

Measurement of the Coulomb Sum Rule and Suppression of the Longitudinal and Enhancement of the Transverse Quasielastic Cross section

From an analysis of all available electron scattering data on Carbon and Oxygen

A. Bodek¹ and M. E. Christy²

¹The University of Rochester, Rochester, NY, USA

Thomas Jefferson National Accelerator Facility, Newport News VA, USA

NuInt 2022 Seoul, Korea

24 Oct 2022, 21:00 Eastern Time 20m

The link is shown below.

<https://indico.cern.ch/event/881216/videoconference/>

passcode: 10242022

or direct link is

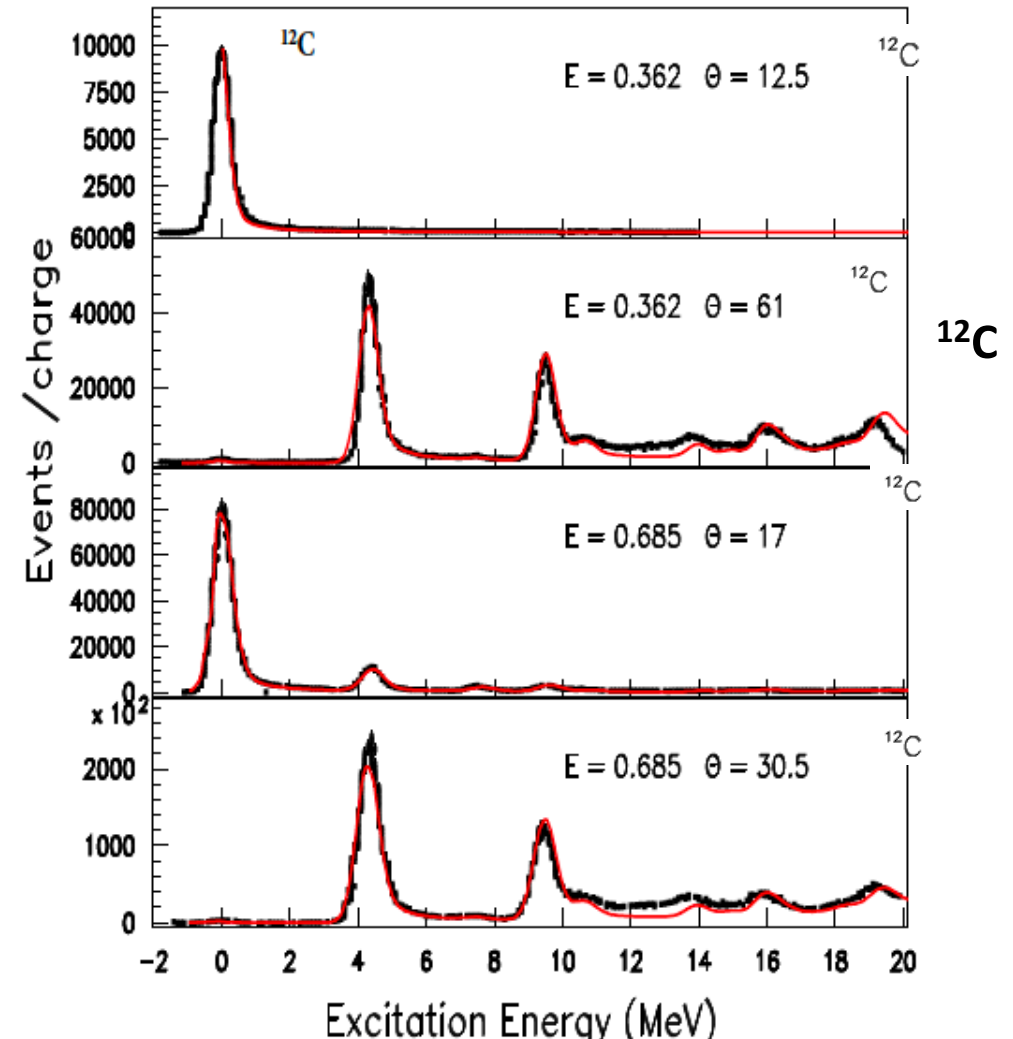
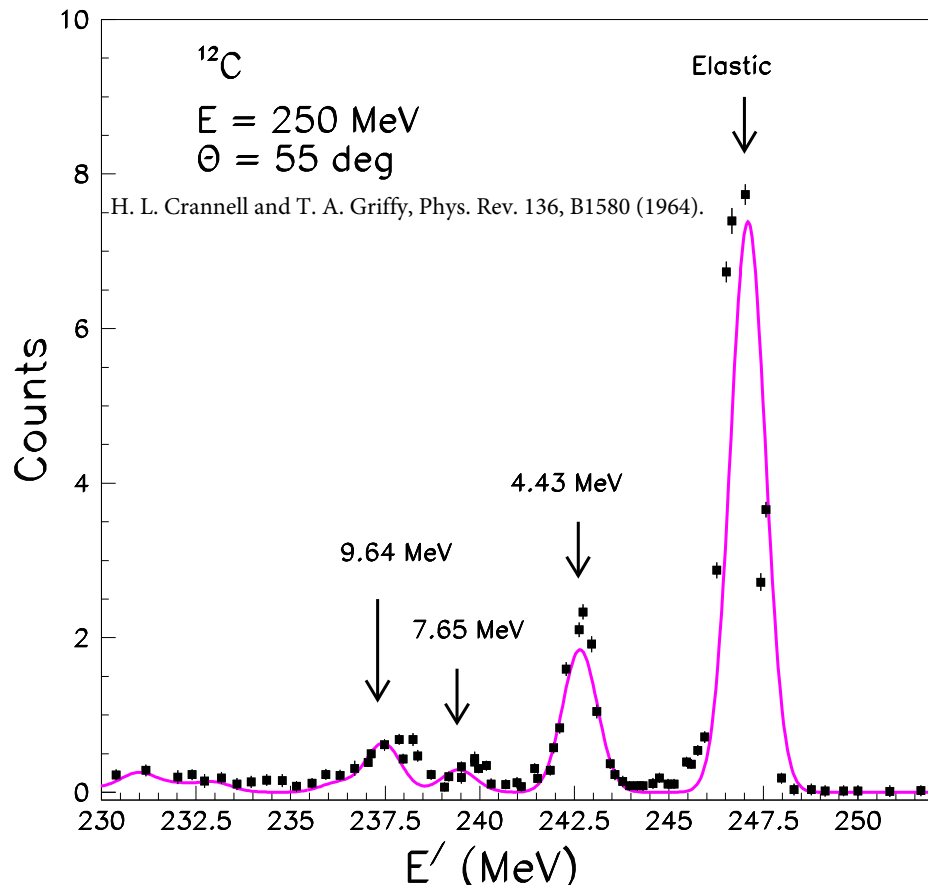
<https://cern.zoom.us/j/63191264477?pwd=NTd0aTFjWmYvdnp1WmswQnArQVhSUT09>

This talk reports on analysis of all available H, D, Carbon and Oxygen electron scattering data (Analysis will be expanded to all nuclei)

- We update the Bosted-Christy fit to **all of the world's electron scattering data** on H, D and nuclear targets to include the **lowest values of energy transfer ν and q^2** (for **C12** we fit **~8000 cross section data** and **250 measurements for O16**).
- We fit for: **QE cross section (including Transverse Enhancement/MEC and longitudinal low q Quenching)**. **Resonance and pion production**, **DIS**, **nuclear excitations**, elastic scattering data. Since the cross sections span a large range of energies and scattering angles, we **extract both the longitudinal RL and transverse RT** contributions, and also get the Coulomb Sum Rule.
- We parameterize both the **Enhancement of the Transverse QE cross section** and the **Quenching of the Longitudinal QE cross section**. **We extract the most precise Coulomb Sum rule as a function of q and compare to theoretical calculations.**
-
- The fit can be used **in lieu of data to benchmark Monte Carlo predictions** (e.g. for e-H, e-D and e-¹²C and e-¹⁶O cross sections, and to is being used **compute radiative corrections for electron scattering experiments.**

