

NuINT 2022

Hoam Faculty House at Seoul

24 October, 2022

National University, Seoul, Korea

QED radiative corrections and nuclear medium effects at GeV energies

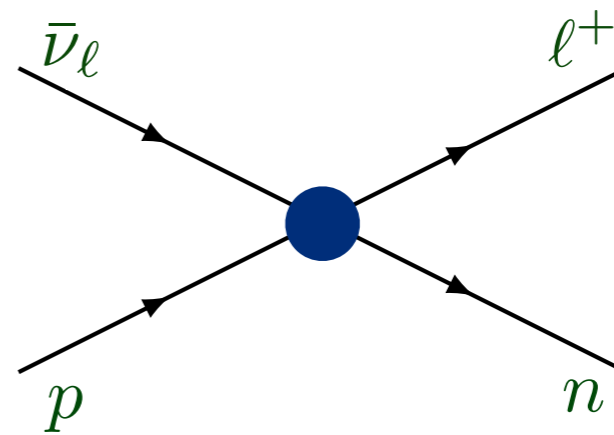
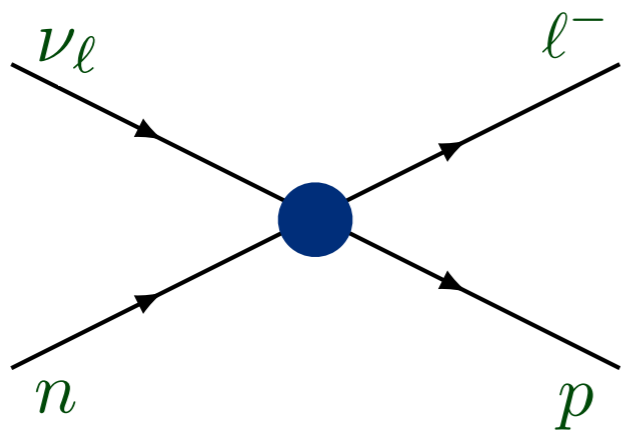


Oleksandr Tomalak

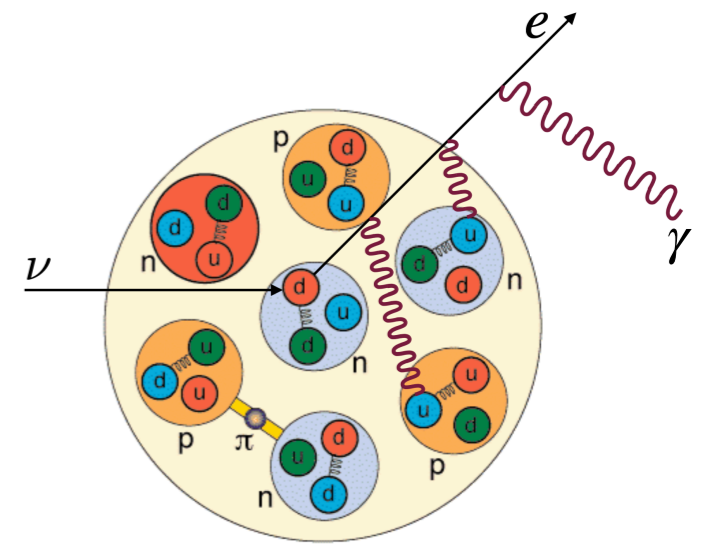
LA-UR-22-29360

Outline

- 1) radiative corrections to
charged-current elastic scattering on nucleons

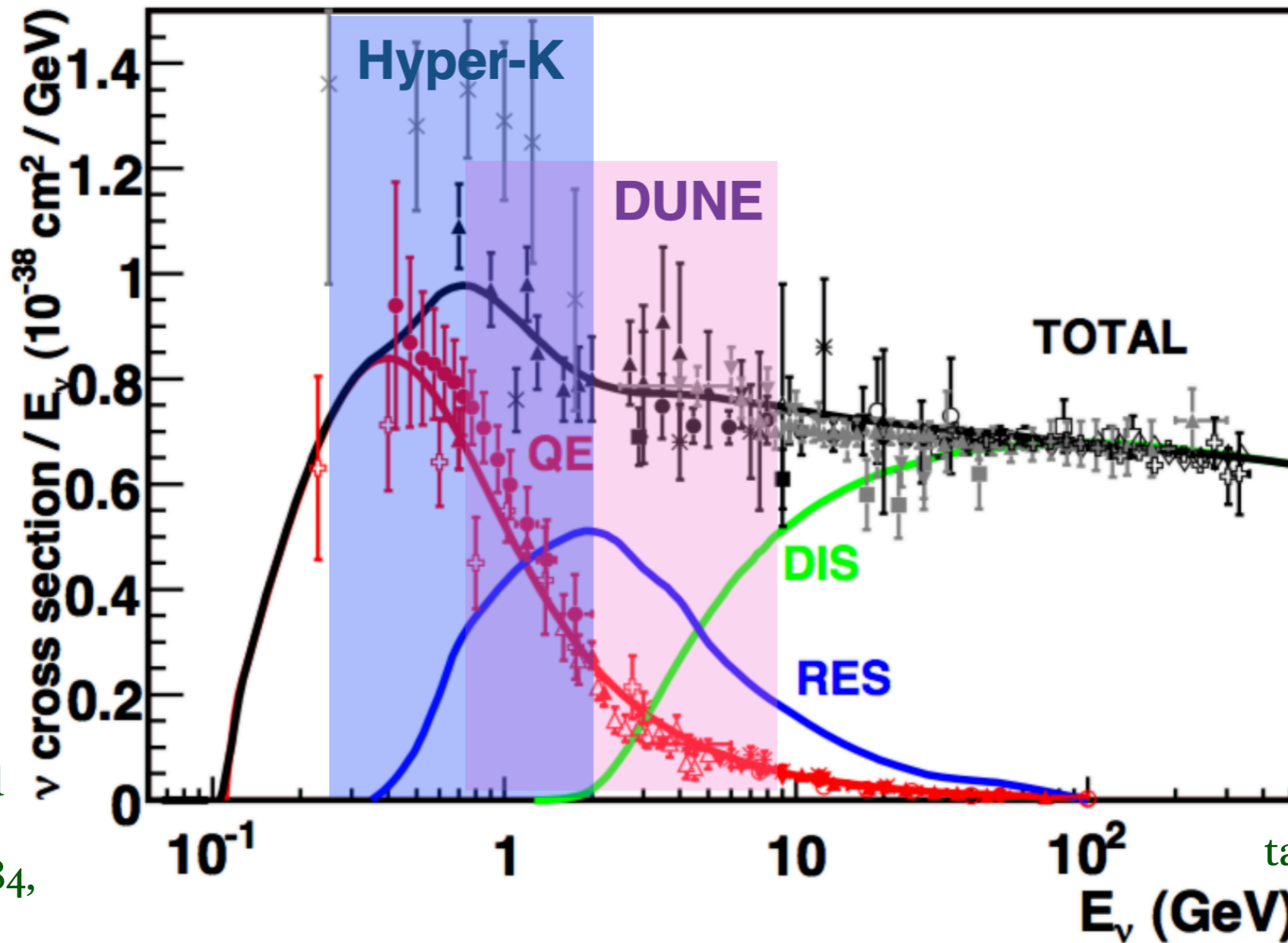


- 2) QED nuclear medium effects in
(anti)neutrino-nucleus and
electron-nucleus scattering



CCQE. Why should we care?

- neutrino-nucleus cross sections and future accelerator-based fluxes

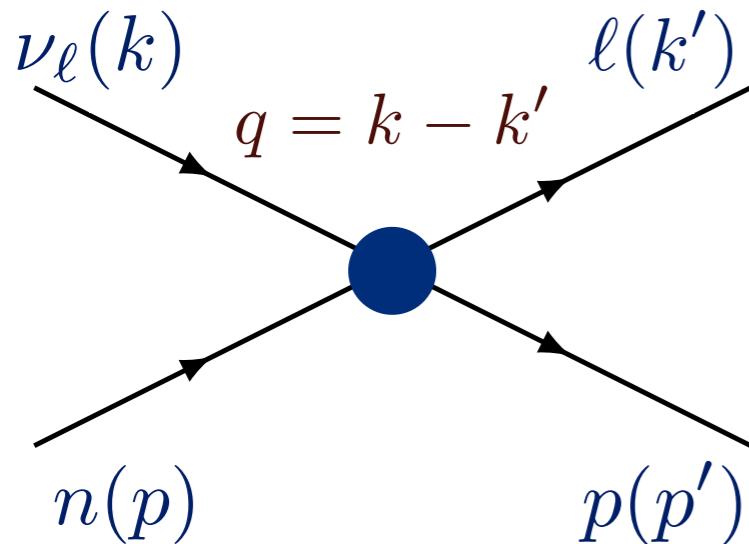


Formaggio and
Zeller
Rev. Mod. Phys. 84,
1307-1341 (2013)

talk by Teppei Katori

- basic process: bulk of events at Hyper-K and DUNE
- channel for reconstruction of neutrino energy

CCQE scattering on free nucleon



neutrino energy

$$E_\nu$$

momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu(Q^2) = \langle p | \bar{u} (\gamma^\mu - \gamma^\mu \gamma_5) d | n \rangle$$

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_P^V(Q^2) + \gamma^\mu \gamma_5 F_A(Q^2) + \frac{q^\mu}{M} \gamma_5 F_P(Q^2)$$

form factors: isovector Dirac and Pauli

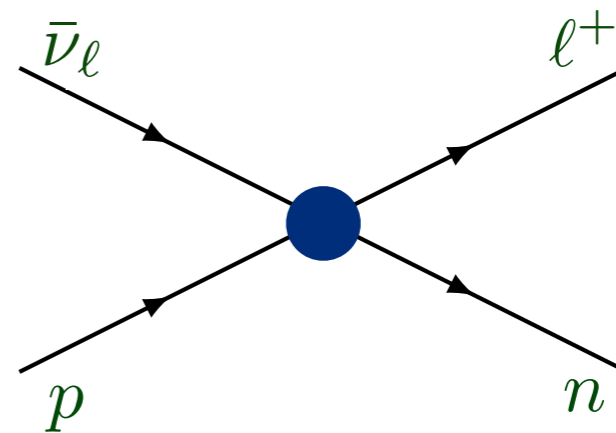
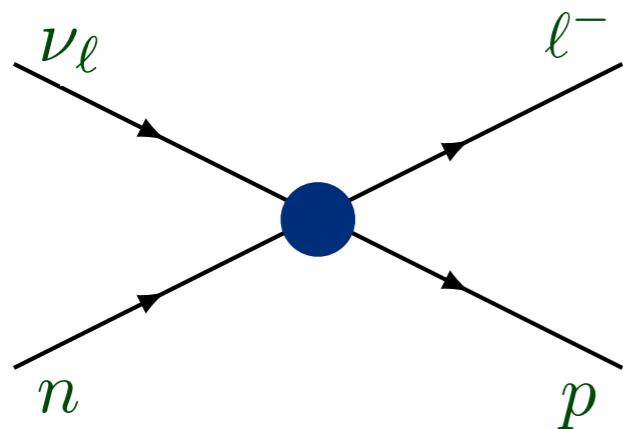
axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

talks by Tejin Cai, Aaron Meyer,
Julia Tena Vidal, Noah Steinberg ...

tree-level amplitude

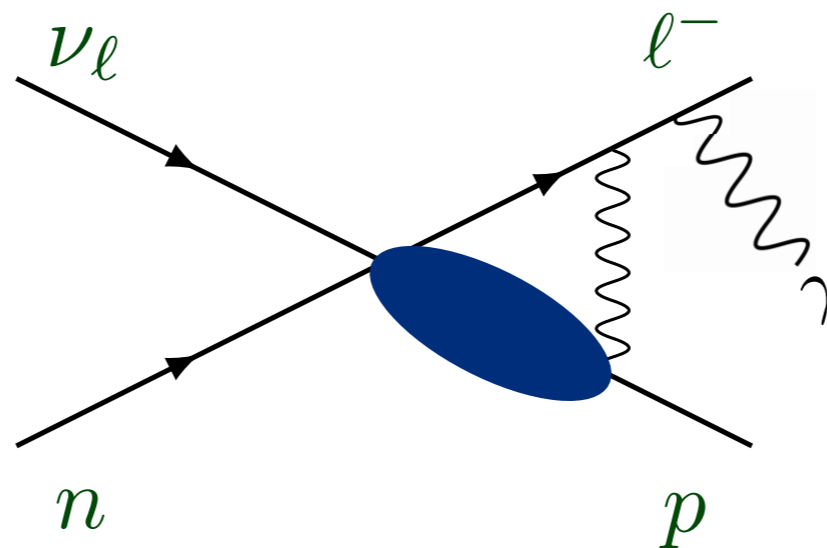
$$T = \frac{G_F V_{ud}}{\sqrt{2}} (\bar{\ell}(k') \gamma_\mu (1 - \gamma_5) \nu_\ell(k)) (\bar{p}(p') \Gamma^\mu(Q^2) n(p))$$



O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

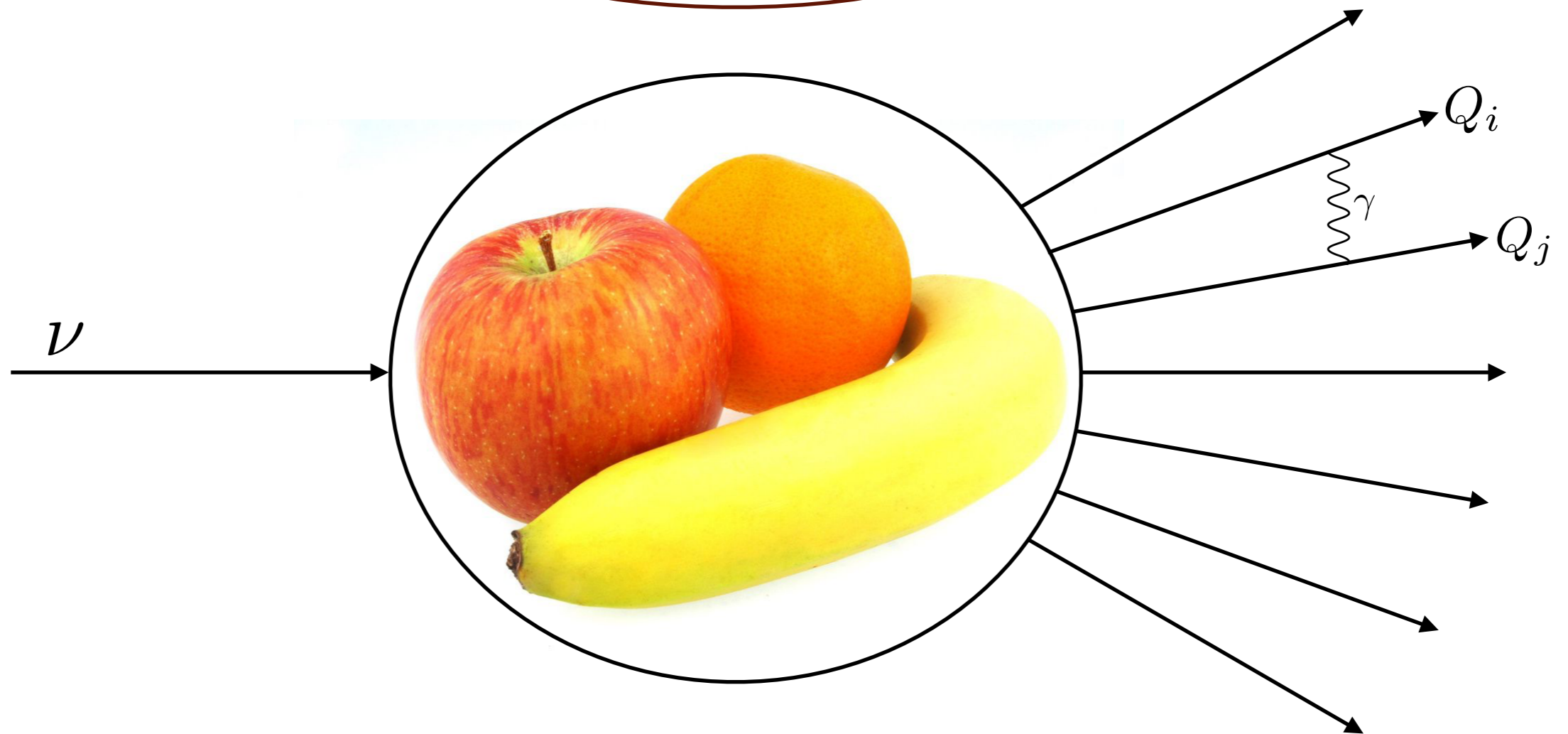
Radiative corrections in charged-current elastic scattering on free nucleons

