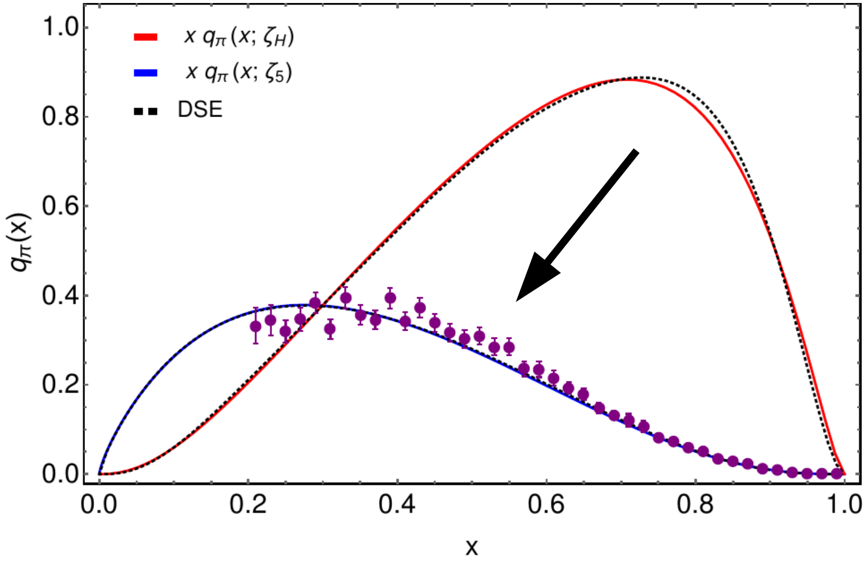


- We use the DSE prediction (**DB**) of the PDF as **benchmark** to get a **realistic LFWF**.



ζ_H	$<x^2>$	$<x^4>$	$<x^6>$
DSE	0.3020	0.1460	0.0857
GPD	0.3020	0.1461	0.0861

- At ζ_H, we see a **small deviation** from the *realistic* PDF.
- **But**, **DGLAP** evolution *sweeps* any difference.



Running coupling and evolution

PI strong running coupling

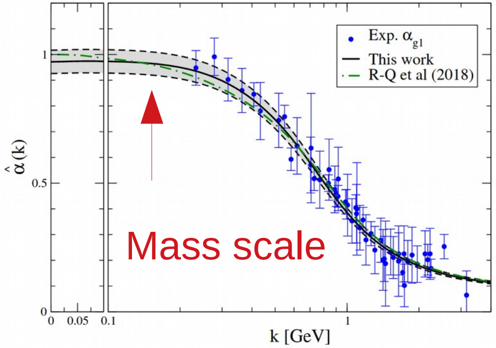
Idea: Define an effective coupling such that the equations below are exact.

$$\frac{d}{dt}q(x;t) = -\frac{\alpha(t)}{4\pi}\int_x^1 \frac{dy}{y}q(y;t)P\left(\frac{x}{y}\right)$$

- or -

$$\frac{d}{dt}M_n(t) = -\frac{\alpha(t)}{4\pi}\gamma_0^n M_n(t)$$

i.e. no LO, NLO, etc: **all orders are there**



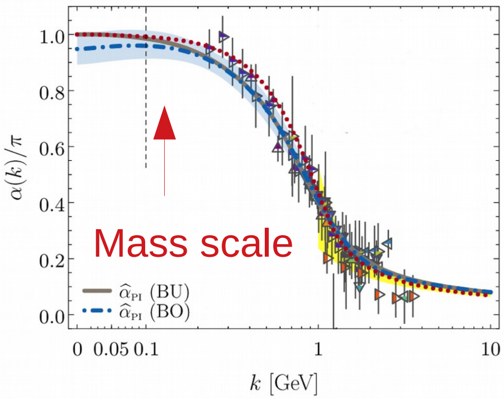
Z-F Cui et al., arXiv:1912.08232

… and define, not tune, the (initial) **hadron scale ζ_H**.

(fully dressed quasiparticles are the correct degrees of freedom)

- The **answer** comes from the PI effective charge.