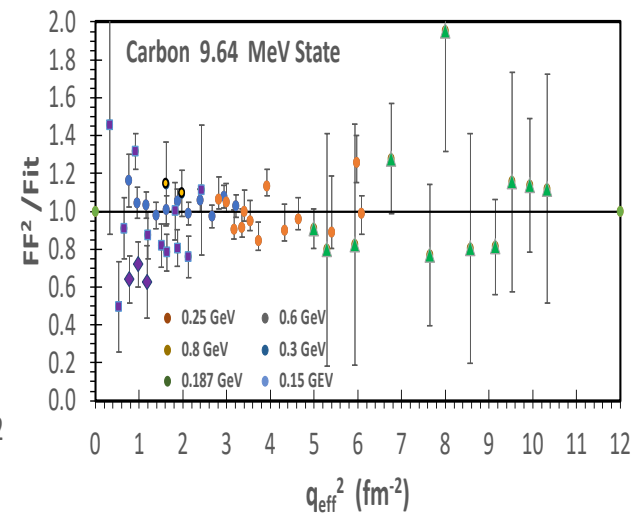
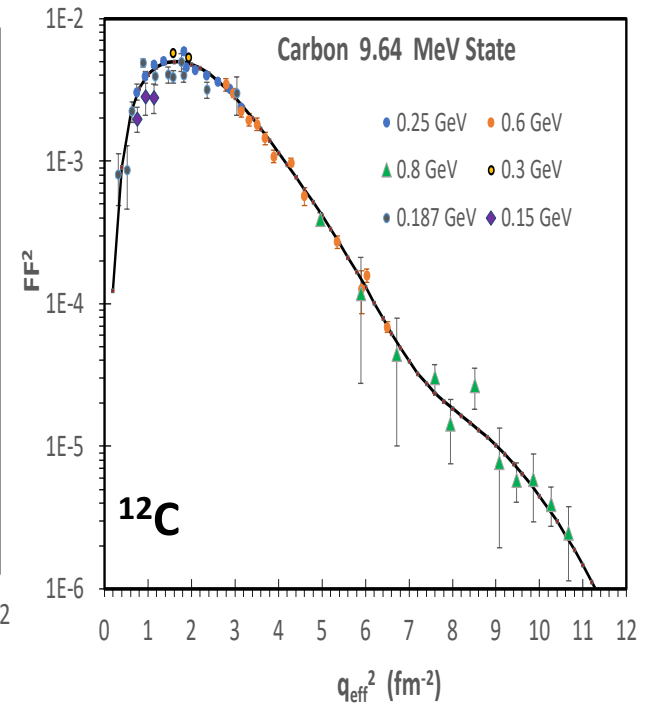
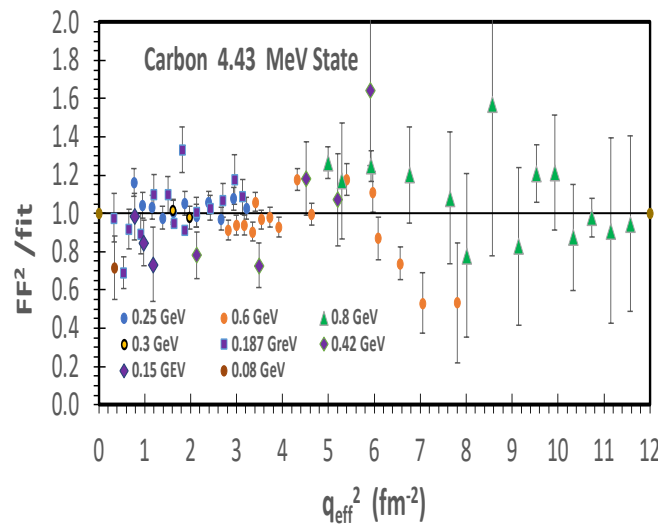
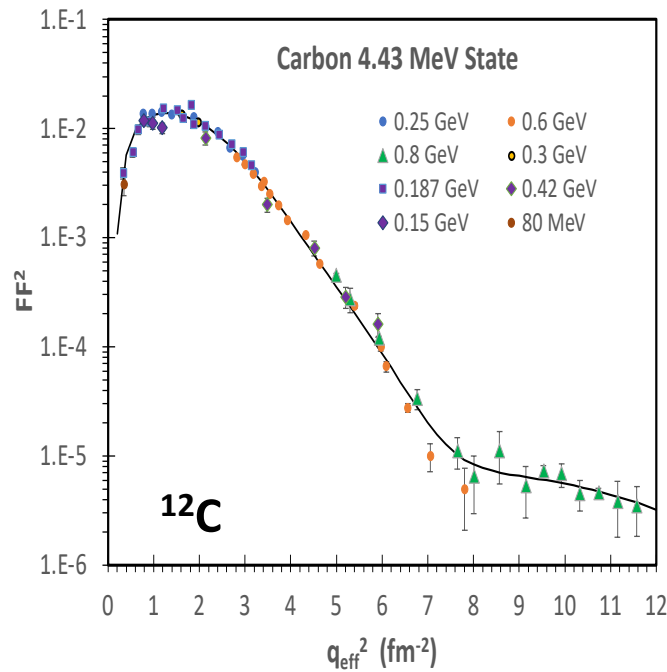
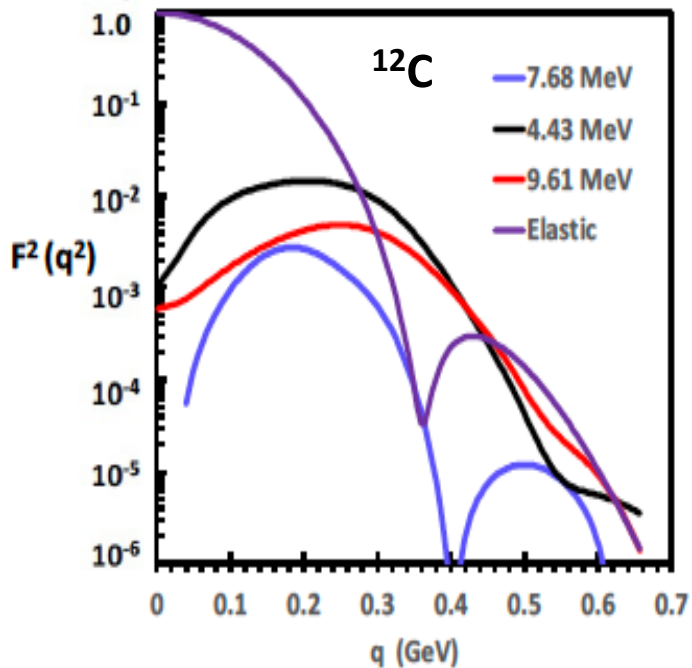
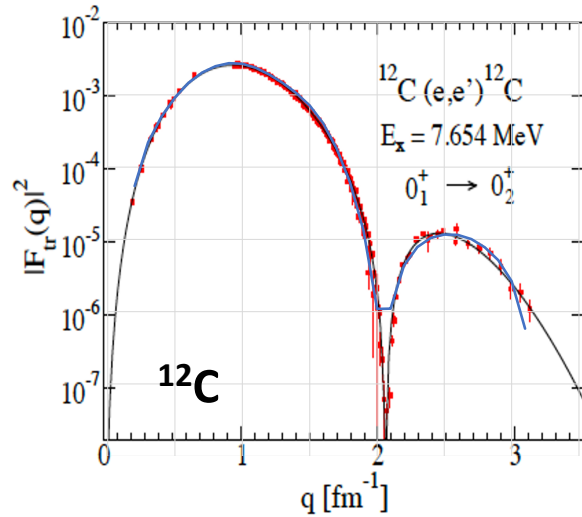
Nuclear excitations in Carbon

Note: Nuclear excitations are significant at low energy transfer ν and contribute up to 30% to the longitudinal Inelastic Coulomb Sum Rule (CSR)



We parameterize form-factors for elastic scattering and for 17 longitudinal and transverse nuclear excitation ($2 < \text{Excitation Energy} < 50 \text{ MeV}$) important since they contribute up to 30% to the Coulomb Sum Rule

Examples: Squares of Elastic form factor and the form factors for the first 3 nuclear excitations (all are longitudinal)



We parameterize form-factors for elastic scattering and for 17 longitudinal and transverse nuclear excitations ($2 < \text{Excitation Energy} < 50 \text{ MeV}$)

Cross sections for excitations less than 10 MeV multiplied by (1/6)

