QUARK MASS FUNCTION FROM A OGE-TYPE INTERACTION IN MINKOWSKI SPACE

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COLLABORATORS AND WORKS

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PAPERS:

Phys. Rev. D 98, 114033 (2018)
Phys. Lett. B764, 38 (2017)
Phys. Rev. D 96, 074007 (2017)
Phys. Rev. D 90, 096008 (2014)
Phys. Rev. D 89, 016005 (2014)

MESON PHENOMENOLOGY — MOTIVATION

- upcoming experiments
  COMPASS++/AMBER (CERN)
  GlueX (JLab),GlueX (JLab)
  PANDA (FAIR-GSI):

  meson properties
  origin of **hadronic mass**
- need better theoretical understanding of $\mathbf{q\bar{q}}$ **mesons**
- study **spectrum** and **structure**

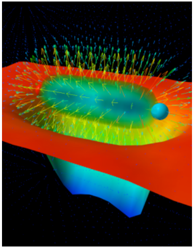


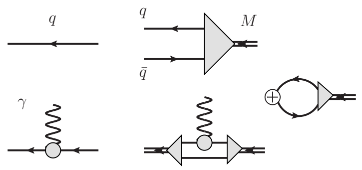
Image credit: D. Leinweber

OBJECTIVES

- unified description for **all** $q\bar{q}$ mesons from pion (0.14 GeV) to $b\bar{b}$ ($>$ 10 GeV)
  ✓ already achieved for **heavy** and **heavy-light** mesons
  Leitão, Stadler, Peña, EB, PLB (2017), PRD (2017)
  × to do: **light** mesons
- spectrum and decay properties ⇒ information about the Lorentz structure of the **confining** interaction
- describe mass-generation mechanism of **dynamical chiral-symmetry breaking** (Talk by C. Roberts)

Calculation of

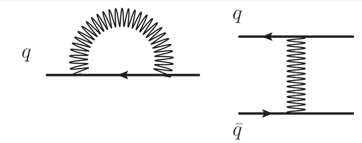
- **dynamical quark mass function**
- vertex functions
- quark-photon vertex
- meson form factors
- meson decay properties



COVARIANT SPECTATOR THEORY (CST)

Guiding principles

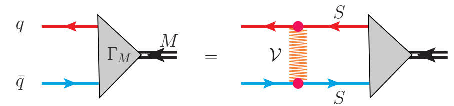
- manifest Lorentz **covariance**
- solved in **Minkowski** space
- **confinement**: covariant interaction kernel that reduces in nonrelativistic limit to 'linear-confining+Coulomb' potential
- **dynamical chiral-symmetry breaking**: axial-vector Ward Takahashi identity
  massless pion in chiral limit
  quark masses dynamically generated through self-interaction with $q\bar{q}$ interaction



GROSS (CST) EQUATION

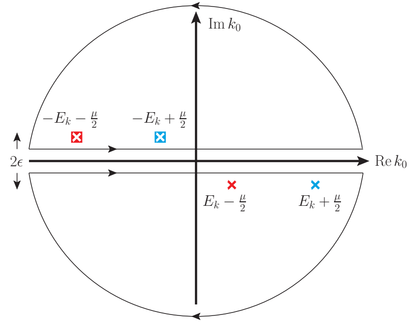
Gross, PR 186 (1969)

Bethe-Salpeter equation (BSE)



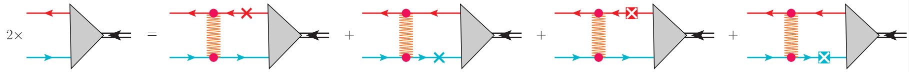
- assume: $\exists$ **real** quark-mass poles
- keep only **quark pole** contributions
- **3D covariant Minkowski** integrations
- beyond RL: correct **1-body** Dirac limit