J. R-Q et al., arXiv:1909.13802.

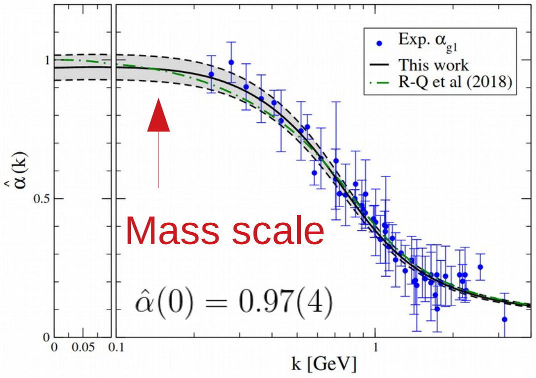


D. B et al., Phys.Rev. D96 (2017) no.5, 054026.
J. R-Q et al., Few Body Syst. 59 (2018) no.6, 121.

PI strong running coupling

‘NEW’ (no parameters)

Mass scale ***directly*** inferred from **gluon propagator**:



$$m_0 = 0.43(1)\ \text{GeV}$$

- We identify:

$$\zeta_H = m_0$$

- Purely ***soft*** dynamics

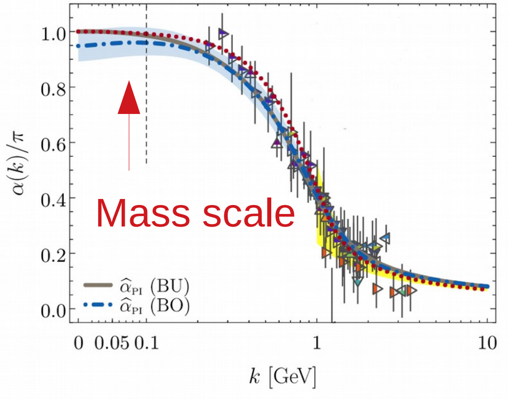
Z-F Cui et al., arXiv:1912.08232

- ***Algebraic*** version was used in the full DSE calculation of pion PDF.
- But, ***both*** forms are entirely **equivalent**.

‘OLD’ (algebraic version)

$$\alpha_{\rm PI}(k^2) \approx \frac{4\pi}{\beta_0 \ln[(m_\alpha^2 + k^2)/\Lambda_{\rm QCD}^2]}$$

Fixed to ***match*** the **saturation** point:



$$m_\alpha = 0.3\ \text{GeV}$$

- We identify:

