Imperial College London

MK single pion production model

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NuInt workshop

Oct. 28, 2022

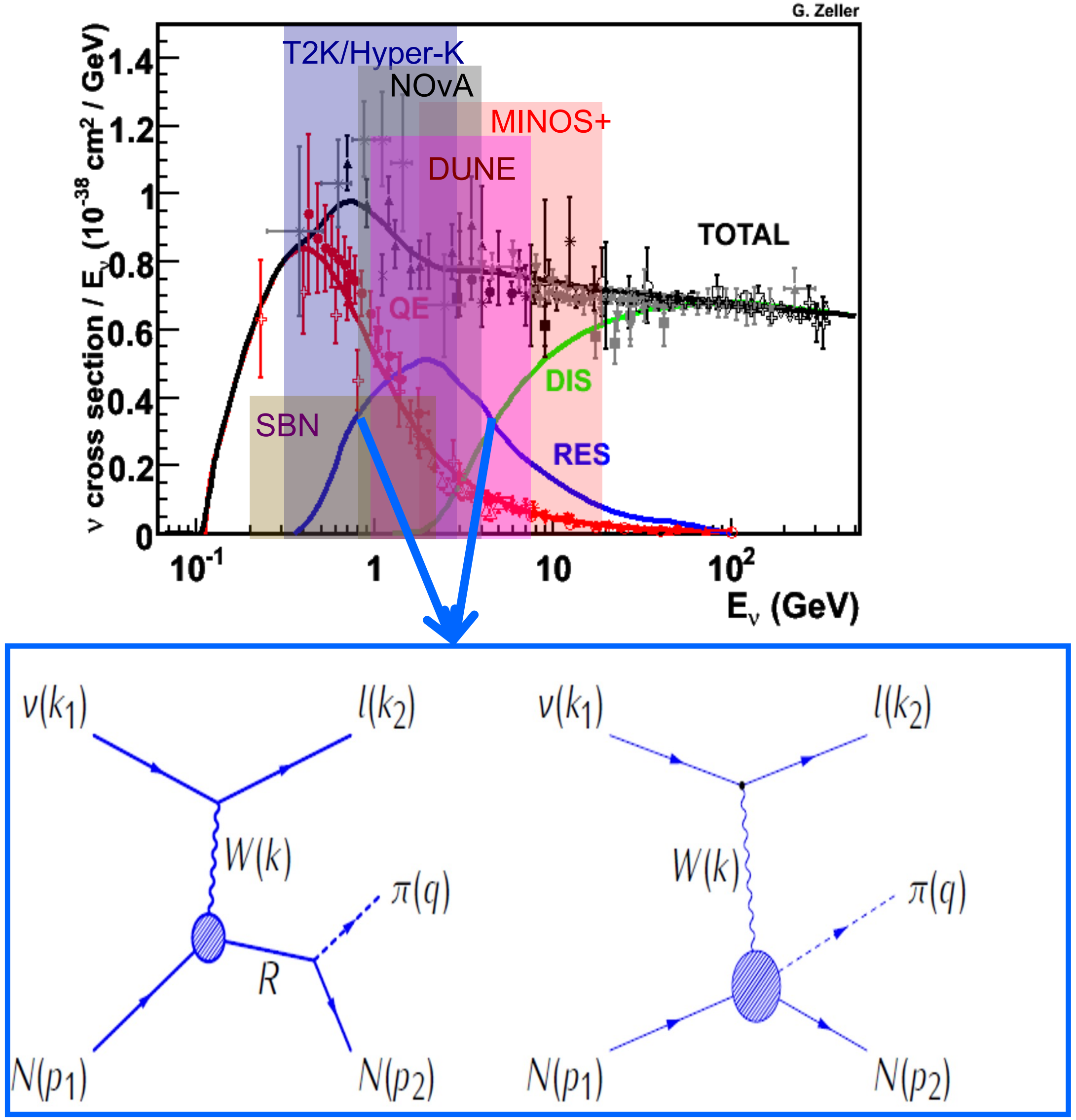
Goal: New approach for building models that have the **maximum impact** on our ability to extract and interpret interesting physics measurements.

1. improving a model through detailed **theoretical calculations**.
2. Transforming the model to something that could be easily and efficiently **incorporated into event generators**.
3. Studying the **systematic uncertainties** of the theoretical model.
4. providing a few adjustable **physics-motivated parameters**, or ”knobs” which can be used in future measurements.

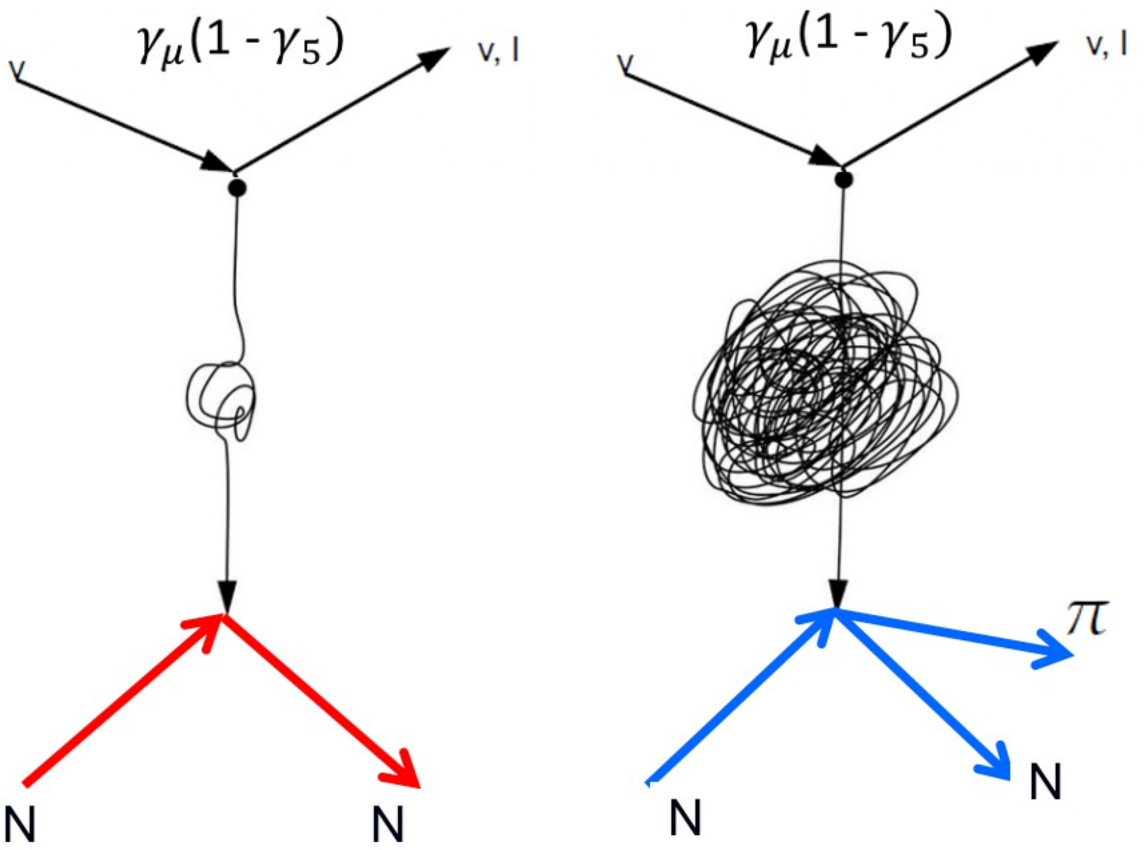
Why Single pion production?

- For electron appearance experiments neutrino must be at difficult intermediate energy (non-perturbative domain) where single pion production has a significant contribution.
- Single pion can be produced via decay of resonance excitations or non-resonant interactions.
- Single pion production overlaps with DIS(QE) at higher (lower) energy.

ν_μ CC cross section per nucleon



Modelling pion production is already difficult at nucleon level!



Elastic or Quasi-elastic

Single Pion Production

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

Rein-Sehgal model (1981)

- It is not a full kinematic model. The helicity amplitudes are **not** a function of pion angles!

$$d\,\sigma/dW\,dQ^2$$

- It does not cover non-resonant interaction.
- It has a simple lagrangian!
- It uses simple form-factors for all resonances.
- Like other phenomenological model is only valid in the resonance region.

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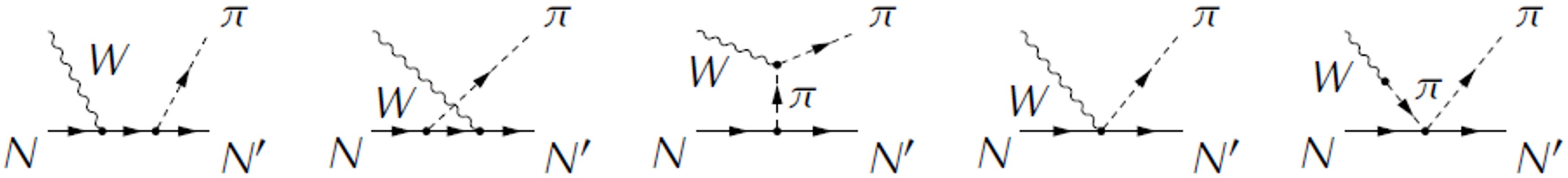
The first version of MK model