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
 editors suggestion in Phys. Rev. D (2022)



QED corrections

$$m_e \ll m_\mu \ll E_\nu$$

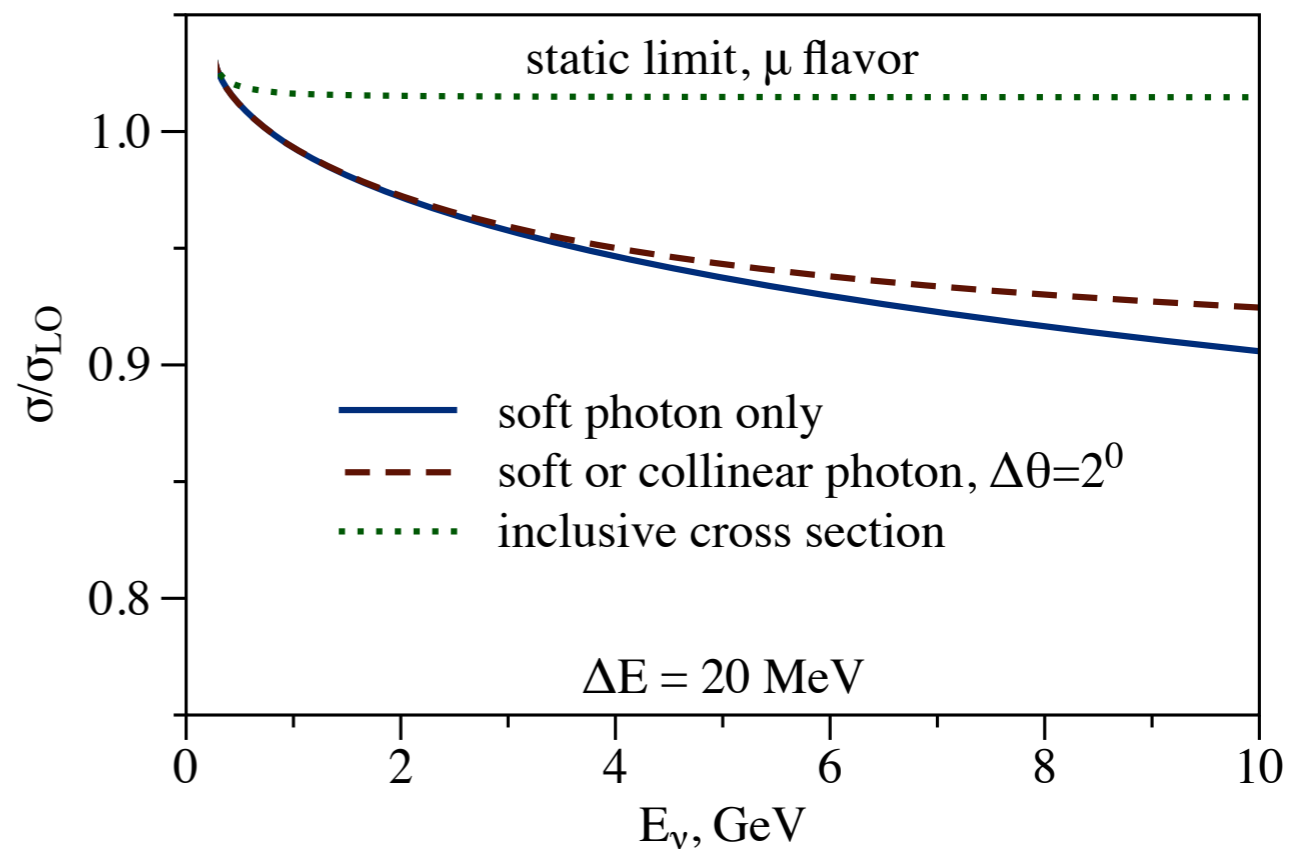
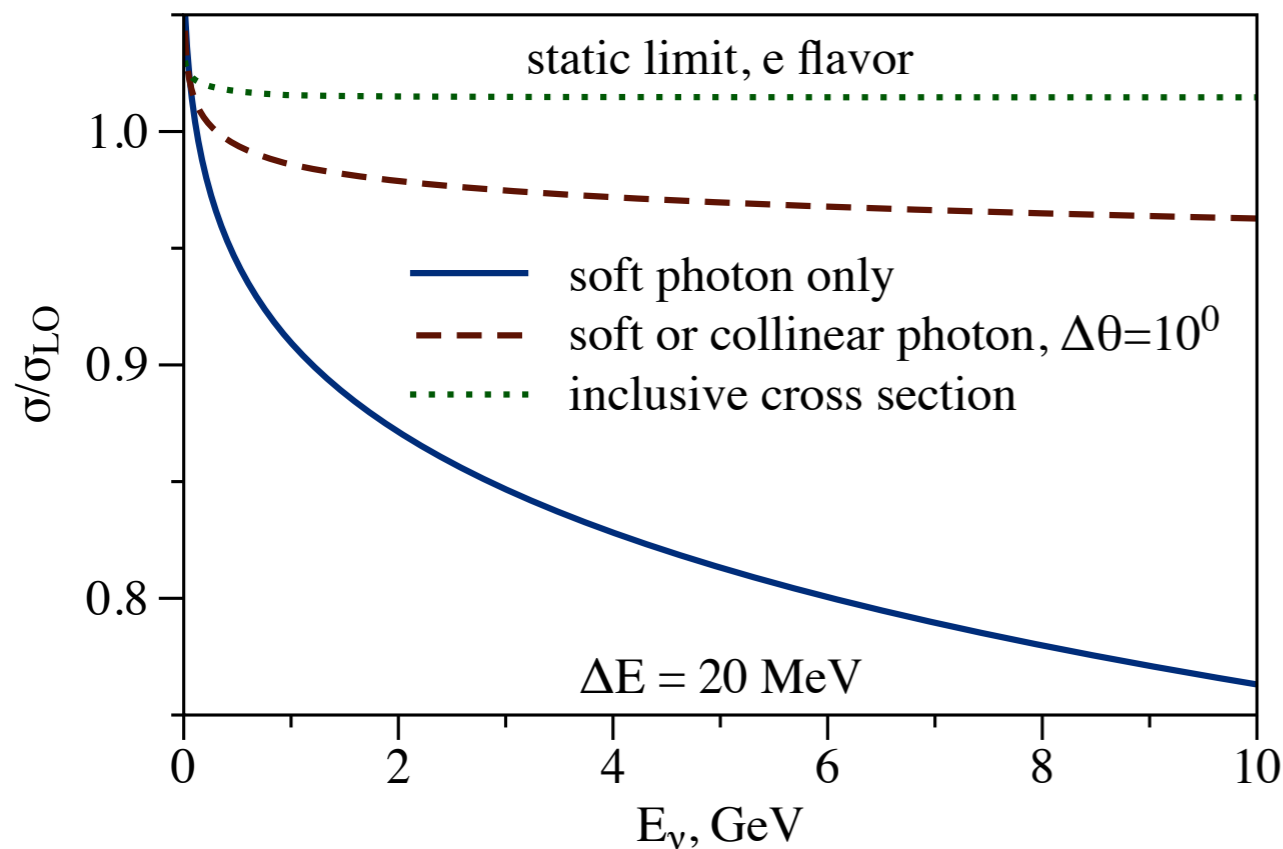


$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \ln \frac{E_\nu}{m_e} \sim 6 - 10 \text{ or } \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

- scale separation introduces large flavor-dependent QED logarithms

Static nucleon limit

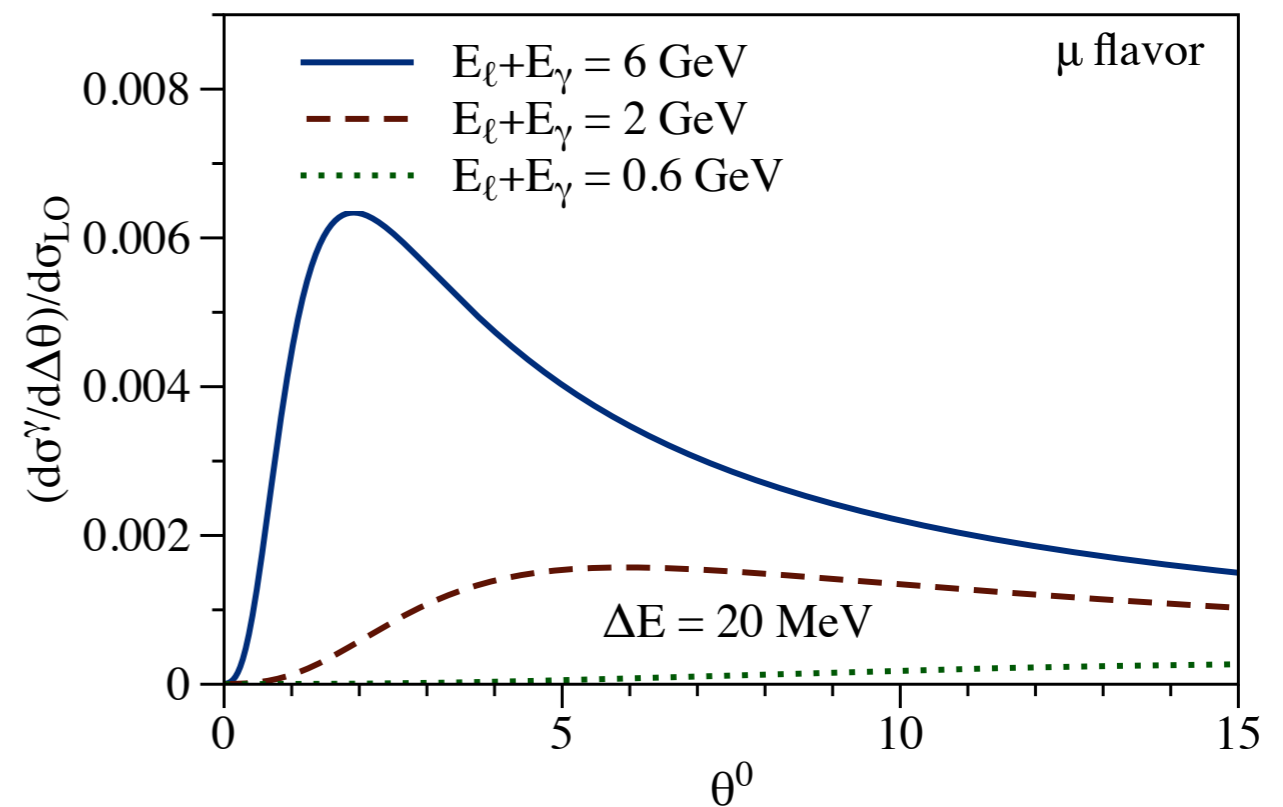
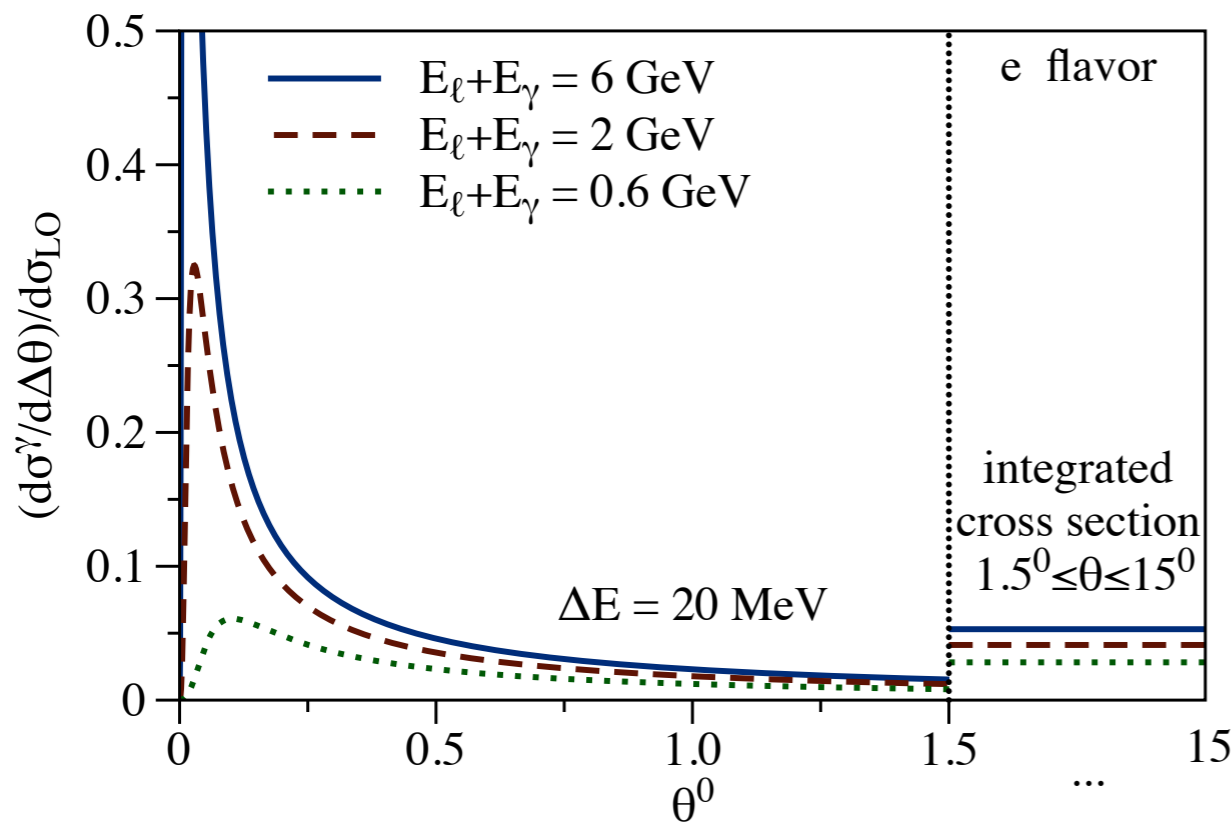
- formal limit of infinitely heavy nucleus $m_\ell \ll E_\ell \ll M$
- provides correct soft and collinear logarithms
- soft-photon energy < 20 MeV, jet size: 10° for electron and 2° for muon



- flavor-dependent effect, same for $\nu_\ell n \rightarrow \ell^- p$ vs $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- collinear observable: cancellation of virtual vs real logs
- inclusive observables (+ γ): few % level, flavor independent

Electron vs muon jets

- factorization for radiation of collinear photons
- cone angle is defined to lepton direction
- photons of energy > 20 MeV, fixed energy in the cone



- flavor-dependent effect, same for $\nu_\ell n \rightarrow \ell^- p$ vs $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- forward-peaked radiation for electron flavor
- negligible radiation for muons with shifted peak position

