Two issues on Pion PDF

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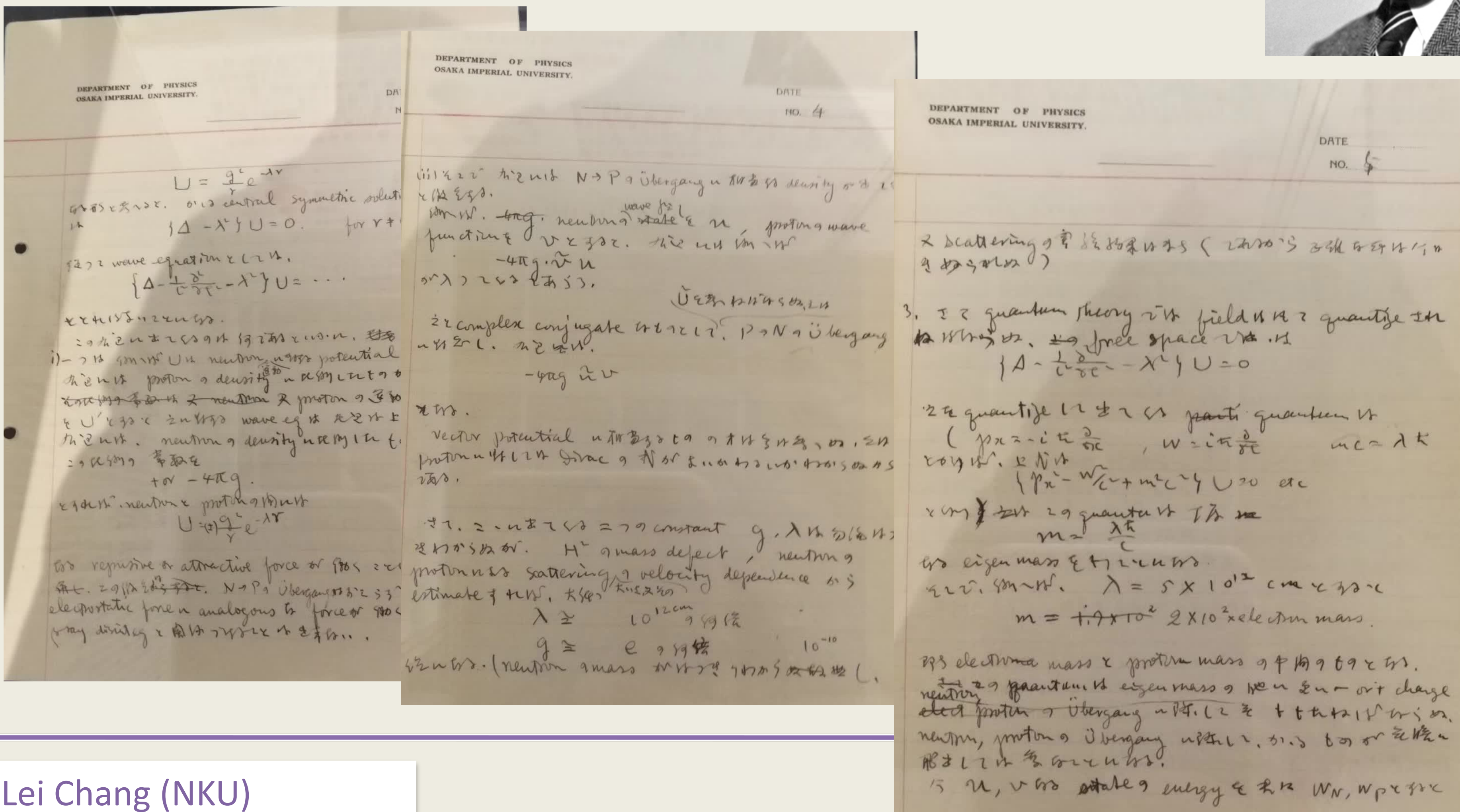
Nankai University

Perceiving the Emergence of Hadron Mass through AMBER@CERN
30/03/2020, https://cern.zoom.us/j/333776400

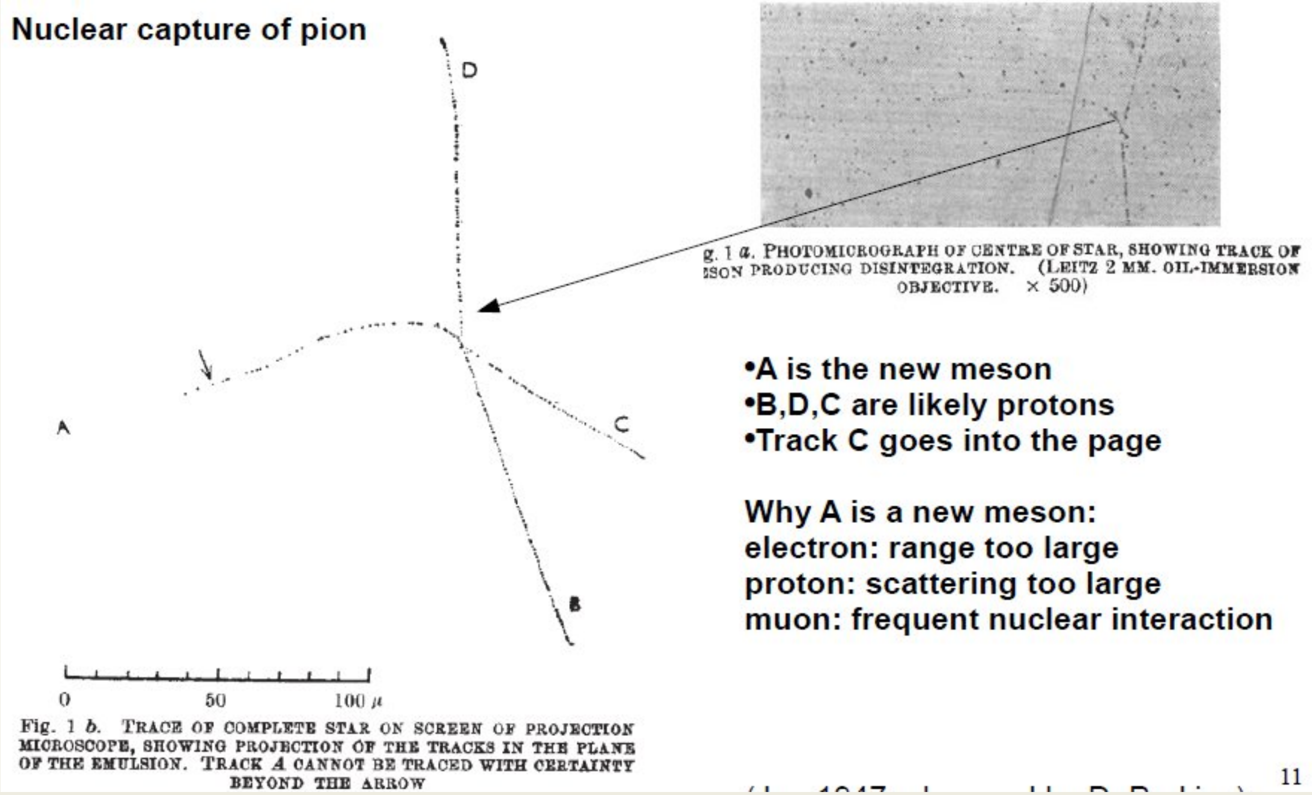
Messager of QCD

- In October 1934**, Hideki Yukawa** predicated the existence of a “heavy quantum” meson, exchanging nuclear force between neutrons and protons.