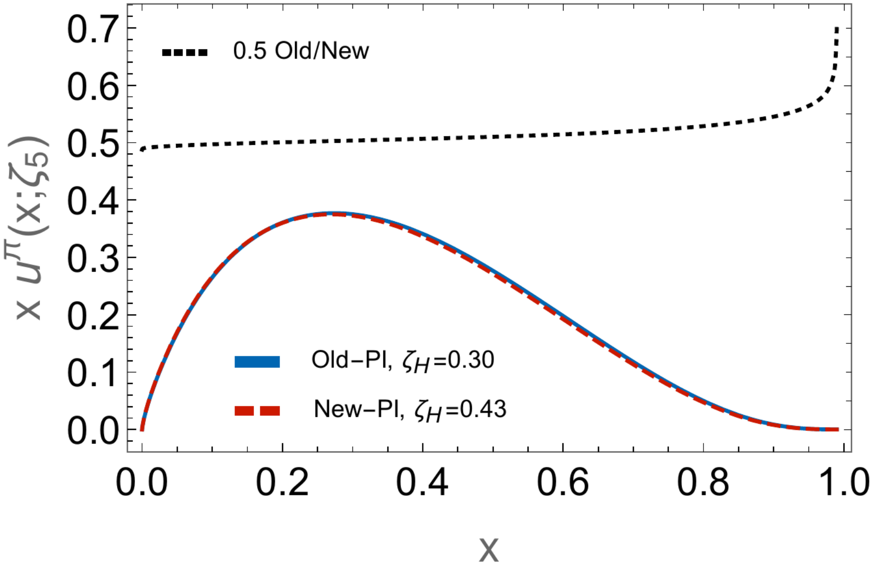
$$\zeta_H = m_\alpha(1 \pm 0.1)$$

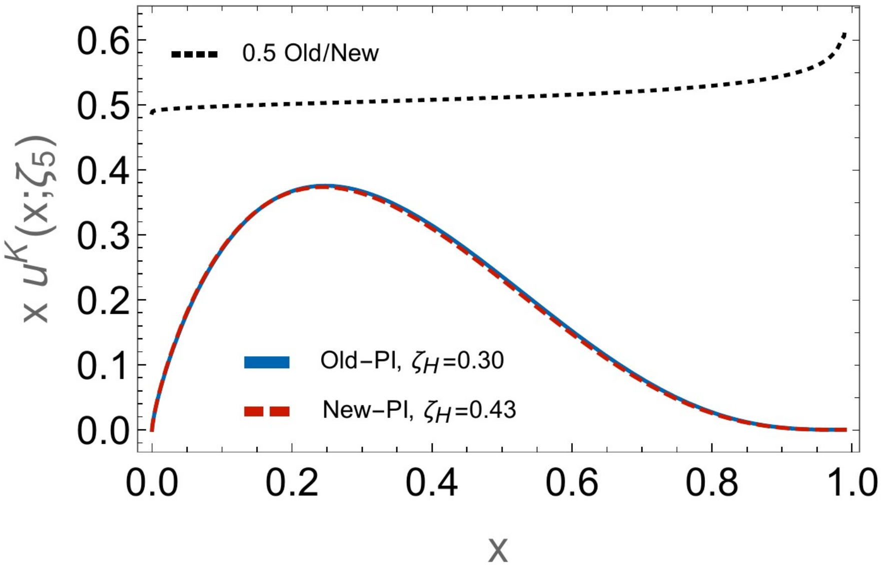
D. B et al., Phys.Rev. D96 (2017) no.5, 054026.
J. R-Q et al., Few Body Syst. 59 (2018) no.6, 121.

PI strong running coupling

- **Amazingly**, the '*old*'-*algebraic* form of the PI charge, yields essentially the **same results** as those obtained with the most sophisticated version.

→ ***Intuition*** and ***numbers*** behind the algebraic one are **correct**.

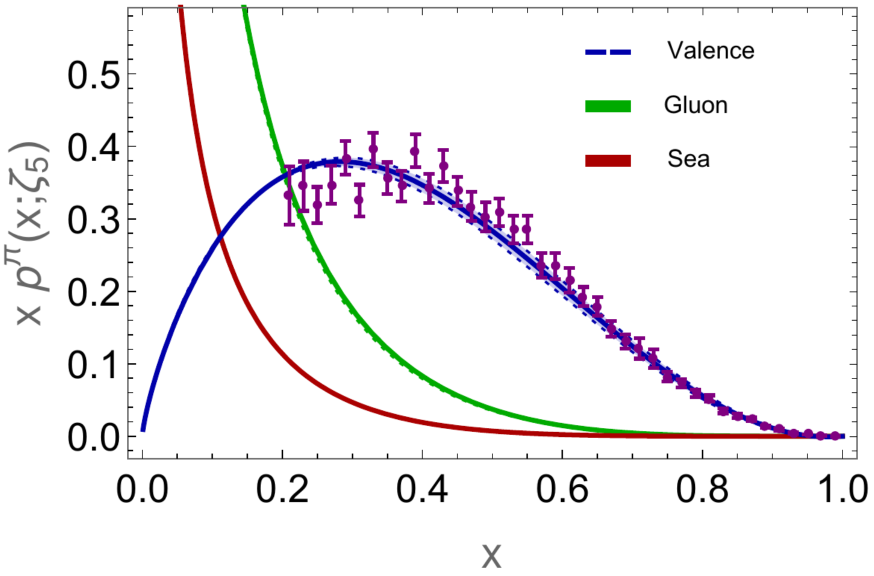




We shall use the most sophisticated.

Parton Distribution Functions

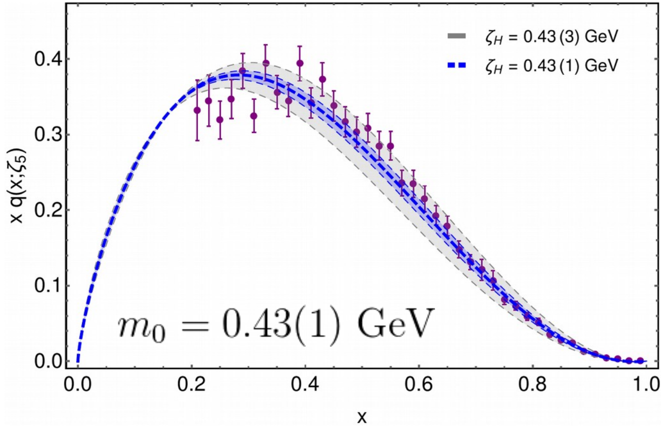
Pion Parton Distribution Functions



ζ	$<x>_q$	$<x>_G$	$<x>_S$
ζ_2	0.243(6)	0.410(7)	0.104(5)
ζ_5	0.208(6)	0.448(6)	0.136(6)