Nuclear excitation region

Ex < 50 MeV

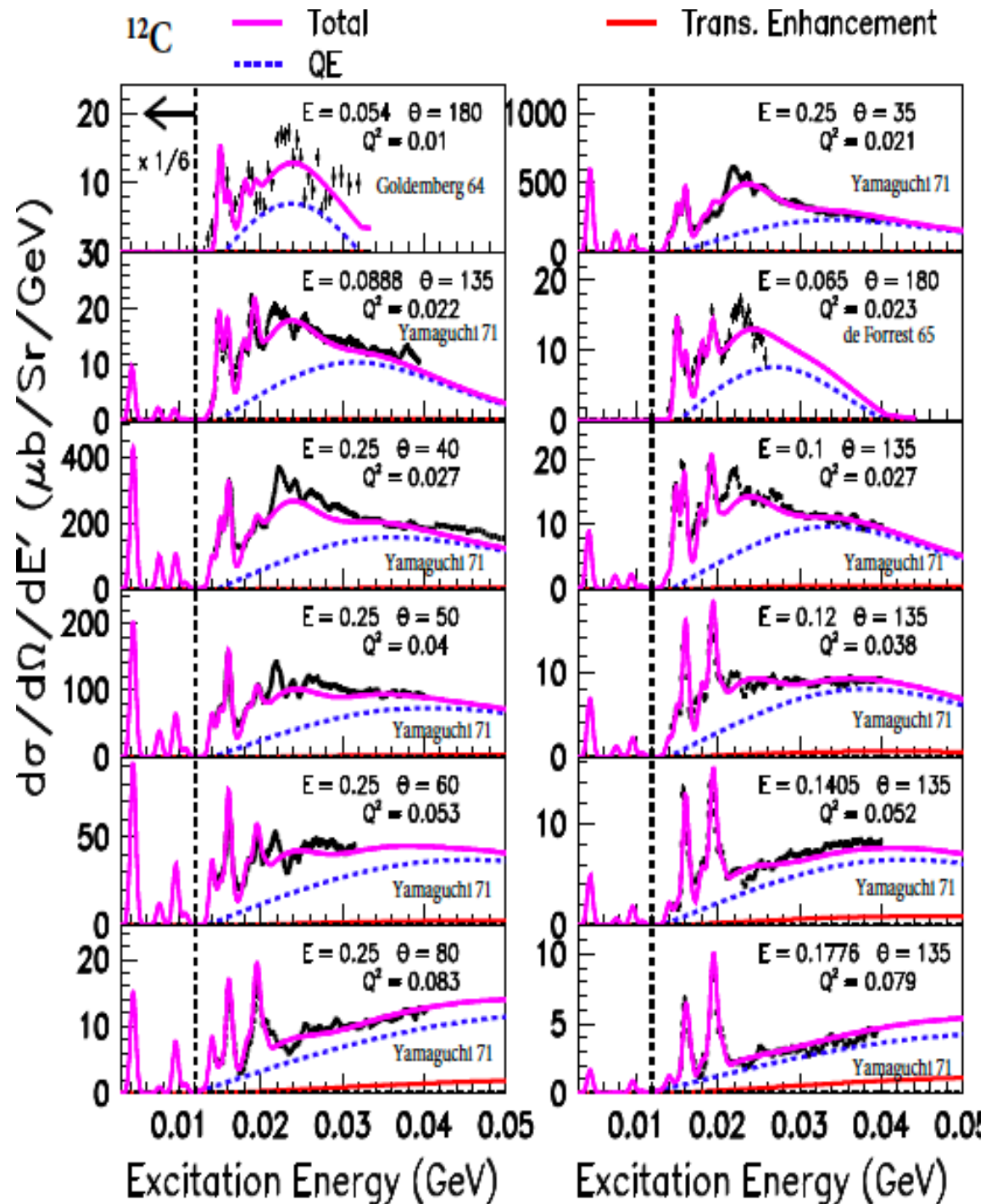
Comparison of our fit to representative e-C12 data for $0.01 < q^2 < 0.08 \text{ GeV}^2$.

Shown: Total including excitations : solid -----

Quasielastic (QE) contribution: dashed-----

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of TTransverse QE response dashed-----

Electron scattering cross sections



Coulomb Sum rule: Contribution of nuclear excitations

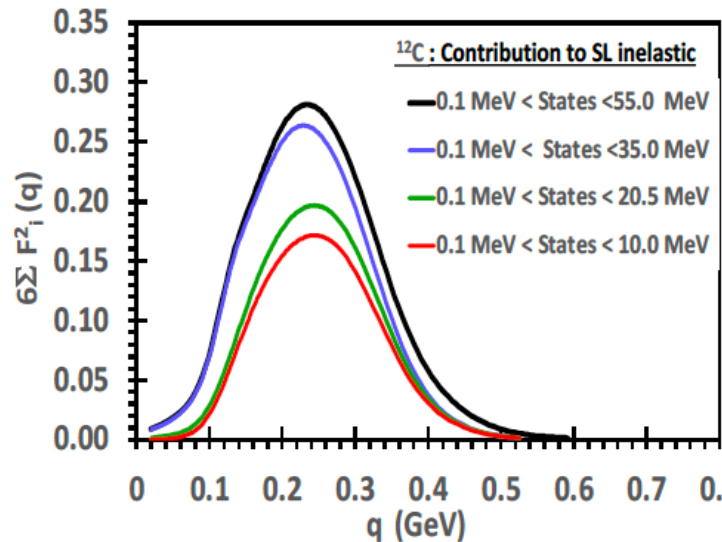
The electron scattering differential cross section can be written in terms of longitudinal ($R_L(\mathbf{q}, \nu)$) and transverse ($R_T(\mathbf{q}, \nu)$) response functions [7]:

$$\frac{d\sigma}{d\nu d\Omega} = \sigma_M [AR_L(\mathbf{q}, \nu) + BR_T(\mathbf{q}, \nu)], \quad (1)$$

$$\sigma_M = \frac{\alpha^2 \cos^2(\theta/2)}{4E_0^2 \sin^4(\theta/2)}$$

At very low q , $SL_{\text{inelastic}} = 0$

At high q expect $SL_{\text{inelastic}} = 1$



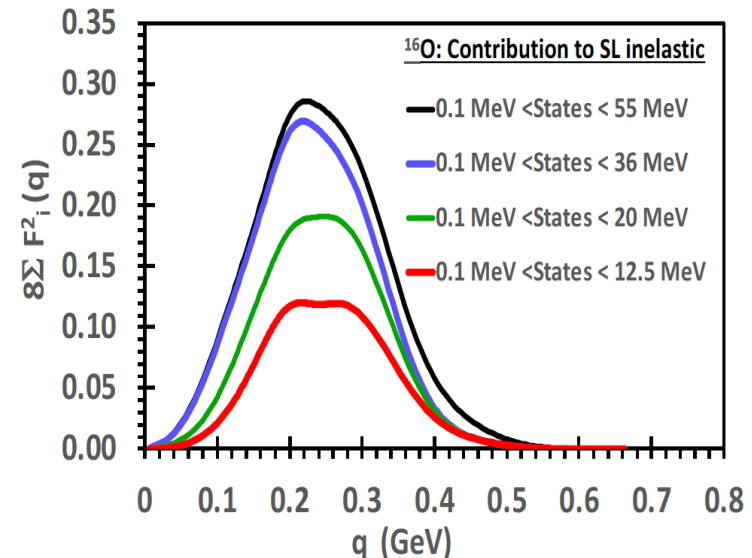
$$\text{CSR}(\mathbf{q}) = \int R_L(\mathbf{q}, \nu) d\nu \quad (3)$$

$$= \int R_L^{QE}(\mathbf{q}, \nu) d\nu + GE'_p(q) \times Z^2 \sum_{\text{all}}^L F_i^2(\mathbf{q})$$

$$= GE'_p(q) \times [Z \int V_L^{QE} d\nu + Z^2 \sum_{\text{all}}^L F_i^2(\mathbf{q})]$$

By dividing by $ZGE'_p(\mathbf{q})$ we obtain the normalized inelastic Coulomb Sum Rule is:

$$SL(\mathbf{q}) = \int V_L^{QE}(\mathbf{q}, \nu) d\nu + Z \sum_{\text{all}}^L F_i^2(\mathbf{q}) \quad (5)$$