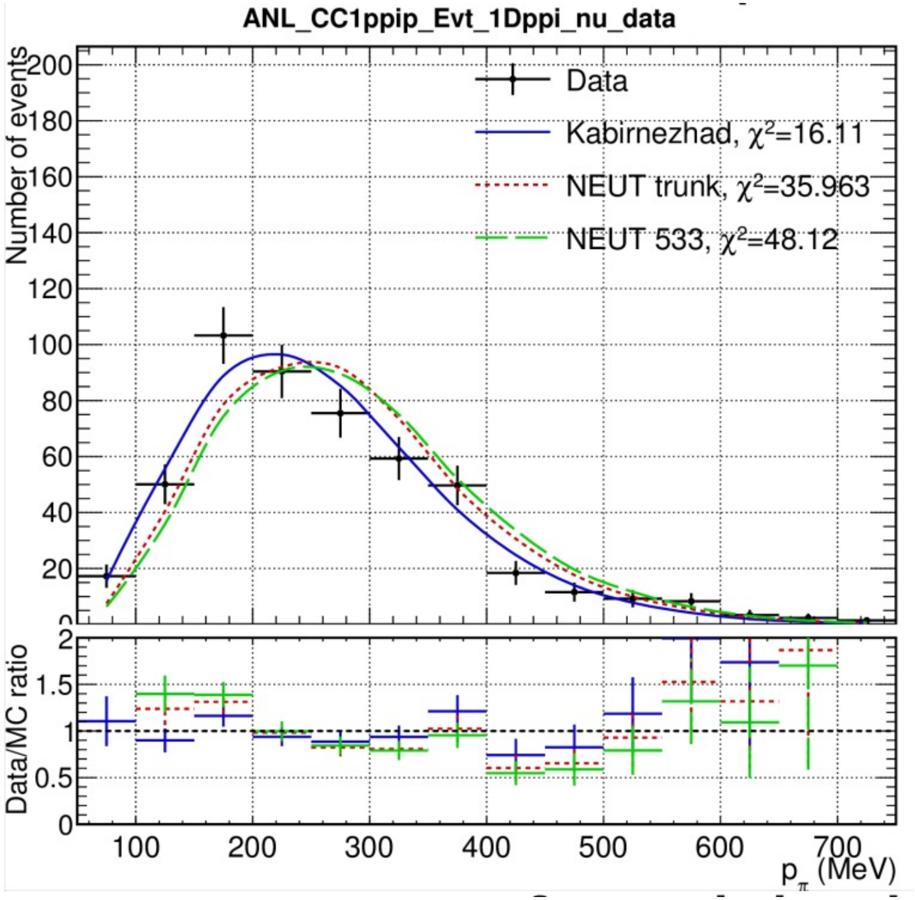
M. Kabirnezhad, Phys. Rev. D **97** (2018)

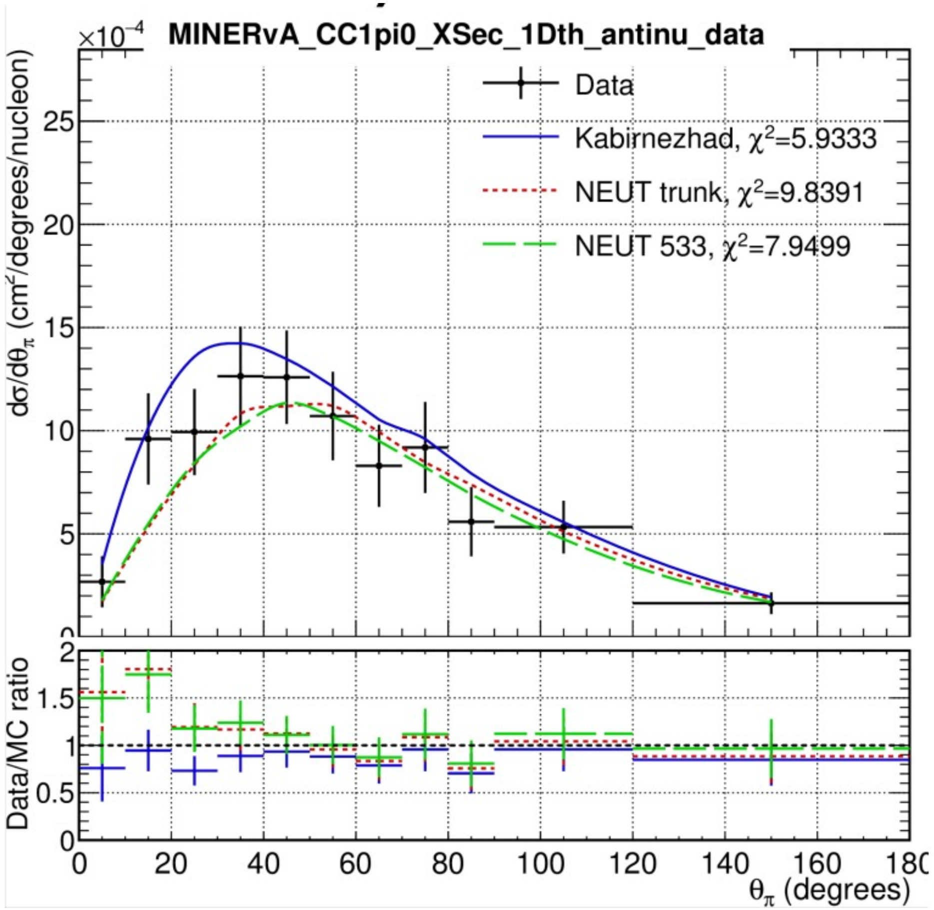


Non-resonant background is only valid at low W

E. Hernandez, J. Nieves and M. Valverde, Phys. Rev. D **76** (2007) 033005

Results: MK model (2018)





From [Clarence Talk at NuInt 2017]

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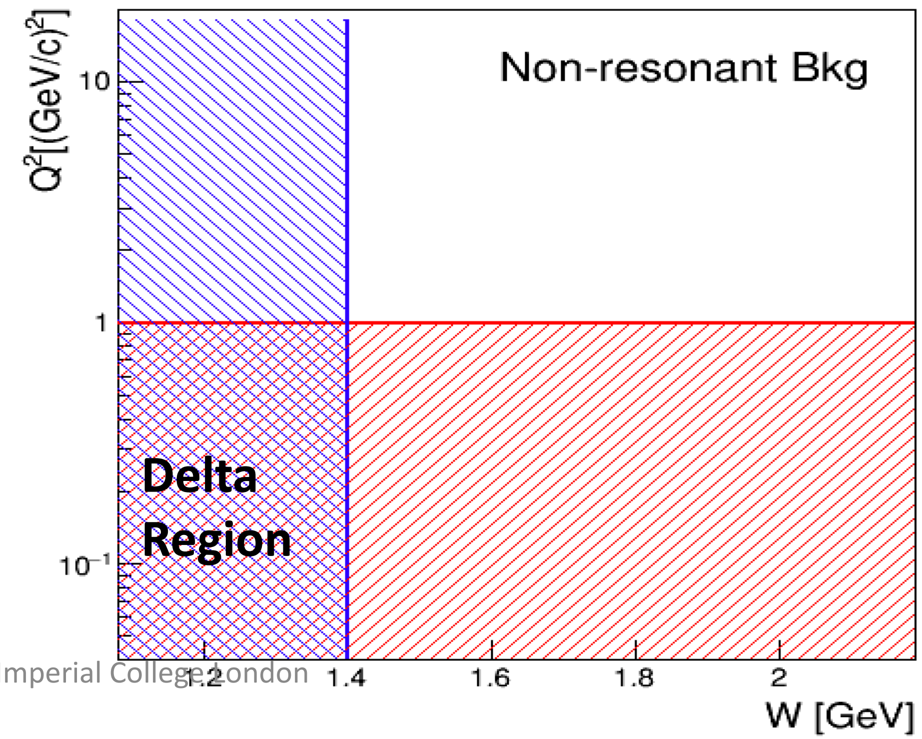
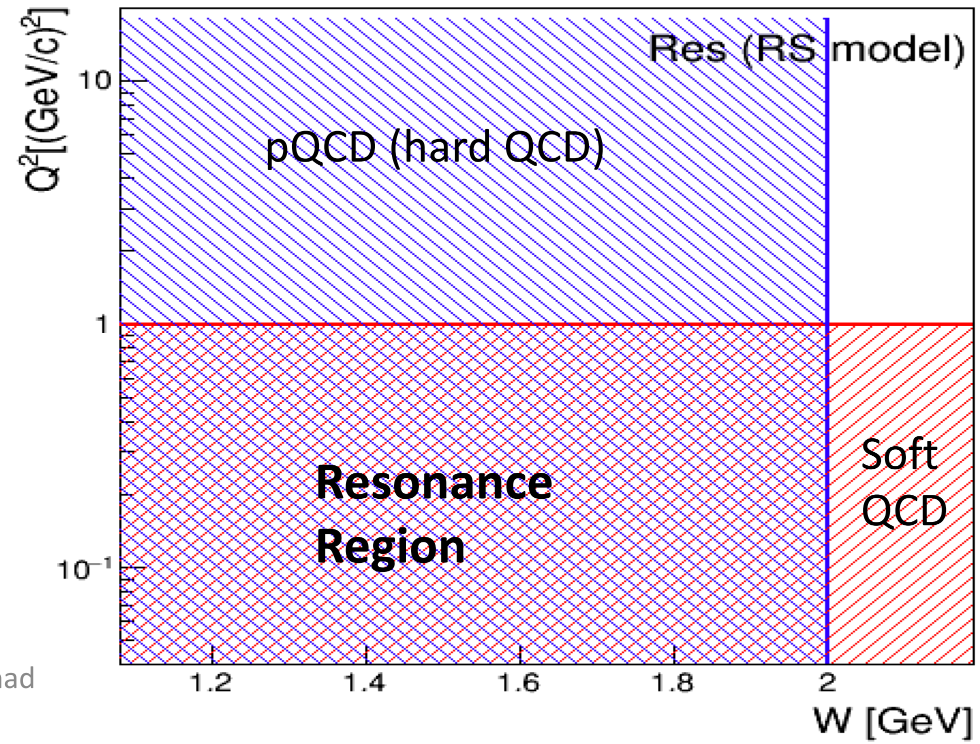
This is improved in the newer versions of the MK model

Models in Resonance region

- Pion production models in neutrino interactions are usually valid inside the Delta → resonance region.
- These models need to be extended to outside of the resonance region to link it to the DIS region (**transition region**) properly.





QCD is the underlying fundamental theory for resonance excitation!

- In the non-perturbative domain (resonance region), QCD was (is?) not able to provide a comprehensive treatment.
- Phenomenological models (EFT) are necessary in the absence of QCD.
- The linking idea of these two methods (QCD and EFT) is **quark-hadron duality**, i.e., the form factor asymptotes calculated in both perturbative QCD and EFT must be the same.

Verifying the model is difficult with limited neutrino data sets!

- Existing neutrino data on “free” nucleon is scarce and it is doubtful that it will be improved.