quark poles in complex k_0 plane

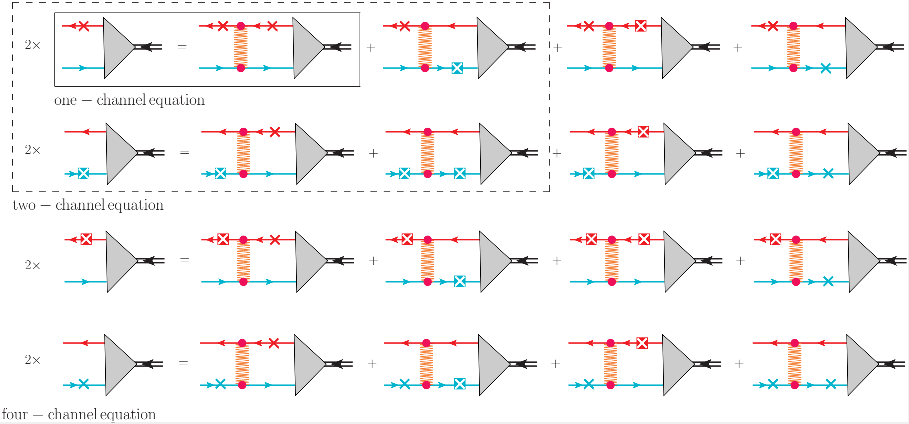


charge-conjugation symmetric Gross equation

Savkli, Gross PRC (2001), EB, Gross, Peña, Stadler PRD (2014)



4-CHANNEL EQUATIONS



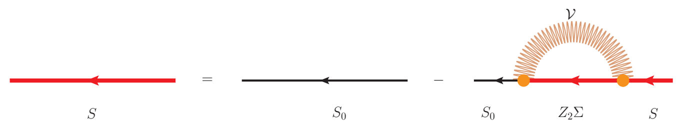
- 1-channel equation: ✓ unequal quark masses, × $\mathcal{C}$-symmetry, × light mesons
- 2-channel equation: ✓ $\mathcal{C}$-symmetry, × light mesons
- 4-channel equation: ✓ all mesons, also light mesons (pion)

All have smooth 1-body (Dirac) and nonrelativistic (Schrödinger) limits

CST DYSON EQUATION

EB, Gross, Peña, Stadler, Leitão, PRD (2018)

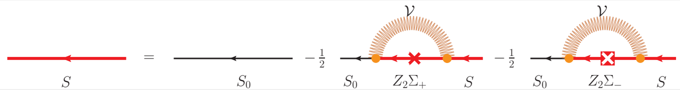
Dyson equation for **dressed** quark propagator (self-energy Σ)



$$S(p) = S_0(p) \sum_n [-Z_2 \Sigma(\not{p}) S_0(p)]^n = \frac{1}{m_0 + Z_2 \Sigma(\not{p}) - \not{p} - \mathrm{i}\epsilon} \equiv \frac{Z(p^2)[M(p^2) + \not{p}]}{M^2(p^2) - p^2 - \mathrm{i}\epsilon}$$

dressed quark **mass function** $M(p^2)$

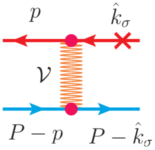
CST: $\boxed{M(m^2) = m}$ pole equation for **constituent quark mass** m



$$\boxed{Z_2\Sigma(\not{p}) = \frac{Z_2^2}{2} \sum_\sigma \int_{\mathbf{k}} (m + \hat{\not{k}}_\sigma) \mathcal{V}(p, \hat{k}_\sigma)}$$ with renormalization $Z_2(m) \equiv \frac{Z(m^2)}{1 - 2mM'(m^2)}$

COVARIANT INTERACTION KERNEL

$$\mathcal{V}(p,\hat{k}_\sigma) = \underbrace{\mathcal{V}_\ell(p,\hat{k}_\sigma)}_{\text{'linear confining'}} + \underbrace{\mathcal{V}_g(p,\hat{k}_\sigma)}_{\text{OGE-type}} + \underbrace{\mathcal{V}_c(p,\hat{k}_\sigma)}_{\text{covariant 'constant'}}$$



reduces to $V^{\rm nr}(r) = \sigma r - \frac{4}{3}\frac{\alpha_{\rm s}}{r} + C$ in nonrelativistic limit

$$\mathcal{V}_\ell(p,\hat{k}_\sigma) = \underbrace{\frac{1}{4}\sum_a \lambda_a \otimes \lambda_a}_{\frac{4}{3}\text{ (color singlets)}} \left[(1-\lambda)(\mathbf{1}\otimes\mathbf{1} + \gamma^5\otimes\gamma^5) - \lambda\gamma^\mu\otimes\gamma_\mu\right] V_\ell(p,\hat{k}_\sigma)$$

covariant off-shell generalization of 'linear **confinement**'