Messager of QCD

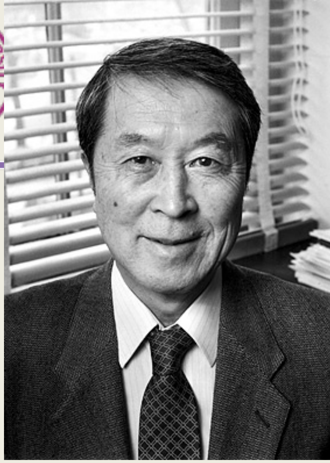
It was discovered by **Cecil Powel** in 1949 in cosmic ray tracks in a photographic emulsion.



Mass measured in scattering

$\approx 250\text{-}350\ m_e$

Messager of QCD



Yoichiro Nambu associated it with CSB in 1960.

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

Nambu and Jona-Lasinio 1961 paper [9] was an amazing breakthrough. Before the word "quark" was invented, and one learned anything about quark masses, it postulated the notion of chiral symmetry and its spontaneously breaking. They postulated existence of 4-fermion interaction, with some coupling G, strong enough to make a superconductor-like gap even in fermionic vacuum. The second important parameter of the model was the cut-off $\Lambda \sim 1\, GeV$, below which their hypothetical attractive 4-fermion interaction operates.

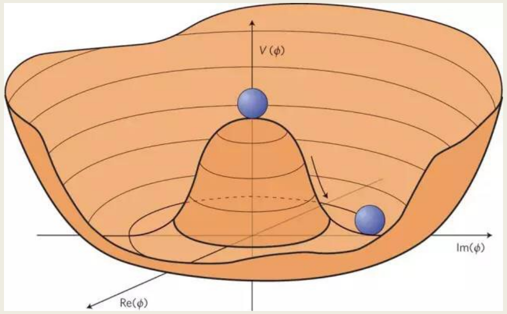
E. Shuryak, arXiv:1908.10270

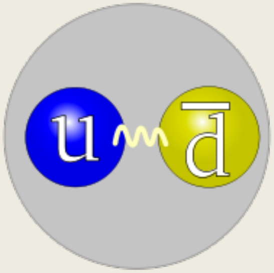
Messager of QCD

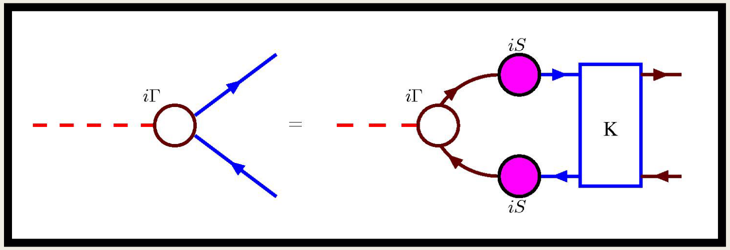
PION's dichotomy

Goldstone Boson

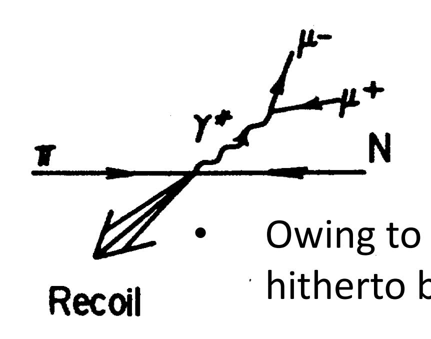
Bound State



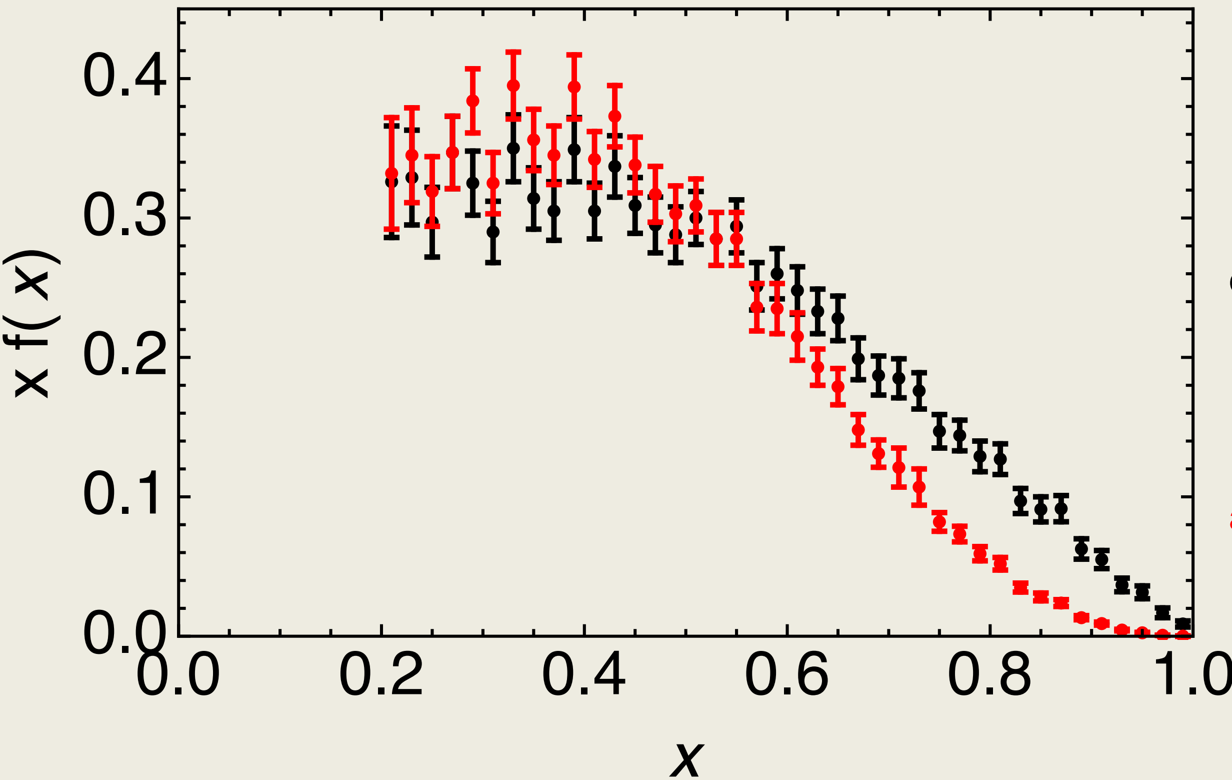




Bethe-Salpeter equation



- Owing to absence of pion targets, the pion's valence-quark distribution functions have hitherto been measured via the Drell-Yan process(CERN(1983&1985), FNAL(1989)):



- ✓ 1989…Conway *et al.* Phys. Rev.D 39 (1989) 92
  Leading-order analysis of Drell-Yan data
- ✓ 2010...Aicher *et al.* Phys. Rev. Lett.105 (2010) 252003
  Consistent next-to-leading order anaylsis