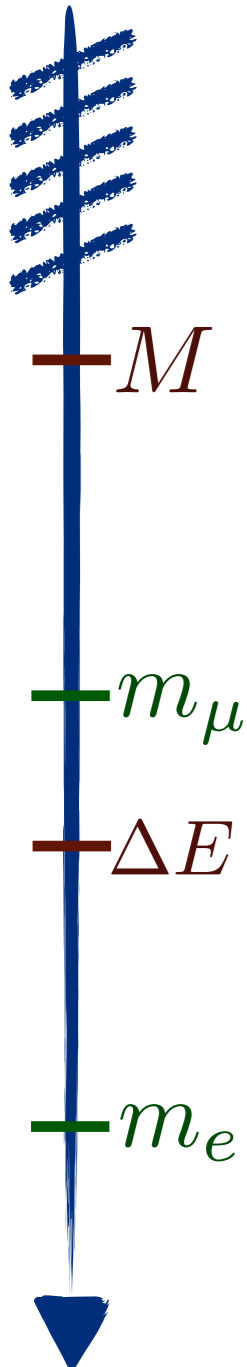
Factorization approach

- cross section is given by **factorization formula**

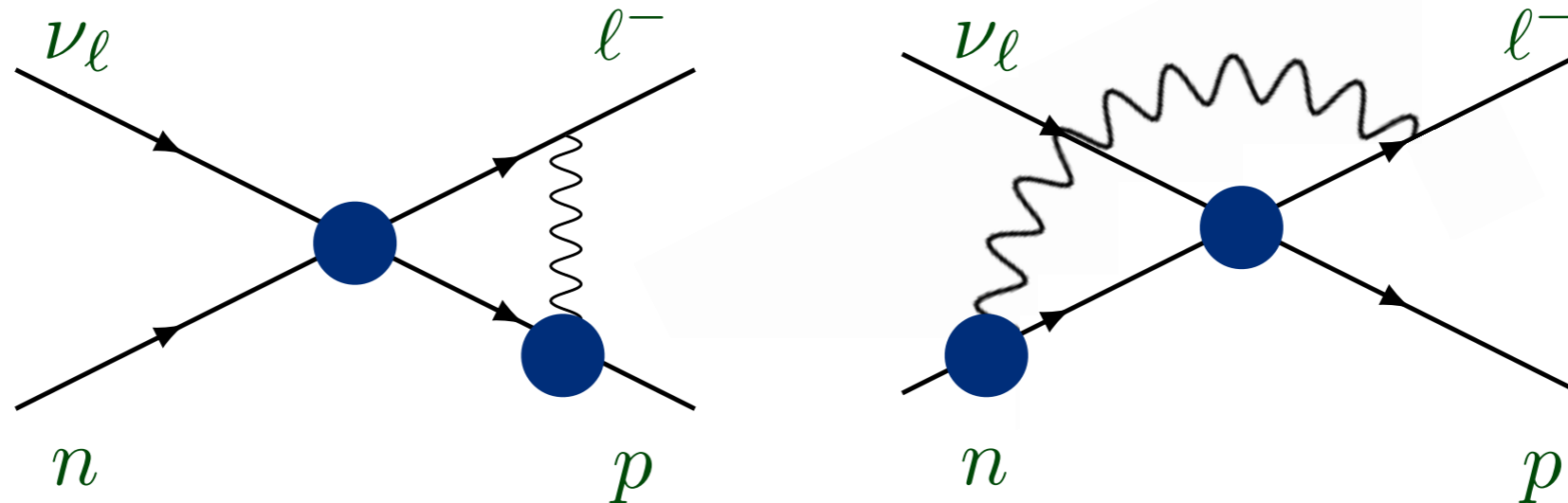
$$d\sigma \sim S \left(\frac{\Delta E}{\mu} \right) J \left(\frac{m_\ell}{\mu} \right) H \left(\frac{M}{\mu} \right)$$

- determine **hard function** at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- soft and collinear functions are evaluated **perturbatively**



Hadronic model at GeV scale




- exchange of photon between the charged lepton and nucleons
- assume **onshell form** for each interaction with dipole form factors
discussed for neutrino-nucleon scattering: Graczyk, Phys. Lett. B 732, 315-319 (2013)
- add **self energy** for charged particles
- reproduce soft and collinear regions of SCET

- best determination of hard function

Factorization approach

- cross section is given by **factorization formula**

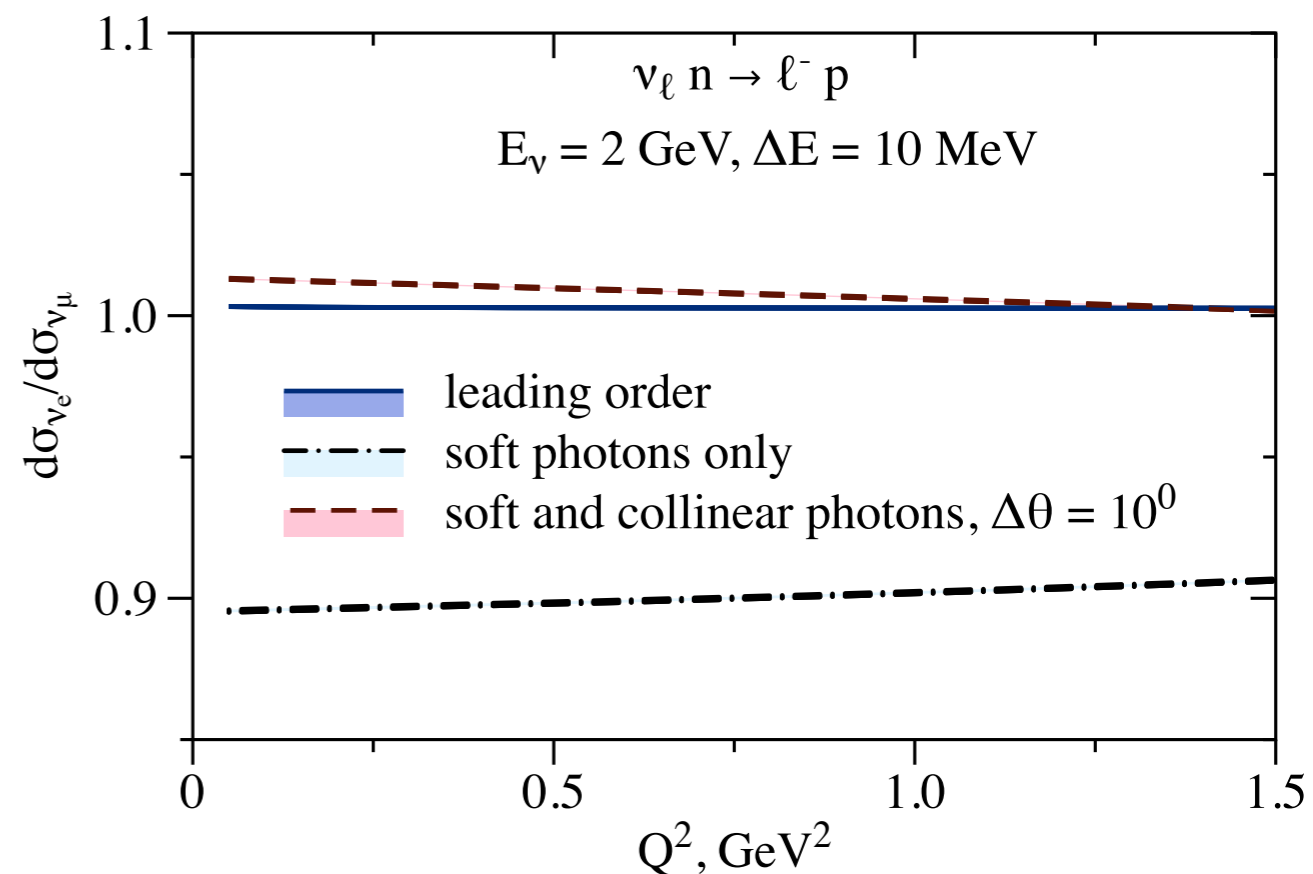
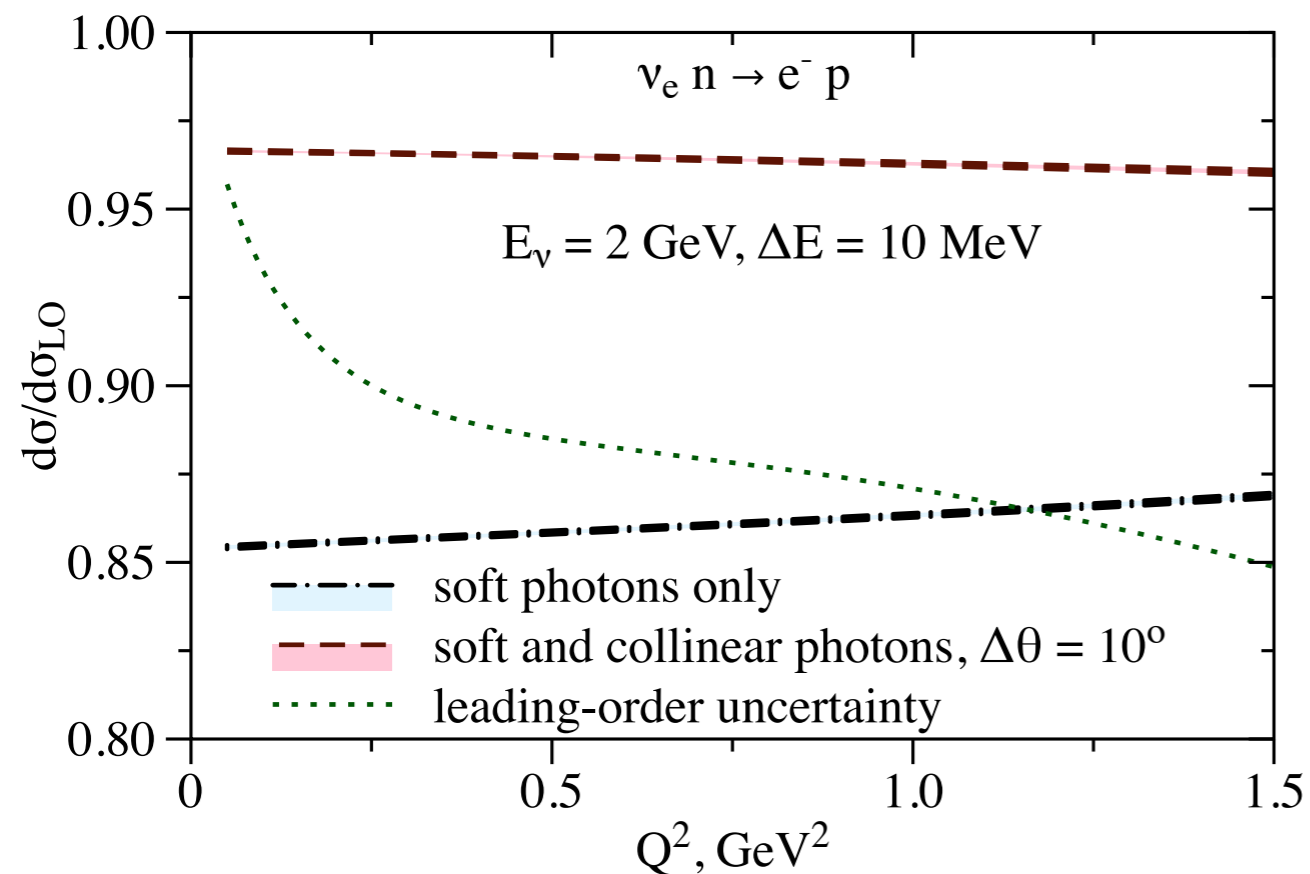
$$d\sigma \sim S \left(\frac{\Delta E}{\mu} \right) J \left(\frac{m_\ell}{\mu} \right) H \left(\frac{M}{\mu} \right)$$

- 
- determine **hard function** at hard scale by matching experiment or **hadronic model** to the theory with heavy nucleon
 - **RGE evolution** of the hard function to scales $\Delta E, m_\ell$
 - **soft and collinear functions** are evaluated **perturbatively**
 - calculate cross section at low energies accounting for **all large logs**
ep scattering with soft radiation only: Richard J. Hill, Phys. Rev. D 95 1, 013001 (2016)

- **soft and collinear functions** determined **analytically**
- **hard function** describes physics at GeV energies

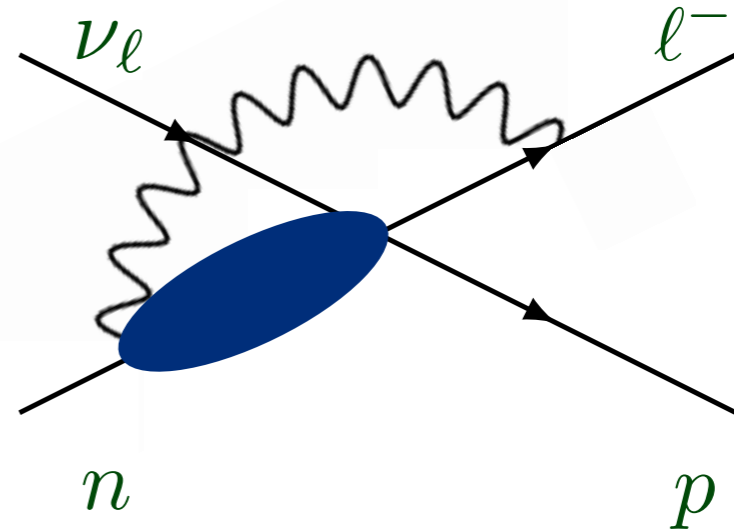
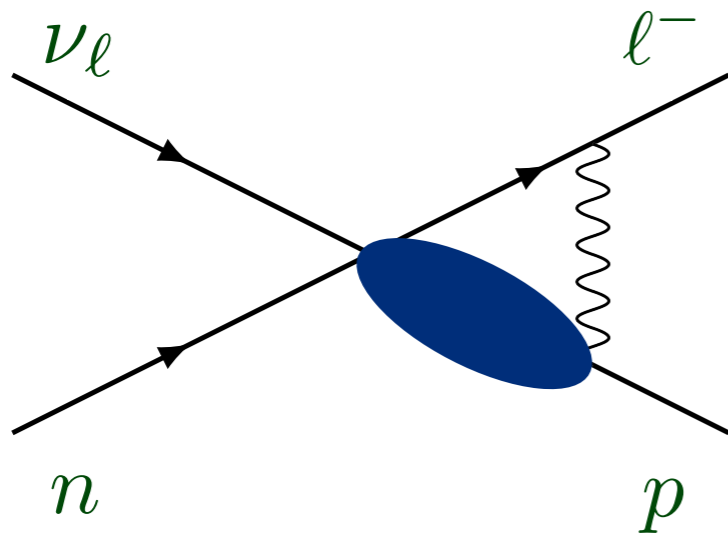
Exclusive observables

