Light Meson Structure from a Basis Light-front Approach

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with
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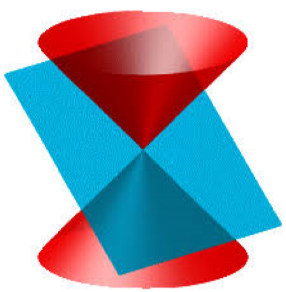


Perceiving the Emergence of Hadron Mass through AMBER@CERN

31 Mar. 2020

Outline

- Basis Light-front Quantization approach

- Application to π/K
  - Valence Fock section only $|q\bar{q}\rangle$
  - With one dynamical gluon $|q\bar{q}\rangle + |q\bar{q}g\rangle$

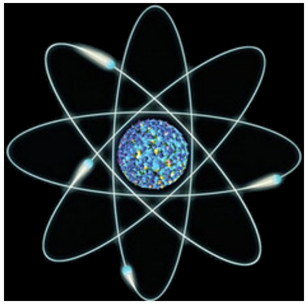
- Observables sensitive to valence contribution in π/K

- Summary and Future Plan

Hamiltonian Formalism

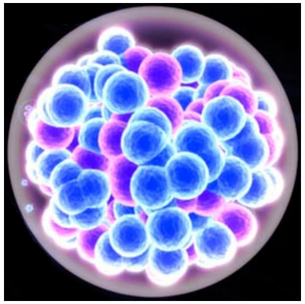
- Schrödinger equation universally describes different physics:

$$H|\psi\rangle = E|\psi\rangle$$



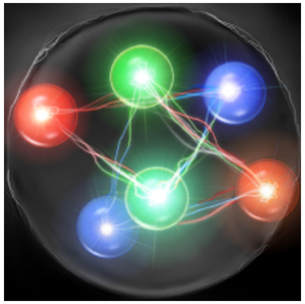
atom

Nonrelativistic, few-body



nucleus

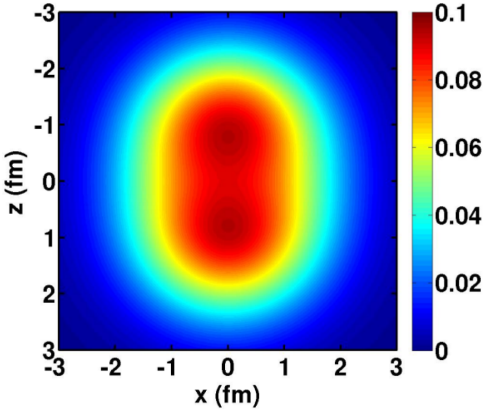
Nonrelativistic, many-body



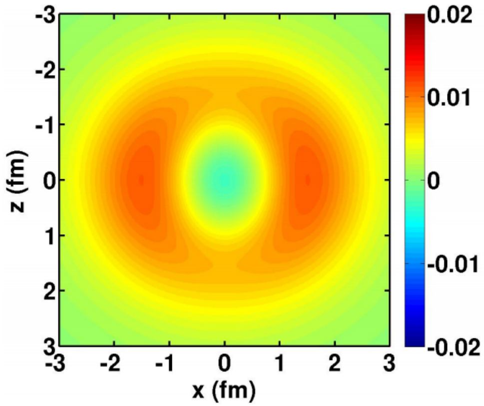
hadron

Relativistic, many-body

- Wave functions encode full information of the system



Proton density



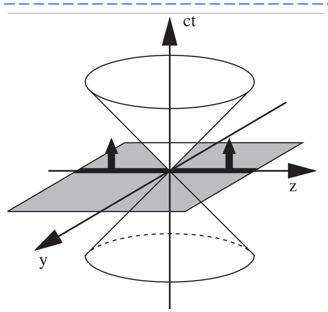
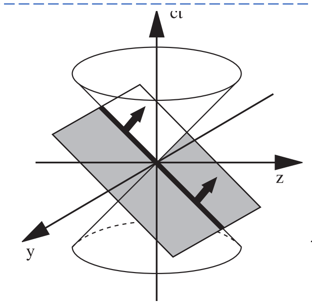
Neutron – proton density

^{9}Be

Adapted from Yang Li's talk

Light-front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3,$ $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3,$ $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

- Not just a coordinate transformation.
- New theory !!!

Advantages:

- Frame-independent wave functions
- Simple vacuum structure
- No square root in Hamiltonian P^-

Basis Light-front Quantization

[Vary et al, 2008]

- Nonperturbative eigenvalue problem

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

  - P^-: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P^-_β: eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\boldsymbol{\pi}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}gg\rangle + d|q\bar{q}q\bar{q}\rangle + \ldots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi^b_{n,m}(\vec{p}_\perp)$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \le N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

Large N_{max} and K : High UV cutoff & low IR cutoff

Application to Light Mesons

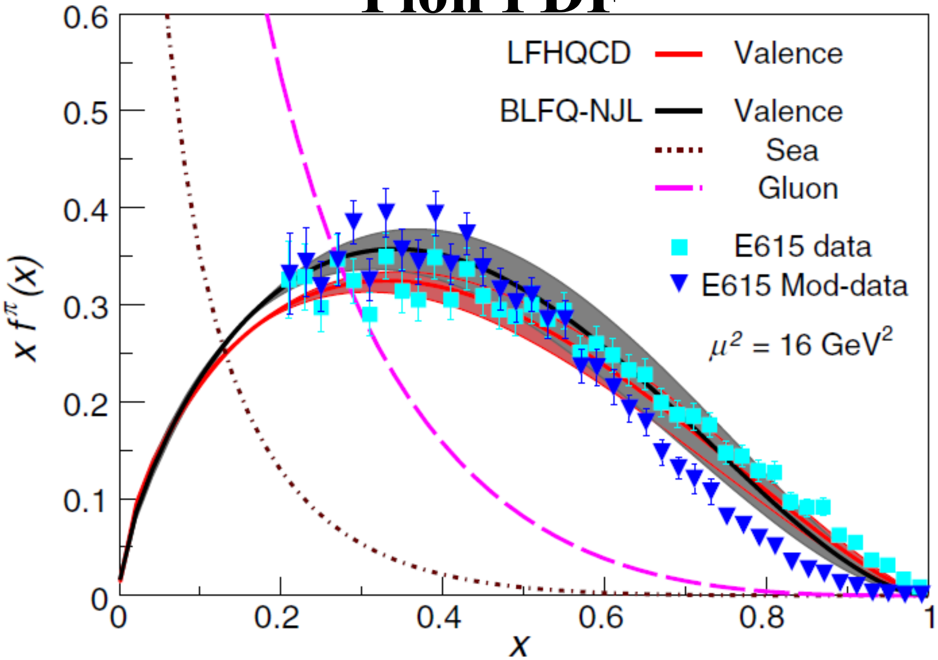
$$|\boldsymbol{\pi}/\boldsymbol{K}\rangle = |q\bar{q}\rangle + \cdots$$