DSEs meet lQCD

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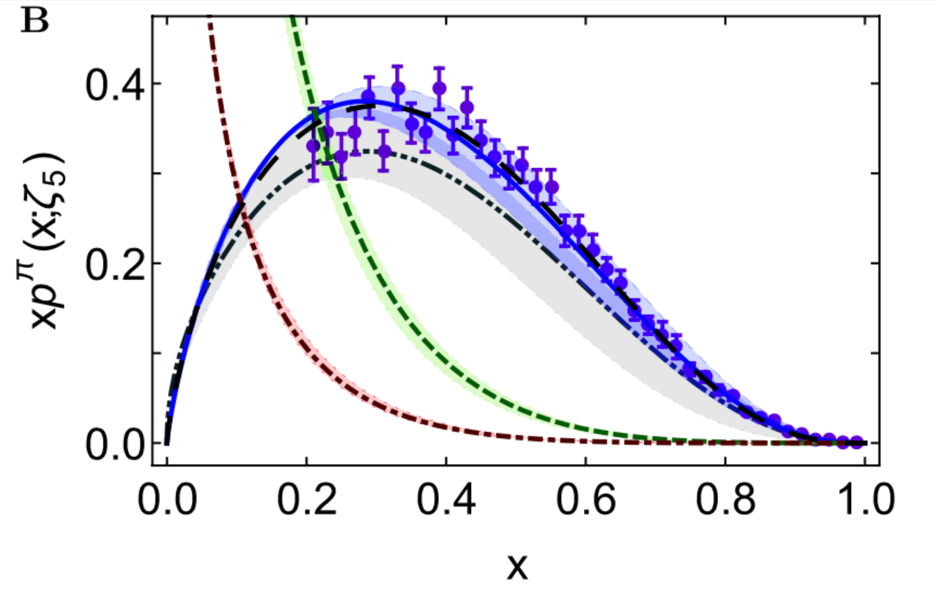
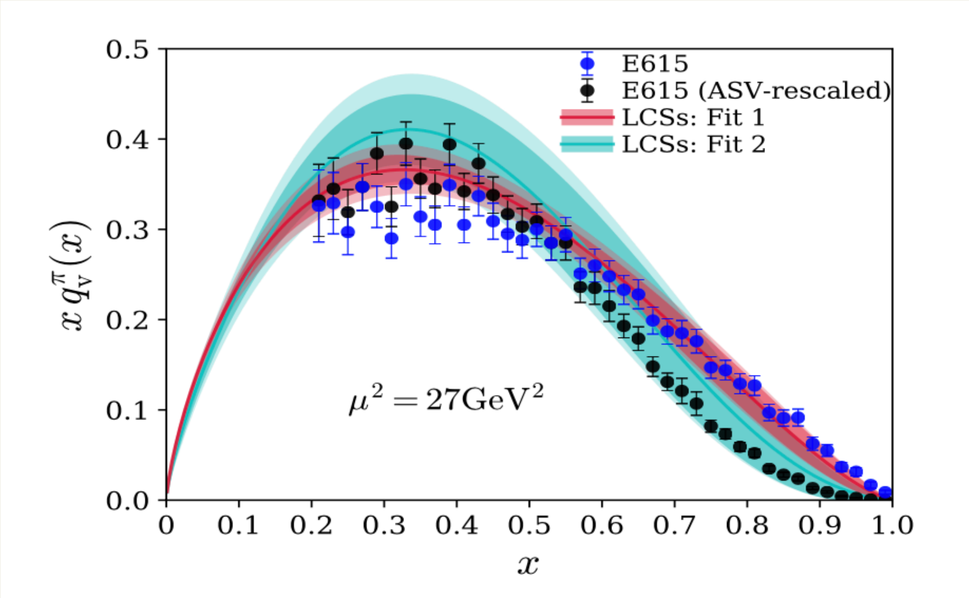
DSEs



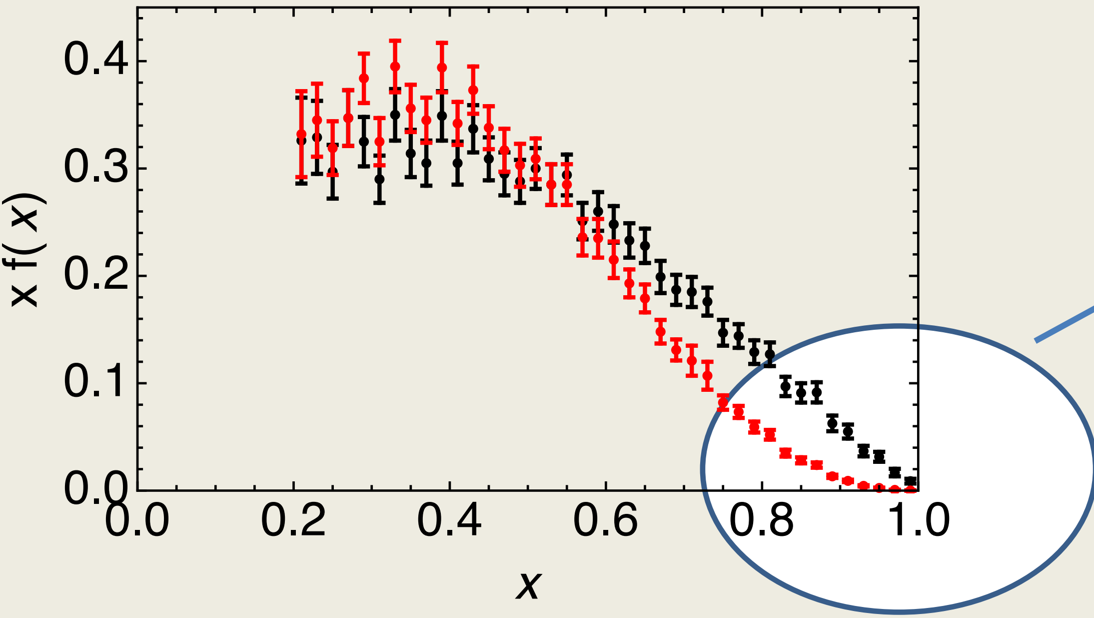
Minghui Ding, *et al.*
CPC 44 (2020) 031002

lQCD



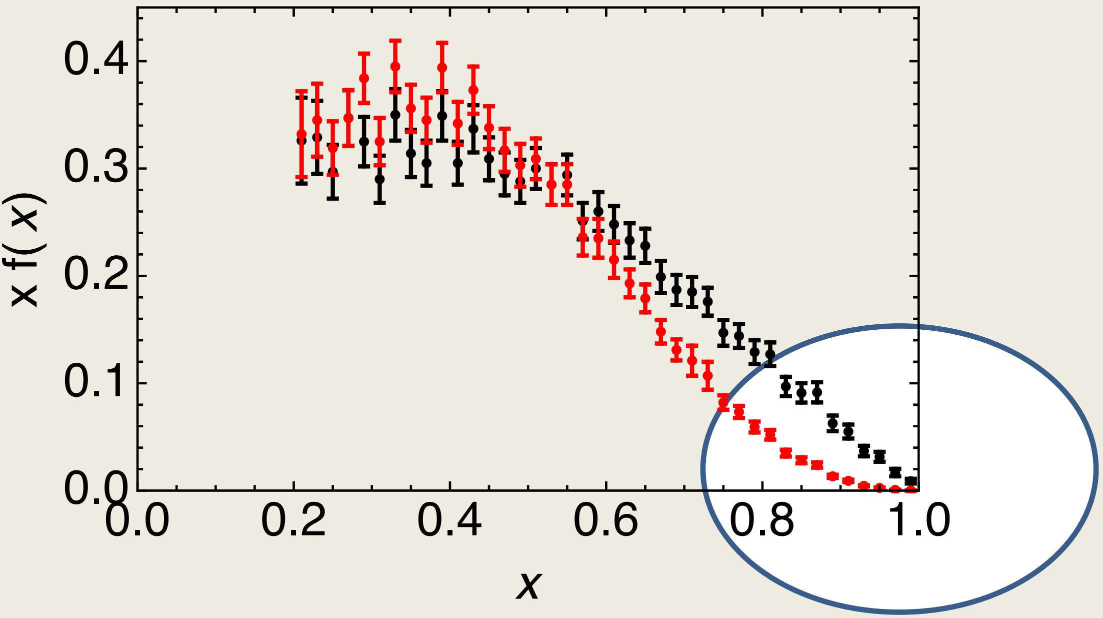
Raza Sabbir Sufian, *et al.*
arXiv: 2001.04960