A solution is to split the hadron current

1. Vector part (electron scattering data)
2. Axial part (pion and neutrino scattering data)

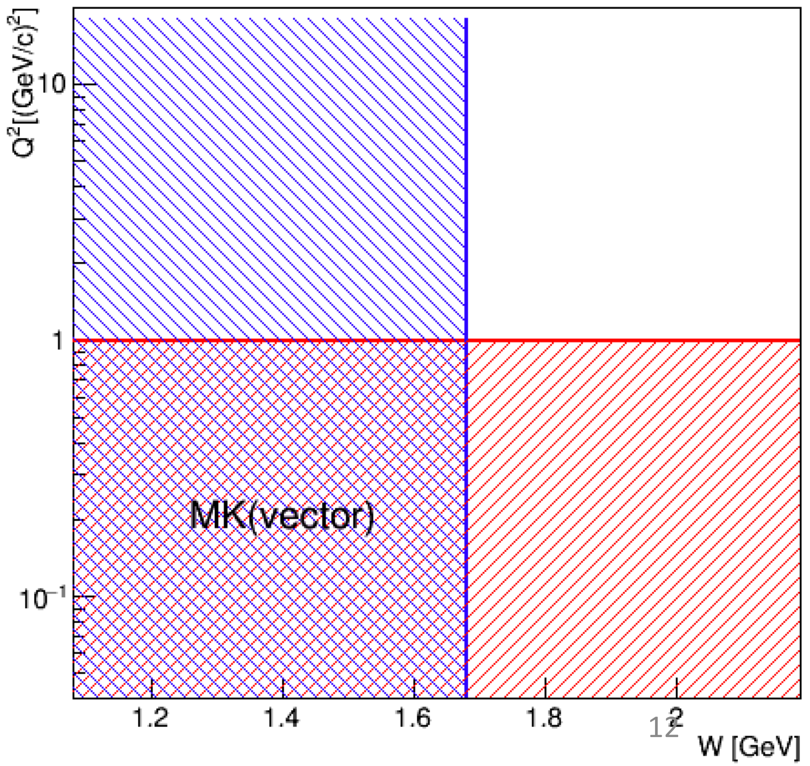
The vector current (past)

M. Kabirnezhad

Phys. Rev. D 102, 053009 (2020)

- Resonance model was updated: Rein-Sehgal → Rarita-Schwinger.

- Vector form-factors are updated.
- Model is only valid at low $Q^2 < 1$ GeV^2.
- Electron scattering data is used to constrain the vector currents.
- J-Lab data on hydrogen target : $ep \rightarrow ep + \pi^0$ & $ep \rightarrow en + \pi^+$ channels 1.1<W<1.68 GeV, and $Q^2 < 1$ GeV^2



The vector current (latest)

M. Kabirnezhad
arxiv:2203.15594[hep-ph]

- The model is extended to higher momentum transferred (Q2) by introducing form factors based on vector meson dominance (VMD) models consistent with the Quantum Chromodynamics (QCD) theory.
- The model is extended to higher invariant mass (W) by using Regge trajectory in non-resonant interactions (following the Hybrid model).
- The form factors are defined to improve agreement with exclusive electron-proton data.
- All the free parameters are fit to the **exclusive J-lab data** .

Electron scattering data

Beam Energy (GeV)	Q^2 Range $(GeV/C)^2$	W Range (GeV)	PID	# data points
1.046	0.16 - 0.32	1.1 - 1.34	$p\pi^0$	650
1.046	0.16 – 0.32	1.1 – 1.34	$p\pi^+$	642
1.515	0.30 – 0.60	1.11 – 1.57	$p\pi^+$	800
1.645	0.40 – 0.90	1.1 – 1.68	$p\pi^0$	1130
2.445	0.65 – 0.90	1.1 – 1.68	$p\pi^0$	890
2.445	0.90 – 1.80	1.1 – 1.68	$p\pi^0$	710
5.499	1.80 – 4.00	1.62 – 2.01	$p\pi^+$	450
5.754	1.72 – 4.16	1.15 – 1.67	$p\pi^+$	1452
5.754	3.00 – 6.00	1.11 – 1.39	$p\pi^0$	568

previous analysis (2020)

The latest analysis

~ **7300 × 3**

data points

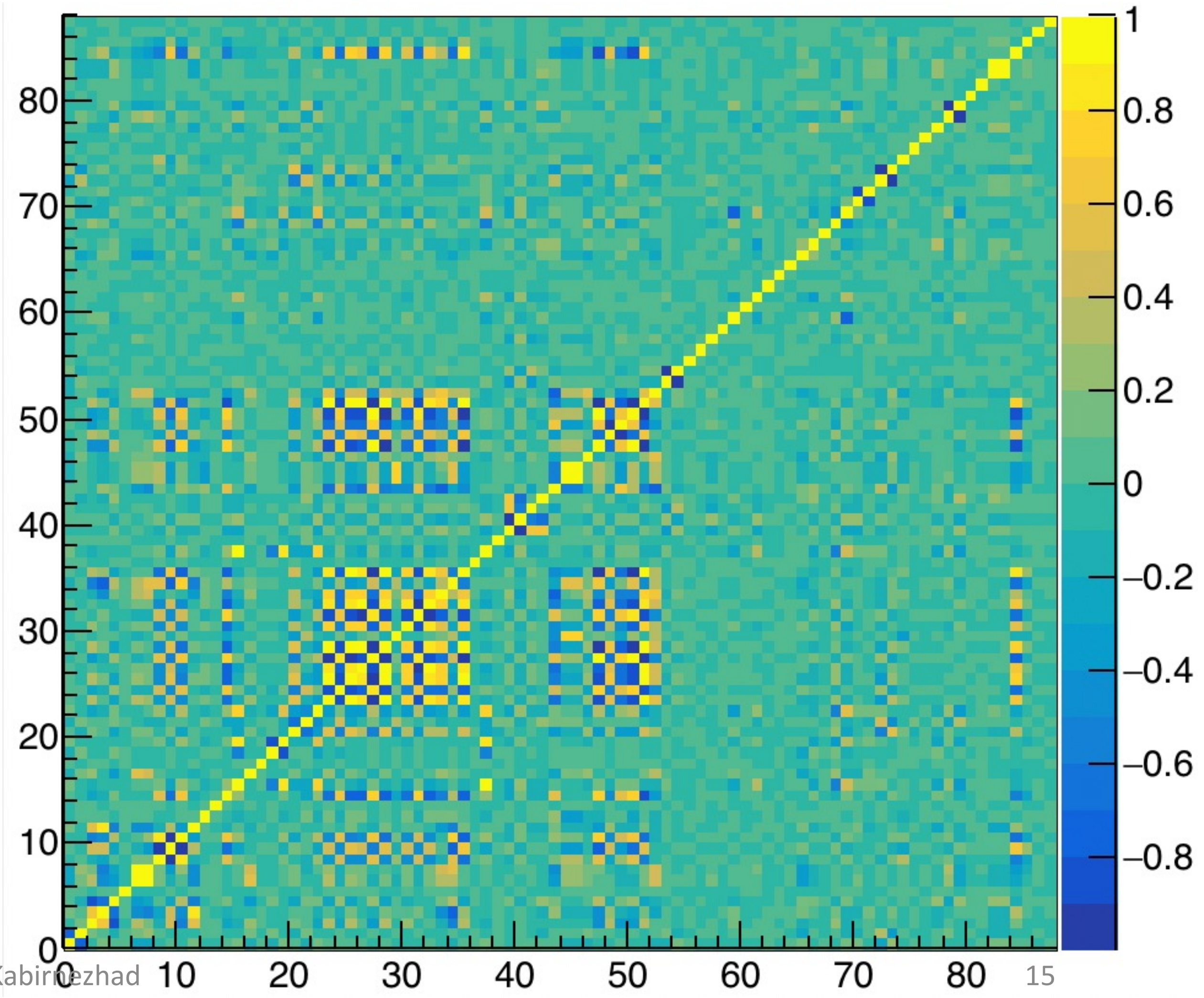
Reduced $\chi 2 \simeq 2.3$

Data sets in a broad range of Q2 (**0.16 to 6 GeV²)** and W (**1.1- 2.01 GeV**)

Systematic uncertainty

Understanding correlation between the parameters will help us to:

- evaluate the systematic uncertainty correctly.
- provide a few knobs to help the neutrino collaborations to estimate the systematic errors based on a set of theory-motivated dials.