Data: M. Aicher et al.,
Phys.Rev.Lett. 105 (2010) 252003

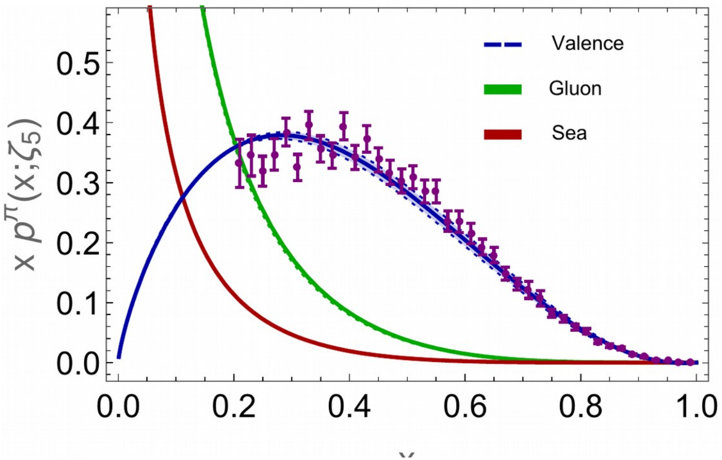
- The refined PI effective charge provides a **reduced uncertainty** (as compared with the algebraic one).

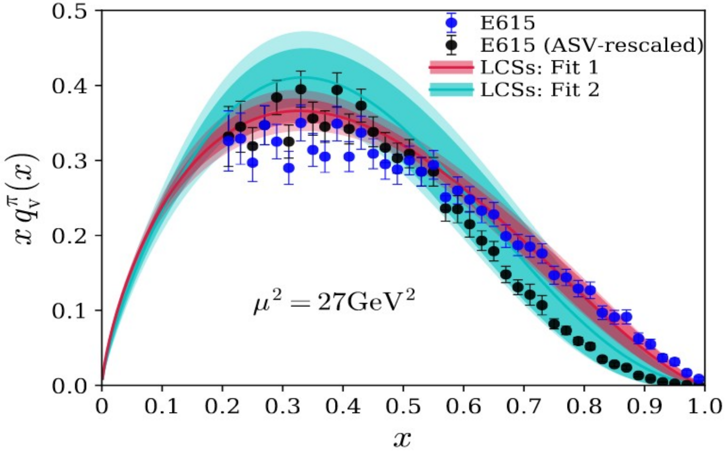


- By construction, it **exhibits** the features of the DSE **prediction** for the pion PDF:

M. Ding et al., Chin.Phys. 44 (2020) no.3, 031002
Phys.Rev. D101 (2020) no.5, 054014

Pion Parton Distribution Functions





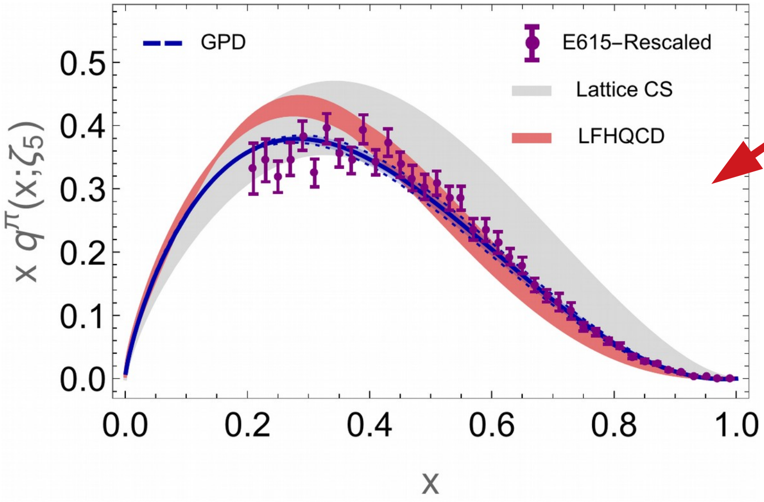
- Excelent agreement with novel **Lattice “Cross-Section”** results:

R.S. Sufian et al., arXiv:2001.04960

+ ‘*Same*’ large-x exponent**:**

> The resultant PDFs obtained are in agreement with the $q_{\mathrm{v}}^{\pi}(x)$ extracted from the experimental data. Our analysis indicates that a $(1-x)^2$-behavior of the $q_{\mathrm{v}}^{\pi}(x)$ at large x is preferred. Future calculations with finer

Pion Parton Distribution Functions



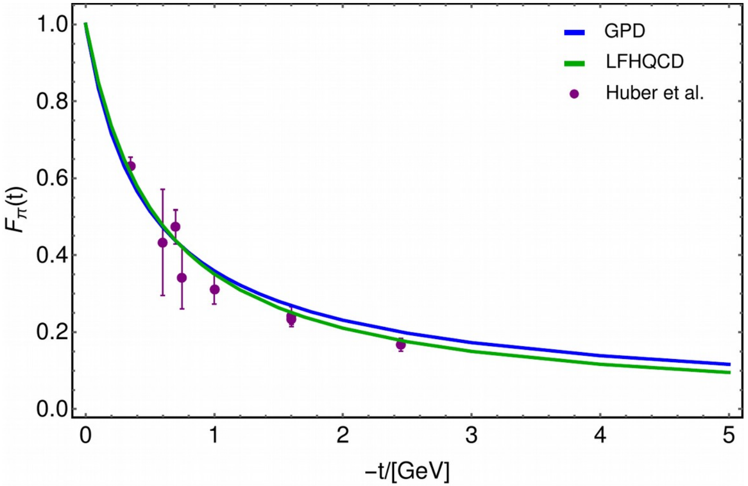
“Rule-2” (L. Chang’s talk)

- Rule-1: $\sim(1-x)^{2\tau-3}$, with $g(\tau)=2$
- Rule-2: $\sim(1-x)^{2\tau-2}$, with $g(\tau)=2+\frac{1}{\tau-1}$

- ***Arbitrariness*** of the universal reparametrization function*:
  - Light-front hamiltonian QCD can also **accomodate** large-x exponent of ‘2’:

    L. Chang et al., arXiv:2001.07352

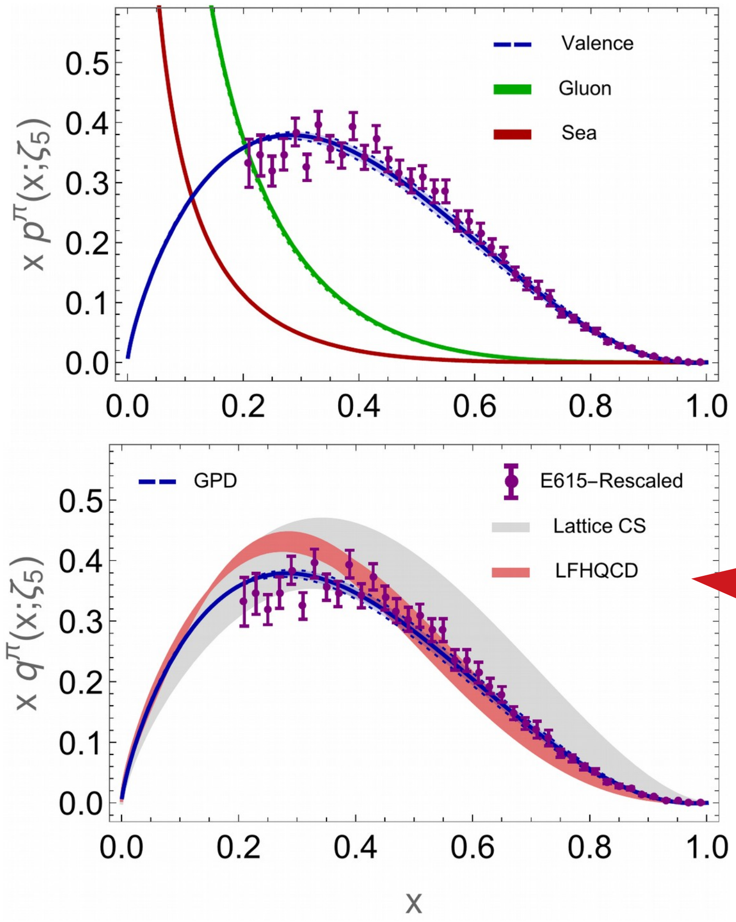
- Agreement goes ***beyond*** the **forward limit**:



(*) HLFHS Coll., Phys.Rev.Lett. 120 (2018) 18, 182001

Data: G. Huber et al., Phys. Rev. C78, 045203 (2008)

Pion Parton Distribution Functions



- Excelent agreement with novel **Lattice “Cross-Section”** results:

  R.S. Sufian et al., arXiv:2001.04960

  + ‘*Same*’ large-x exponent**:**

  > The resultant PDFs obtained are in agreement with the $q_{\rm v}^\pi(x)$ extracted from the experimental data. Our analysis indicates that a $(1-x)^2$-behavior of the $q_{\rm v}^\pi(x)$ at large x is preferred. Future calculations with finer

- Light-front hamiltonian QCD can also **accomodate** such behavior:

  L. Chang et al., arXiv:2001.07352

“Rule – 2”

(L. Chang’s talk)

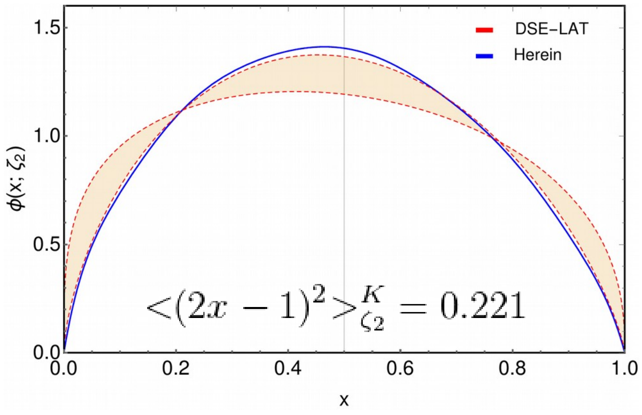
- Different treatments arrive at, essentially, the ***same*** prediction. Such **confluence** cannot go unnoticed.

Kaon and Pion PDF

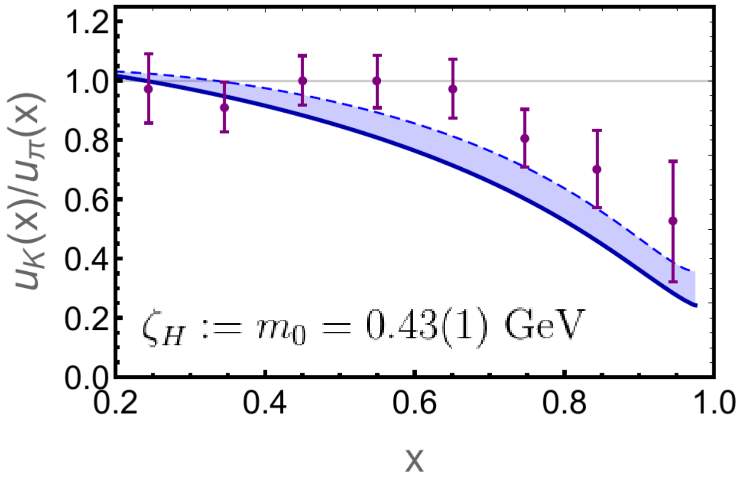
- Kaon ***exploratory*** DSE computation motivated our **BSWF model**:

	$<x>$	$<x^2>$	$<x^3>$
DSE	0.458	0.256	0.160
GPD	0.461	0.252	0.154

- While also keeping an ***acceptable*** **PDA**:



DSE-LAT: J. Segovia et al., Phys.Lett. B731 (2014) 13-18



- **Momentum fractions:**

ζ	$<x>_u$	$<x>_s$	$<x>_G$	$<x>_S$
ζ_2	0.224(5)	0.302(5)	0.378(6)	0.096(4)
ζ_5	0.191(5)	0.269(5)	0.414(5)	0.126(5)

✓ **Same** initial **scale** for both pion and kaon.