Issue-1



Large-x dependent

Issue-1



Model and Model

- Nambu – Jona-Lasinio model, translationally invariant regularisaion
  $q^{\pi}(x) \sim (1\text{-}x)^{0}$,
  which becomes "1" after evolving from a low resolution scale
- NJL models with a hard cutoff & also some duality arguments:
  $q^{\pi}(x) \sim (1\text{-}x)^{1}$
- Relativistic constituent quark models:
  $q^{\pi}(x) \sim (1\text{-}x)^{0\ldots2}$
  depending on the form of model wave function
- Instanton-based models
  $q^{\pi}(x) \sim (1\text{-}x)^{1\ldots2}$

Drell-Yan-West relation

PRL24(1970)181, PRL24(1970)1206

Form factor

$$F_1^p(t) \sim \frac{1}{(-t)^{\tau-1}} ,$$

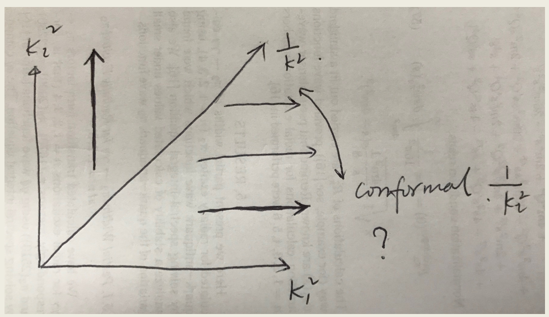
Structure function

$$u^p(x) \sim (1-x)^{2\tau-3} .$$

Spin-1/2

Z. F. Ezawa, Nuovo Cim. A 23 (1974)271

BSA ∝



The Bethe-Salpeter ampltidue of Pion decrease as

$$\Gamma(k_1^2; k_2^2) \to \frac{1}{k_1^2}$$

as $k_1^2 \to \infty$ with k_2^2 fixed finite, or with $\frac{k_1^2}{k_2^2}$ fixed.

Form factor

$$F_1^p(t) \sim \frac{1}{(-t)^{\tau-1}} ,$$

Structure function

$$u^\pi(x) \sim (1-x)^{2\tau-2} .$$

Spin-0

τ the number of quarks inside the hadron

LFHQCD

Nankai University

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond,[1] Tianbo Liu,[2,3] Raza Sabbir Sufian,[2] Hans Günter Dosch,[4] Stanley J. Brodsky,[5] and Alexandre Deur[2]

(HLFHS Collaboration)

A way to parameterize GPD/PDF

Starting Point

$$F_\tau(t) = \frac{1}{N_\tau}\int_0^1 dy(1-y)^{\tau-2} y^{-t/4\lambda-\frac{1}{2}}$$

Change $y = \omega(x)$ $\quad w_\tau(0)=0;\ w_\tau(1)=1;\ \frac{\partial w_\tau(x)}{\partial x} \geq 0\,.$

$$F_\tau(t) = \frac{1}{N_\tau}\int_0^1 dx(1-w_\tau(x))^{\tau-2} w_\tau(x)^{-t/4\lambda-\frac{1}{2}}\frac{\partial w_\tau(x)}{\partial x} \quad + \quad F_\tau(t) = \int_0^1 dx\ H_\tau(x,t)$$

$$H(x,t) = q_\tau(x) e^{t f_\tau(x)}$$

Parton Distribution Function

$$q_\tau(x) = \frac{1}{N_\tau}(1-w_\tau(x))^{\tau-2} w_\tau(x)^{-\frac{1}{2}}\frac{\partial w_\tau(x)}{\partial x}$$

LFHQCD

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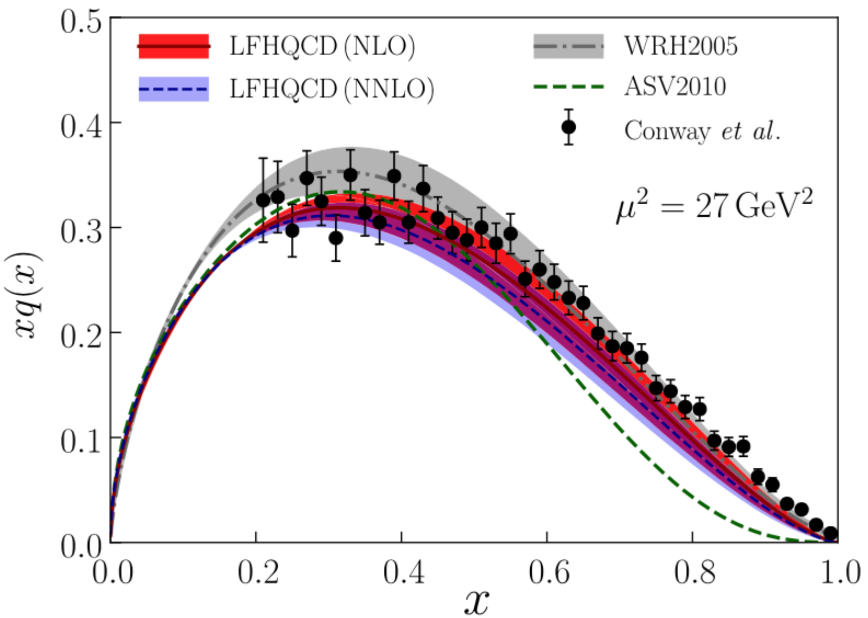

FIG. 4. Comparison for $xq(x)$ in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale $\mu_0 = 1.1\pm0.2$ GeV at NLO and the initial scale $\mu_0 = 1.06\pm0.15$ GeV at NNLO.

To study the behavior of $w(x)$ at large x, we perform a Taylor expansion near $x = 1$

$$w(x) = 1 - (1-x)w'(1) + \frac{1}{2}(1-x)^2 w''(1) + \cdots. \quad (12)$$

Upon substitution of (12) in (9), we find that the leading term in the expansion, which behaves as $(1-x)^{\tau-2}$, vanishes if $w'(1) = 0$. Hence, setting

$$w'(1) = 0 \quad \text{and} \quad w''(1) \neq 0, \quad (13)$$

we find $q_\tau(x) \sim (1-x)^{2\tau-3}$, which is precisely the Drell-Yan inclusive counting rule at $x \to 1$ [63–65], corresponding to the form factor behavior at large Q^2 (3).

rates the ρ Regge trajectory determined in LFHQCD. It could give further insights in understanding the quark-hadron duality and hadron structure. The falloff of the pion PDF at large x is an unresolved issue [89].