- cancellation of uncertainties from hard function for e/μ and ratio to LO

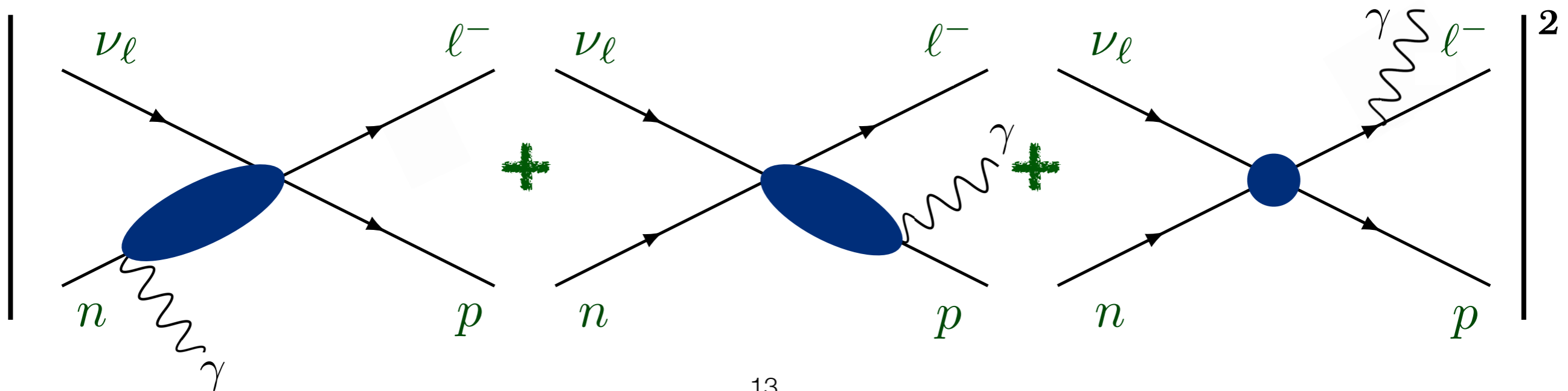


- ratios: cancellation of uncertainty from hard function

Inclusive observables

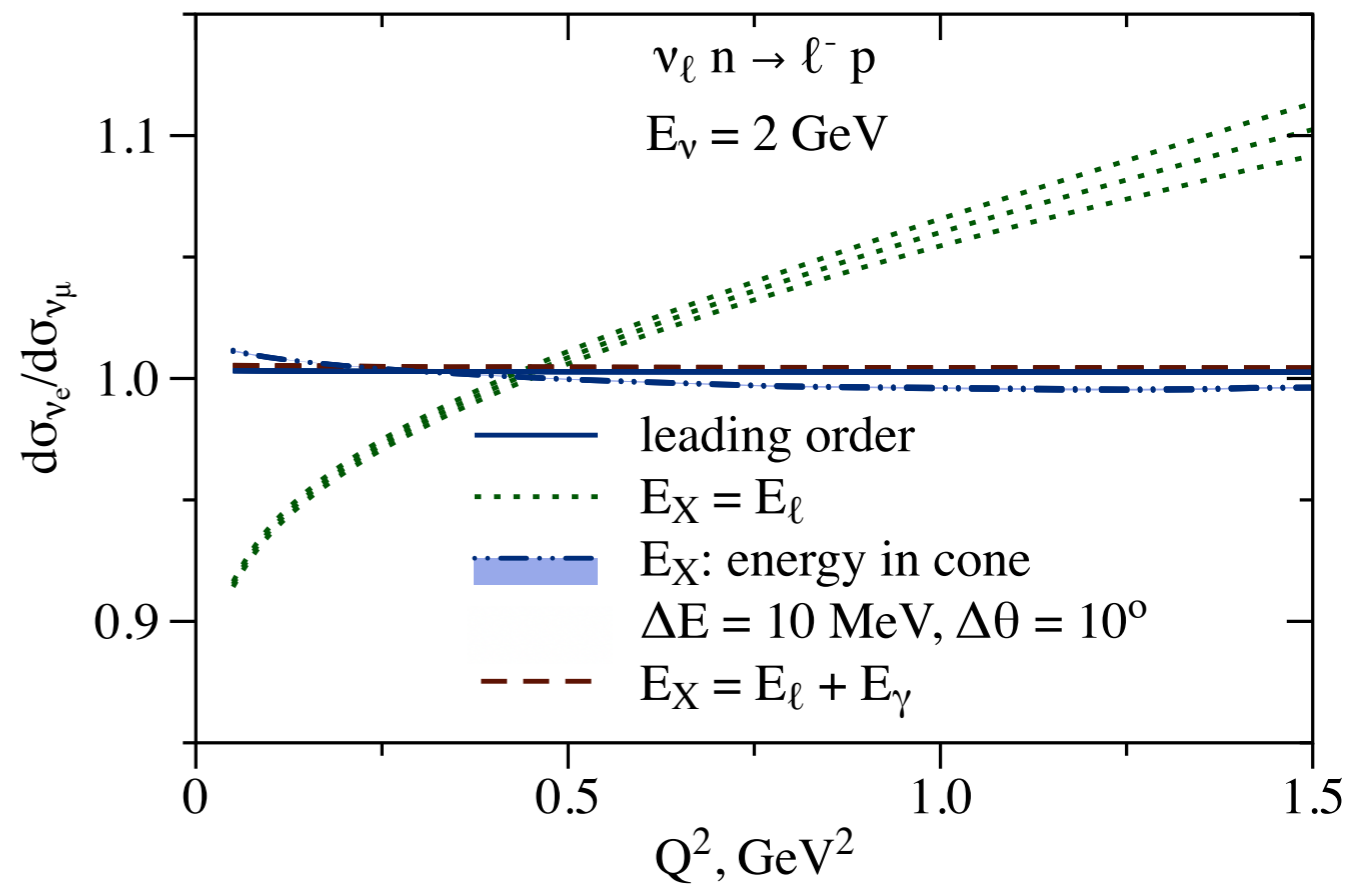
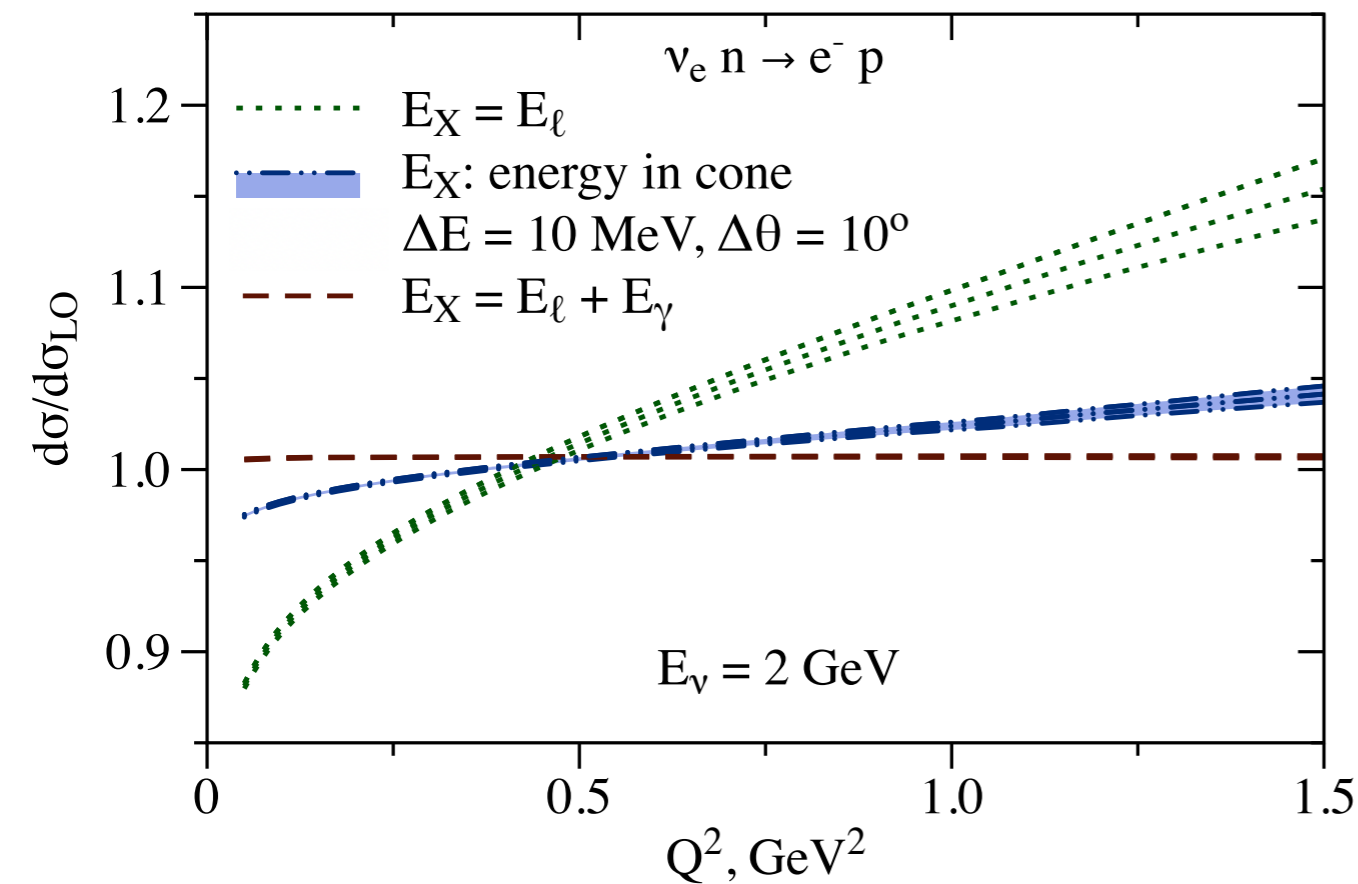


- the same gauge-invariant model for the real radiation
- arbitrary hard photons are part of the observable



Inclusive observables

- kinematics $Q^2 = 2M(E_\nu - E_X)$ is reconstructed with 3 different E_X



- dependence on reconstruction of kinematics and cuts
- predict σ_{ν_e} from σ_{ν_μ} measurements with neutrino beam



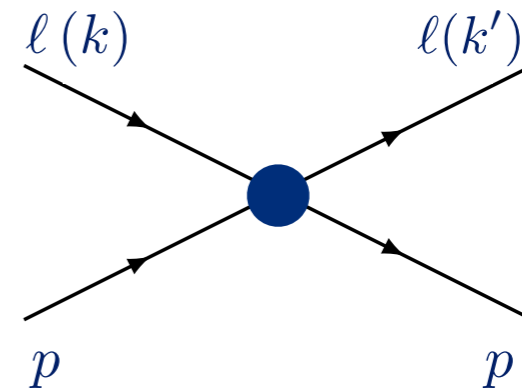
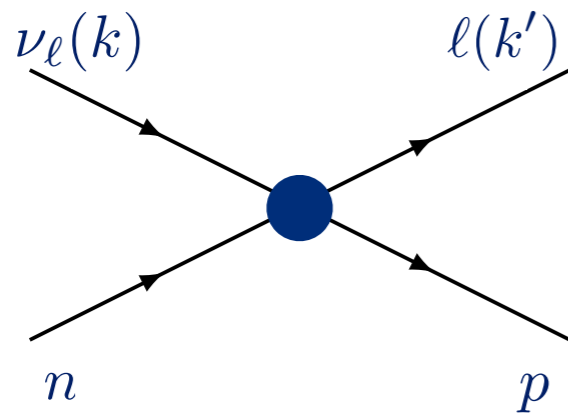
Electron/muon ratio

	E_ν , GeV		$\left(\frac{\sigma_e}{\sigma_\mu} - 1\right)_{\text{LO}}$, %	$\frac{\sigma_e}{\sigma_\mu} - 1$, %
T2K/HyperK	0.6	ν	2.47 ± 0.06	$2.84 \pm 0.06 \pm 0.37$
		$\bar{\nu}$	2.04 ± 0.08	$1.84 \pm 0.08 \pm 0.20$
NOvA/DUNE	2.0	ν	0.322 ± 0.006	$0.54 \pm 0.01 \pm 0.22$
		$\bar{\nu}$	0.394 ± 0.003	$0.20 \pm 0.01 \pm 0.19$

TABLE II: Inclusive electron-to-muon cross-section ratios for neutrinos and antineutrinos without kinematic cuts. Uncertainties at leading order are from vector and axial nucleon form factors. For the final result, we include an additional hadronic uncertainty from the one-loop correction to the first uncertainty, and provide a second uncertainty as the magnitude of the radiative correction.

$$\frac{\sigma(m_\ell \rightarrow 0)}{\sigma(m_\ell = 0)} \approx 1 + Am_\ell^2 + \alpha Bm_\ell^2 \ln m_\ell$$

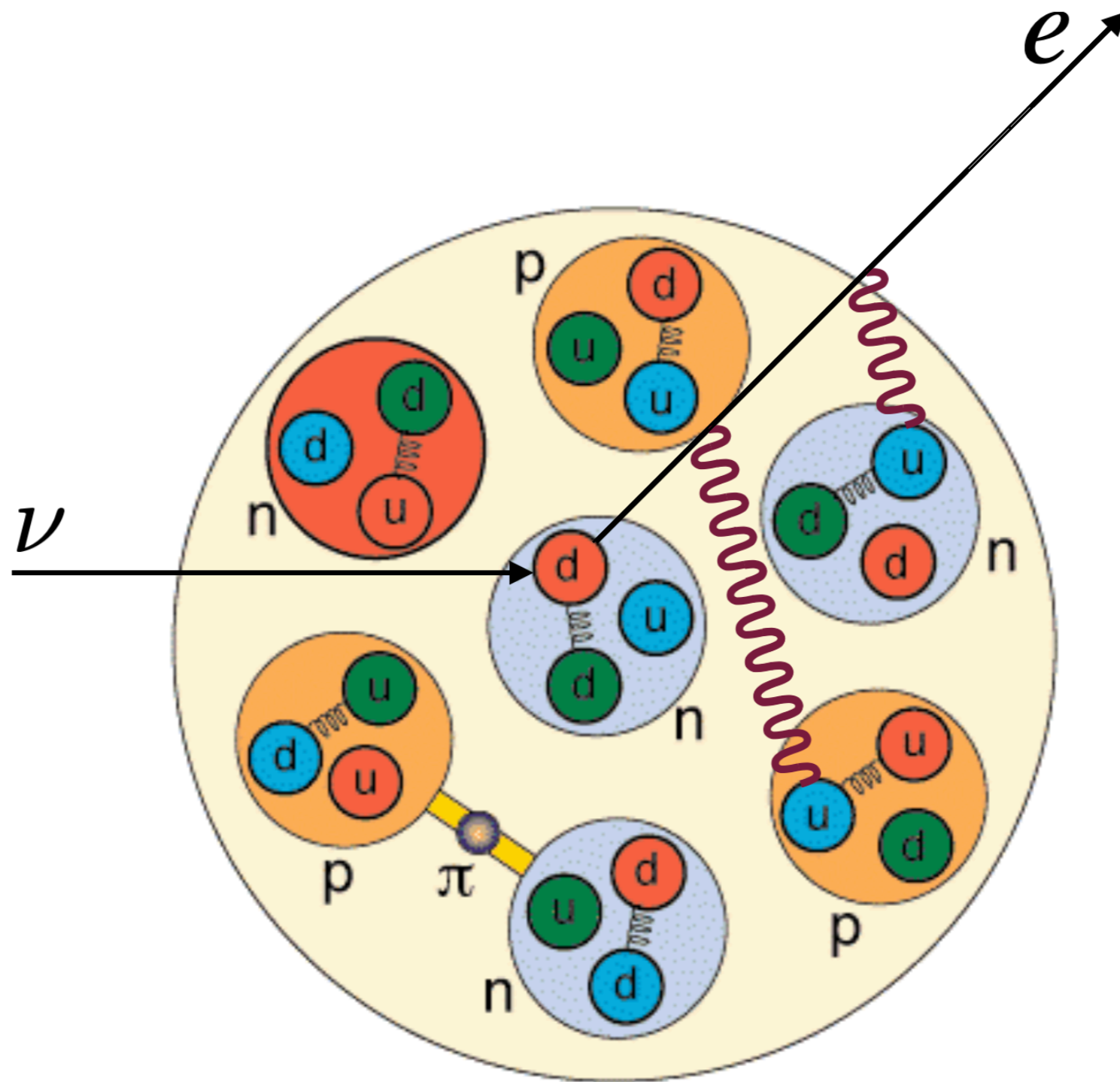
- inclusive cross sections and flavor ratios determined by KLN
- nuclear effects: suppressed by expansion parameters squared



QED nuclear medium effects in neutrino-nucleus and electron-nucleus scattering

O. T. and Ivan Vitev, Phys. Lett. B 805, 135466 (2022)

QED medium effects

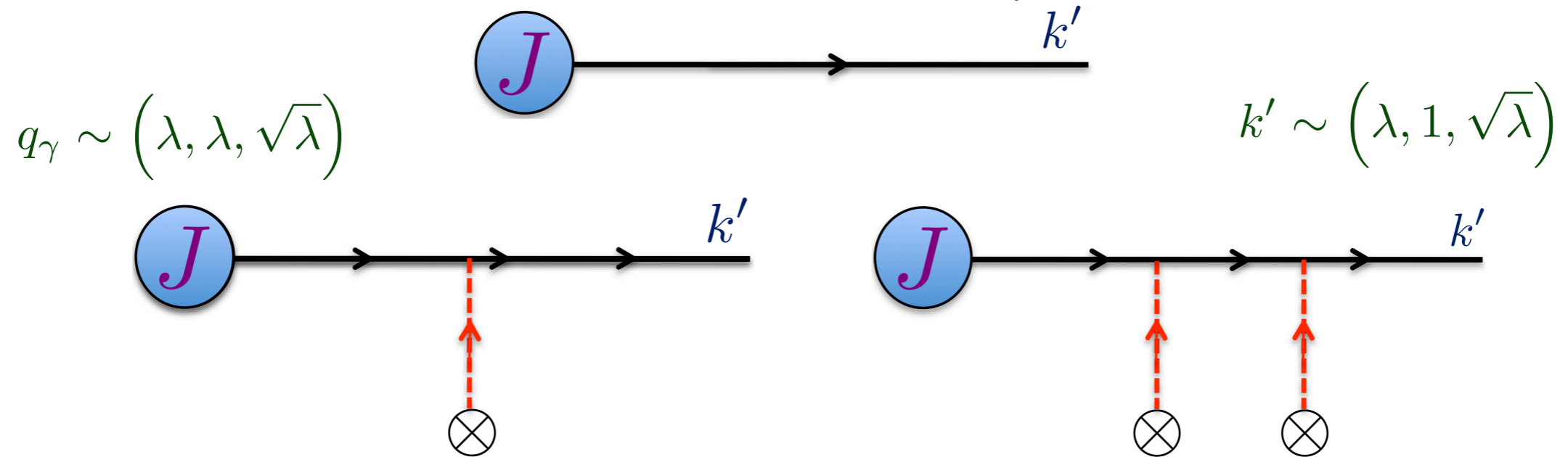


- charged lepton exchanges photons with nuclear medium

SCET_G formulation

- forward scattering is dominant process
- Glauber photons exchanged with a nuclear charge distribution