Nuclear excitations are significant at low energy transfer ν and contribute up to 30% to the longitudinal Inelastic Coulomb Sum Rule (CSR)

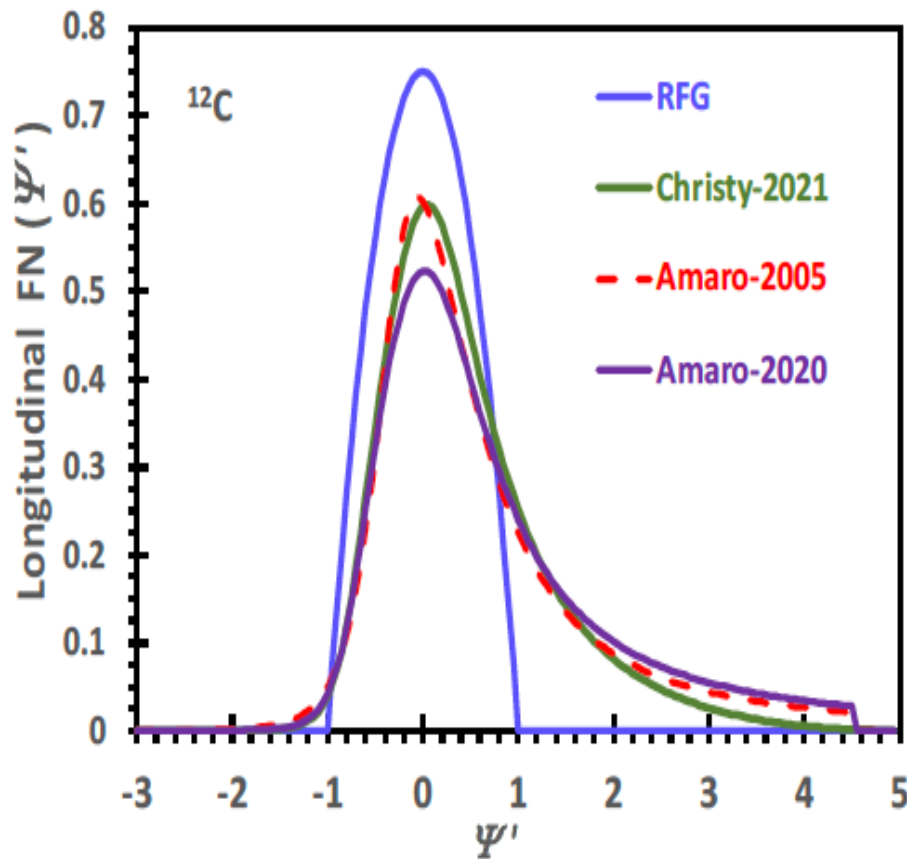
Modeling QE:

Use superscaling- Fit for the longitudinal scaling function parameters in the overall fit

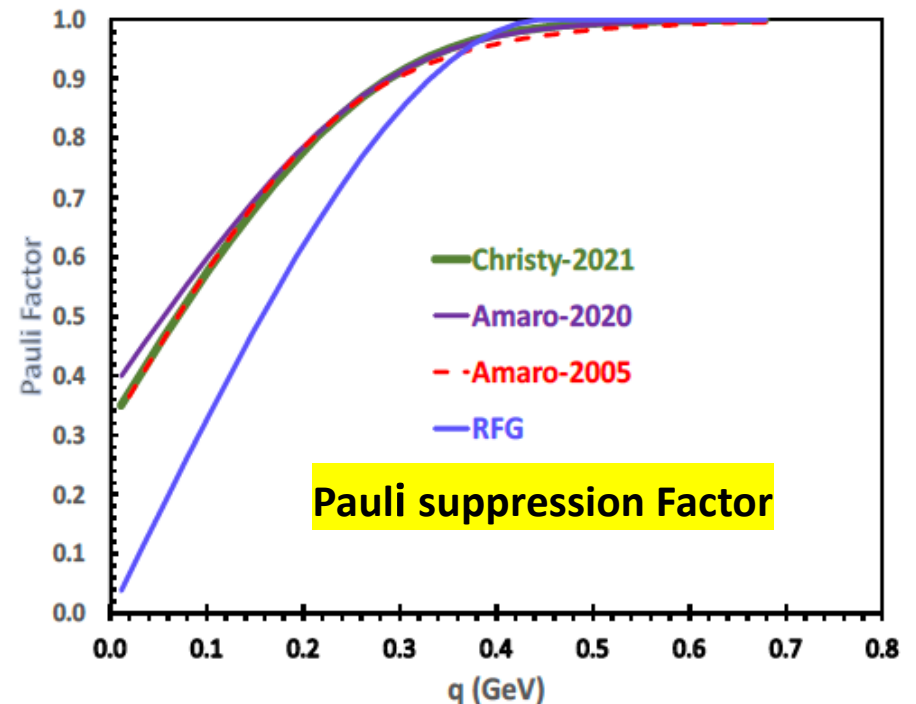
The ψ' superscaling variable is given by the following expression:

$$\psi' \equiv \frac{1}{\sqrt{\xi_F}} \frac{\lambda' - \tau'}{\sqrt{(1 + \lambda')\tau' + \kappa\sqrt{\tau'(1 + \tau')}}}, \quad (16)$$

where $\xi_F \equiv [\sqrt{1 + \eta_F^2} - 1]$, $\eta_F \equiv K_F/M_n$, $\lambda \equiv \nu/2M_n$, $\kappa \equiv |q|/2M_n$ and $\tau \equiv |Q^2|/4M_n^2 = \kappa^2 - \lambda^2$.