PHYSICAL REVIEW LETTERS **120,** 182001 (2018)

Pion Parton Distribution Function in Light-Front Holographic QCD
Lei Chang, Khépani Raya, Xiaobin Wang (Nankai U.). Jan 21, 2020. 4 pp.
e-Print: arXiv:2001.07352 [hep-ph] | PDF

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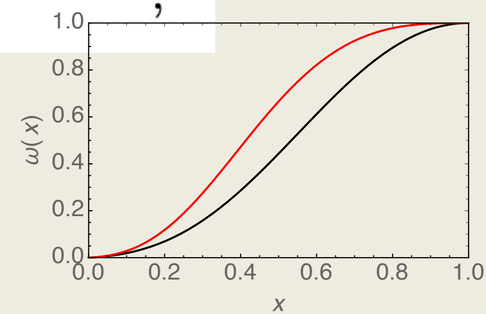
Note： ω could be τ dependent generally

$$w_\tau(x) = x^{(1-x)^{g(\tau)}} e^{-a_\tau (1-x)^{g(\tau)}},$$



- Rule-1: $\sim(1-x)^{2\tau-3}$, with $g(\tau) = 2$
- Rule-2: $\sim(1-x)^{2\tau-2}$, with $g(\tau) = 2 + \frac{1}{\tau-1}$

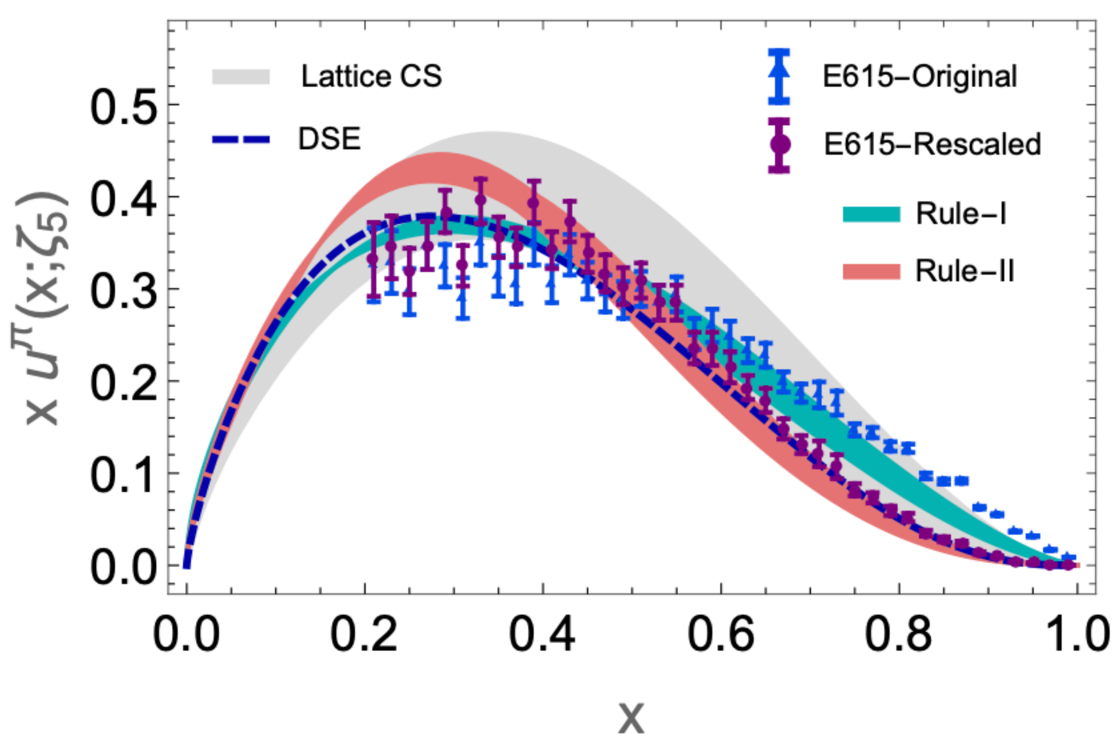
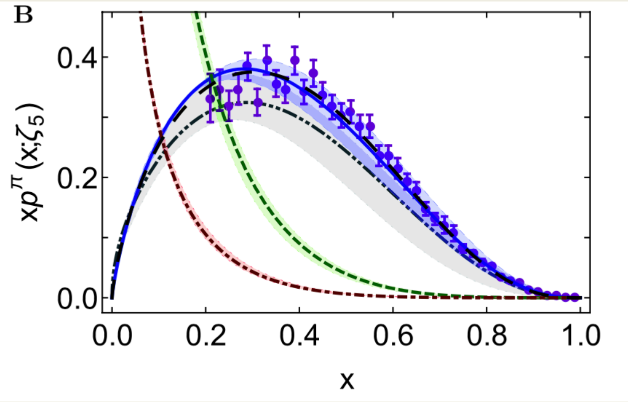
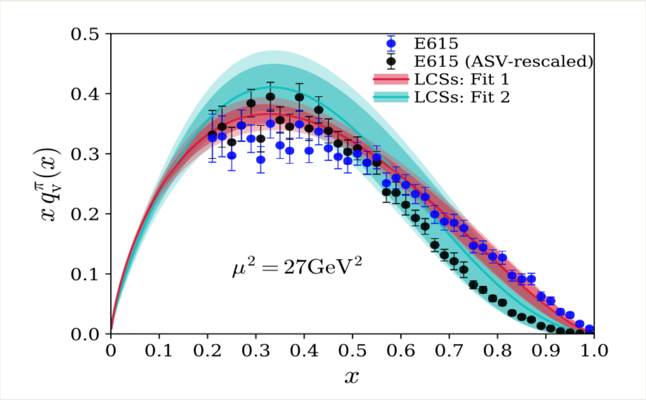


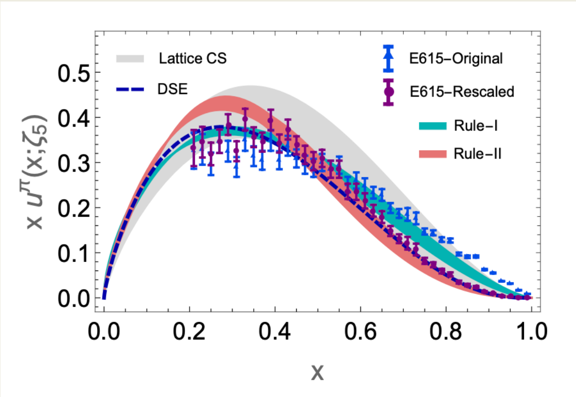
FIG. 1. *Valence-quark pion PDF.* Obtained NLO results at $\zeta_5 = 5.2$ GeV, from the rules in (18). The corresponding (blue and red) error bands account for the uncertainty in the initial scale, $\zeta_1 = 1.1 \pm 0.2$ GeV and the variation of $a_4/a_2 = 0.1$ to 1. The broadest, gray band, corresponds to the novel lQCD "CS" result from [20] and the dashed-line depicts the DSE result [18, 19]. **Data points:** (triangles) LO extraction "E615-Original" [13] and (circles) the NLO analysis "E615-Rescaled" of Ref. [15].