QCD: G. Ovanesyan and I. Vitev, JHEP 06 080 (2011)



- change: integral along final lepton direction over charge and potential

$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho(z) dz \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0(\vec{k}, \vec{k}' - \vec{q}_\perp) - \sigma_0(\vec{k}, \vec{k}') \right)$$

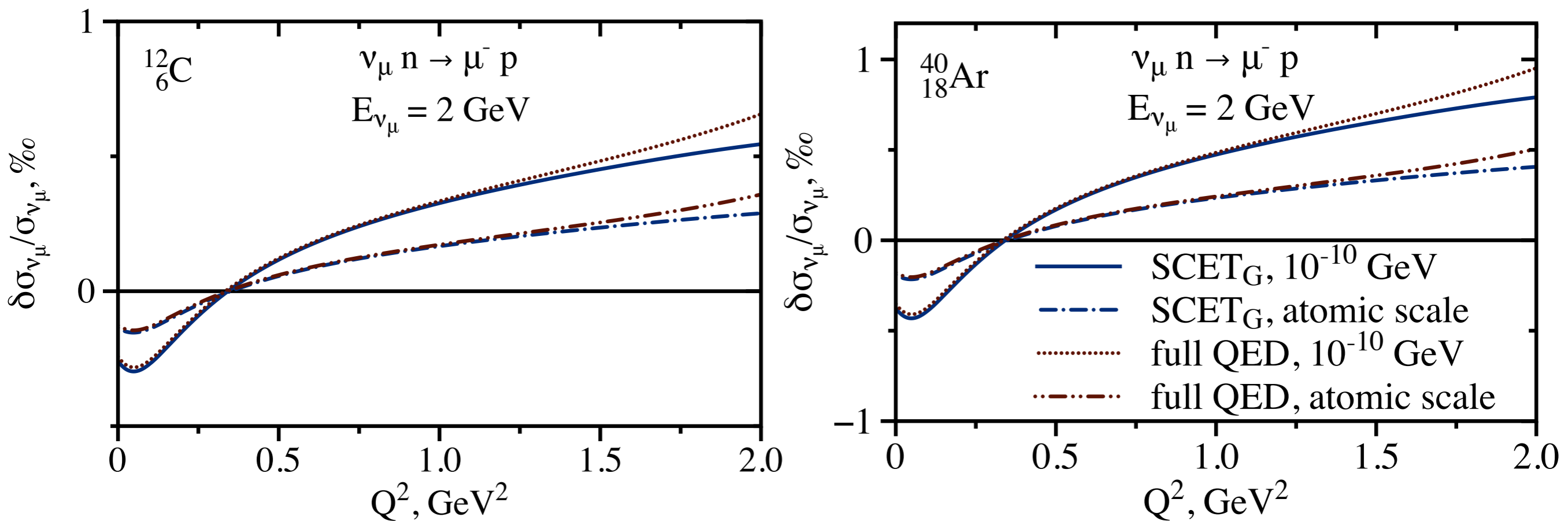
- leading-order cross sections are distorted
- EFT and full QED calculations are performed

Neutrino scattering

IR regularization

- relative correction per nucleon

$$v(q_{\perp}^2) = \frac{e^2}{q_{\perp}^2 + \lambda^2}$$



flavor-independent at GeV energies

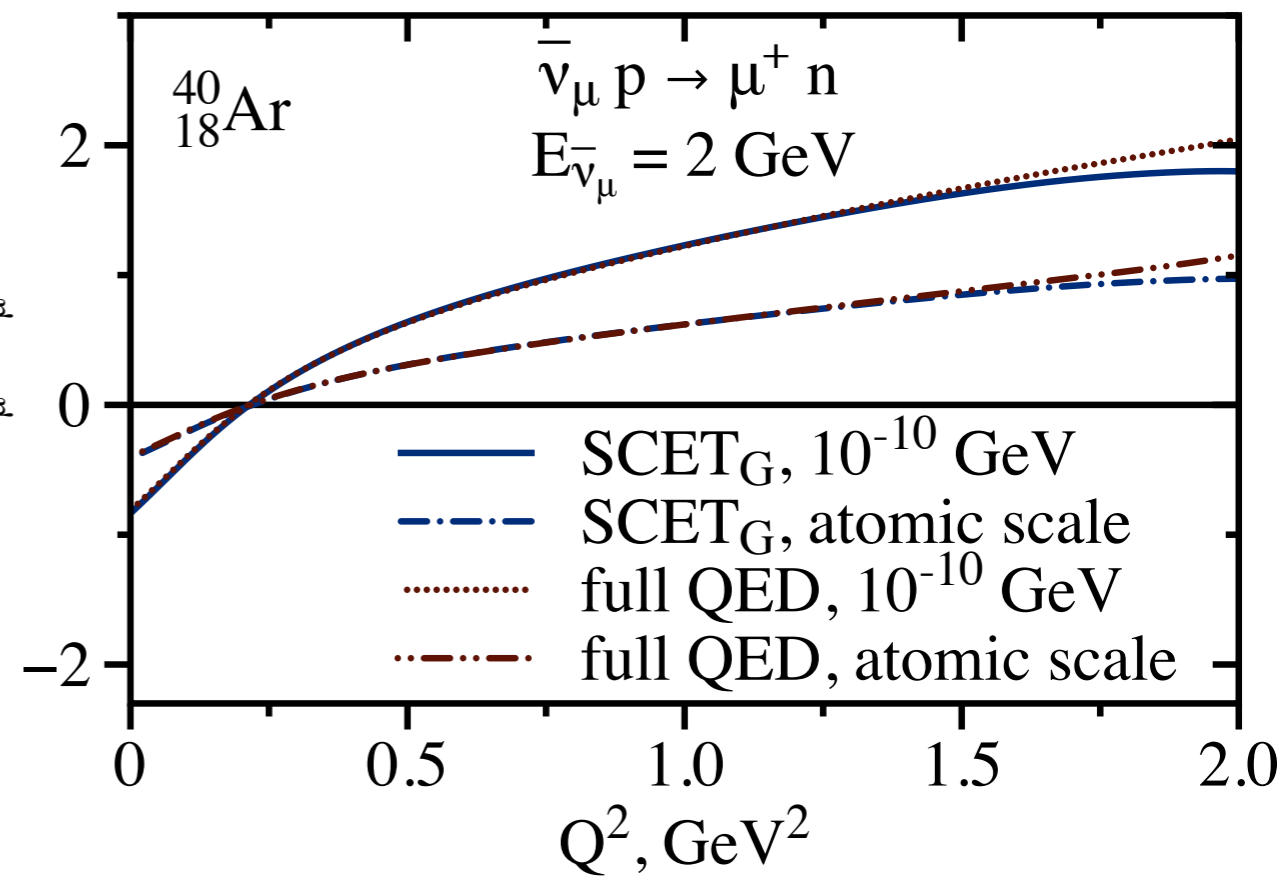
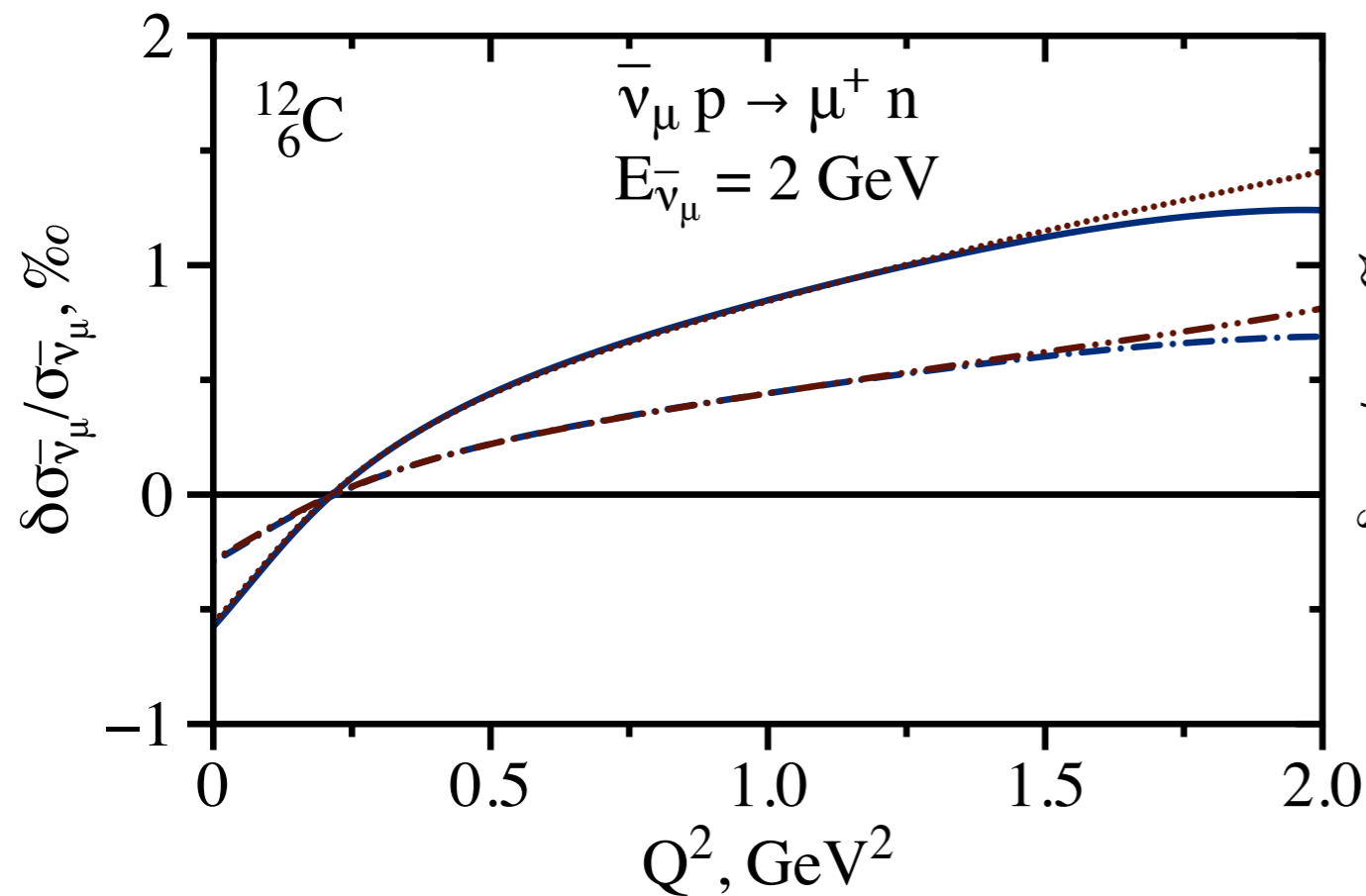
- permille-level distortion of cross sections: $\mathcal{O}(\alpha^2)$ correction
- smaller correction to inclusive cross section

Antineutrino scattering

- relative correction per nucleon

IR regularization

$$v(q_{\perp}^2) = \frac{e^2}{q_{\perp}^2 + \lambda^2}$$



flavor-independent at GeV energies

- permille-level distortion of cross sections: $\mathcal{O}(\alpha^2)$ correction
- larger correction than for neutrino scattering

SCET_G formulation

- forward scattering is dominant process
- Glauber photons exchanged with a nuclear charge distribution
- add initial-state exchanges, no interference with final-state exchanges
- change: integral along initial lepton direction over charge and potential

$$\delta\sigma_i \sim \int_{\text{lepton line}}^{\text{initial}} \rho(z) dz \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0(\vec{k} + \vec{q}_\perp, \vec{k}') - \sigma_0(\vec{k}, \vec{k}') \right)$$

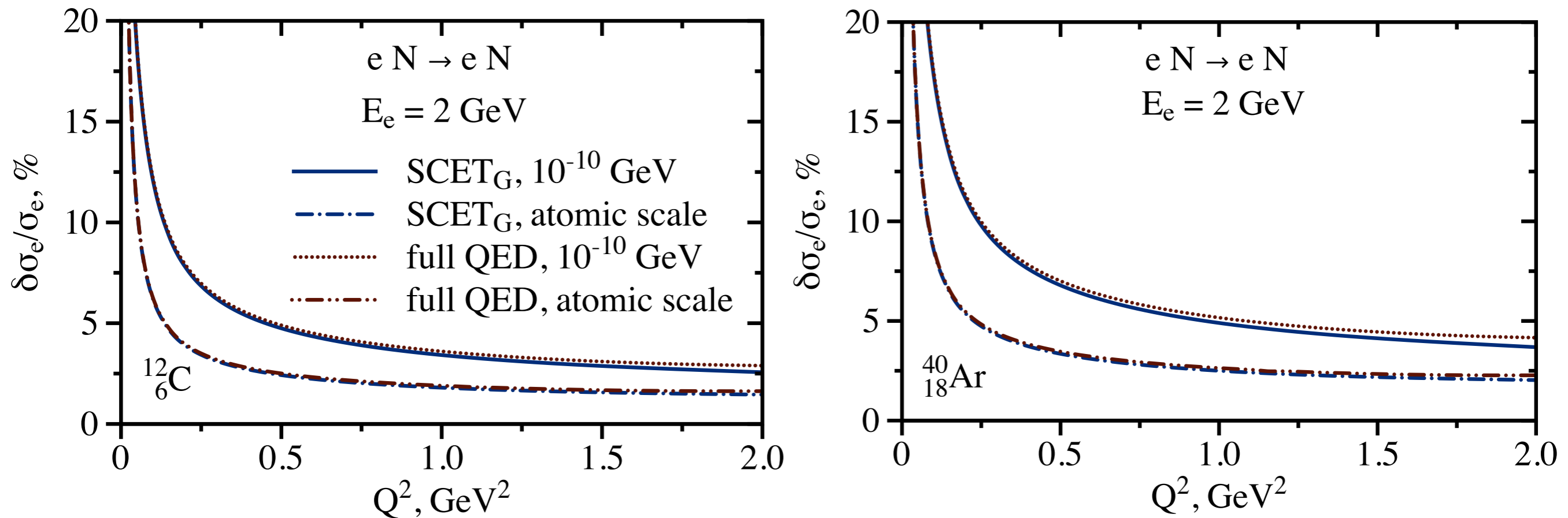
- change: integral along final lepton direction over charge and potential

$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho(z) dz \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0(\vec{k}, \vec{k}' - \vec{q}_\perp) - \sigma_0(\vec{k}, \vec{k}') \right)$$

- leading-order cross sections are modified
- EFT and full QED agree at GeV energies and above