We include Rosenfelder Pauli suppression



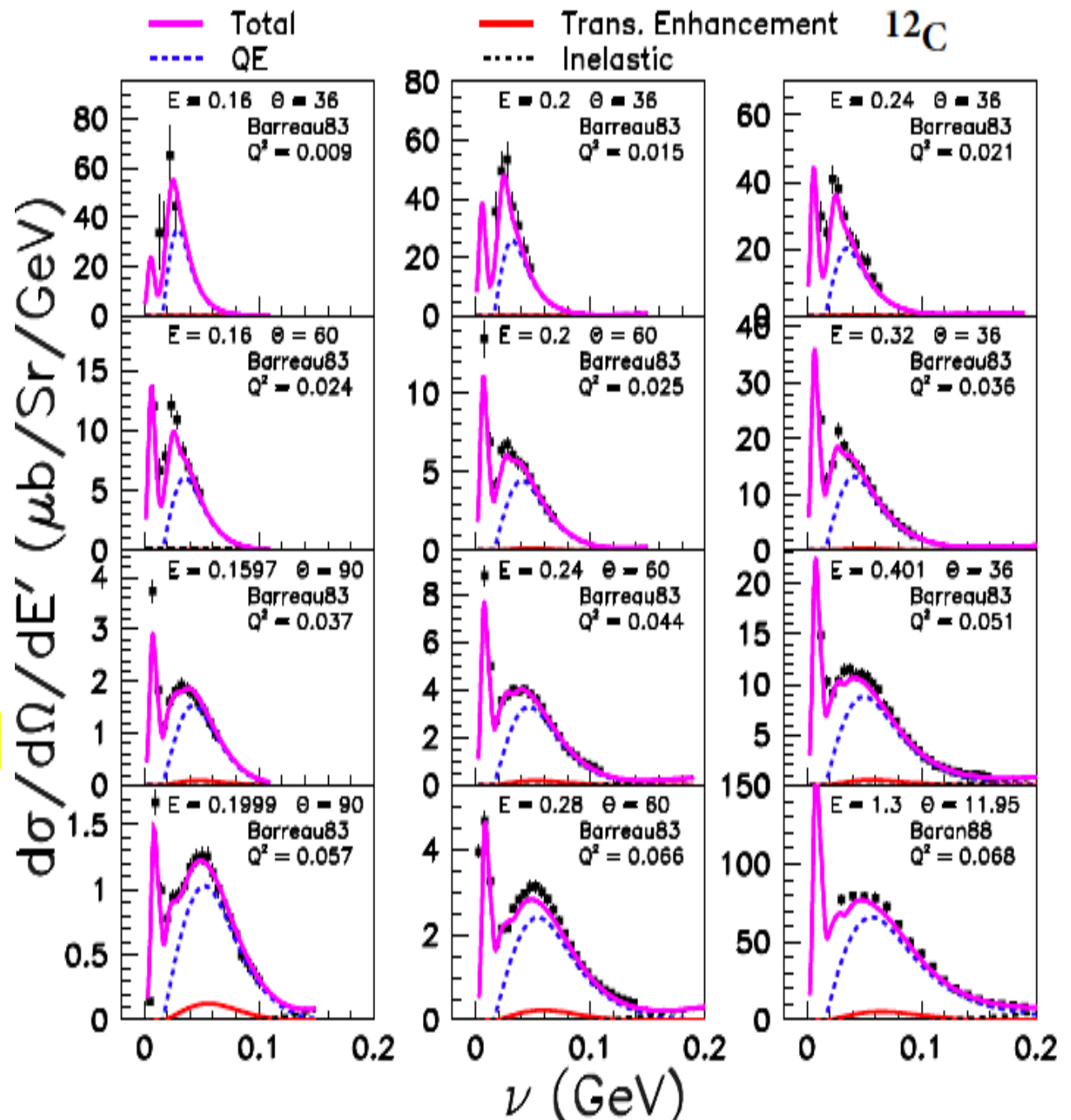
Quasielastic (QE) Region-I

Comparison of our fit to representative e-C12 data
 For $\nu < 0.2$ GeV and $0.01 < q^2 < 0.068$ GeV².

Shown: Total including excitations solid -----

Quasielastic (QE) contribution dashed -----

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed-----



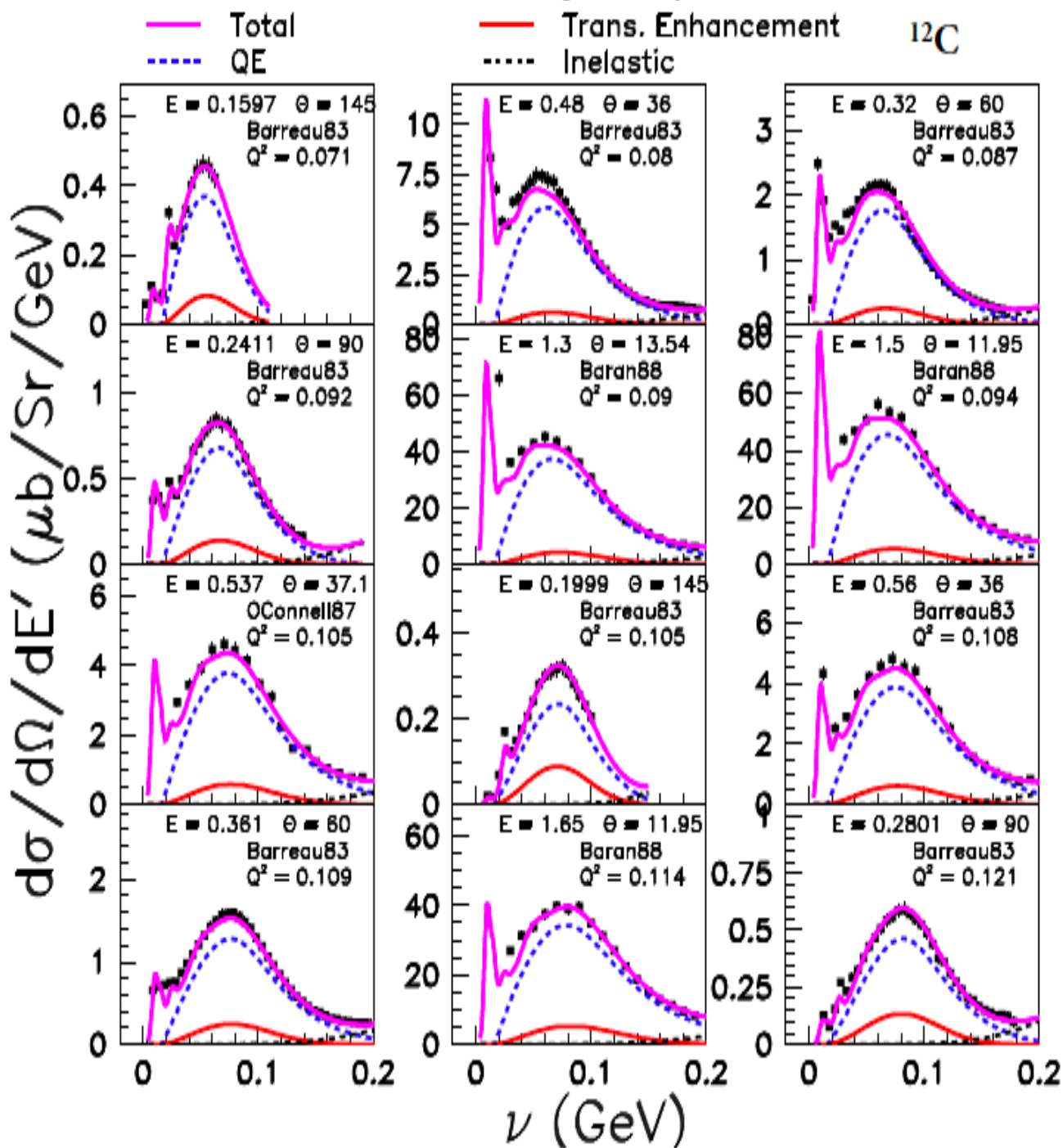
Quasielastic (QE) Region II

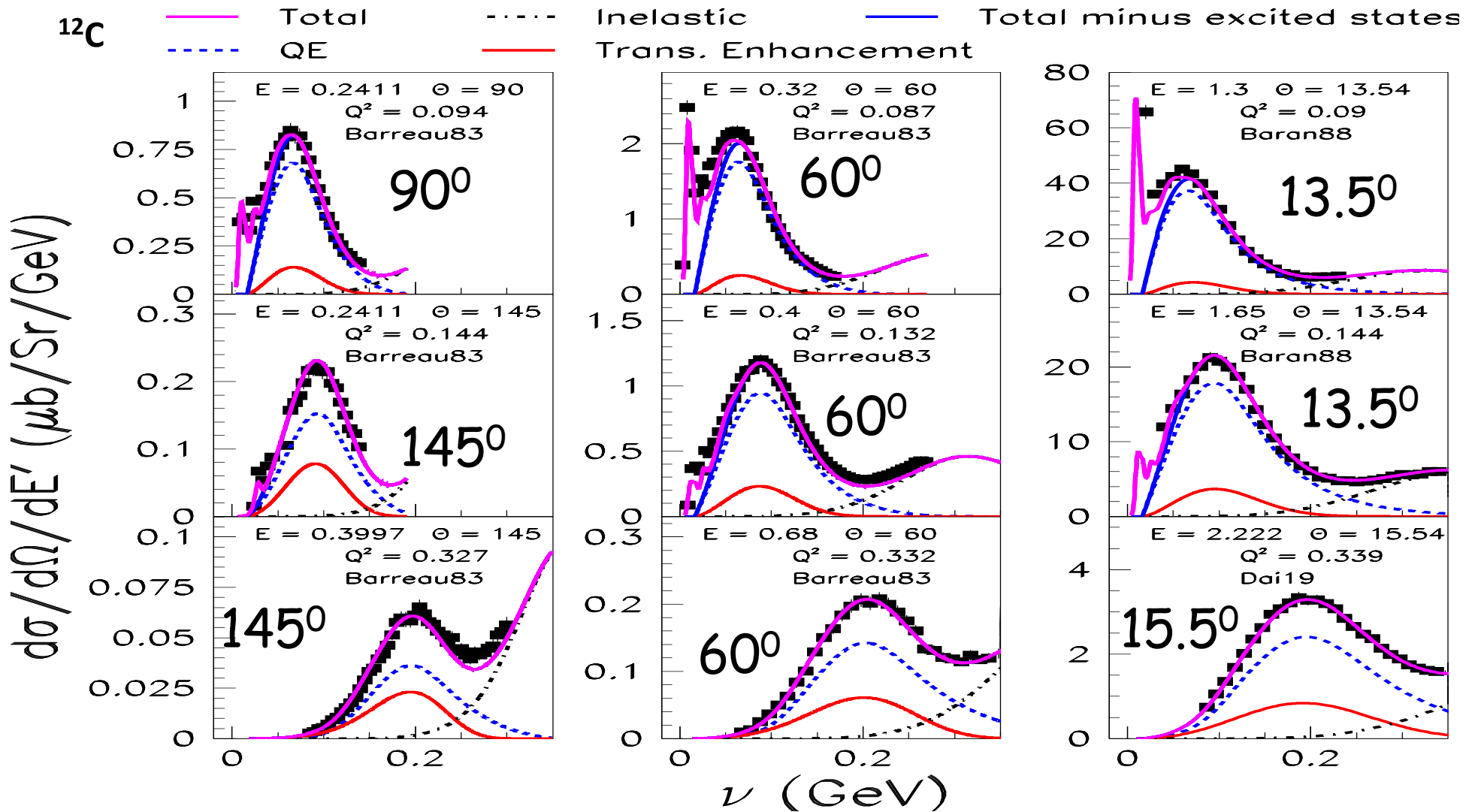
Comparison of our fit to representative e-C12 data for $< 0.2 \text{ GeV}$ and $0.071 < q^2 < 0.121 \text{ GeV}^2$.