DSEs



Lattice



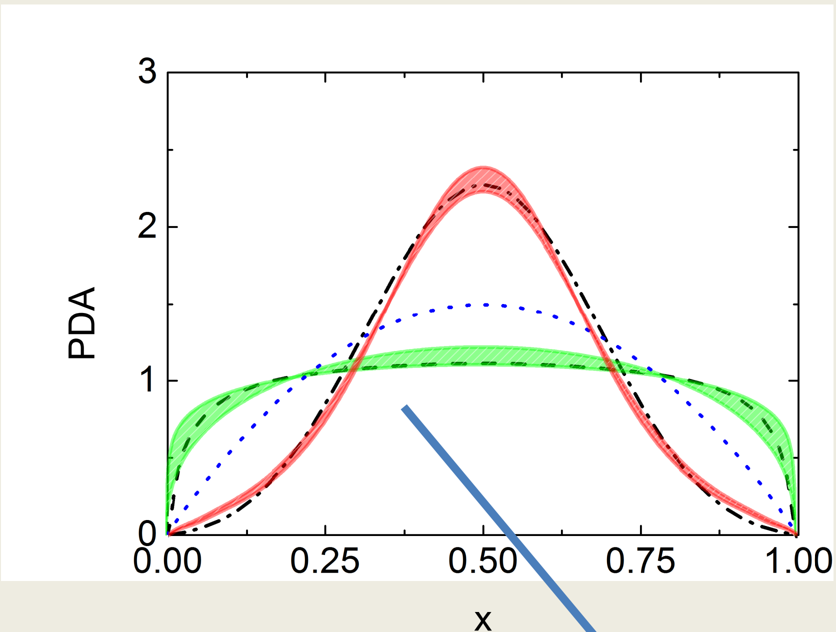
LFHQCD

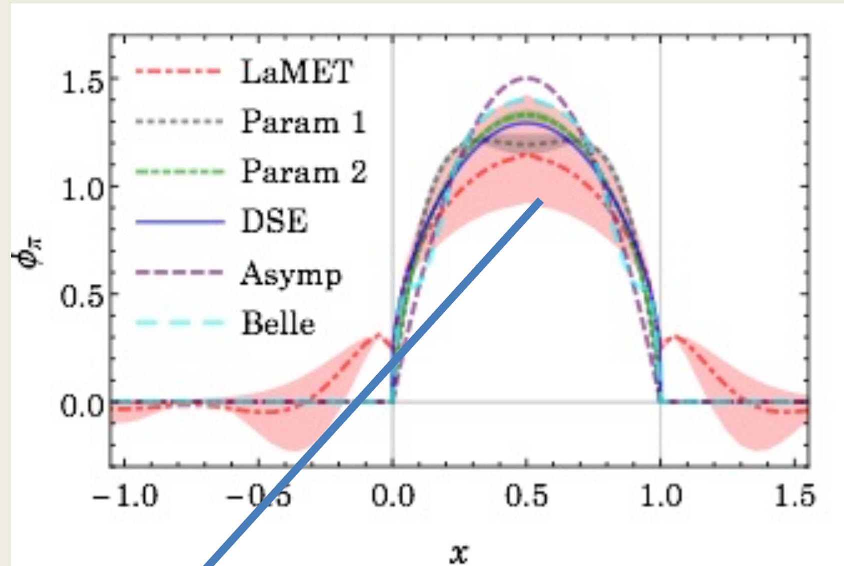
$$(1-x)^2 \quad !$$

Issue-2

Bayesian Extraction of the Parton Distribution Amplitude from the Bethe-Salpeter Wave Function

Fei Gao[b,c], Lei Chang[a], Yu-xin Liu[b,c,d]





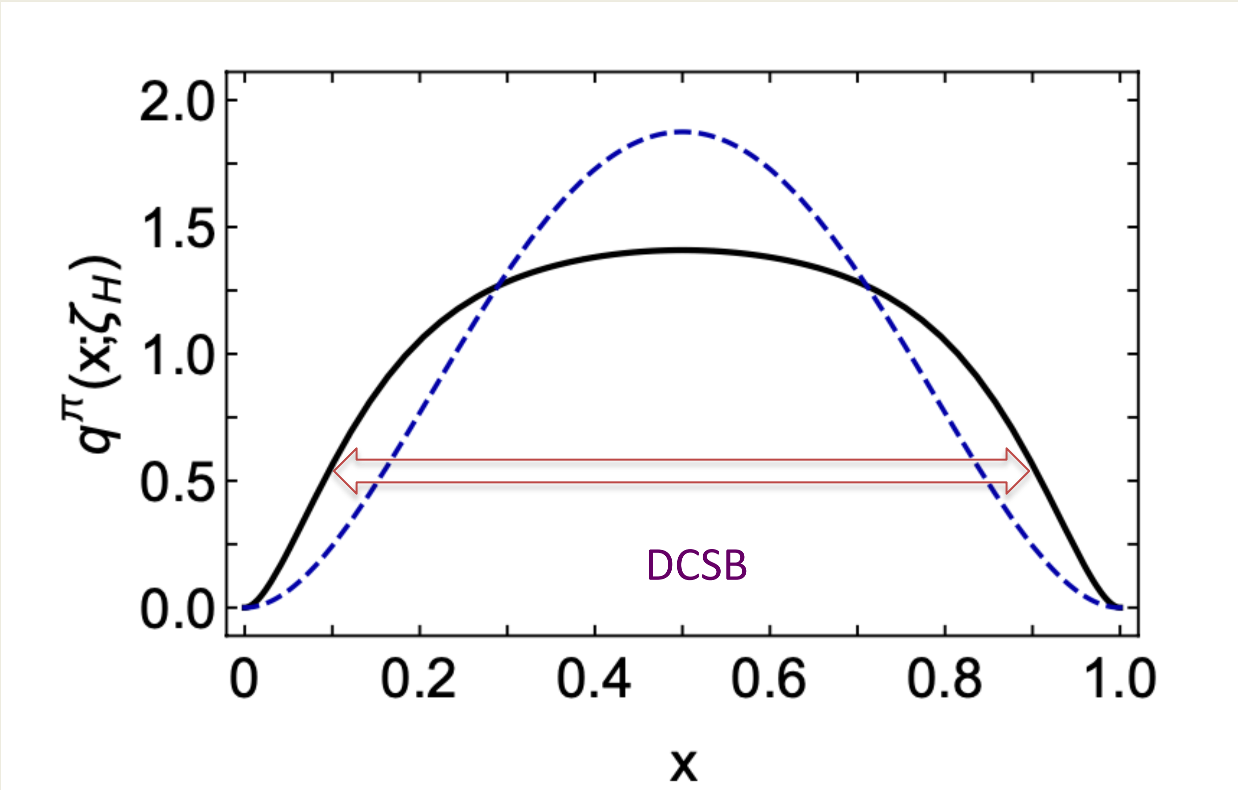
Pion Distribution Amplitude from Lattice QCD

Jian-Hui Zhang,[1,*] Jiunn-Wei Chen,[2,3,†] Xiangdong Ji,[4,5,‡] Luchang Jin,[6,§] and Huey-Wen L

BROADER

PDF at the hadronic scale(DSEs)

Minghui Ding, *et al*, arXiv:1905.05208, PRD 101 (2020) 054014



DCSB

↓

Mass

↙ ↘

Wave Function | Vertex Function

↓

PDA, PDF, EMFF,…

The cause is the same, the valence-quark distribution function is hardened owing to DCSB, which is a realisation of the mechanism responsible for the emergence of mass in the Standard Model. Emergent mass is expressed in the momentum-dependence of all QCD Schwinger functions. It is therefore manifest in the pointwise behaviour of wave functions, elastic and transition form factors, etc.; and as we have now displayed, also in parton distributions.

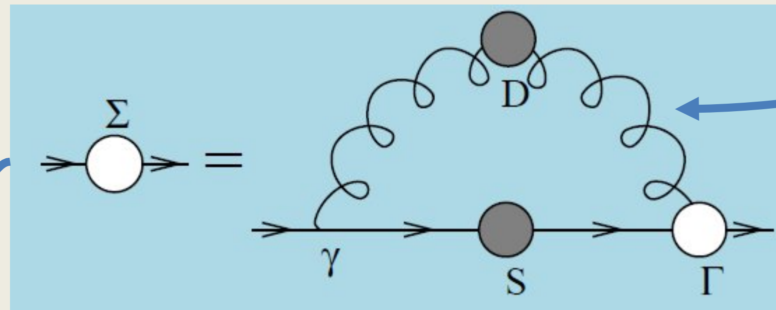
Model Independent?