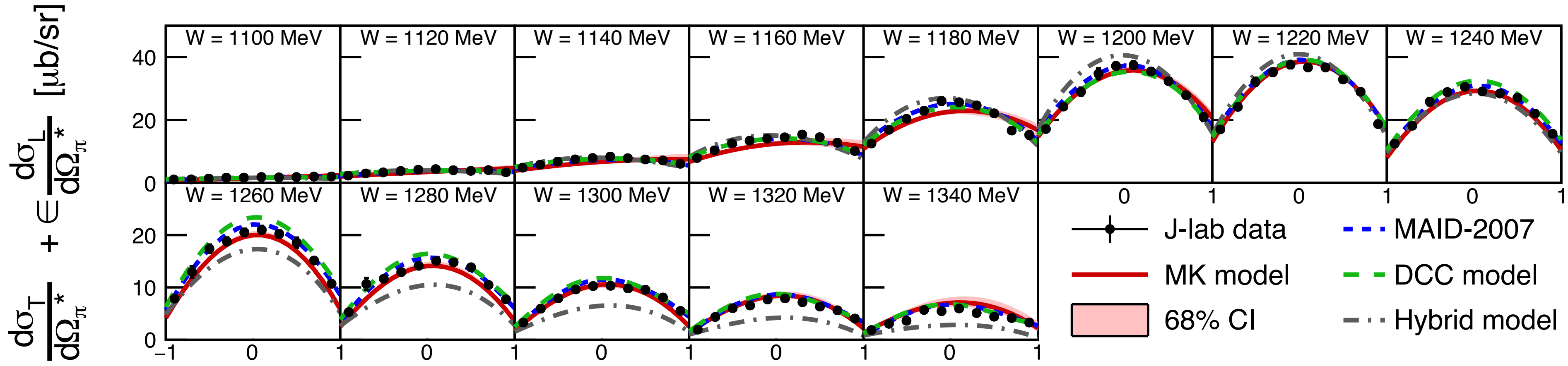


Single pion electro-production models

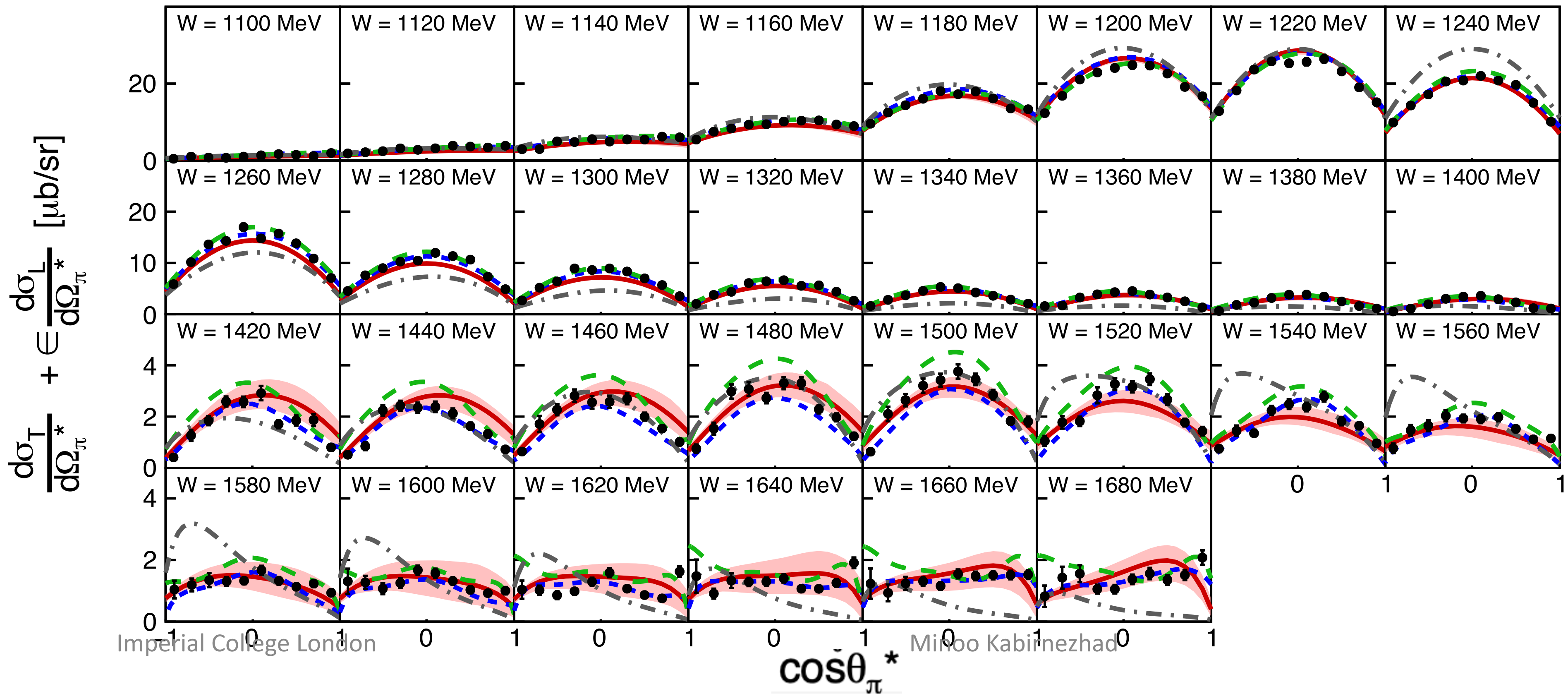
- **MAID-2007** is the latest Unitary Isobar model which is valid in $Q^2 < 5.00\ (GeV/c)^2$ region.
- **DCC** is a dynamical coupled-channeled model developed by theOsaka group in 2015. It is valid in $Q^2 < 3.00\ (GeV/c)^2$ region.
- **Hybrid** model which is valid at low Q^2 .
- MAID and DCC models used electron data to fit their form-factors.
- Hybrid model use Lalakulich-Paschos form-factors which is a fit to the MAID results.

Model comparisons (low Q^2)