- $\boldsymbol{q}^2 \to -q_\sigma^2 = -(p-\hat{k}_\sigma)^2 \Rightarrow \int_{\mathbf{k}} V_\ell(p,\hat{k}_\sigma)\psi(\hat{k}_\sigma) = -\sigma \int_{\mathbf{k}} \frac{\psi(\hat{k}_\sigma)-\psi(\hat{k}_{R\sigma})}{q_\sigma^4}$

  confinement: meson vertex function vanishes if **both** *quarks are on-shell!* ✓

  Savkli, Gross PRC (2001)

- $\mathcal{V}_\ell$ consistent with **D**χ**SB** (AVWTI satisfied) ✓

  EB, Peña, Ribeiro, Stadler, Gross PRD (2014)

- $\lambda = 0 \Rightarrow \mathcal{V}_\ell$ does not contribute to self-energy, consistent with meson spectrum ✓

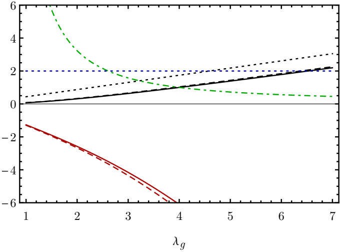
  Leitão, Stadler, Peña, EB, PLB (2017), PRD (2017)

OGE-TYPE KERNEL

EB, Gross, Peña, Stadler, Leitão, PRD (2018)

$$\mathcal{V}_{\rm g}(p,\hat{k}_\sigma) = 4\pi\alpha_{\rm s}\, g(y)\, \gamma_\mu \otimes \gamma_\nu \frac{1}{M_{\rm g}^2+|q_\sigma^2|}\left[g^{\mu\nu} - (1-\xi)\frac{q_\sigma^\mu q_\sigma^\nu}{q^2}\right] \frac{1}{4}\sum_a \lambda_a \otimes \lambda_a$$

- finite **gluon mass** $M_g = 0.6$ GeV and **prescription** $q^2 \to -|q^2|$
  $\Rightarrow$ **removes** singularity in gluon propagator ✓
- form factor $g(y) = \frac{\lambda_{\rm g}^{4n}}{\lambda_{\rm g}^{4n}+(y^2-1)^n}$ (with $y^2 = \frac{E_k^2}{m^2}$, $\lambda_{\rm g} = \frac{\Lambda_{\rm g}}{m}$)
  regularizes $\int_{\mathbf{k}}$ (also at $p^2 = 0$)✓
- self-energy $Z_2\Sigma_{\rm g}(\not{p}) = \frac{1}{4}(3+\xi)Z_2 A_{\rm g}(p^2) + \not{p}\frac{1}{2}(3-\xi)Z_2 B_{\rm g}(p^2) - \not{p}(1-\xi)Z_2 R_{\rm g}(p^2)$
  where $A_{\rm g}, B_{\rm g}, R_{\rm g} \propto Z_2\alpha_{\rm s}\int_{\mathbf{k}}\cdots \Rightarrow$ **renormalized** coupling $\boxed{\alpha_{\rm s}^{\rm r}(m) \equiv Z_2^2(m)\alpha_{\rm s}}$
- mass pole equation for m is gauge **independent** ✓



$m_0 = 0$: $\boxed{\alpha_{\rm s}^{\rm r} T_{\rm g} = 1}$ $[Z_2\alpha_{\rm s}T_g \equiv \frac{A_g(m^2)}{m} + B_g(m^2)]$