PDF with QCD Evolution

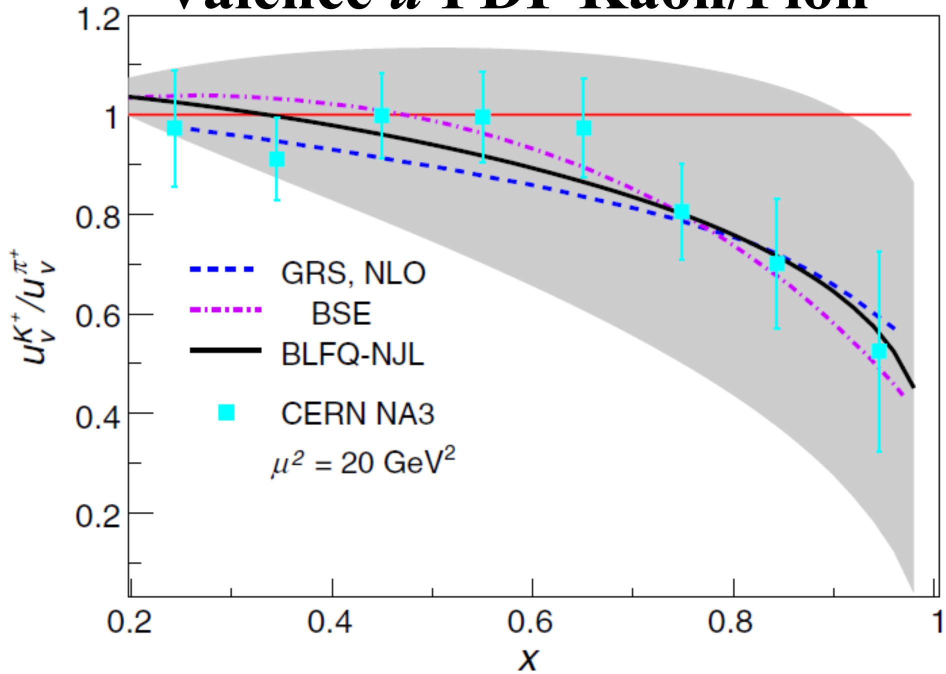
$$H_{\rm LF} = \underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF Kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^{\,2}}_{\text{Transverse}} - \underbrace{\frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x\left(x(1-x)\partial_x\right)}_{\text{Longitudinal}} + H_{\rm NJL}^{\rm eff}$$

- Diagonalizing $H_{\rm LF}$ ➡ LF wavefunctions ➡ PDFs

Pion PDF



Valence *u* PDF Kaon/Pion

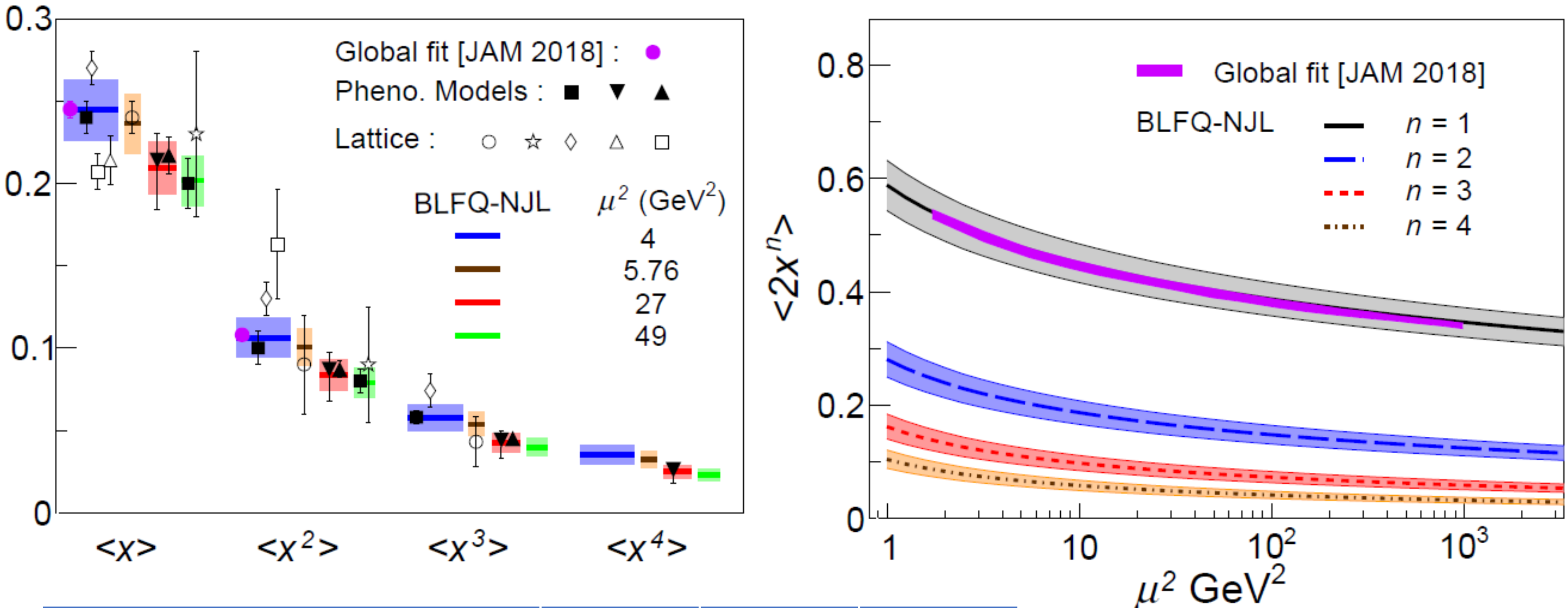


[Lan, Mondal, Jia, Zhao, Vary, PRL122, 172001(2019)]

Agree with experimental results

The moments of pion valence quark PDF

$$\langle x^n \rangle = \int_0^1 dx\ x^n f_v^{\pi/K}(x, \mu^2),\ \ n = 1, 2, 3, 4.$$



$\langle x \rangle$ @ 4 GeV^2	Valence	Gluon	Sea
BLFQ-NJL	**0.489**	**0.398**	**0.113**
[Ding *et. al.*, BSE model 2019']	0.48(3)	0.41(2)	0.11(2)