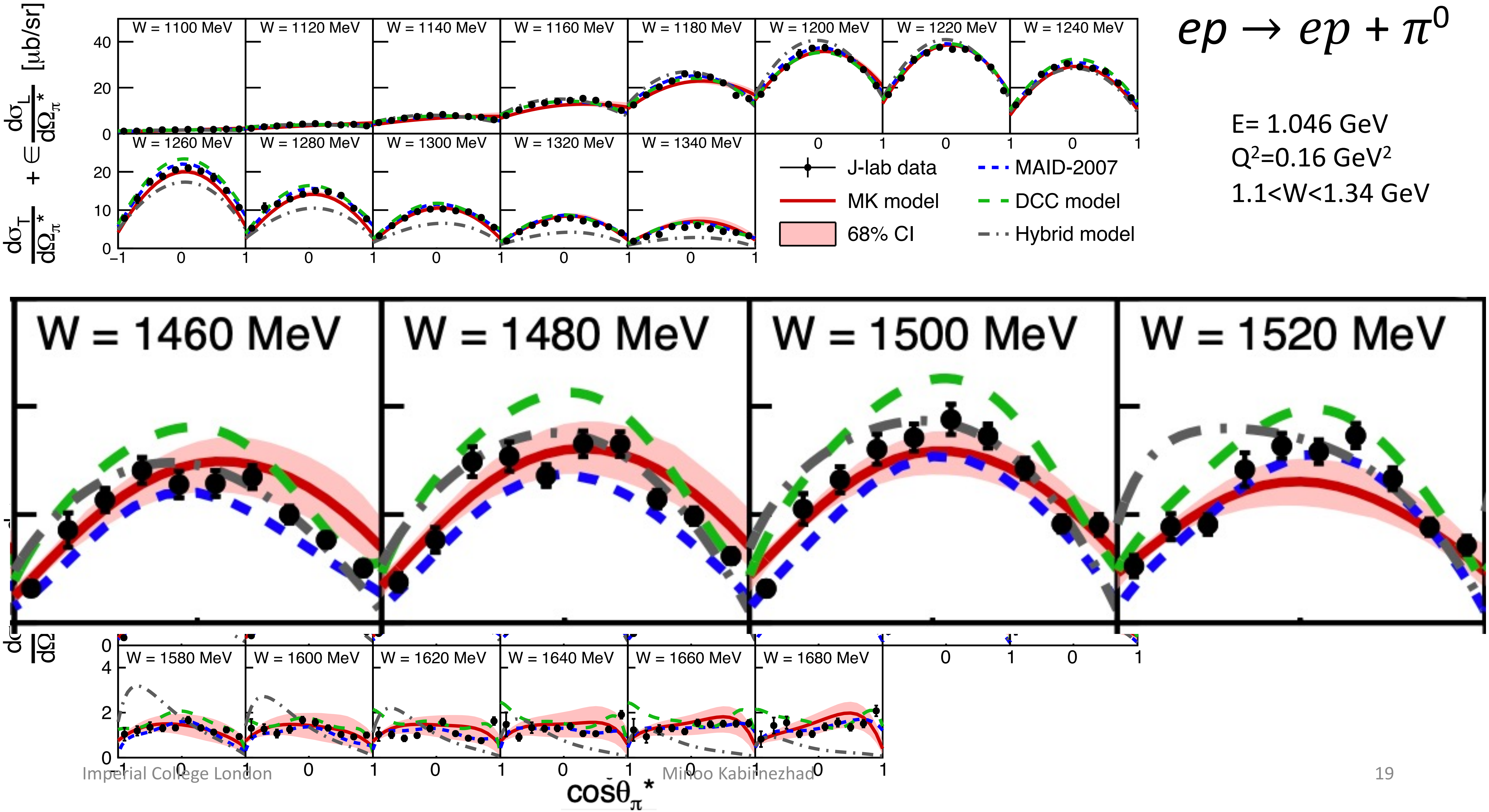
$ep \rightarrow ep + \pi^0$



E= 1.046 GeV
Q^2=0.16 GeV^2
1.1<W<1.34 GeV



E= 1.645 GeV
Q^2 =0.4 GeV^2
1.1<W<1.68 GeV



$ep \rightarrow ep + \pi^0$

E= 1.046 GeV

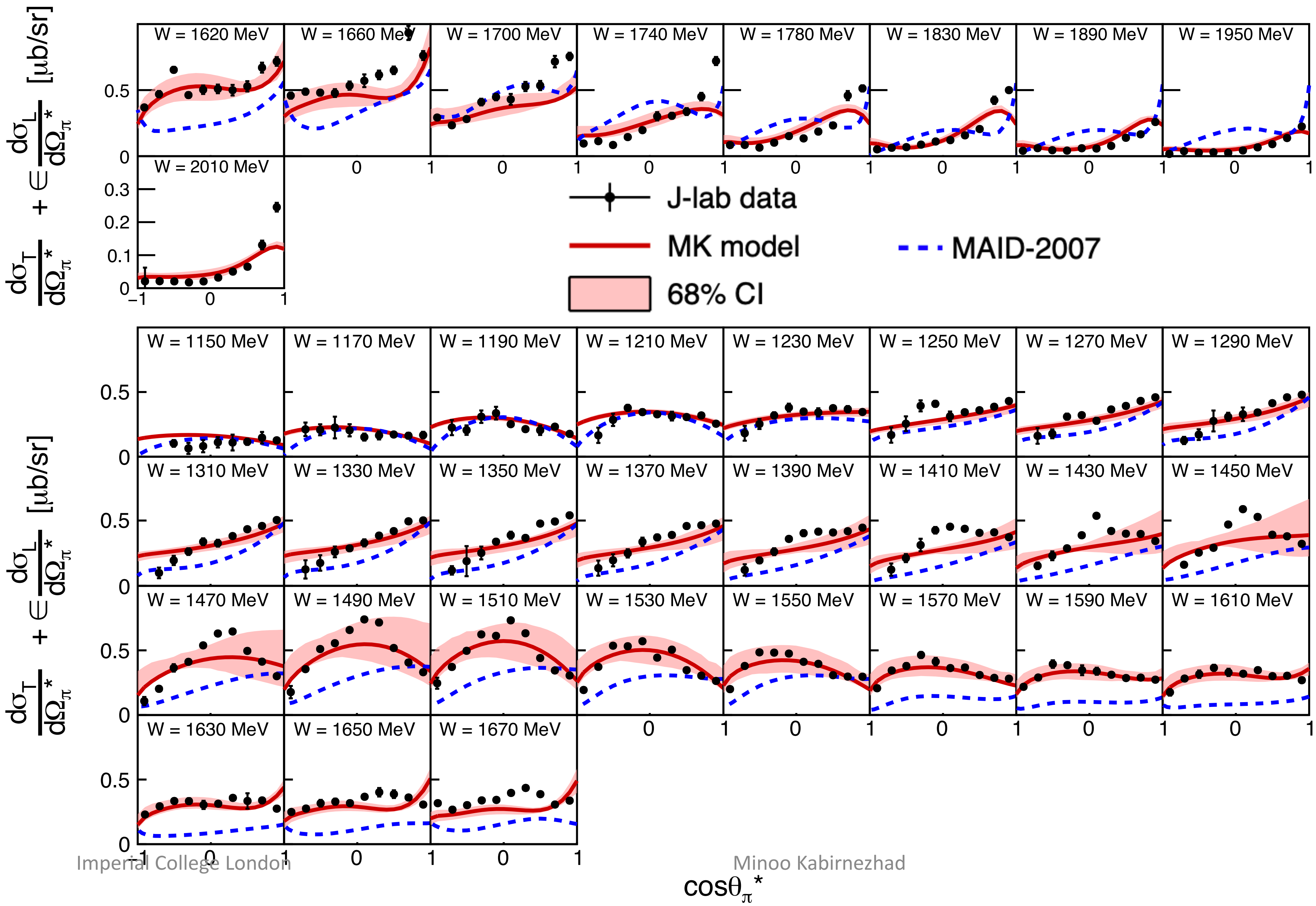
Q^2=0.16 GeV^2

1.1<W<1.34 GeV

Model comparisons (high Q^2)

Only MAID model predicts results. High Q^2 region is out of the valid region of DCC and Hybrid model.

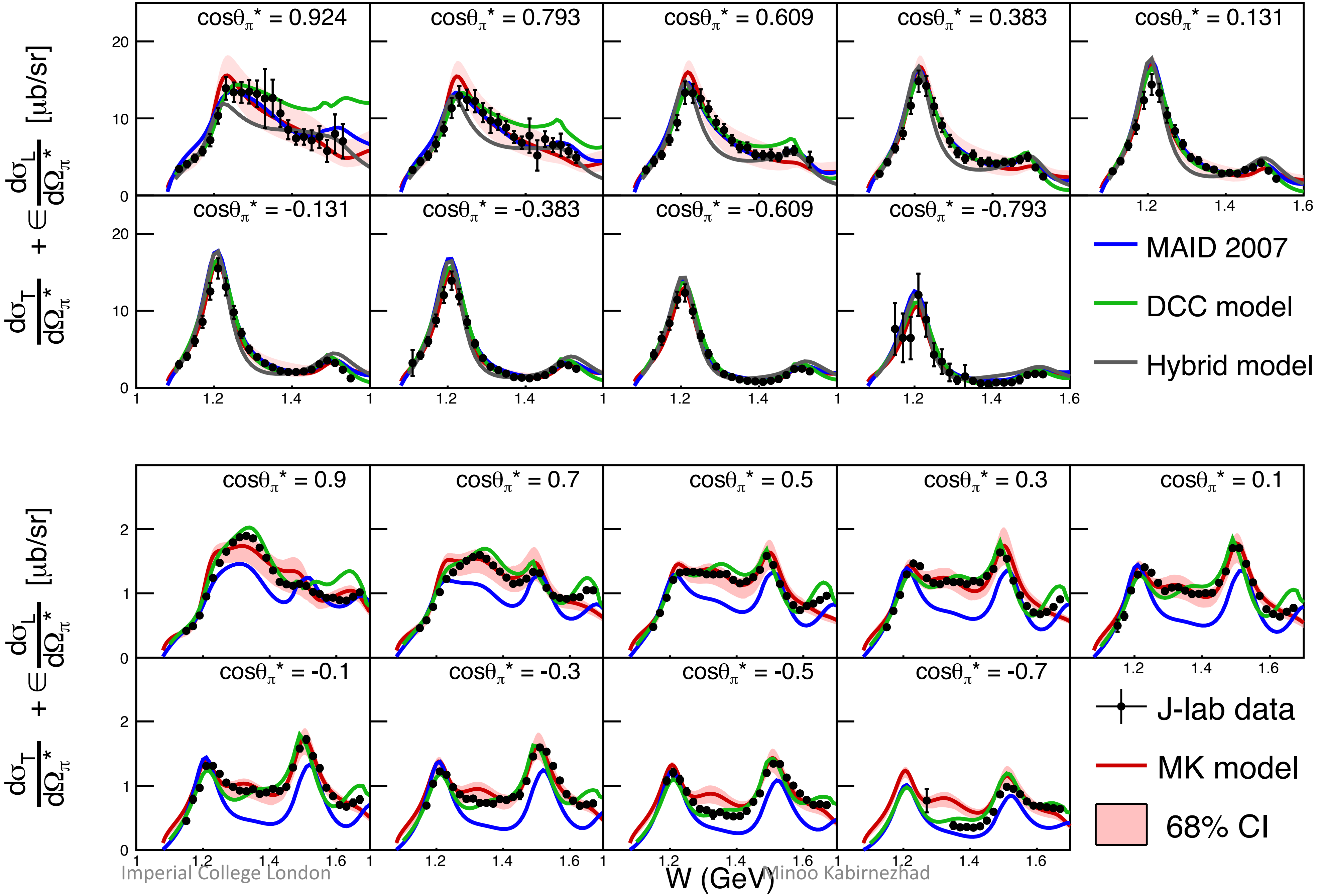
$ep \rightarrow en + \pi^+$



E= 5.449 GeV
Q^2=2.6 GeV^2
1.62 <W< 2.01 GeV
Third resonance region

E= 5.754 GeV
Q^2=3.48 GeV^2
1.15<W<1.67 GeV

W distribution in low and medium Q^2



$ep \to en + \pi^+$

E= 1.515 GeV
Q^2=0.4 GeV^2
1.1<W<1.41 GeV

E= 5.754 GeV
Q^2=2.05 GeV^2
1.15<W<1.67 GeV

Remarks on model comparison

- **At low Q2**, the MAID and the DCC results have a good agreement for both channels. However the Hybrid model shows some discrepancy with data in the second resonance region and in the transition between the first and the second regions.

- **At high Q2**, the MAID result show the data agreement for ep → epπ0 channel is better than the ep → enπ+ channel, which shows a bias in their analysis since in theory, the only difference between the two channels are the Clebsch-Gordan coefficients.

Neutron form-factors

Next steps to develop a neutrino model is to use **electron-neutron** data to fit the neutron form-factors. J-lab data will be available for **the first time** soon!

Isospin relations for the vector form factors.

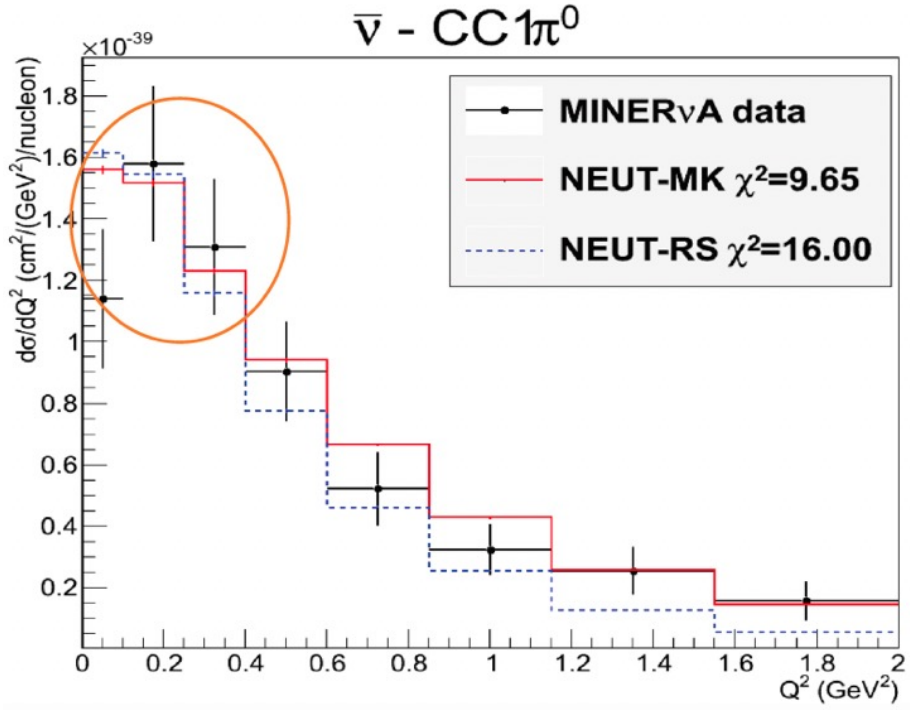
	for $I = 1/2$	for $I = 3/2$
$e^- p \to e^- R^+$	F_i^p	F_i^N
$e^- n \to e^- R^0$	F_i^n	F_i^N
$\nu p \to \ell^- R^{++}$	-	$\sqrt{3} F_i^V = -\sqrt{3} F_i^N$
$\nu n \to \ell^- R^+$	$F_i^V = F_i^p - F_i^n$	$F_i^V = -F_i^N$
$\nu p \to \nu R^+$	$\tilde{F}_i^p = (\frac{1}{2} - 2\sin^2\theta_W) F_i^p - \frac{1}{2} F_i^n - \frac{1}{2} F_i^s$	$\tilde{F}_i^N = (1 - 2\sin^2\theta_W) F_i^N$
$\nu n \to \nu R^0$	$\tilde{F}_i^n = (\frac{1}{2} - 2\sin^2\theta_W) F_i^n - \frac{1}{2} F_i^p - \frac{1}{2} F_i^s$	$\tilde{F}_i^N = (1 - 2\sin^2\theta_W) F_i^N$

From T. Leitner thesis

Improving the axial current

- **PCAC** relation allow us to use pion scattering data at Q^2=0. At low Q^2(<0.2 GeV), the axial current has the main contribution (due to the conservation of vector current).