Shown: Total including excitations solid -----

Quasielastic (QE) contribution dashed -----

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed-----

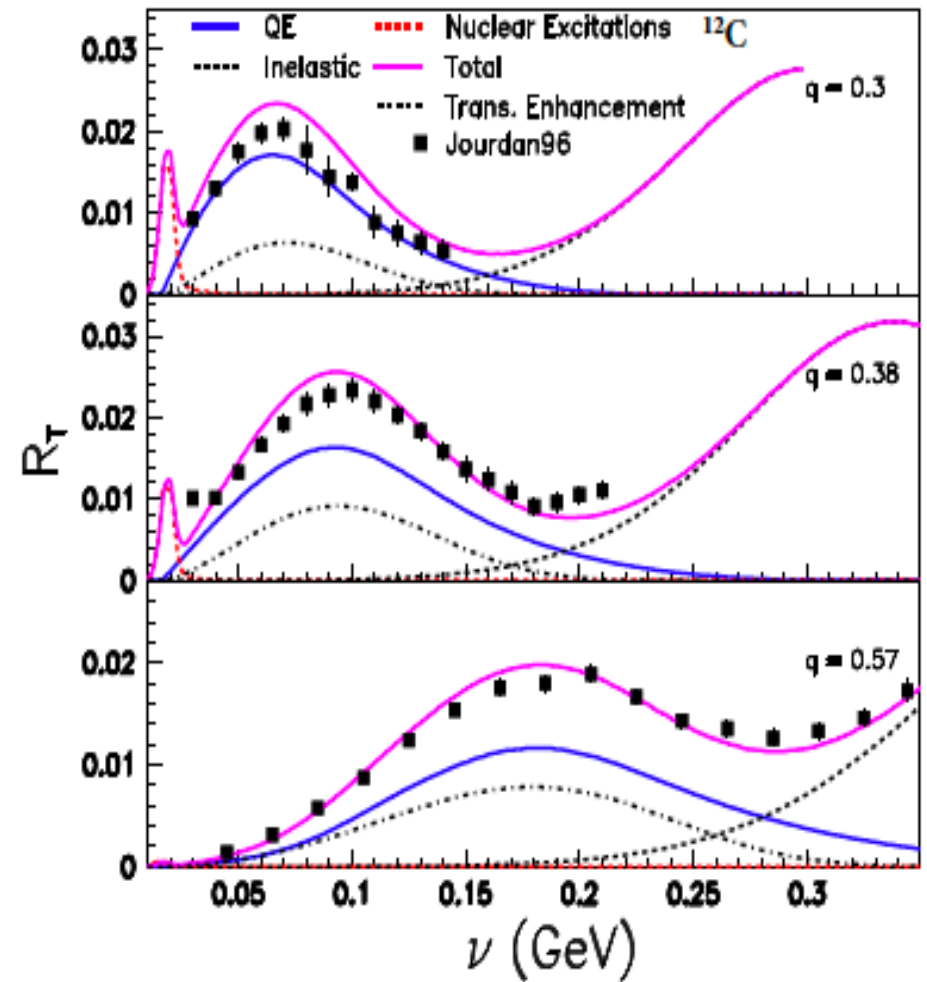
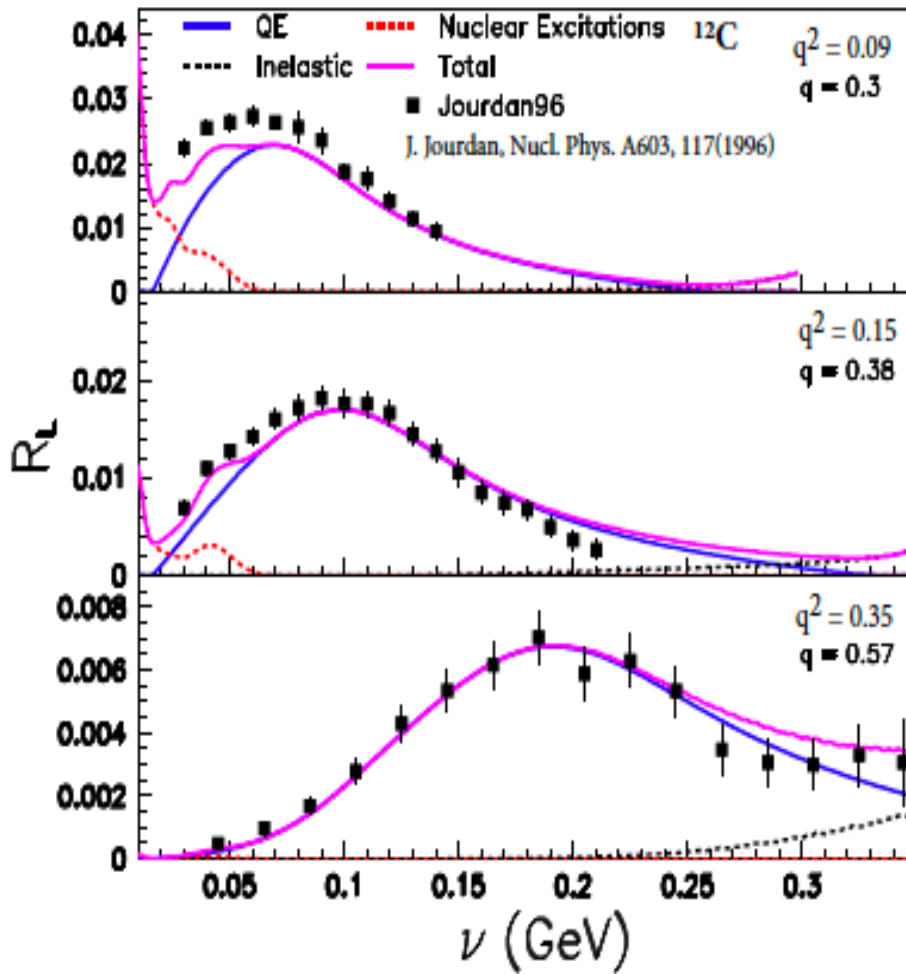




The overall fit provides R_L and R_T at all values of q

Shown are **large and small angle cross sections** at the same q that provide the major contribution to the extraction of R_L and R_T at

$q^2=0.09, 0.15$ and 0.35 GeV^2 ($q=0.3, 0.38$ and 0.57 GeV)



Comparison of our R_L and R_T from our universal fit to (~ 8000 cross sections) to previous extraction by Jourdan at $q^2=0.09, 0.15$ and 0.35 GeV^2 . ($q=0.3, 0.38$ and 0.57 GeV)

Our extraction is more reliable since we include all of the world's data in the fit

- **At low q** the contribution of the **nuclear excitations** important.
- The **superscaling fit function** describes the QE distribution at higher ν .
- **Resonance region** is modeled with **Fermi Smeared H and D data**.

Note

- The 1p1h cross section is enhanced when 2 body currents are included in the calculations. This is the largest contribution to the Transverse Enhancement and increases the cross section in the region of the QE peak.
- The 2 body currents also result in 2p2h final states. It increases the cross section between the QE peak and pion production threshold.

Pauli Suppression Factor

We use the Rosenfelder method.