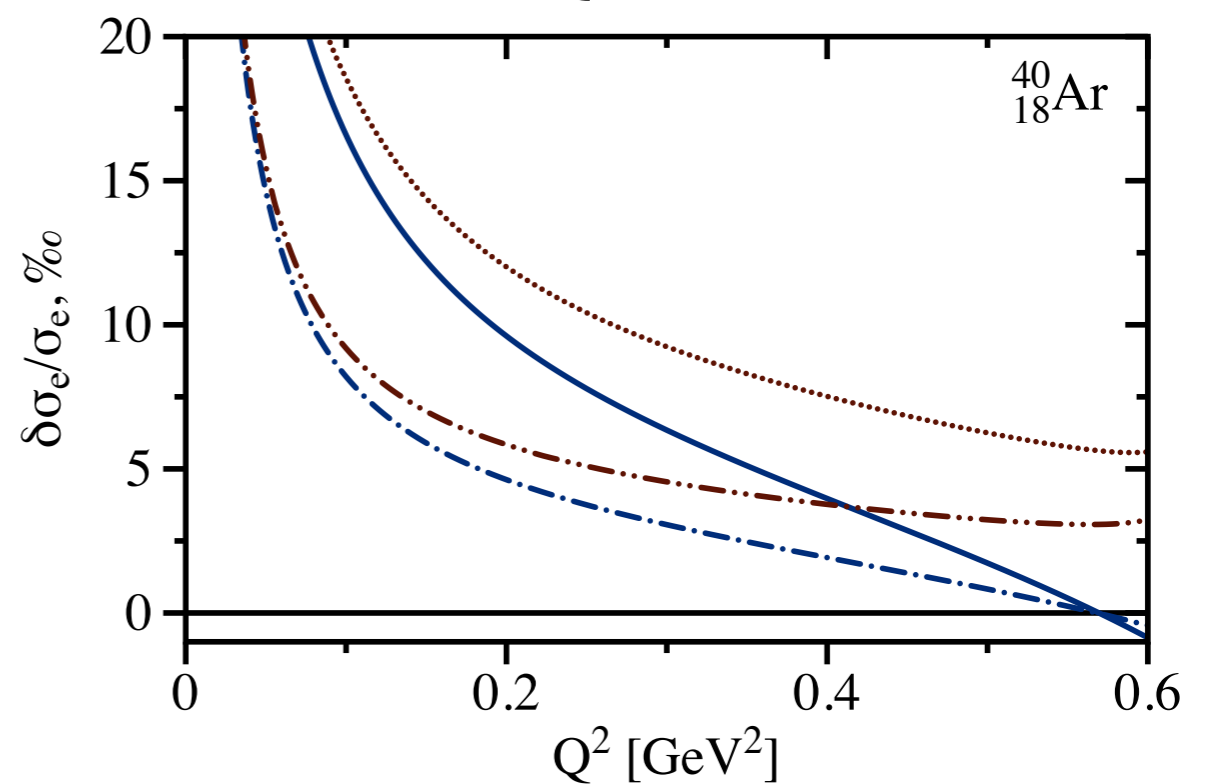
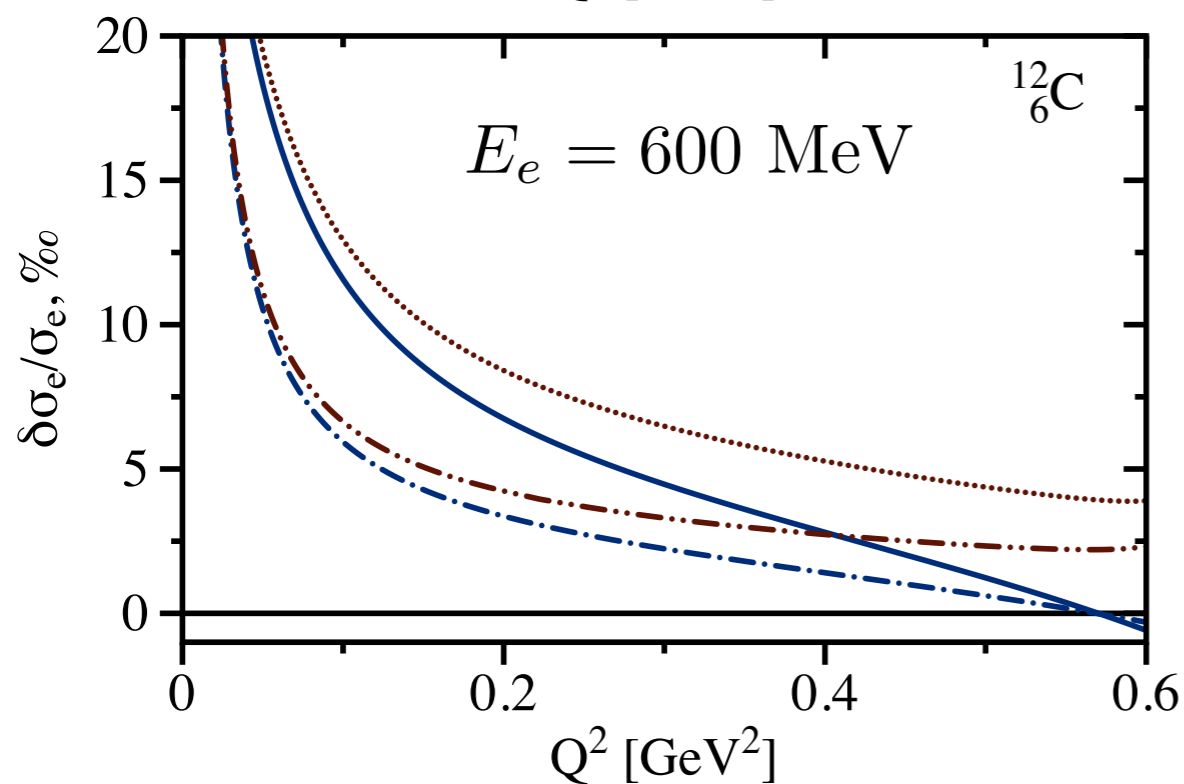
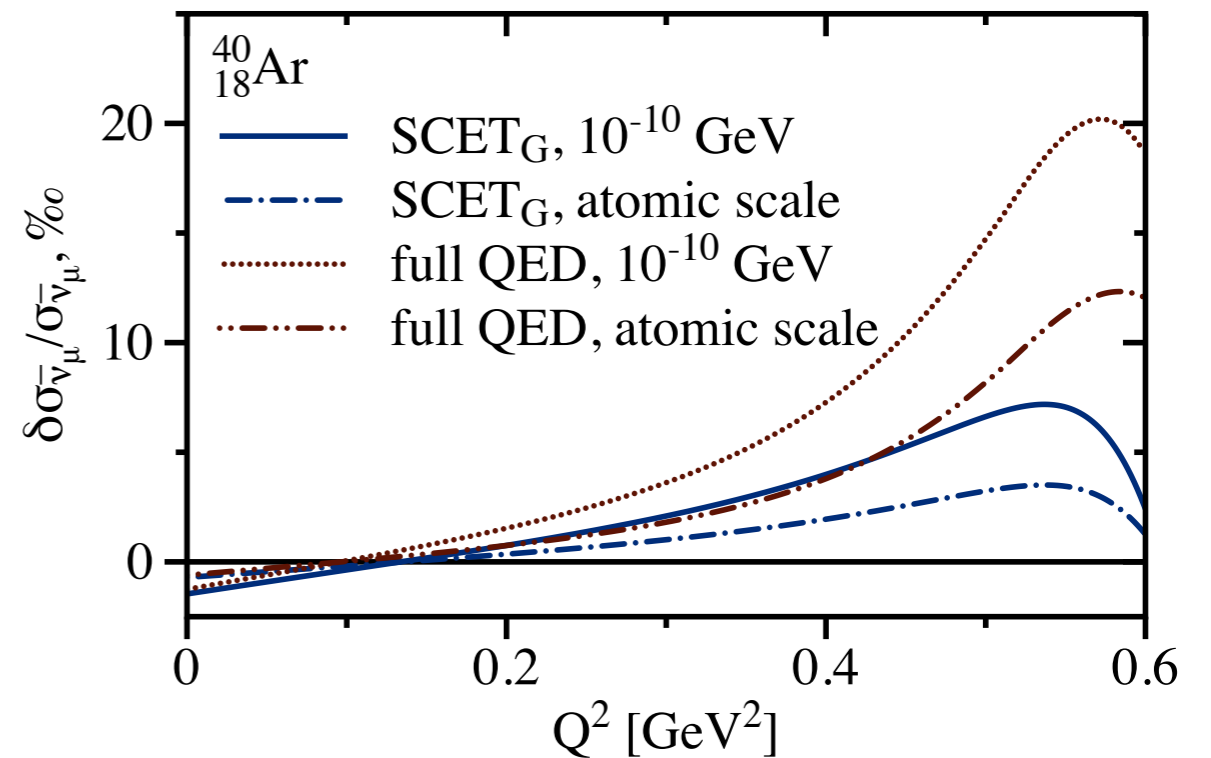
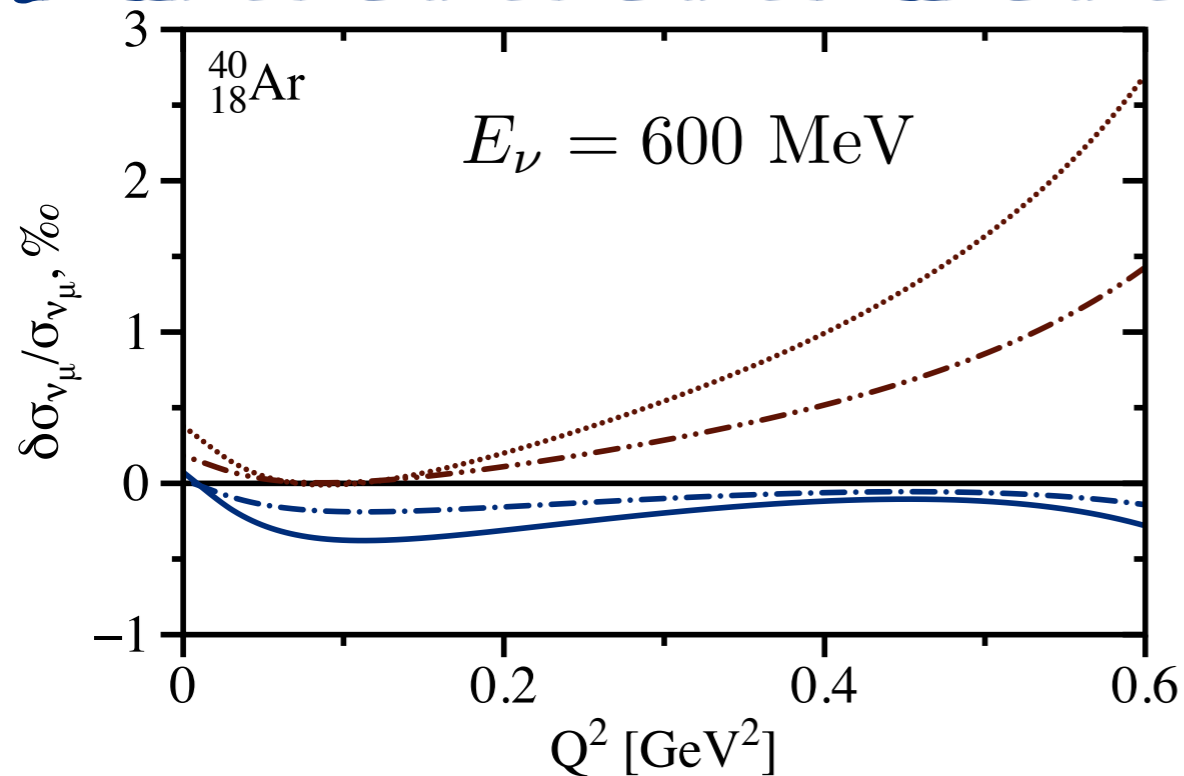
Electron scattering

- relative correction per nucleus after incoherent sum over nucleons



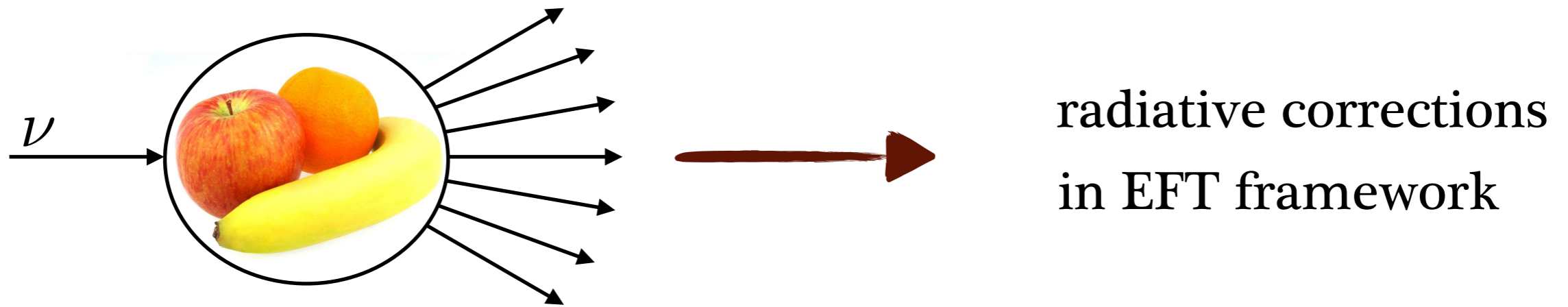
- **percent-level** at low momentum transfers: $O(\alpha^2)$ correction
- **critical new effect** for electron scattering experiments

QED nuclear medium effects at low energy



- EFT breaks: mass and kinematic effects

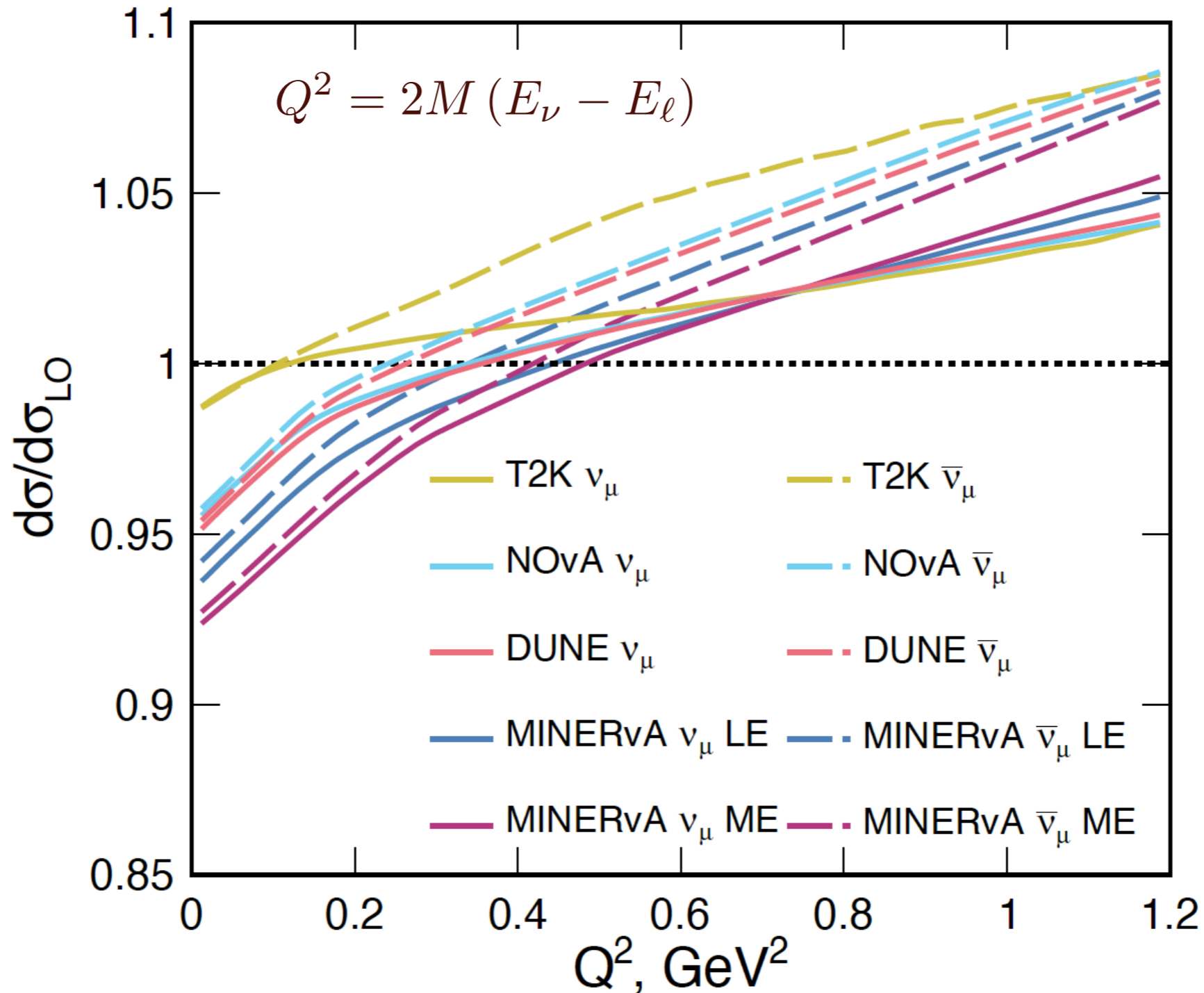
Conclusions



- radiative corrections to neutrino-nucleon cross sections formulated in factorization framework
- charged-current elastic electron vs muon cross-section ratios evaluated from theory with sub-percent uncertainty
- permille-level QED nuclear medium effects in neutrino scattering
- permille- to percent-level QED corrections in electron scattering

Thanks for your attention !!!