Mainly axial contributions



Improving the axial current

- **PCAC** relation allow us to use pion scattering data at $Q^2=0$. At low Q^2(<0.2 GeV), the axial current has the main contribution (due to the conservation of vector current).

- **Using neutrino data:** very limited measurements exist on neutrino-nucleon interaction.

PCAC (Adler) relation

$$\left.\frac{d\sigma^{\nu,\bar{\nu}}}{dQ^2 dW}\right|_{Q^2\to 0} = \frac{G_F^2 \cos^2\theta_c f_\pi^2 E_\mu^L}{2\pi^2 |\mathbf{k}| E_\nu^L} \cdot \sigma_{\text{off}}(\pi^\pm p \to \pi^\pm p).$$

cross section of virtual pions carry the four-momentum.

In order to use pion scattering data we need to:

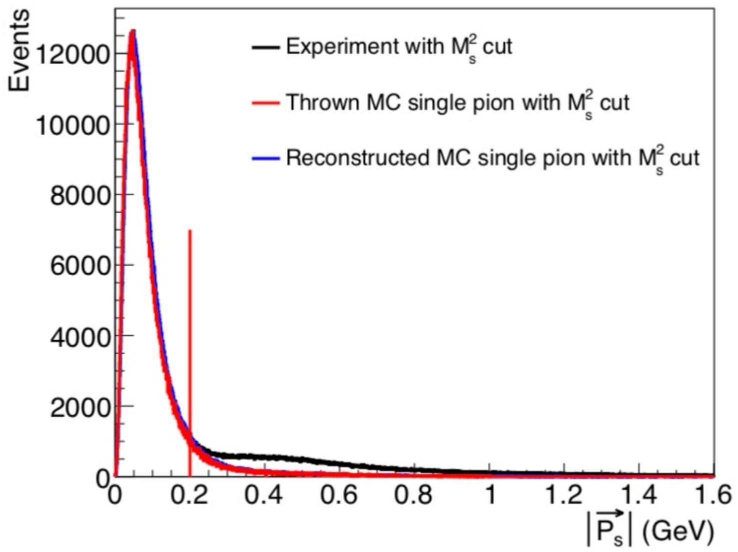
1. Relate the σ_{off} to σ_{on}
2. Extrapolate the relation to higher $Q^2 \gg m_\pi^2$

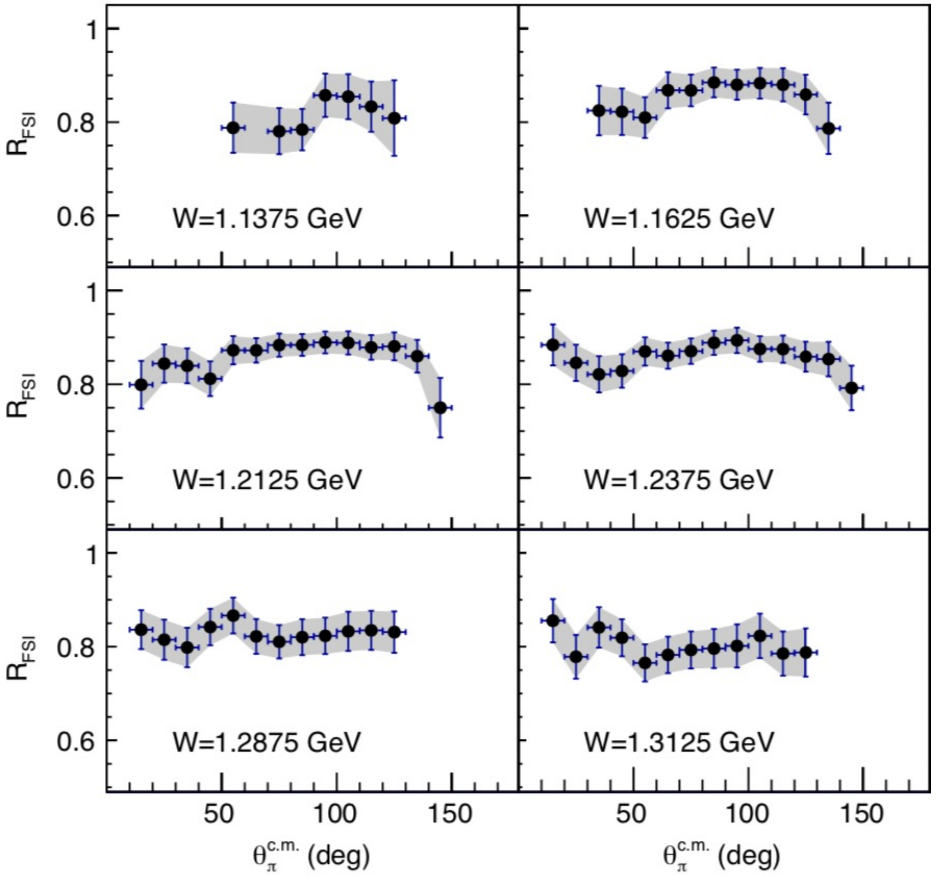
Neutrino data: ANL & BNL measurements

- In the ANL experiment, data were initially taken with a hydrogen fill of the bubble chamber, and then data were taken with a deuterium fill for the remainder of the experiment.
- Event rates are only available as a **combination** of both hydrogen (30%) and deuterium fills of the detector.
- In the BNL experiments results are **separated** into hydrogen and deuterium measurements.
- There is no measurements for single pion production on hydrogen.

Deuterium effects

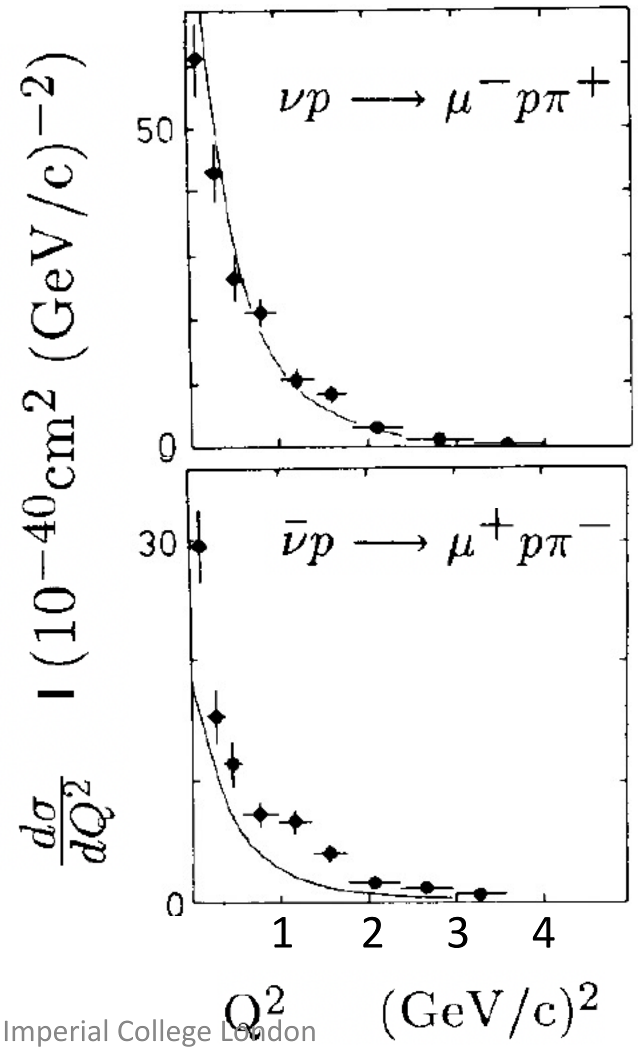
- Recent J-lab measurements show the nuclear effects on deuterium is different from what we expected!
- Measurement of full exclusive cross section $\gamma^* n(p) \rightarrow p\pi^- (p)$ vs quasi-free cross section for $0.6 < Q^2 < 0.8\ (GeV/C)^2$