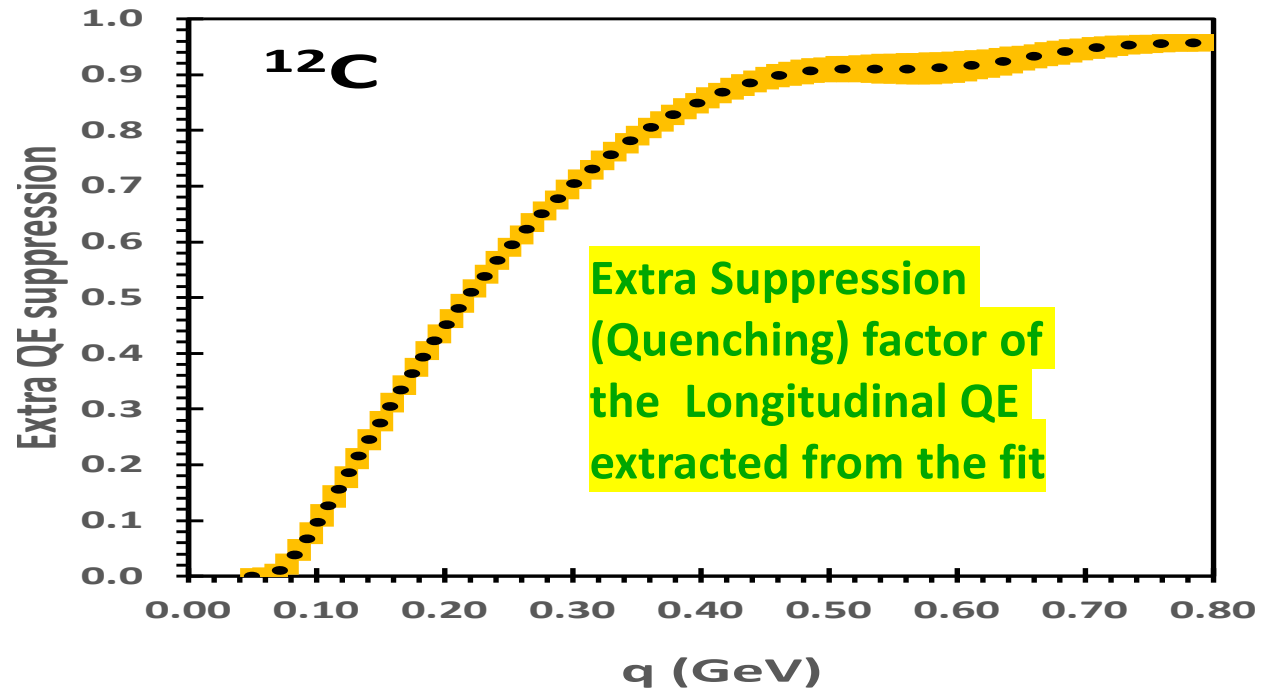
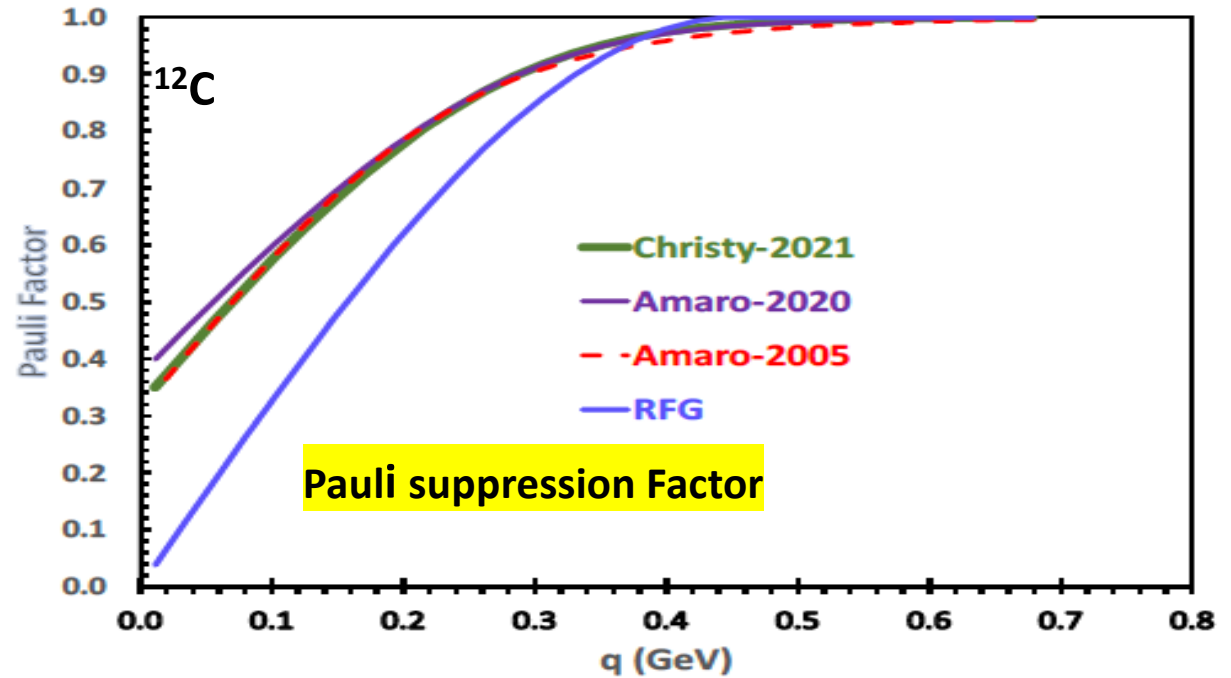
P_1 = Pauli factor for the Christy 2021 QE superscaling model.

P_2 = Pauli factor for another QE model (e.g. Amaro-2020)

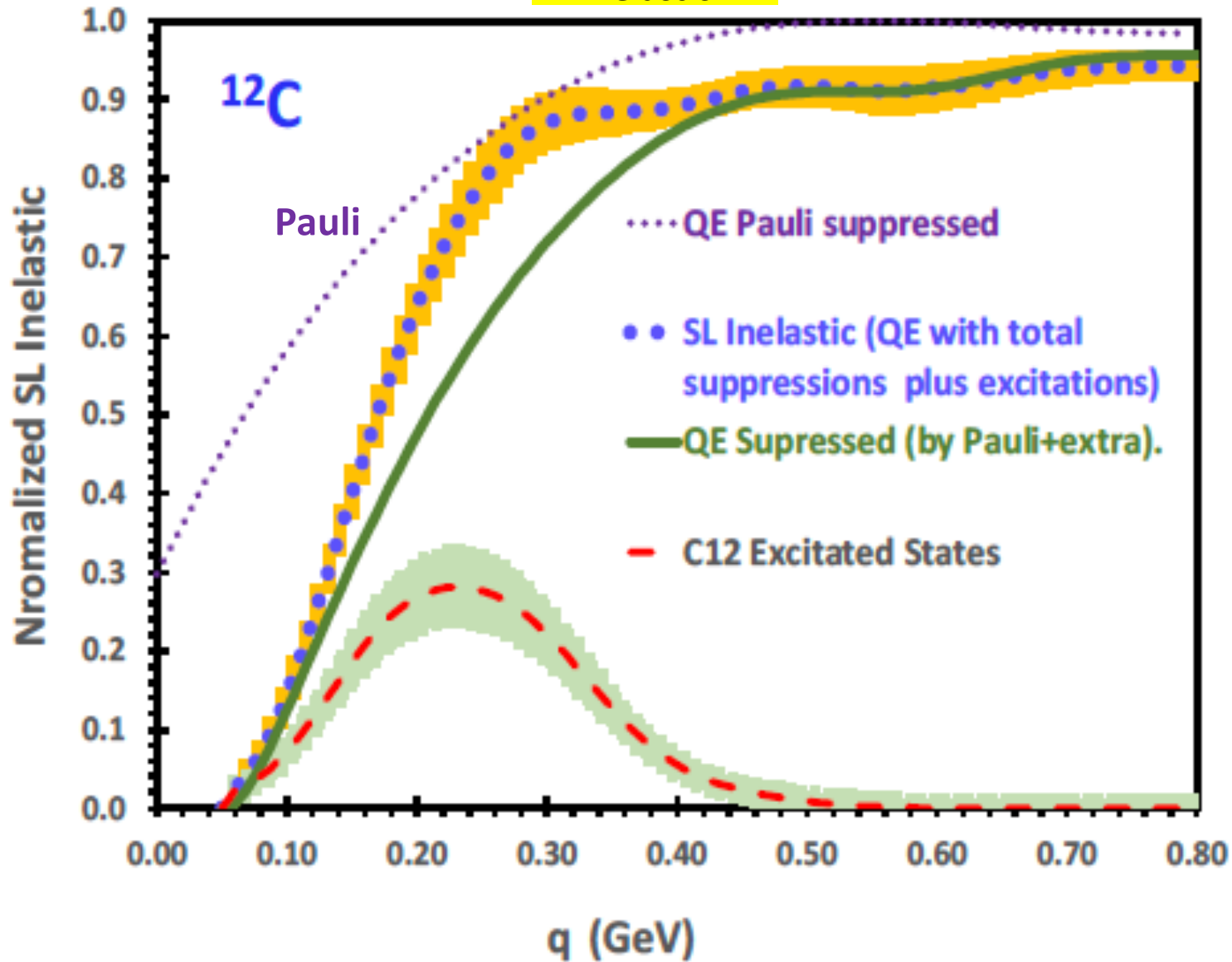
ES_1 = The Extra Suppression (Quenching) of the longitudinal QE cross section (in addition to Pauli) extracted from our fit