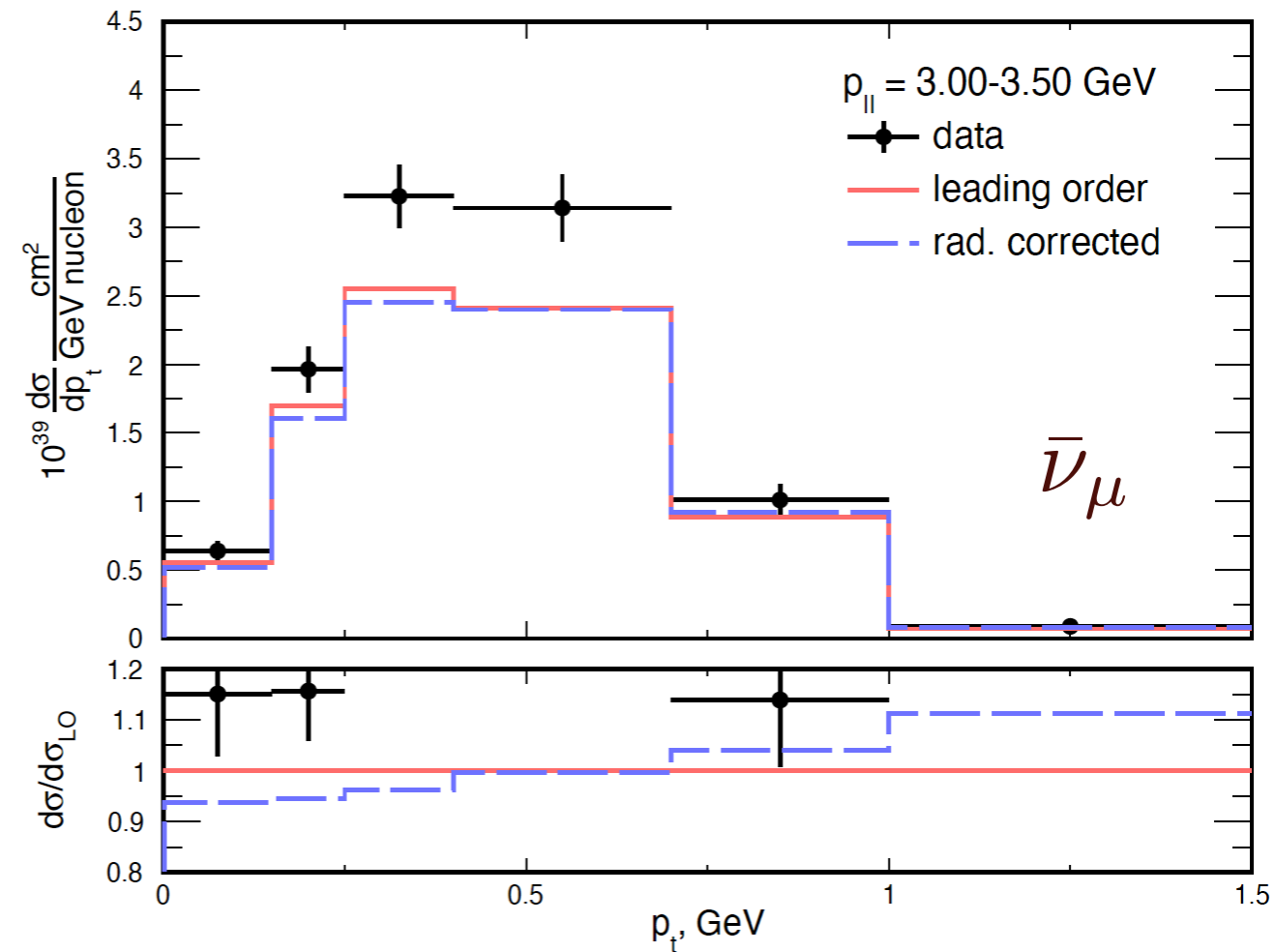
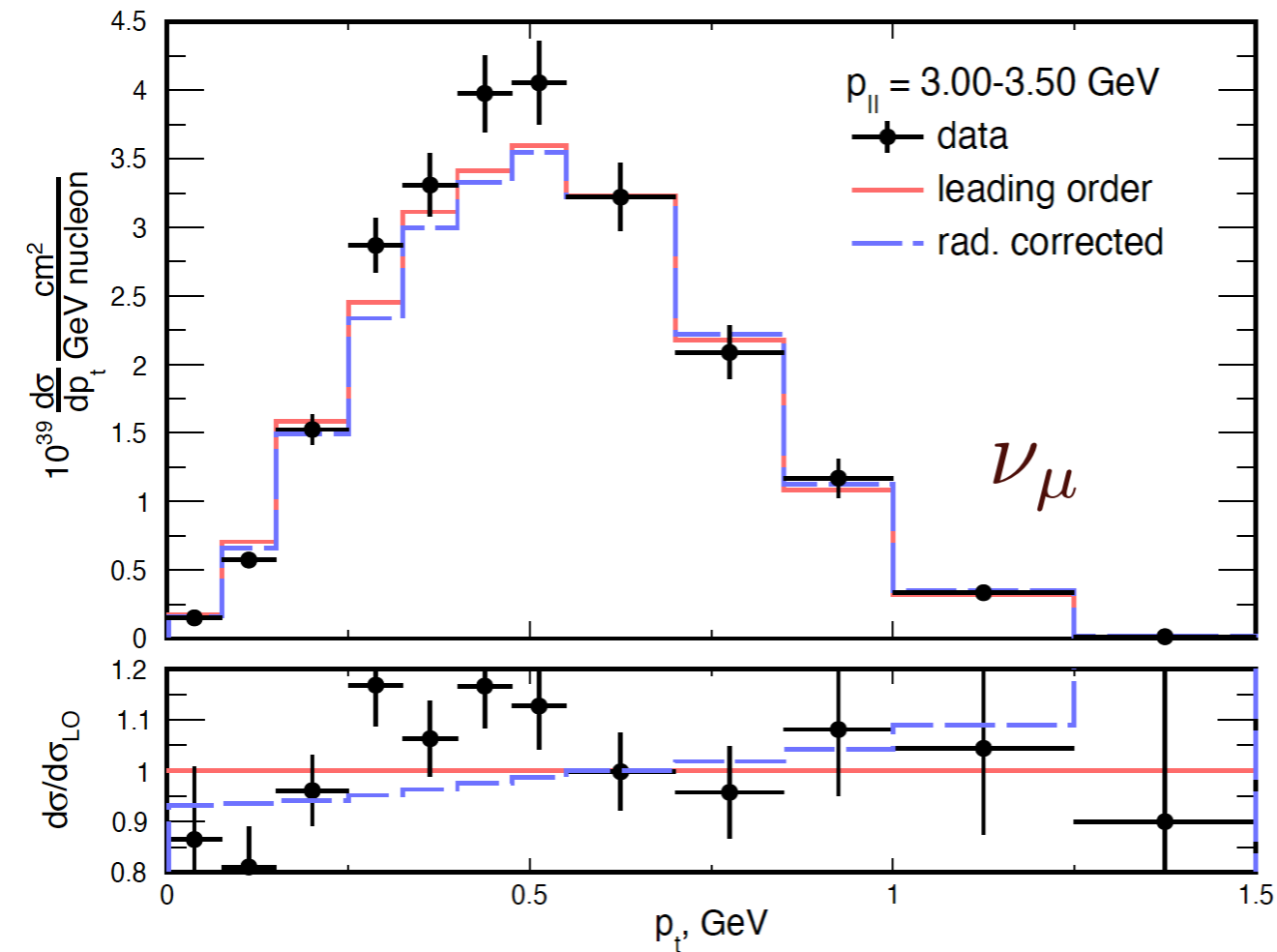
Comparison to data



- lepton energy spectra with kinematics from the lepton leg
- generator NEUT + flux average over typical experiments

Comparison to data

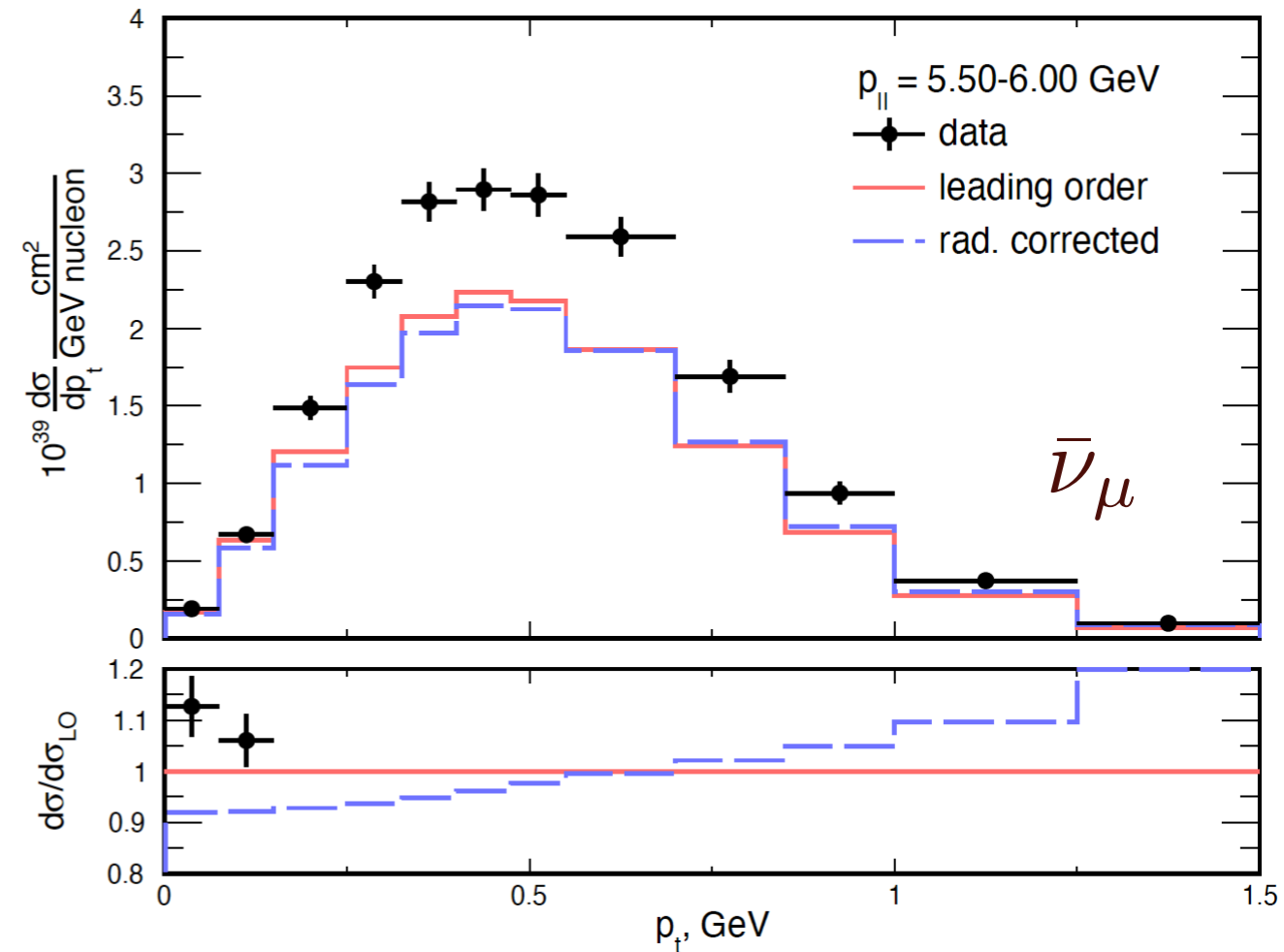
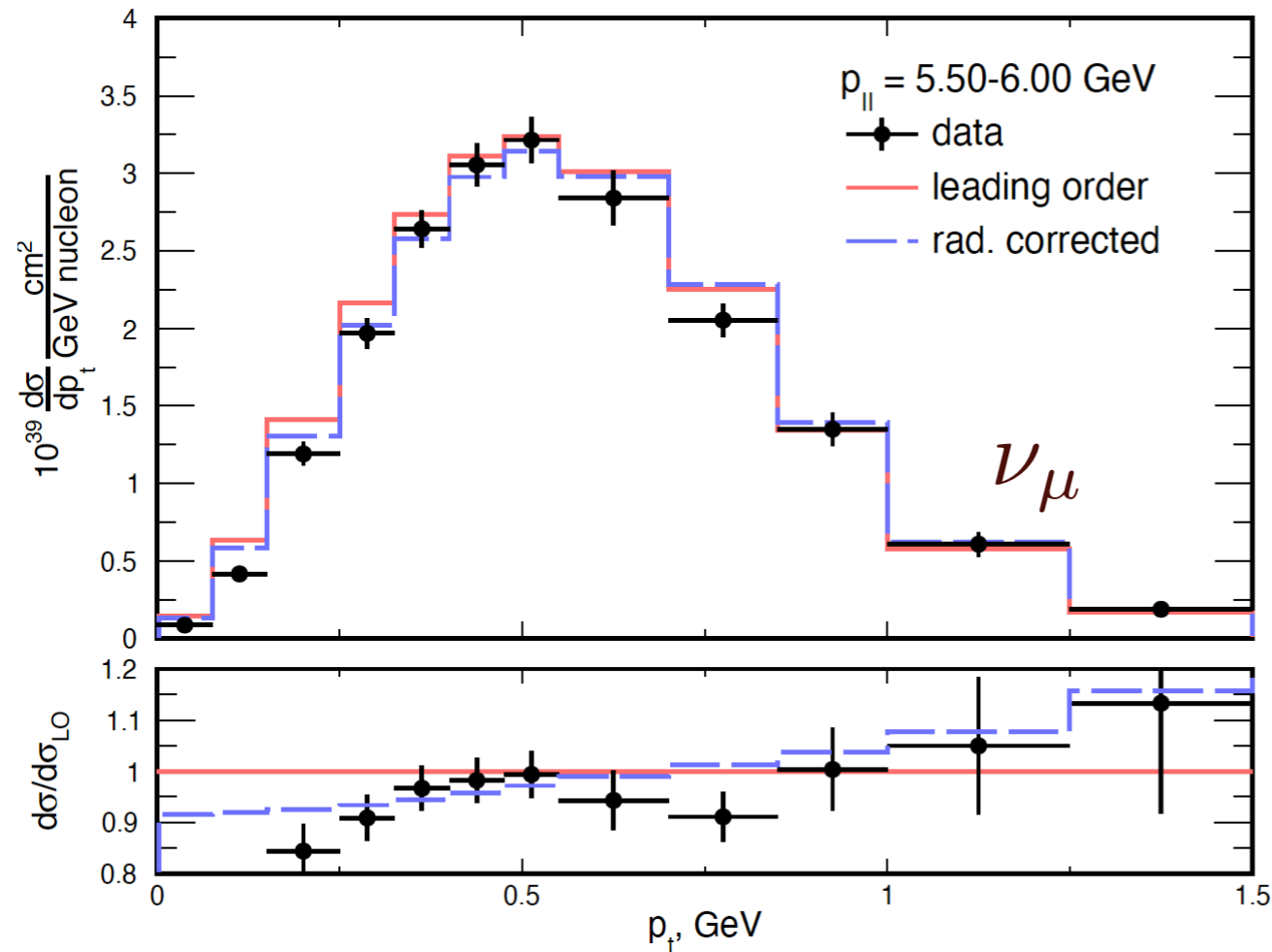
- low-energy flux data from MINERvA@FERMILAB