Agree with other results

[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

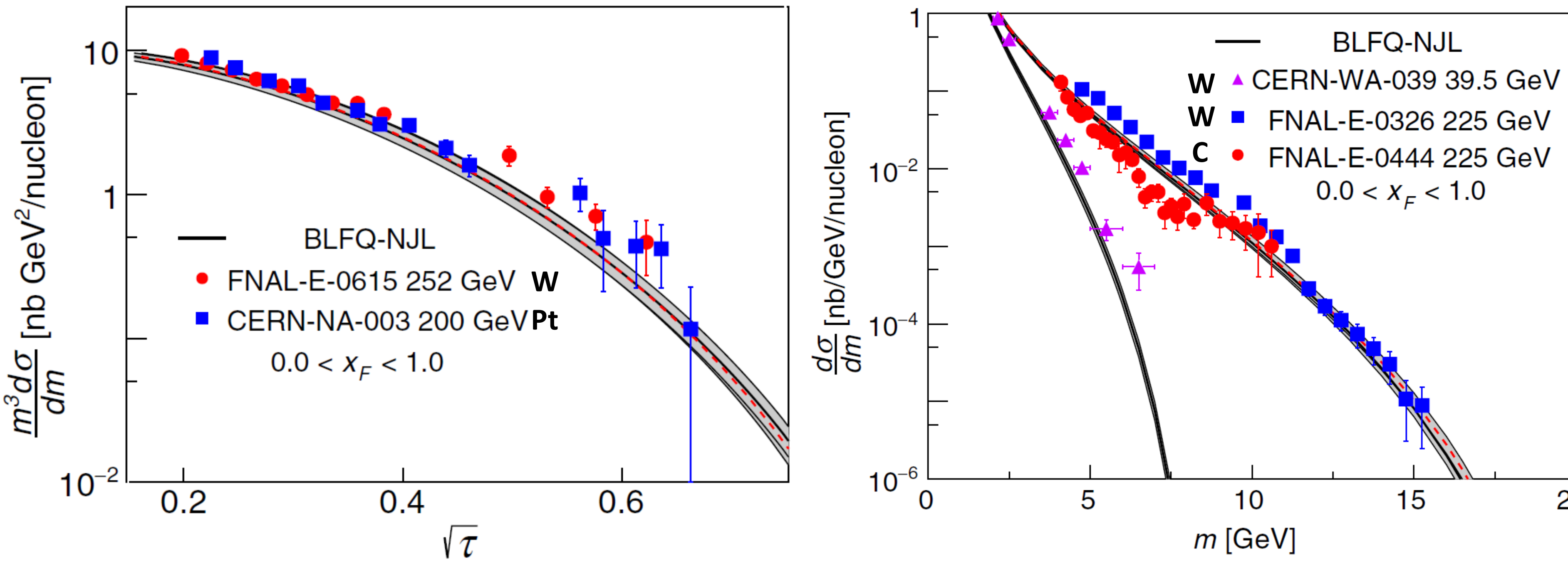
Drell-Yan cross section

$$\pi^- \ \text{Nucleus} \rightarrow \mu^+ \ \mu^- \ X$$

[nCTEQ 2015]

[S. D. Drell and T.-M. Yan, PRL (1970)]
[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]
[C. Anastasiou et al, PRL91(2003)182002]

$$\frac{m^3 d^2\sigma}{dmdY} = \frac{8\pi\alpha^2}{9}\frac{m^2}{s}\sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/\pi}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$



Agree with experimental data (**FNAL E615**, **326**, **444**, & **CERN NA3**, **WA-039**).

[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

πN Drell-Yan cross section

$$u^{\pi^+}_{val} = u^{\pi^+} - \bar{u}^{\pi^+} \qquad d^{\pi^-}_{val} = d^{\pi^-} - \bar{d}^{\pi^-}$$

$$u^{\pi^+}_{val} = \bar{d}^{\pi^+}_{val} = \bar{u}^{\pi^-}_{val} = d^{\pi^-}_{val}$$

$$\bar{u}^{\pi}_{sea} = u^{\pi}_{sea} = \bar{d}^{\pi}_{sea} = d^{\pi}_{sea} = \bar{s}^{\pi}_{sea} = s^{\pi}_{sea}$$

$$\pi^{\pm}\ N \xrightarrow{\text{Drell−Yan}} \mu^+\mu^-\ X$$

$$\sigma_{\pi^+N} \sim \frac{4}{9}(u^{\pi}_{vs}\bar{u}^{N}_{s} + \bar{u}^{\pi}_{s}u^{N}_{vs}) + \frac{1}{9}\left(\bar{d}^{\pi}_{vs}d^{N}_{vs} + d^{\pi}_{s}\bar{d}^{N}_{s} + \bar{s}^{\pi}_{s}s^{N}_{s} + s^{\pi}_{s}\bar{s}^{N}_{s}\right)$$

$$\sigma_{\pi^-N} \sim \frac{4}{9}(u^{\pi}_{s}\bar{u}^{N}_{s} + \bar{u}^{\pi}_{vs}u^{N}_{vs}) + \frac{1}{9}\left(\bar{d}^{\pi}_{s}d^{N}_{vs} + d^{\pi}_{vs}\bar{d}^{N}_{s} + \bar{s}^{\pi}_{s}s^{N}_{s} + s^{\pi}_{s}\bar{s}^{N}_{s}\right)$$

Target with nucleus of A=2Z:

$$\sigma_{\pi^-N} - \sigma_{\pi^+N} \sim \frac{4}{9}\bar{u}^{\pi}_{v}u^{N}_{v} - \frac{1}{9}\bar{d}^{\pi}_{v}d^{N}_{v} \xrightarrow{\text{for } C \text{ target}} \sim \frac{3}{9}\bar{u}^{\pi}_{v}u^{N}_{v}$$

contribution from Valence

$$4\sigma_{\pi^+N} - \sigma_{\pi^-N}$$

Contribution from Valence cancel out!

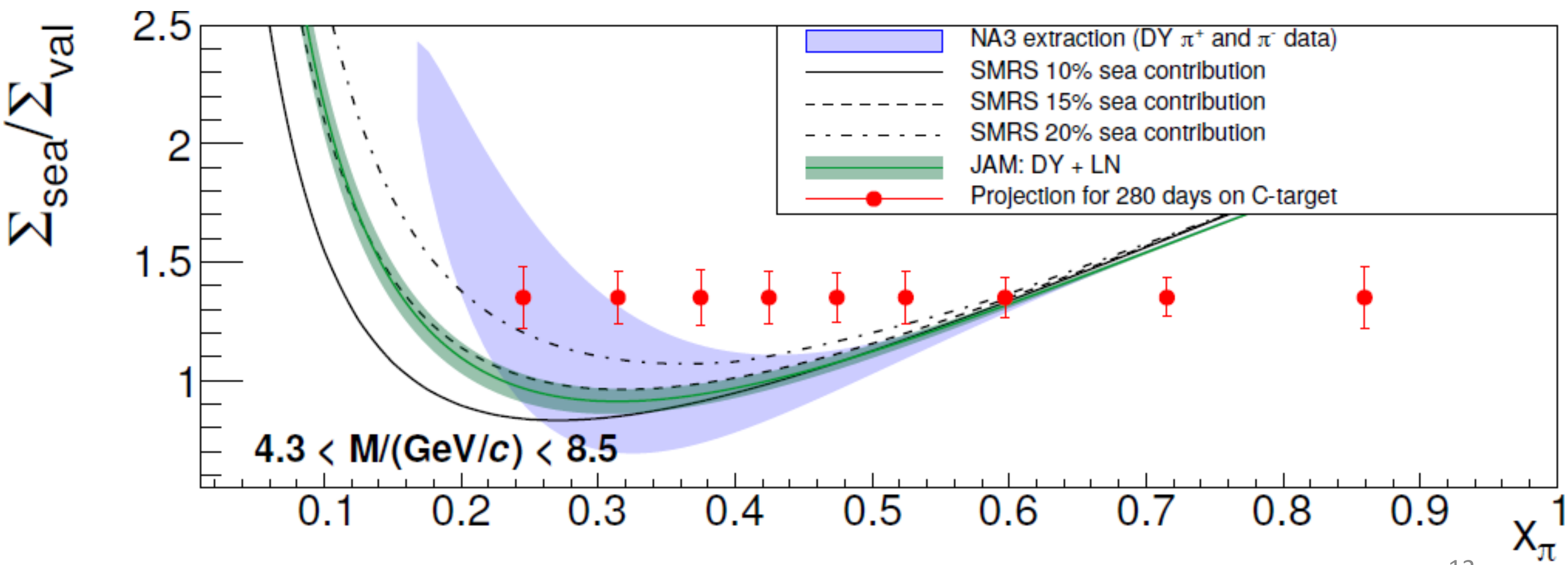
Table 6: PYTHIA Drell-Yan cross sections in LO, for the dimuon mass range $4.3 < M_{\mu\mu}/(\text{GeV}/c^2) < 8.5$.