ES_2 = Extra Suppression (Quenching) for another QE model

$$ES_2 = ES_1 (P_1/P_2)$$

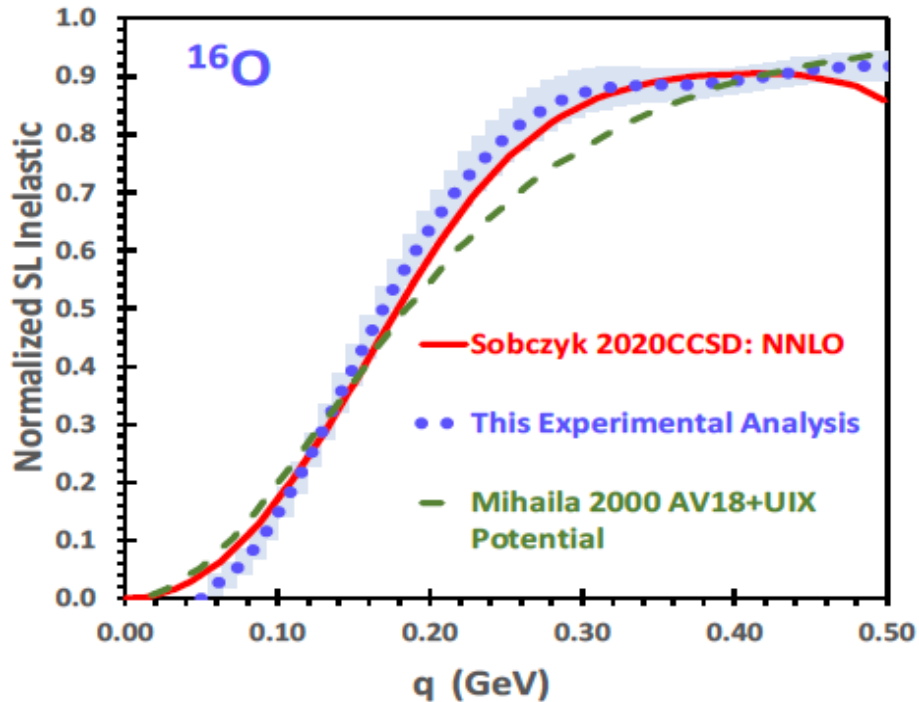
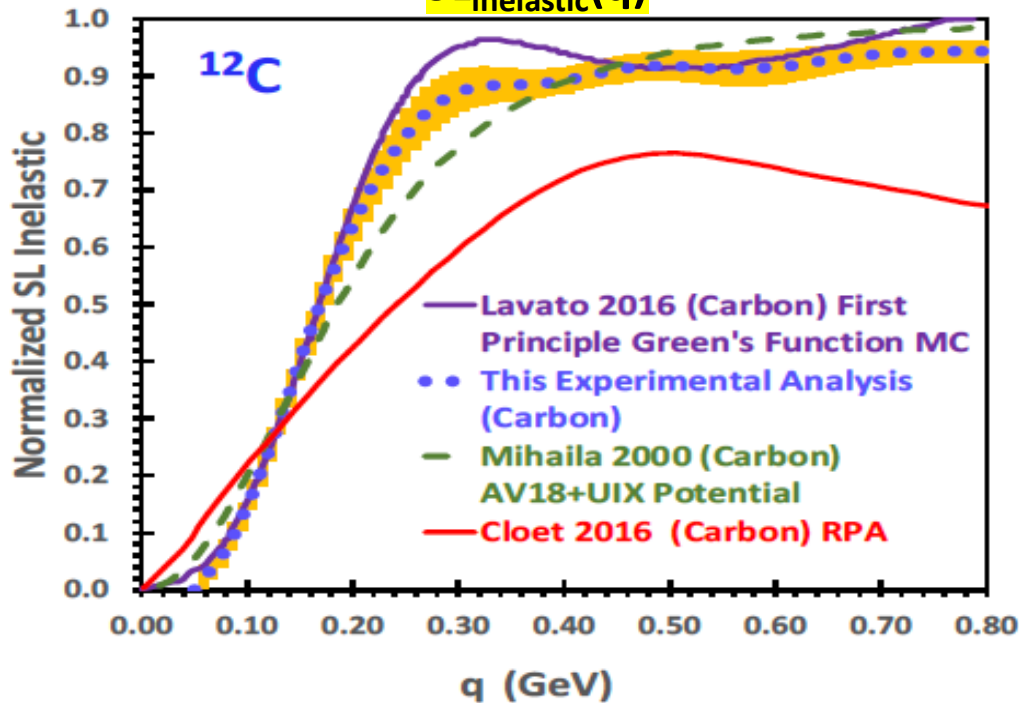


$SL_{\text{inelastic}}(q)$



The different contribution from our fit to the extracted Inelastic Coulomb sum Rule: $SL_{\text{inelastic}}(q)$

$SL_{\text{inelastic}}(q)$



Comparison to theory for Coulomb sum Rule: $SL_{\text{inelastic}}(q)$ ^{12}C

1. Reasonable agreement with Lavato 2016 First Principle Green's function MC.
2. Poor agreement with Mihaila 2010 Coupled Cluster AV18-UIX potential
3. Poor agreement with Cloet 2016 (RPA) A. Lovato et. al, Phys. Rev. Lett. 117, 082501 (2016)

Ian C. Cloet, Wolfgang Bentz, Anthony W. Thomas, Phys. Rev. Lett. 116, 032701 (2016) (arxiv.org/abs/1506.05875).

Bogdan Mihaila and Jochen H. Heisenberg, Phys. Rev. Lett. 84 (2000) 1403. 2009.01761 [nucl-th]

Comparison to theory for Coulomb sum Rule: $SL_{\text{inelastic}}(q)$ ^{16}O

1. Reasonable agreement with Sobczyk 2000 CCSD:NNLO
2. Poor agreement with Mihaila 2010 Coupled Cluster AV18-UIX potential

J. E. Sobczyk, B. Acharya, S. Bacca, and G. Hagen Phys.Rev.C 102 (2020) 064312 (arXiv: 2009.01761 [nucl-th])

Conclusions

- **We fit all existing e-H, e-D, e-¹²C and e-¹⁶O data including elastic, nuclear excitations, Quasielastic, Resonance and Inelastic region.**
- **Fit provides a benchmark** to test electron and neutrino MC generators. (all parameters will be published).
- **The contributions of nuclear excitations is important at low q** and should be added to electron and neutrino MC generators.
- For the QE structure functions, we find that the **QE Longitudinal cross section is Quenched by an Extra Suppression in addition to Pauli blocking** and the **Transverse cross section is Enhanced**. We provide a parameterizations of the **Longitudinal Quenching** and **Transverse Enhancement**.
- We extract the inelastic Coulomb sum rule and compare to theoretical calculations. Since all available e-¹²C data is included in the fit, **this is the best extraction of the Inelastic Coulomb Sum Rule from all the world's data on ¹²C.**