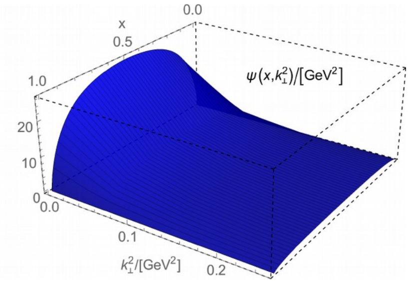
Summary

Summary: Pion

- Using **DSE prediction** of pion PDF as **benchmark**, we modeled the pion **BSWF**.

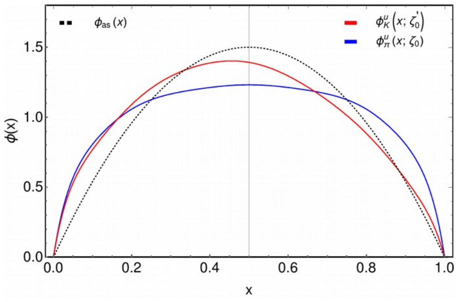
DSE: Chin.Phys. 44 (2020) no.3, 031002
Phys.Rev. D101 (2020) no.5, 054014

BSWFs

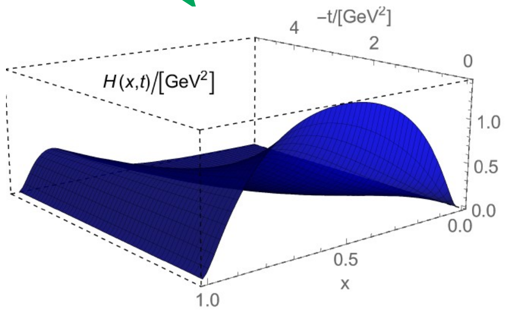


LFWF

$\int dk_\perp$



PDA



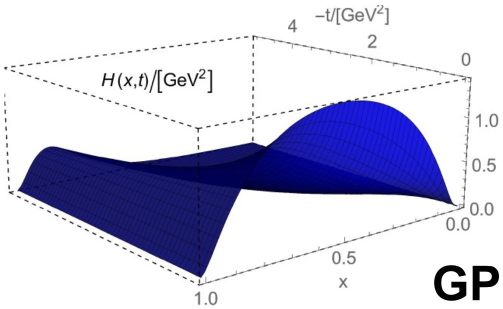
GPD

- **Consistent** features of the **PDA**:
  - Broad and concave at real world scales.
  - Agreement with Lattice and DSE results.

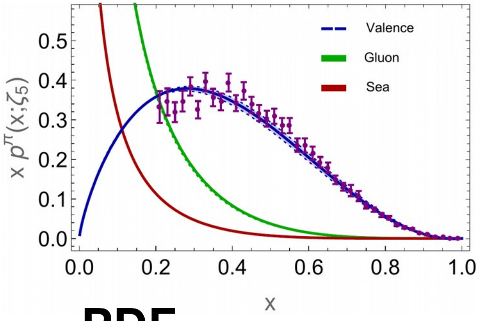
- The **GPD** obtained from the **overlap** representation.
  - Limited to the **DGLAP** region.
  - Compatible with the diagram approach.

(J. Rodriquez-Quintero's talk)

Summary: Pion



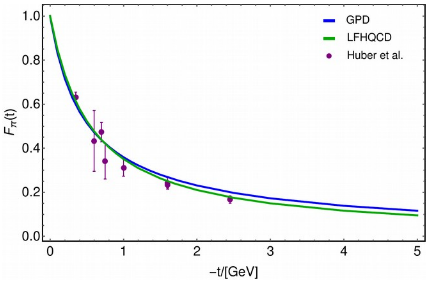
GPD



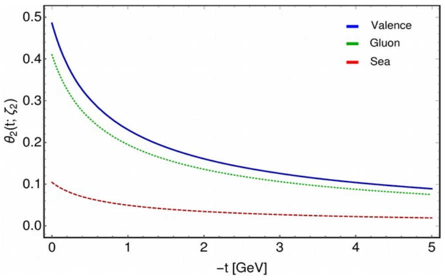
PDF

Red: $\int dx$

Purple: $t = 0, \xi = 0$



EFF



GFF

Connection of the **PDF** with **DSE predictions** implies:

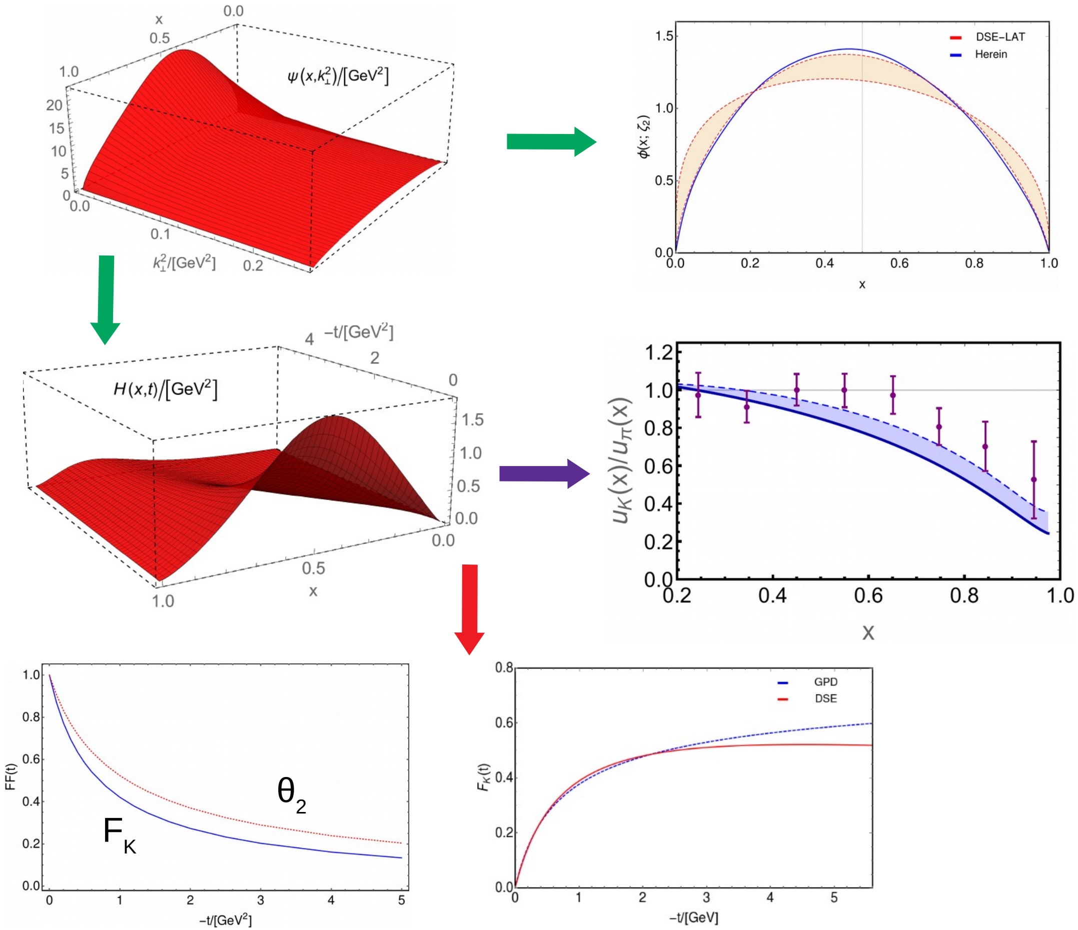
- ✔ Keen agreement with reanalized data.
- ✔ Large-x behavior as predicted by pQCD.
- ✔ Compatible with Lattice and LFHQCD.

- **EFF *consistent*** with empirical data.
  - ➔ One can trust the off-forward quantities.
  - ✗ ERBL region still needed.

- Intimate ***connection*** with the **running coupling**:
  - ✔ PI effective charge = effective charge for PDF evolution.
  - ✔ ***Unambiguous*** definition of the hadron scale.

➔ Both **LFWF** and **GPD** are **excellent candidates** to be the true objects.

Summary: Kaon



- **Qualitative features** of the computed functions are **correctly captured**.
- Specific numbers <u>can be improved</u>.
  - Numerical DSE computations of **PDF moments** could provide **feedback**.
  - The same for the **PDA**.
  - Feedback can also happen the other way around.

→ Then we can achieve the same degree of confidence as for the pion.

→ **Kaon** demands more work.