COMPASS++/AMBER Proposal

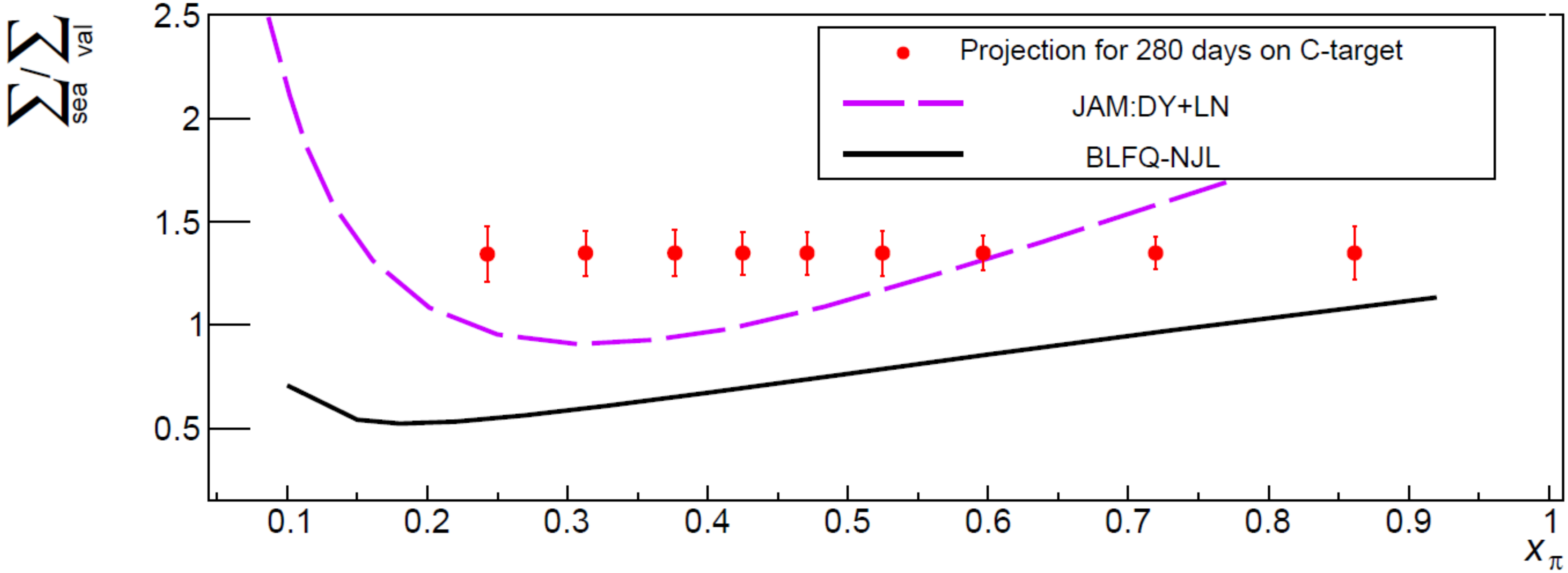
beam and target	$\sigma_{\text{LO}}^{\text{DY}\to\mu\mu}$ (nb)
$\pi^+ p$	0.026
$\pi^+ n$	0.039
$\pi^- p$	0.106
$\pi^- n$	0.051

$$\Sigma_{val}^{\pi D} = -\sigma^{\pi^+ D} + \sigma^{\pi^- D}$$

$$\Sigma_{sea}^{\pi D} = 4\sigma^{\pi^+ D} - \sigma^{\pi^- D}$$



BLFQ-NJL result



$$|\pi\rangle = |q\bar{q}\rangle + \cdots$$

- Only consider the valence Fock sector.
- Smaller sea contribution compared to JAM global fit
- Valence Fock sector not enough?

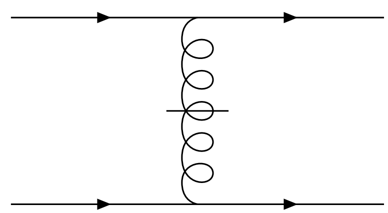
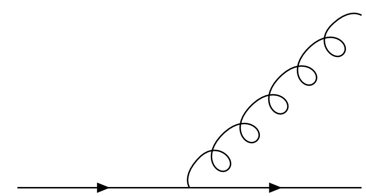
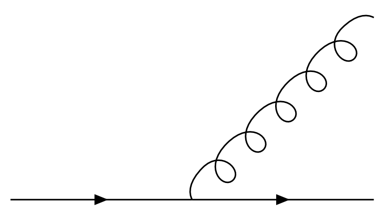
$$|\boldsymbol{\pi}/\boldsymbol{K}\rangle = |q\bar{q}\rangle + \cdots$$

$$|\boldsymbol{\pi}/\boldsymbol{K}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Interaction Part of Hamiltonian

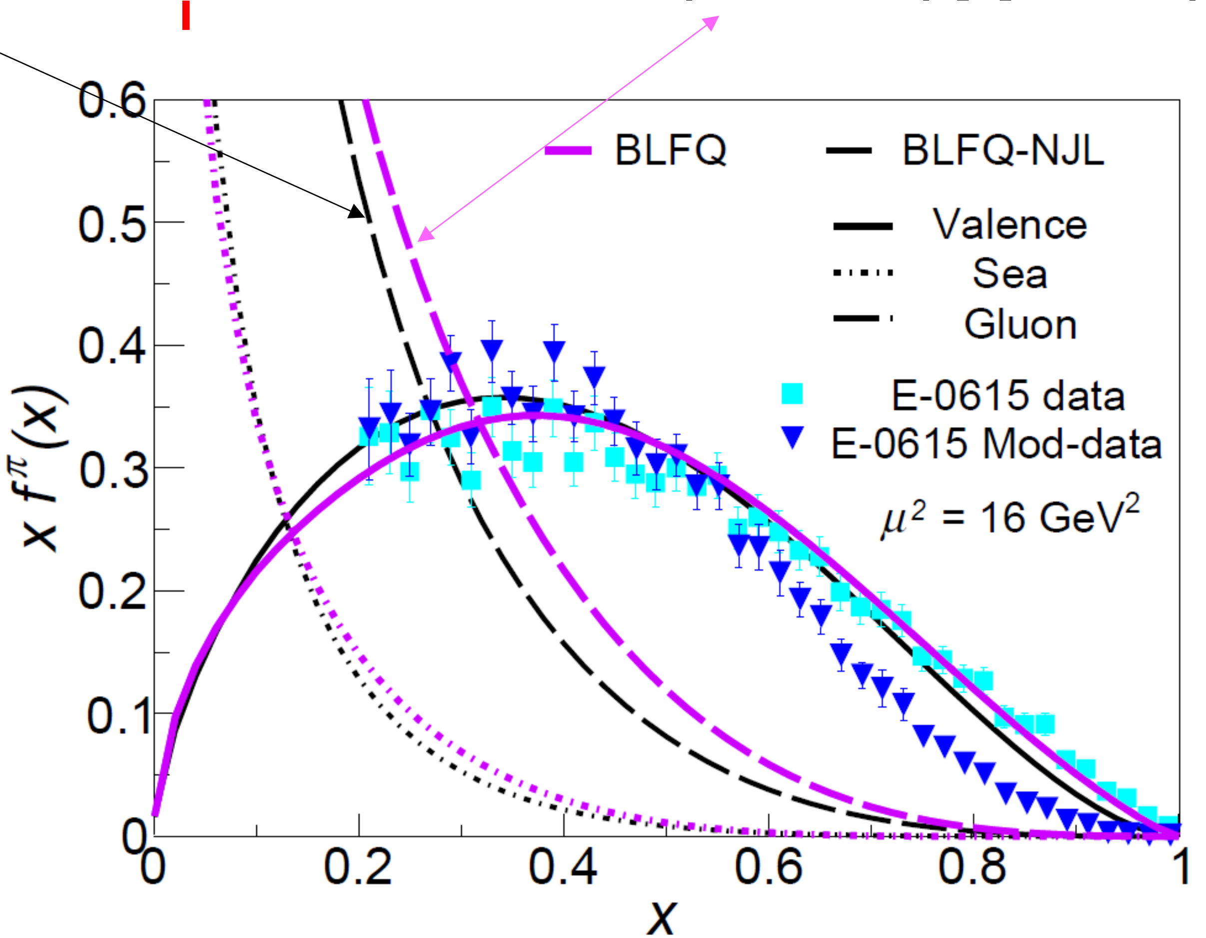
$$|\boldsymbol{\pi}/\boldsymbol{K}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + \ldots.$$