$T_{\rm g}$ with/without $|q^2|$-prescription

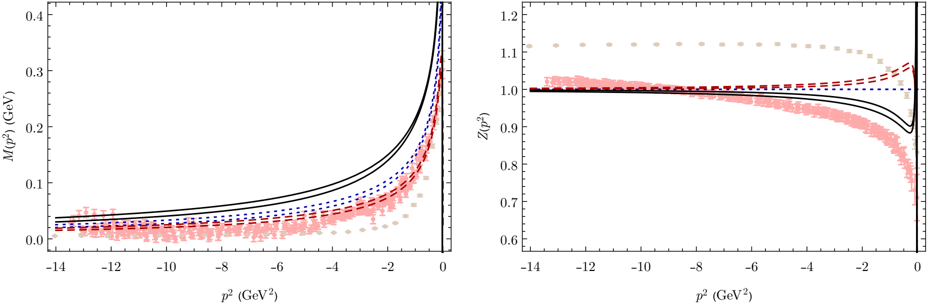
(solid: $n = 4$, dashed: $n = 3$,

dotted: $M_{\rm g} = 0$), dotdashed: $\alpha_{\rm s}^{\rm r}$

$\Rightarrow$ only **with** $|q^2|$-prescription:

$T_{\rm g} \simeq 2$, $\alpha_{\rm s}^{\rm r} \simeq 0.5 \Rightarrow \lambda_{\rm g} \simeq 7$

$M(p^2)$ AND $Z(p^2)$ FOR SPACELIKE p^2 AND LATTICE DATA



lattice data: **Bowman et al PRD (2005)**, **Oliveira et al, PRD (2019)**

Landau gauge ($\xi = 0$)

Feynman-'t Hooft gauge ($\xi = 1$)

Yennie gauge ($\xi = 3$)

$m = 0.3$ GeV, $m_0 = 0$:
$\lambda_g = 5$ ($\alpha_s^r = 0.722$)
$\lambda_g = 3$ ($\alpha_s^r = 1.577$)
$\Rightarrow$ **reasonable** behaviour similar to LQCD (no fitting)

for $\alpha_s^r \simeq 0.5 \Rightarrow$ need constant kernel

COVARIANT OFF-SHELL CONSTANT KERNEL IN GENERAL GAUGE

Covariant off-shell generalization of **constant** potential:

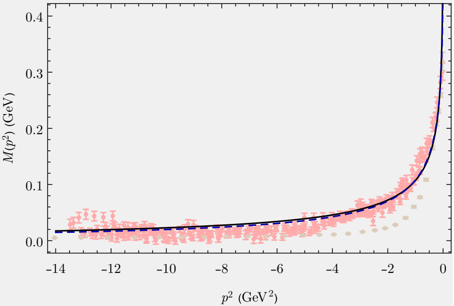
$$\mathcal{V}_{\rm c}(p,\hat{k}_\sigma) = \frac{CE_k}{2m}(2\pi)^3\delta^3\left(\mathbf{k} - \frac{m}{\sqrt{p^2}}\,\mathbf{p}\right)h(p^2)h(m^2)\gamma_\mu\otimes\gamma_\nu\left[g^{\mu\nu} - (1-\xi)\frac{q_\sigma^\mu q_\sigma^\nu}{q_\sigma^2}\right]\frac{1}{4}\sum_a \lambda_a\otimes\lambda_a$$

- nonrelativistic limit: $\mathcal{V}_{\rm c} \to \mathcal{V}_{\rm c}^{\rm nr} \propto C\delta^3(\mathbf{k}-\mathbf{p})$ ($\equiv$ constant potential)
- **correction** to OGE part: strong quark form factor $h(p^2) \equiv \frac{A_{\rm g}(p^2)}{A_{\rm g}(m^2)}$

self-energy

- satisfy same mass pole eq. as $\mathcal{V}_{\rm g}$: $C \to \frac{3m\,\alpha_{\rm s}\,T_g}{3+\xi}$
- contributes only to scalar part of self-energy: $Z_2\Sigma_c(\not{p}) = m\frac{A_{\rm g}(p^2)}{A_{\rm g}(m^2)}$
- $\boxed{Z(p^2) = 1,\ M(p^2) = m\frac{A_{\rm g}(p^2)}{A_{\rm g}(m^2)}}$