Ultraviolet behavior of quark propagator(Landau Gauge)

$$x = k^2, y = q^2$$

$$\alpha(x) = \frac{g^2(x)}{4\pi} \xrightarrow[x\to\infty]{} \frac{\pi\gamma_m}{\ln x} = \frac{12\pi}{(33-2N_f)\ln x}$$

INPUT



REDUCED TO

$$A(x) \equiv 1$$

$$B(x) = \frac{\alpha(x)}{\pi x}\int_{y_0}^{x} \frac{yB(y)dy}{y+B^2(y)} + \int_{x}^{\Lambda^2} \frac{\alpha(y)B(y)dy}{\pi\,(y+B^2(y))}$$

$$B(x) \sim c_1 x^{-1} (\ln x)^{\gamma_m - 1}$$

Condensate, DCSB

Ultraviolet behavior of Pion BS amplitude

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$$\Gamma_\pi(k;P) = \int \frac{d^4q}{(2\pi)^4} K(k,q)\chi_\pi(q;P)$$

$$\chi_\pi(k;P) = \gamma_5\left[i\mathcal{E}_\pi(k;P) + \not{P}\mathcal{F}_\pi(k;P) + \not{k}(k\cdot P)\mathcal{G}_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu \mathcal{H}_\pi(k;P)\right]$$

$$\Gamma_\pi(k;P) = \gamma_5\left[iE_\pi(k;P) + \not{P}F_\pi(k;P) + \not{k}(k\cdot P)G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)\right]$$

tediously long…

$$E_\pi(k;P) = E_{\pi 0}(x) + (k\cdot P)^2\, E_{\pi 1}(x) + \cdots$$

The ultraviolet behaviors of BSA can be analysed step by step. The important information from chiral symmetry and its breaking could be summarized:

$$E_{\pi 0}(x) \xrightarrow{x\to\infty} c_3 x^{-1}(\ln x)^{\gamma_m - 1} \quad \text{vs} \quad B(x) = f_\pi\, E(x)\ \ (WTI)$$

$$E_{\pi 1}(x) \xrightarrow{x\to\infty} \frac{1}{2} c_3 x^{-3}(\ln x)^{\gamma_m - 1}$$

- c_3 related to DCSB;
- ½ c_3 deriven by DCSB;
- Demonnstrate DCSB on BSE in a model independent way!

Ultraviolet behavior of Pion BS amplitude

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$$E_\pi(k;P) = E_{\pi 0}(x) + (k\cdot P)^2\, E_{\pi 1}(x) + \cdots \qquad +\mathit{Limit\ information}: \frac{E_{\pi 1}}{E_{\pi 0}} \xrightarrow{x\to\infty} \frac{1}{2}\frac{1}{x^2}$$

Introduce an integral representation

$$E(k;P) = \aleph \int_{-1}^{1} dz \left\{ \rho(z) \frac{M^2}{(k+\frac{z}{2}P)^2 + M^2} \right\}$$

$$\frac{E_{\pi 1}}{E_{\pi 0}} \xrightarrow{x\to\infty} \frac{1}{2}\frac{1}{x^2} \Longrightarrow \int_{-1}^{1} dz\rho(z)z^2 = \frac{1}{2}\int_{-1}^{1} dz\,\rho(z) = \frac{1}{2}$$

Note: $\int_{-1}^{1}\frac{3}{4}(1-z^2)dz = \frac{1}{5}$

Ultraviolet behavior of Pion BS amplitude

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$$E_\pi(k;P) = E_{\pi 0}(x) + (k\cdot P)^2\, E_{\pi 1}(x) + \cdots \qquad +\textit{Limit information}: \frac{E_{\pi 1}}{E_{\pi 0}} \xrightarrow{x\to\infty} \frac{1}{2}\frac{1}{x^2}$$

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Note: $\int_{-1}^{1} \frac{3}{4}(1-z^2)dz = \frac{1}{5}$

Model-I

A perspective on Dyson-Schwinger equation: toy model of Pion
Lei Chang (Adelaide U.). 2016. 5 pp.
Published in **EPJ Web Conf. 113 (2016) 05001**
DOI: 10.1051/epjconf/201611305001
Conference: C15-05-18.5 Proceedings

$$\rho(z) = \frac{1}{\pi}\frac{1}{\sqrt{1-z^2}} \Longrightarrow$$

PDA$\propto \frac{8}{\pi}\sqrt{x(1-x)}$
PDF$\propto 6x(1-x)$

AdsQCD

Ultraviolet behavior of Pion BS amplitude

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$$E_\pi(k;P) = E_{\pi 0}(x) + (k\cdot P)^2\, E_{\pi 1}(x) + \cdots \qquad +Limit\ information{:}\ \frac{E_{\pi 1}}{E_{\pi 0}} \xrightarrow{x\to\infty} \frac{1}{2}\frac{1}{x^2}$$

Introduce an integral representation

$$E(k;P) = \aleph \int_{-1}^{1} dz \left\{ \rho(z) \frac{M^2}{(k+\frac{z}{2}P)^2 + M^2} \right\}$$

$$\frac{E_{\pi 1}}{E_{\pi 0}} \xrightarrow{x\to\infty} \frac{1}{2}\frac{1}{x^2} \Longrightarrow \int_{-1}^{1} dz \rho(z) z^2 = \frac{1}{2}\int_{-1}^{1} dz\, \rho(z) = \frac{1}{2}$$

Note asymptotically: $\int_{-1}^{1} \frac{3}{4}(1-z^2)dz = \frac{1}{5}$

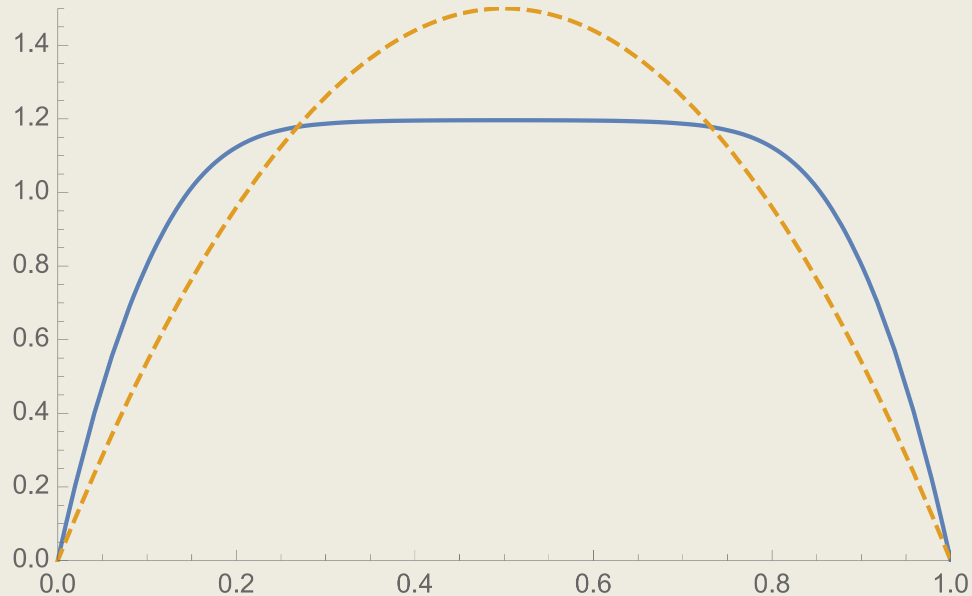
Model-II

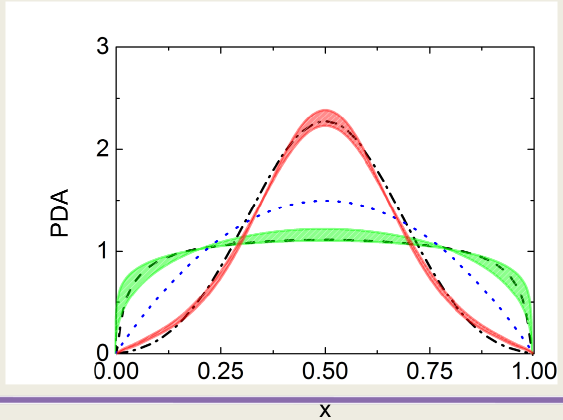
Pion and kaon valence-quark parton quasidistributions
Shu-Sheng Xu (Nanjing U.), Lei Chang (Nankai U.), Craig D. Roberts (Argonne), Hong-Shi Zong (Nanjing U.). Feb 26, 2018. 10 pp.
Published in **Phys.Rev. D97 (2018) no.9, 094014**