$$H_{\text{eff}} = \frac{\overrightarrow{k_\perp^2} + m_q^2}{x} + \frac{\overrightarrow{k_\perp^2} + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^{\,2} - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\left(x(1-x)\partial_x\right) + \boldsymbol{H}_{\text{int}}$$

$\boldsymbol{H}_{\text{int}}$	$\lvert q\bar{q}\rangle$	$\lvert q\bar{q}g\rangle$
$\langle q\bar{q}\rvert$		
$\langle q\bar{q}g\rvert$		0

Pion PDF with high Fock sector

$$|\pi\rangle = |q\bar{q}\rangle + \cdots \quad \Rightarrow \quad |\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

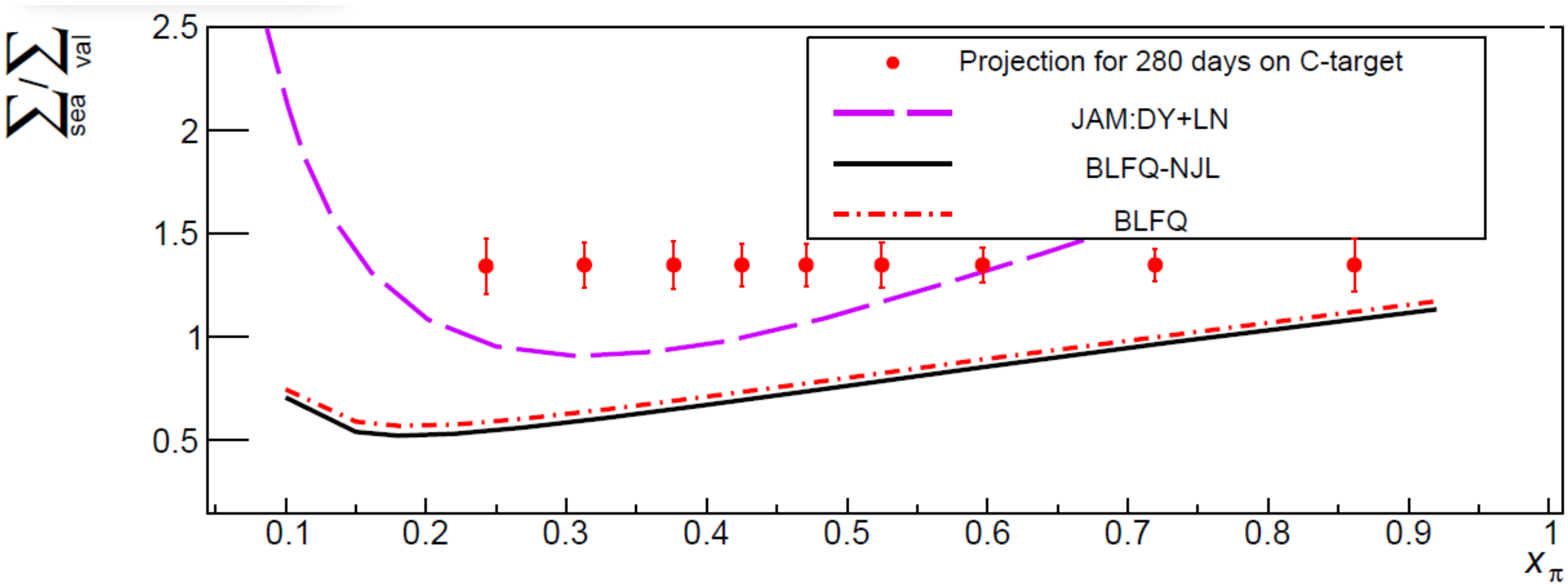


$N_{\max} = 8, K_{\max} = 9, M_J = 0, m_e = 0.33$ GeV,
$m_g = 0.60$ GeV, $\kappa = 0.77$ GeV, $b = 0.49$ GeV,
$\alpha = 1.74$ GeV, $m_f = 3.38$ GeV, $binst = 28$ GeV

[Lan, et al, in preparation]

Ratio of cross section with high Fock sector

$$|\pi\rangle = |q\bar{q}\rangle + \cdots \Rightarrow |\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