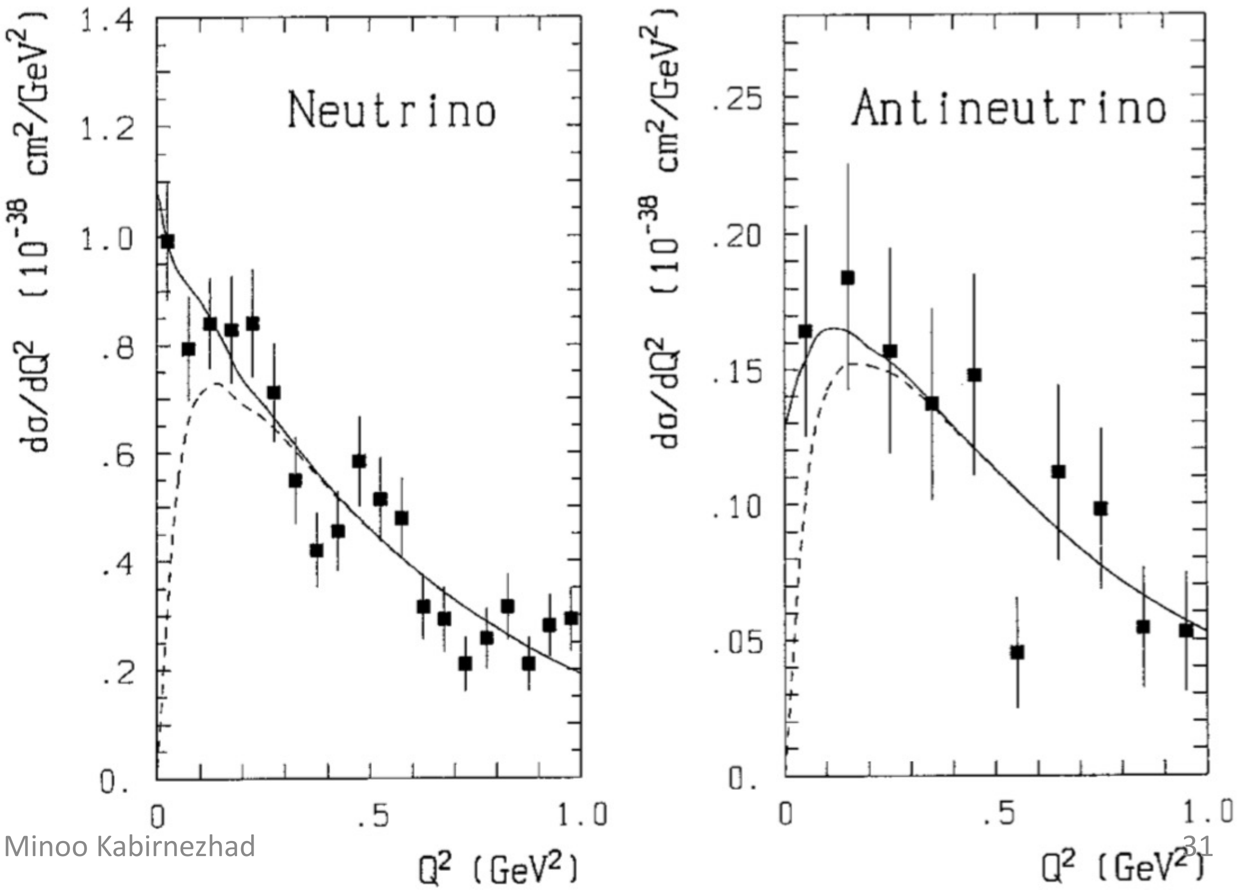


$$R\,(W, Q^2, \cos\theta_\pi^{c.m.}, \phi_\pi^{c.m.}) = \frac{\frac{d^2\sigma^{qf}}{d\Omega_\pi^{c.m.}}}{\frac{d^2\sigma^{ex}}{d\Omega_\pi^{c.m.}}}$$

BEBC measurements (D_2 vs LH_2), W<1.4 GeV



Liquid hydrogen



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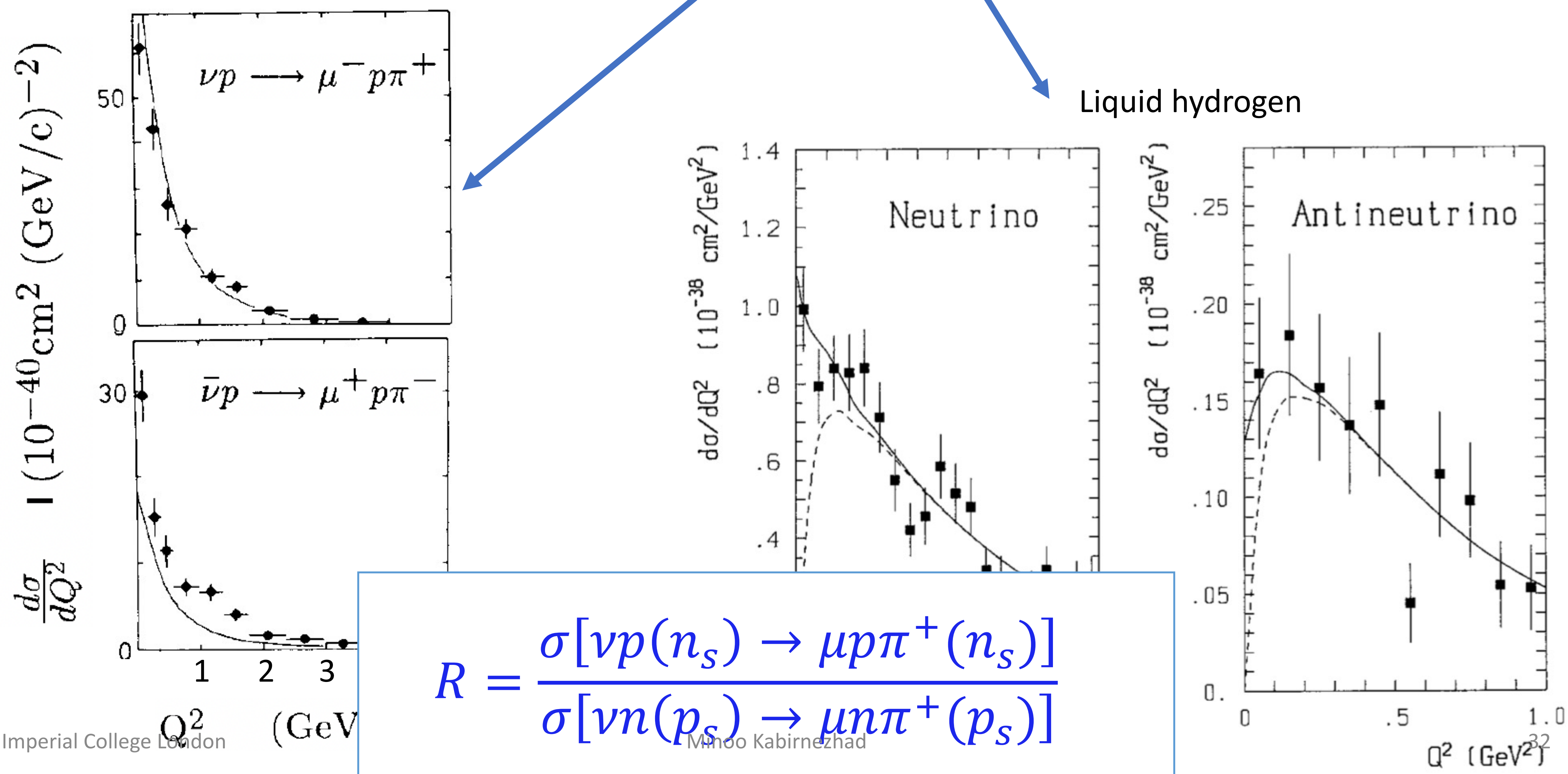


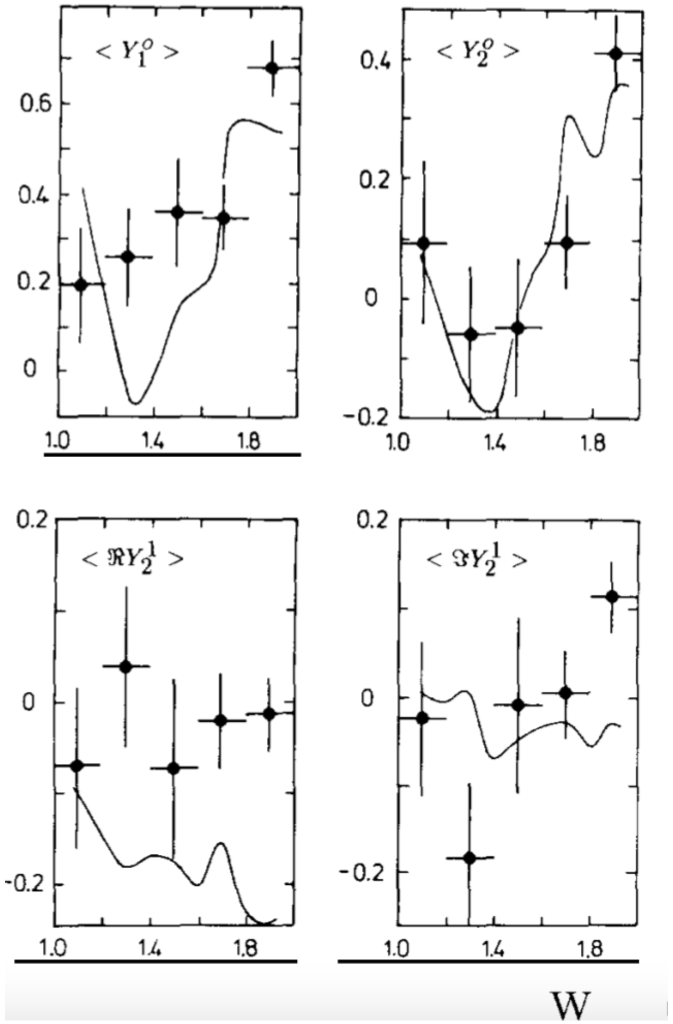
$$R = \frac{\sigma[\nu p(n_s) \to \mu p \pi^+(n_s)]}{\sigma[\nu n(p_s) \to \mu n \pi^+(p_s)]}$$

Averaged value of Spherical Harmonic (BEBC)

$$\frac{d\sigma^{\nu,\bar{\nu}}}{d\Omega\, dW} = a_{00}^{\nu,\bar{\nu}}(W)\, Y_0^0(\Omega) + a_{10}^{\nu,\bar{\nu}}(W)\, Y_1^0(\Omega)$$
$$+ a_{20}^{\nu,\bar{\nu}}(W)\, Y_2^0(\Omega) + \dots$$
$$+ a_{11}^{\nu,\bar{\nu}}(W)\, \mathrm{Re}\, Y_1^1(\Omega) + a_{21}^{\nu,\bar{\nu}}(W)\, \mathrm{Re}\, Y_2^1(\Omega)$$
$$+ a_{22}^{\nu,\bar{\nu}}(W)\, \mathrm{Re}\, Y_2^2(\Omega) + \dots$$
$$+ b_{11}^{\nu,\bar{\nu}}(W)\, \mathrm{Im}\, Y_1^1(\Omega)$$
$$+ b_{21}^{\nu,\bar{\nu}}(W)\, \mathrm{Im}\, Y_2^1(\Omega) + \dots$$

$$< \Re\, Y_i^k > = a_{ik} / a_{00}$$

$$< \Im\, Y_i^k > = b_{ik} / a_{00}$$



Conclusion and prospects

- From the latest T2K OA paper:

"The modelling of the so-called "**transition region**" between single-pion production off a nucleon and shallow- and deep-inelastic scattering is **an unsolved theoretical problem**."

- I am happy to announce that this problem is solved (so far for the vector current).
- The goodness of fit and the comparison of the model to the data in a **broad range of Q^2 ($\in$ [0.16 – 6.00] $(GeV/c)^2$) and W ($\in$ [1.1 – 2.01] GeV)** shows that the model covers the data.