- electron flavor: measurements are uncertain
- muon flavor: comparable to experimental precision

Comparison to data

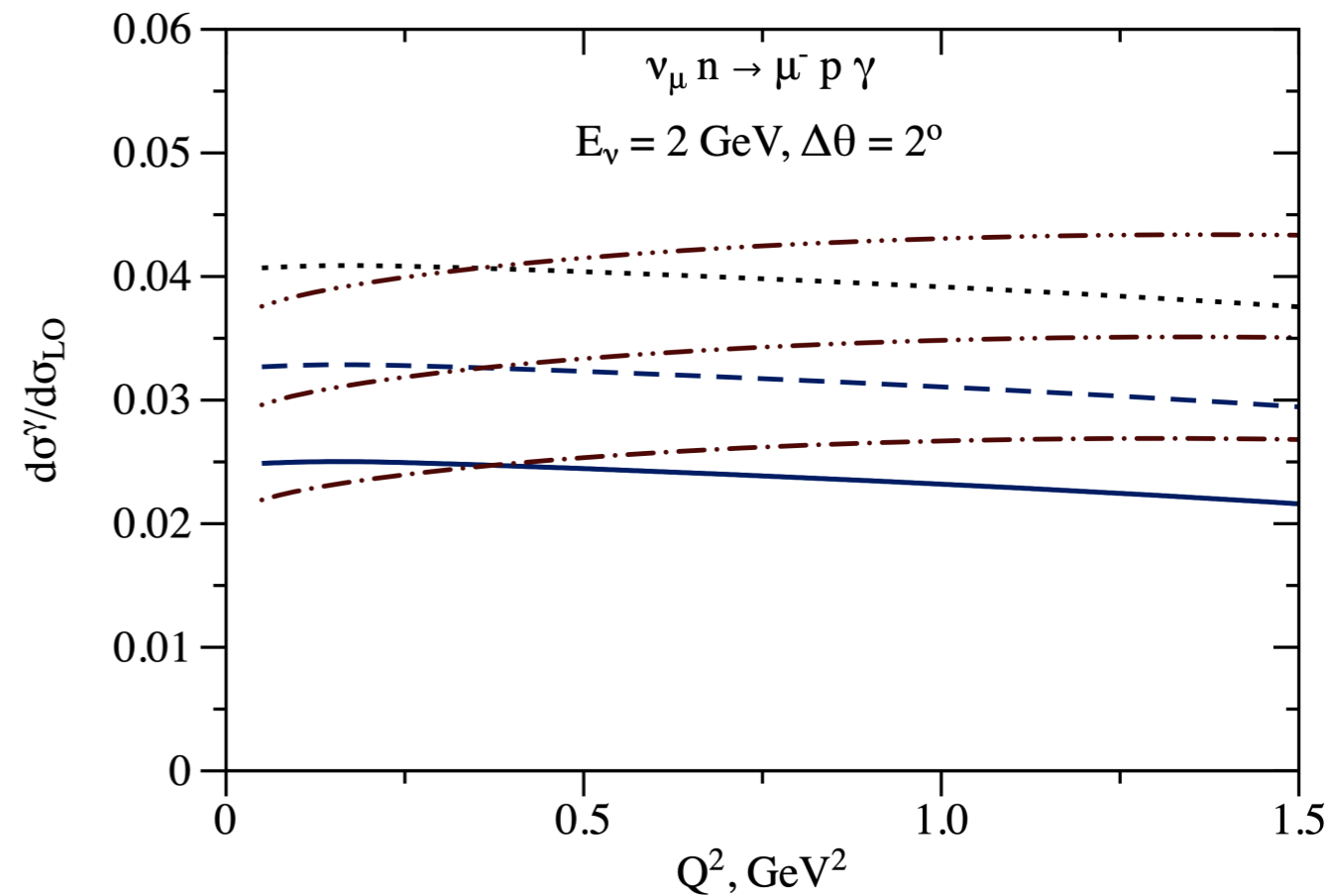
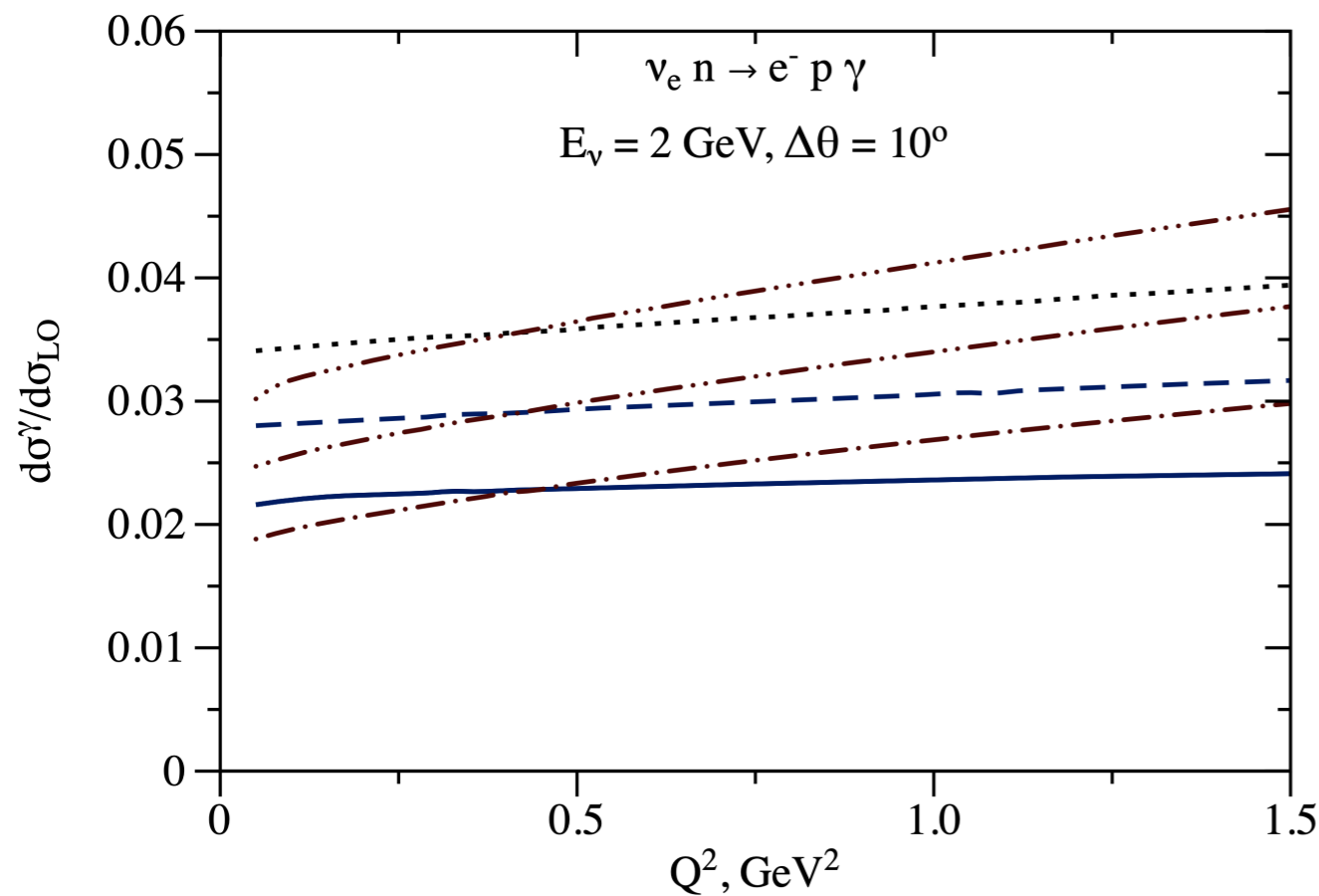
- medium-energy flux data from MINERvA@FERMILAB



- electron flavor: measurements are uncertain
- muon flavor: comparable to experimental precision

Radiation of hard photons

- model-dependent description for radiation of hard photons

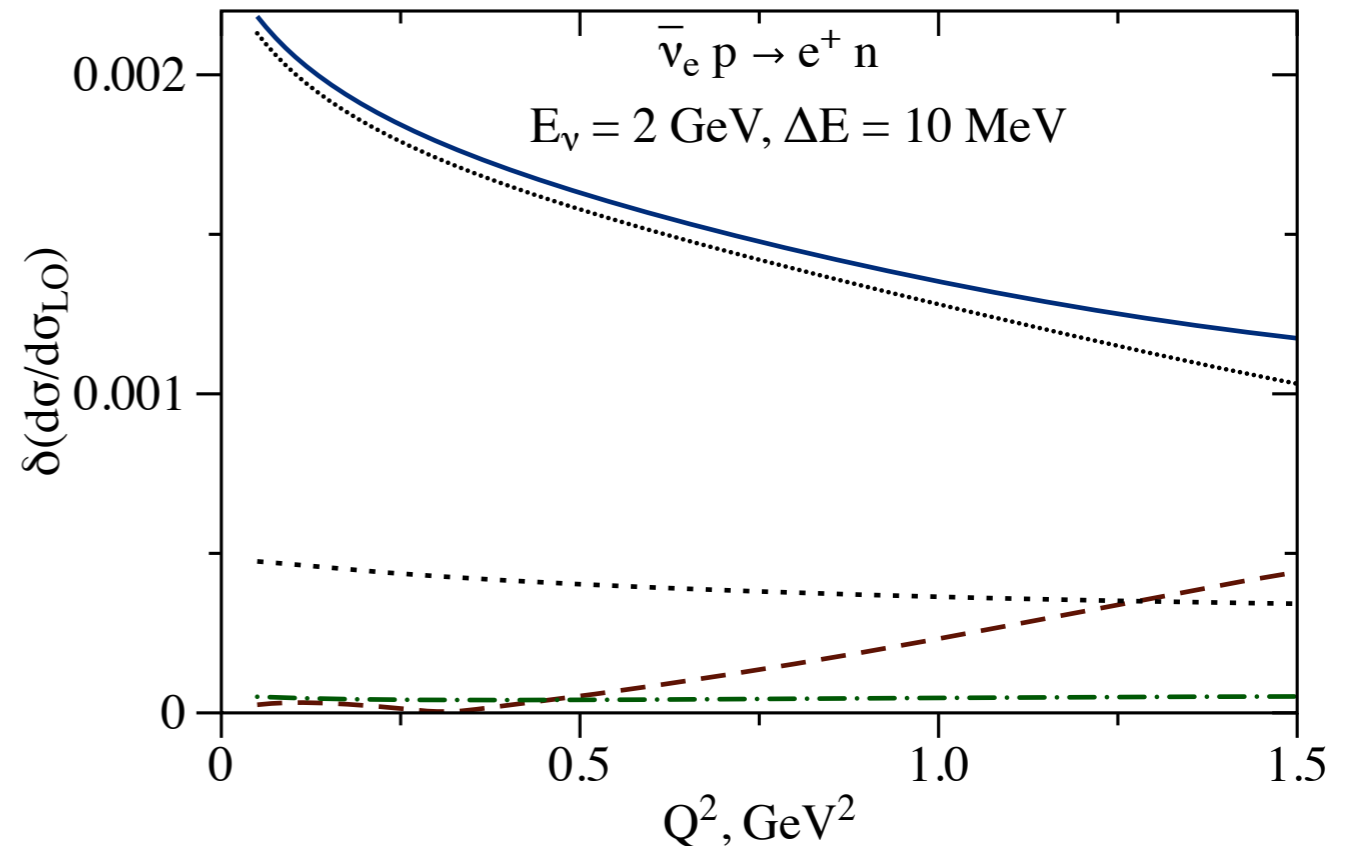
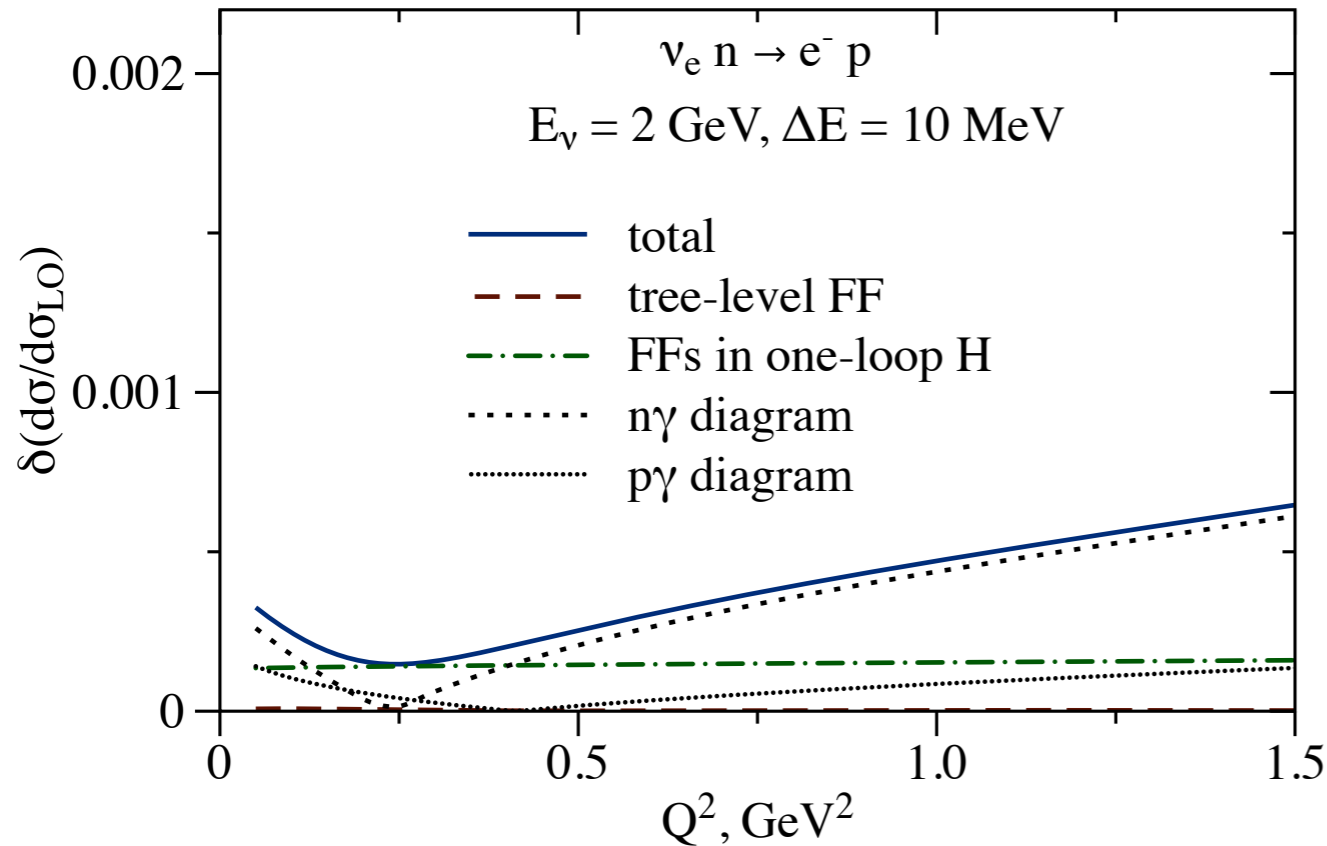


- photon energies are above 20, 40, and 80 MeV: default vs “Blunden calculation”
 “SIFF”
 “hadronic model”

- %-level radiation of non-collinear hard photons
- 10^{-4} flavor misidentification rate for NOvA&T2K kinematics

Error budget

- uncertainties from hard function



A.S. Meyer, M. Betancourt, R. Gran and R.J. Hill, Phys. Rev. D 93 11, 11305 (2016)

- nucleon form factors

Kaushik Borah, Gabriel Lee, Richard J. Hill and O.T., Phys. Rev. D 102 7, 074012 (2020)

- add perturbative uncertainty by variation of scale

- uncertainty of permille level for the ratio to LO result