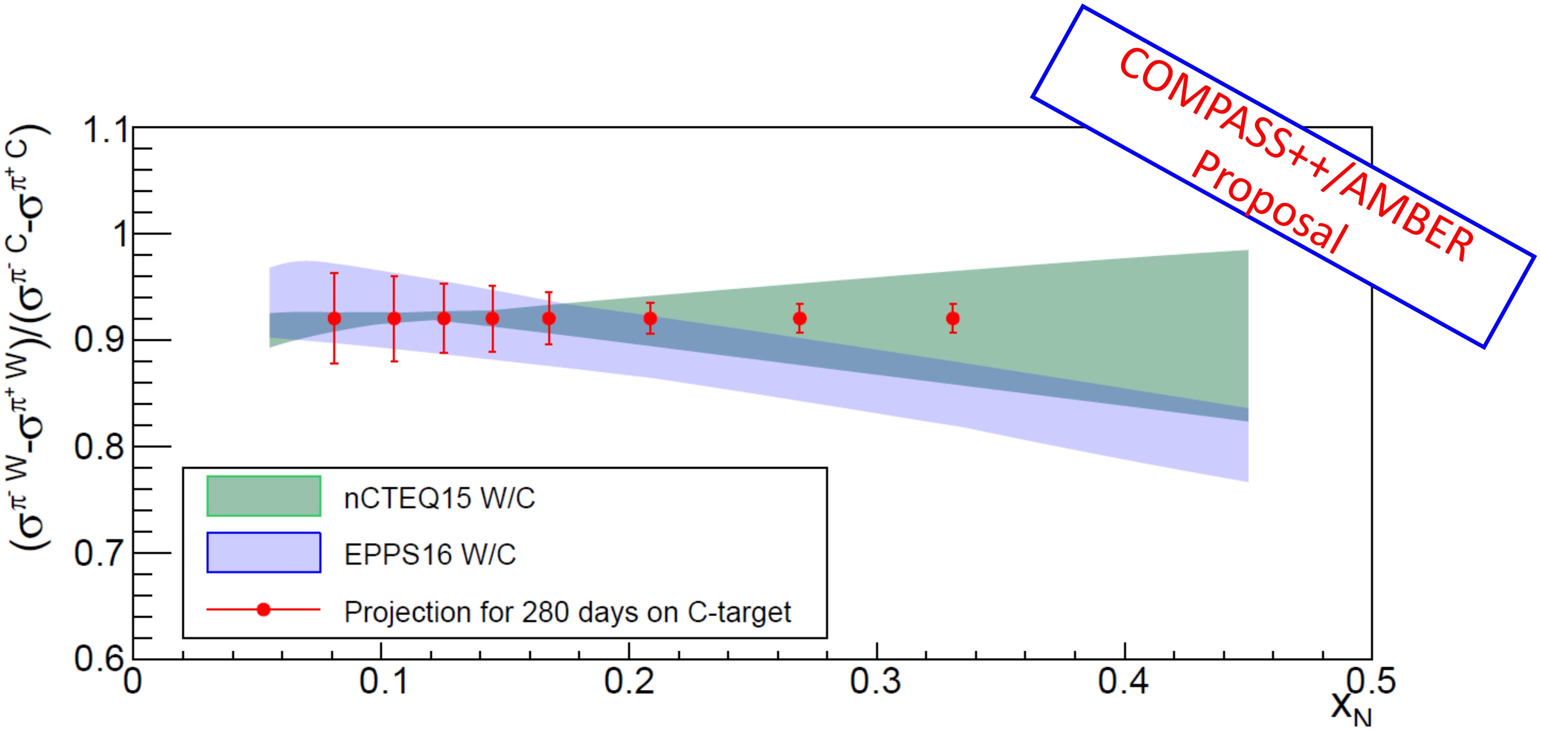


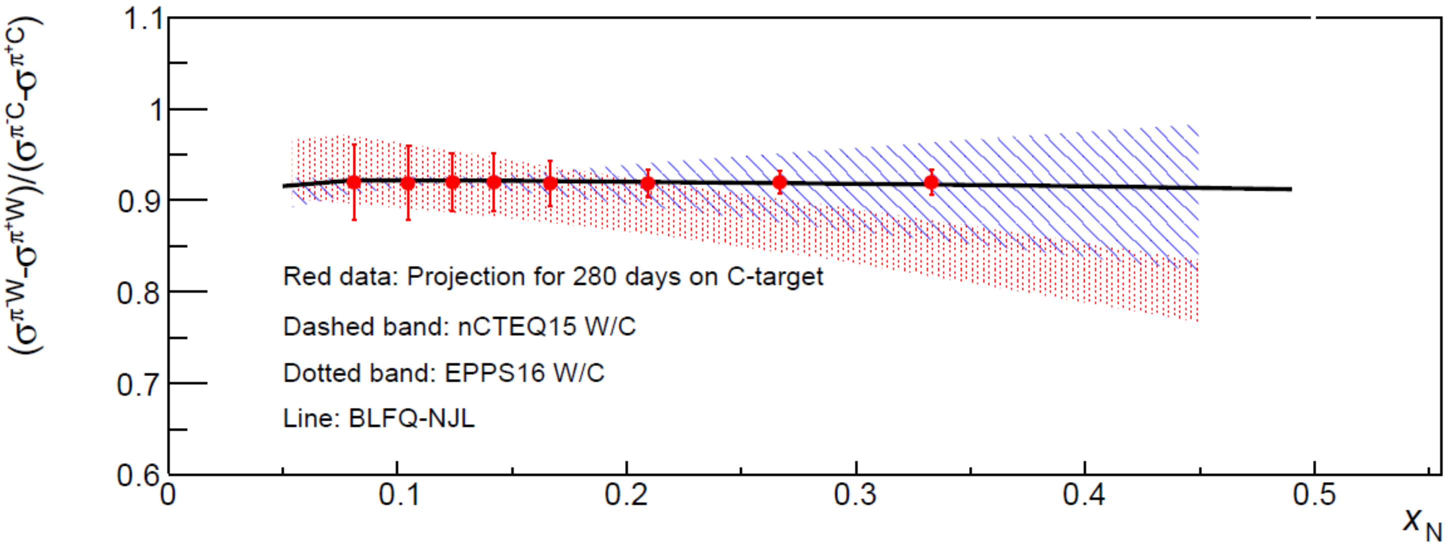
With $|q\bar{q}g\rangle$, the contribution from sea quark increases, but still not enough?

$|q\bar{q}q\bar{q}\rangle$ sector is needed?

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}\bar{q}q\rangle + \cdots$$

Valence contribution





Our results agree well with data.

Observables discriminating the valence distribution between π/K

KN Drell-Yan cross section

Jiangshan Lan

Relations between cross sections and the valence PDFs

1. $$\frac{3}{4}\frac{\sigma_{K^-C}-\sigma_{K^+C}}{\sigma_{\pi^-C}-\sigma_{\pi^+C}} \sim \bar{u}_v^K/\bar{u}_v^\pi$$

2. $$\frac{\sigma_{\pi^-C}-\sigma_{K^+C}}{\sigma_{\pi^-C}-\sigma_{\pi^+C}} \sim \frac{4}{3} + \frac{4\bar{u}_s^C(u_v^\pi - u_v^K) + \bar{d}_s^C(d_v^\pi - s_v^K)}{3\bar{u}_v^\pi u_v^C}$$

The detail expressions for the cross sections

$$\sigma_{\pi^+N} \sim \frac{4}{9}(u_{vs}^\pi \bar{u}_s^N + \bar{u}_s^\pi u_{vs}^N) + \frac{1}{9}(\bar{d}_{vs}^\pi d_{vs}^N + d_s^\pi \bar{d}_s^N + \bar{s}_s^\pi s_s^N + s_s^\pi \bar{s}_s^N)$$

$$\sigma_{\pi^-N} \sim \frac{4}{9}(u_s^\pi \bar{u}_s^N + \bar{u}_{vs}^\pi u_{vs}^N) + \frac{1}{9}(\bar{d}_s^\pi d_{vs}^N + d_{vs}^\pi \bar{d}_s^N + \bar{s}_s^\pi s_s^N + s_s^\pi \bar{s}_s^N)$$

$$\sigma_{K^+N} \sim \frac{4}{9}(u_{vs}^K \bar{u}_s^N + \bar{u}_s^K u_{vs}^N) + \frac{1}{9}(\bar{s}_{vs}^K s_s^N + s_s^K \bar{s}_s^N + \bar{d}_s^K d_{vs}^N + d_s^K \bar{d}_s^N)$$

$$\sigma_{K^-N} \sim \frac{4}{9}(u_s^K \bar{u}_s^N + \bar{u}_{vs}^K u_{vs}^N) + \frac{1}{9}(\bar{s}_s^K s_s^N + s_{vs}^K \bar{s}_s^N + \bar{d}_s^K d_{vs}^N + d_s^K \bar{d}_s^N)$$