  (gauge independent)
- curves: $\lambda_{\rm g} = 5$, $\lambda_{\rm g} = 3$



CONSTANT AND OGE SELF-ENERGY

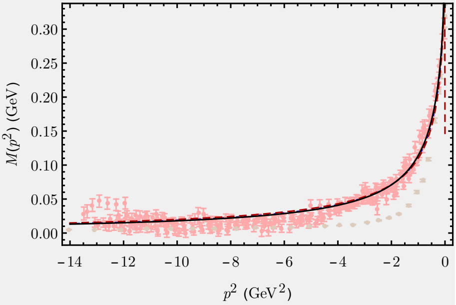
IDEA: each contribution to self-energy satisfies gap equation ⇒ also linear combination

$$Z_2\Sigma(\not p) = \underbrace{\eta Z_2 \Sigma_{\rm g}(\not p)}_{\text{OGE}} + \underbrace{(1-\eta) m \frac{A_{\rm g}(p^2)}{A_{\rm g}(m^2)}}_{\text{'constant'}}$$

fix parameters in chiral limit where $m_0 = 0$

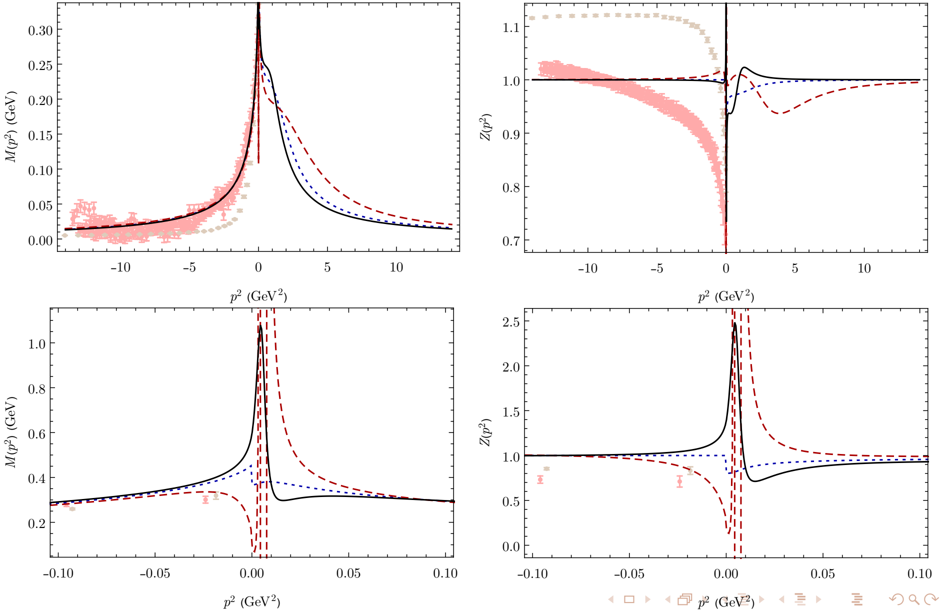
- held **fixed**: $m \to m_\chi = 0.3$ GeV, $M_{\rm g} = 0.6$ GeV, $n = 4$, $\alpha_s^{\rm r}(m_\chi) \to \eta\alpha_s^{\rm r}(m_\chi) \equiv \alpha_s^{\rm p}(m_\chi) = 0.5$ ⇒ 'constant' **decreases** strength of OGE
- roughly **adjust** $\lambda_{\rm g}$ to agree with LQCD

ξ	0	1	3
$\lambda_{\rm g}$	3	2	1.5
$\eta(\lambda_{\rm g})$	0.317	0.155	0.087
$(1-\eta)\frac{Z_2^2 C}{m}$	0.911	0.845	0.608



RESULTS

Landau, **Feynman-'t Hooft**, **Yennie**; lattice data: **Bowman et al PRD (2005)**, **Oliveira et al, PRD (2019)**



SUMMARY AND OUTLOOK

- Covariant Spectator Theory: dynamical quark model in Minkowski space with **confinement** and **dynamical chiral-symmetry breaking**
- $q\bar{q}$ interaction kernel
  2-body equation: very good description of **heavy and heavy-light** meson spectrum
  1-body equation: reasonable dressed **quark mass function**

Outlook and work in progress:

1. mass function for finite bare quark masses
2. running quark-gluon coupling
3. running gluon mass
4. $\lambda \neq 0$: vector structures for V_ℓ in mass function calculation
5. quark mass function into bound-state calculations

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THANK YOU!