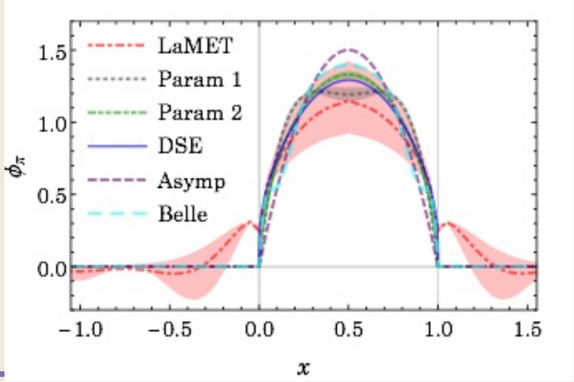
$$\rho_\pi = N\frac{1}{2t}\left[\left(e^{\frac{-z+z_0}{2t}} + e^{\frac{z-z_0}{2t}}\right)^{-2} + \left(e^{\frac{-z-z_0}{2t}} + e^{\frac{z+z_0}{2t}}\right)^{-2}\right]$$

No oscillation on PDA

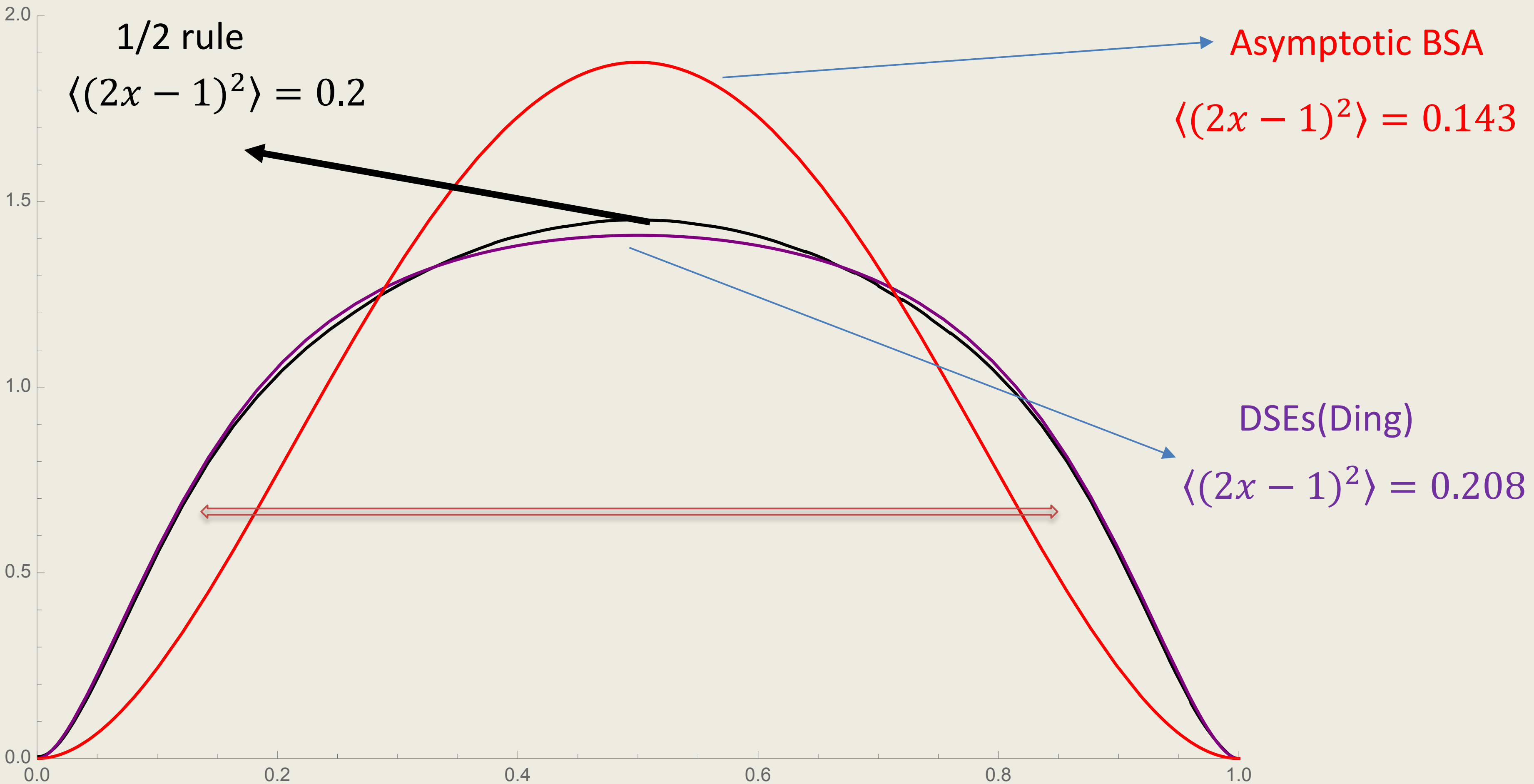
Parton Distribution Amplitude







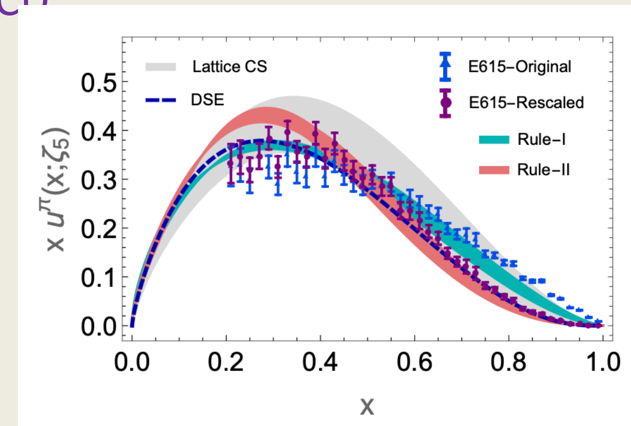
Parton Distribution Function



✓ Accommodate proton and pion DYW relation in LFHQCD

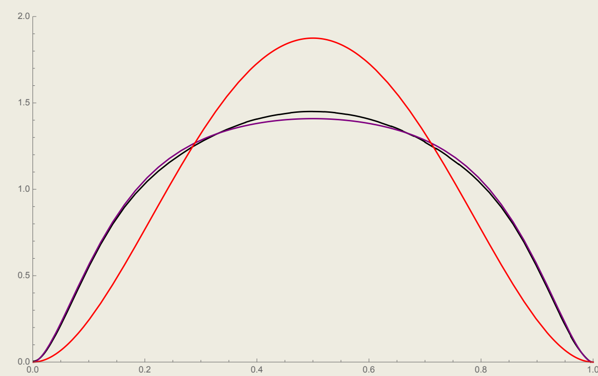
❖ Rule-1: $\sim(1-x)^{2\tau-3}$, with $g(\tau)=2$

❖ Rule-2: $\sim(1-x)^{2\tau-2}$, with $g(\tau)=2+\frac{1}{\tau-1}$



✓ Model independent illustrating the broad character of PDA and PDF

$$DCSB \rightarrow \frac{E_{\pi 1}}{E_{\pi 0}} \xrightarrow{x\to\infty} \frac{1}{2}\frac{1}{x^2}$$



Thanks for your attention