$$\pi^\pm\ N \xrightarrow{\text{Drell-Yan}} \mu^+\mu^-\ X$$

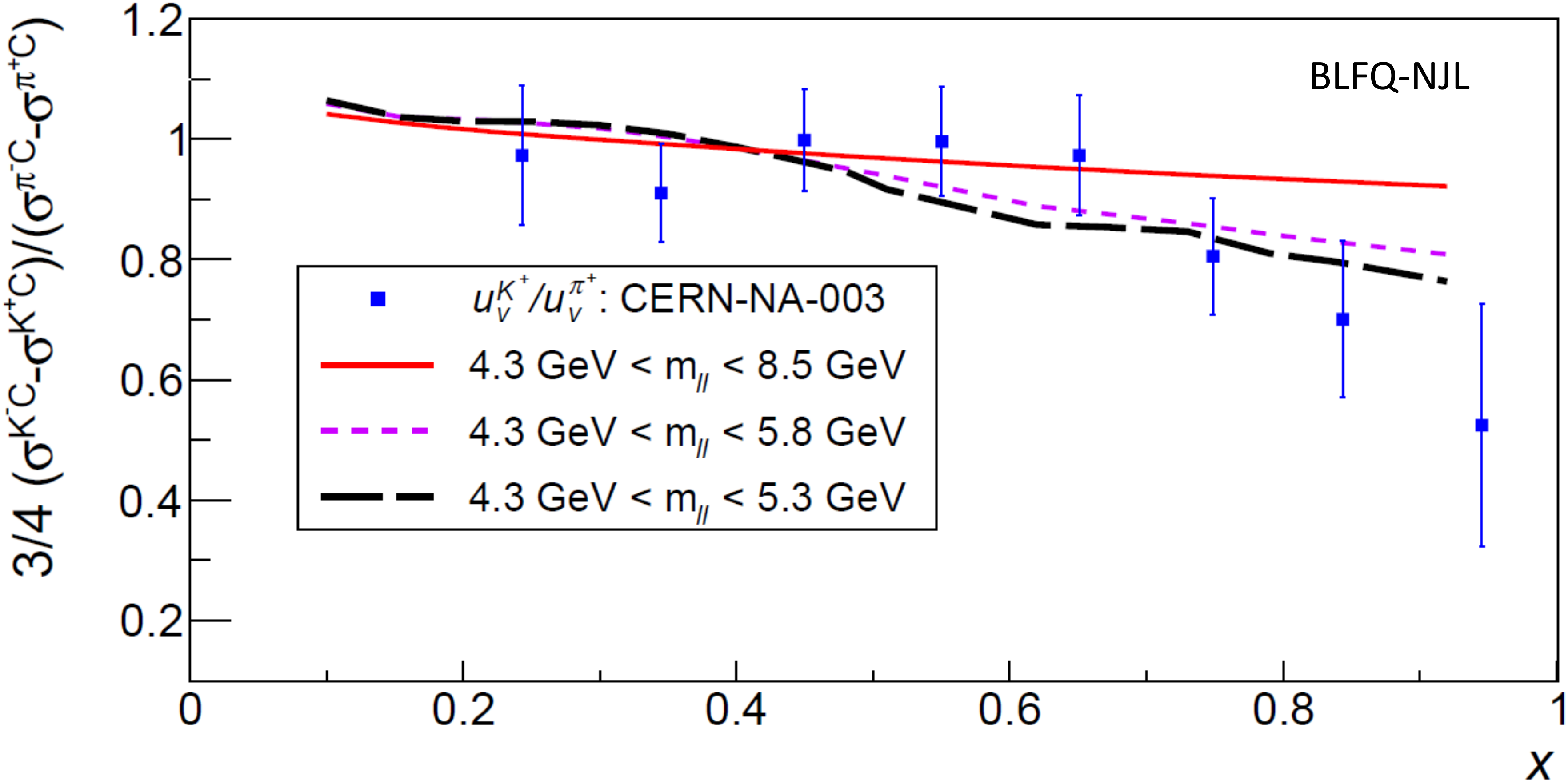
$$K^\pm\ N \xrightarrow{\text{Drell-Yan}} \mu^+\mu^-\ X$$

$$\sigma_{\pi^-N} - \sigma_{\pi^+N} \sim \frac{4}{9}\bar{u}_v^\pi u_v^N - \frac{1}{9}\bar{d}_v^\pi d_v^N \xrightarrow{\text{for } C \text{ target}} \sim \frac{3}{9}\bar{u}_v^\pi u_v^C$$

$$\sigma_{K^-N} - \sigma_{K^+N} \sim \frac{4}{9}\bar{u}_v^K u_v^C$$

First relation

① $\frac{3}{4}\frac{\sigma_{K^-C}-\sigma_{K^+C}}{\sigma_{\pi^-C}-\sigma_{\pi^+C}} \sim \bar{u}_v^K/\bar{u}_v^\pi$



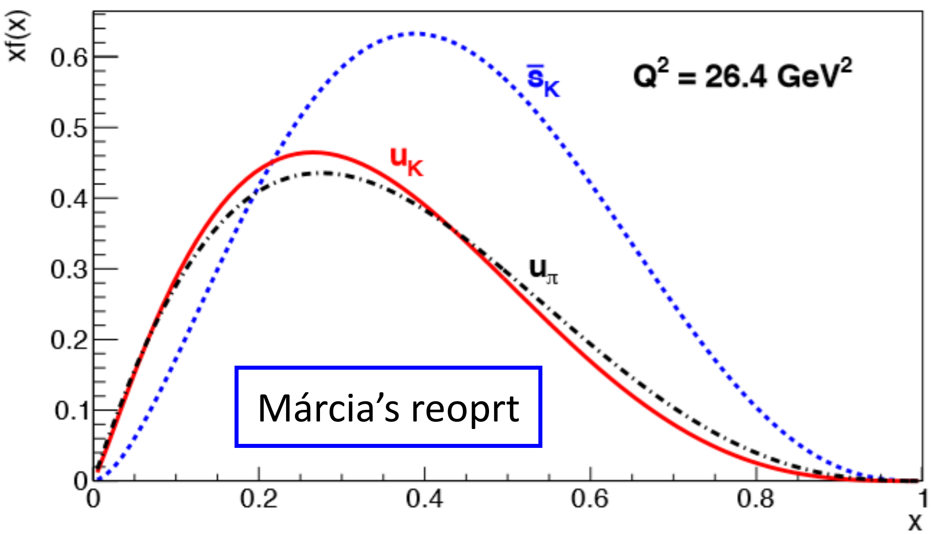
With decreasing the range of invariant mass of leptonic pair, our results tend to agree with extracted data.

Second relation

2 $$\frac{\sigma_{\pi^-C}-\sigma_{K^+C}}{\sigma_{\pi^-C}-\sigma_{\pi^+C}} \sim \frac{4}{3} + \frac{4\bar{u}_s^C(u_v^\pi - u_v^K)+\bar{d}_s^C(d_v^\pi - s_v^K)}{3\bar{u}_v^\pi u_v^C}$$

How much difference for $< x_v >$ between valence u and valence s in the kaon?

$< x_v^u >:< x_v^s > \sim 2:3$



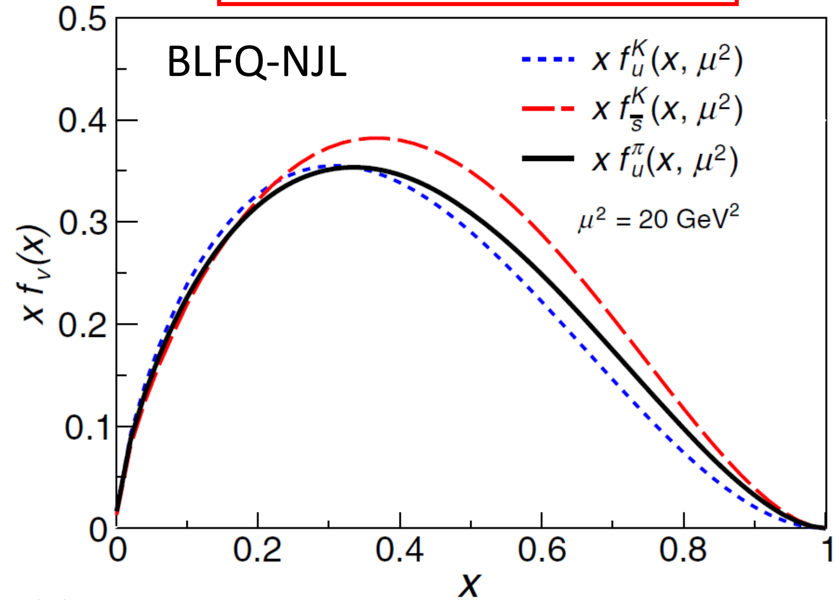
distributions calculated in the framework of the Dyson-Schwinger Equations (DSE)

Phys. Rev. D93 (7) (2016) 074021

DSE: momentum carried by $\bar{s}^K > u^K$

u^K and u^π are almost same

$< x_v^u >:< x_v^s > \sim 7:8$



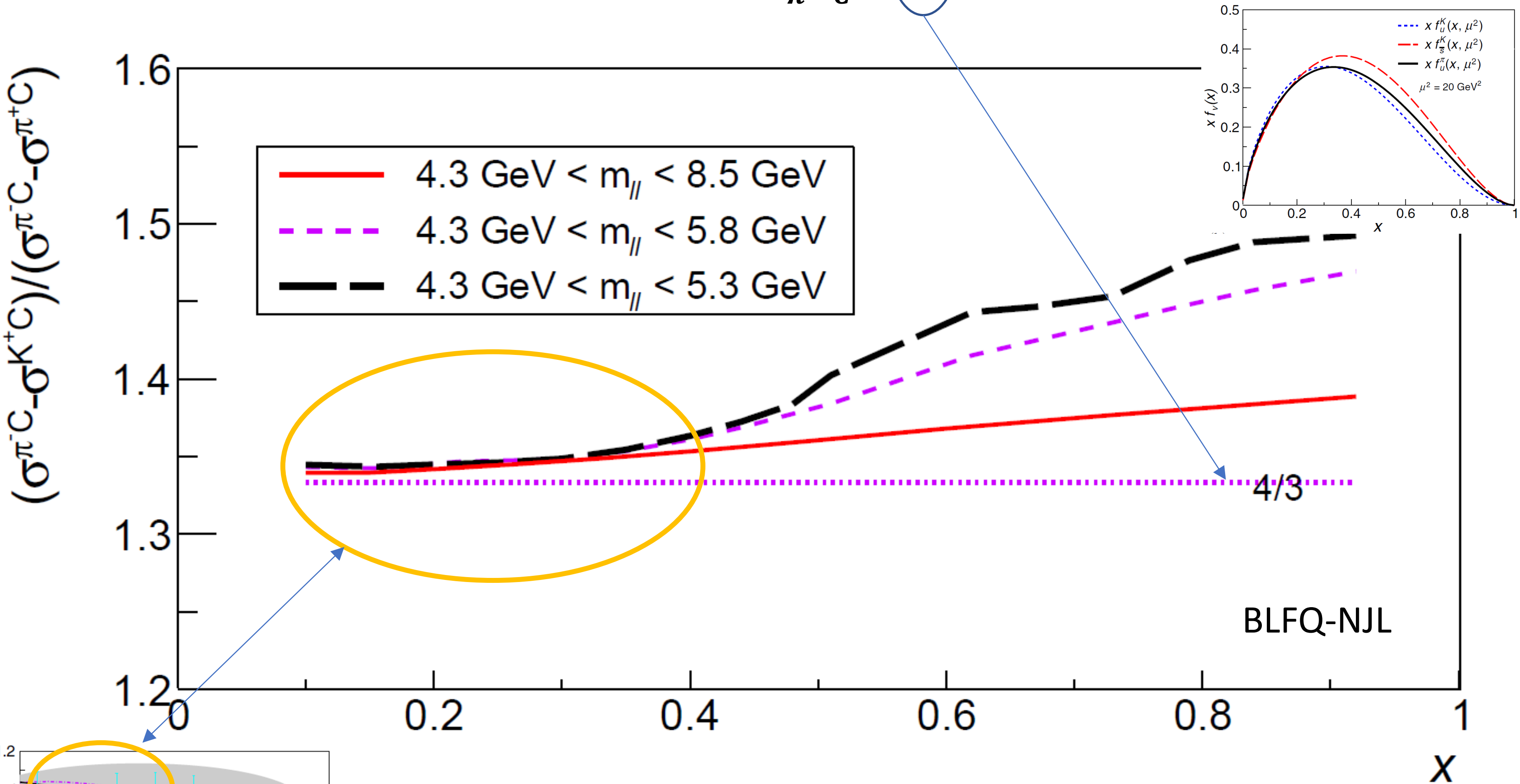
[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

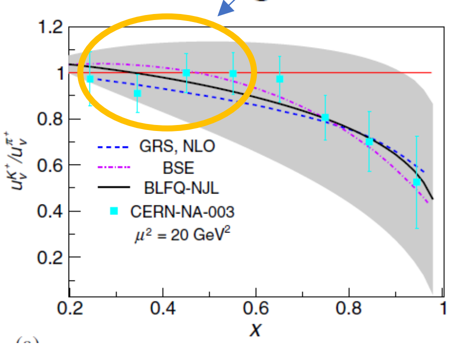
BLFQ-NJL: momentum carried by valence quarks of pion and kaon are comparable

Second relation

2

$$\frac{\sigma_{\pi^- C}-\sigma_{K^+ C}}{\sigma_{\pi^- C}-\sigma_{\pi^+ C}} \sim \frac{4}{3} + \frac{4\bar{u}_s^C(u_v^\pi - u_v^K)+\bar{d}_s^C(d_v^\pi - s_v^K)}{3\bar{u}_v^\pi u_v^C}$$





$(u_v^\pi - u_v^K) \sim \mathbf{0}$ within $0.2 < x < 0.4$

- Large difference between d_v^π and s_v^K ➡ Second relation $< 4/3$
- Small difference between d_v^π and s_v^K ➡ Second relation $> 4/3$

Conclusions

- Basis Light-front Quantization: nonperturbative approach to relativistic bound states
- Light-front Hamiltonian ⟶ Wavefunction ⟶ Observables
- Systematically expandable by including higher Fock sectors

$$|\boldsymbol{\pi}/\boldsymbol{K}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + \ldots$$

Future Plans

- Meson: heavy quarkonia ➡ heavy-light meson ➡ strange meson
- Baryon: nucleon ➡ excited nucleon ➡ baryons with s, b, c quarks...
- Expansion in Fock sectors:
  - $|\mathrm{Baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle + \cdots$
  - $|\mathrm{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}q\bar{q}\rangle + \cdots$
  - $|\mathrm{Exotic\ hadrons}\rangle = |q\bar{q}q\bar{q}\rangle + \cdots$
- Evaluation of observables: FF, PDF, DY cross section, PDA, GPD, TMD, GTMD...
- Tremendous amount of possibilities...

Thank you !