Conclusion and prospects

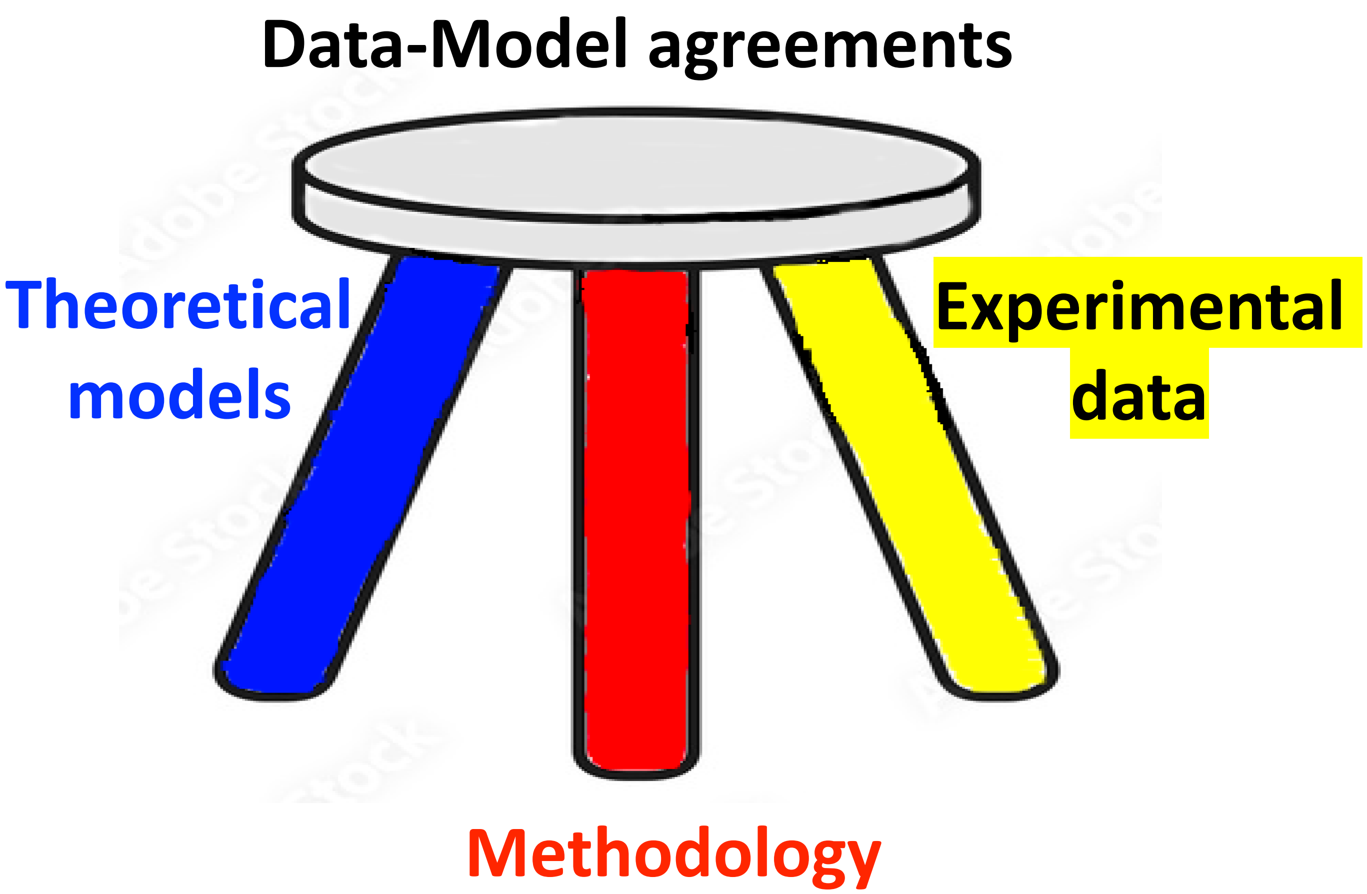
- From the latest T2K OA paper:

“A robust interaction uncertainty model is required to assess the significance of the results. ”

- I developed appropriate systematic uncertainties within the model which can be used in future T2K measurements.
- The cross-section evaluations include covariance matrix of the uncertainties.
- Only a few parameters have freedom that can be used to estimate the systematic uncertainties.

Final remarks (long term plans)

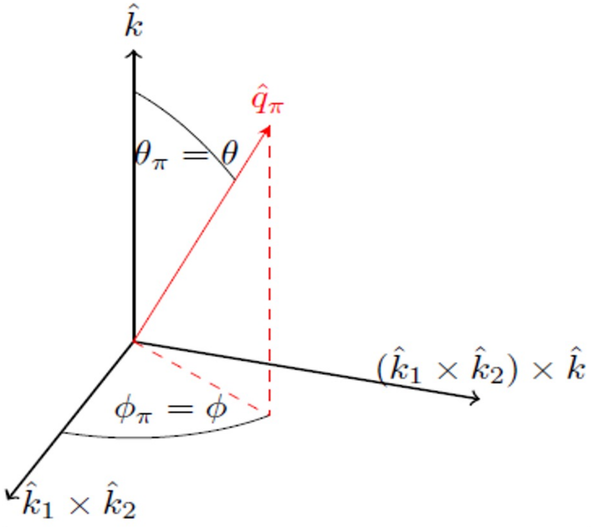
- The data-model agreements is due to model improvements, the use of significant amount of experimental data and developments in the methodology of analysis and evaluation.
- Next plan is to add nuclear effects to the MK model by using the same principles.



Backup

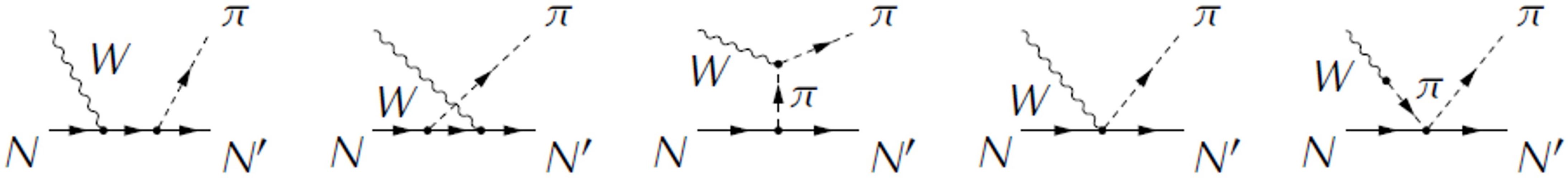
MK-model

M. Kabirnezhad,
Phys. Rev. D **97**, 013002



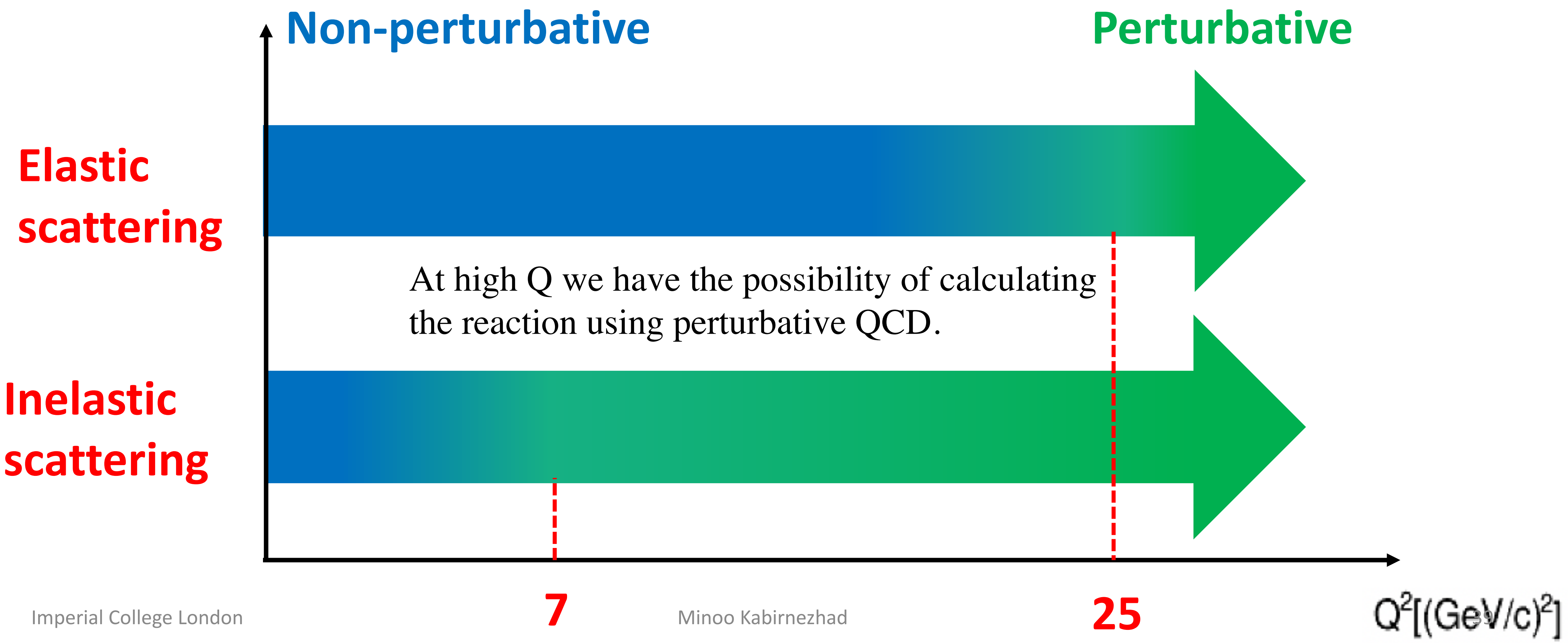
- MK model is a model for single pion production i.e. resonant and nonresonant interactions including **the interference effects**.
- Uses Rein-Sehgal model with Graczyk-Sobczyk form-factor to describe resonant interaction (17 resonances) up to W=2 GeV.
- Lepton mass is included.
- Non-resonant background is defined by a set of diagrams determined by HNV model.

E. Hernandez, J. Nieves and M. Valverde, Phys. Rev. D **76** (2007) 033005



Perturbative vs Non-perturbative domains

the region of the perturbative QCD applicability is estimated differently in the **elastic** and **inelastic** scattering.



Analysis of electron-induced exclusive data

- The standard cross-section formula for the single pion electro-production

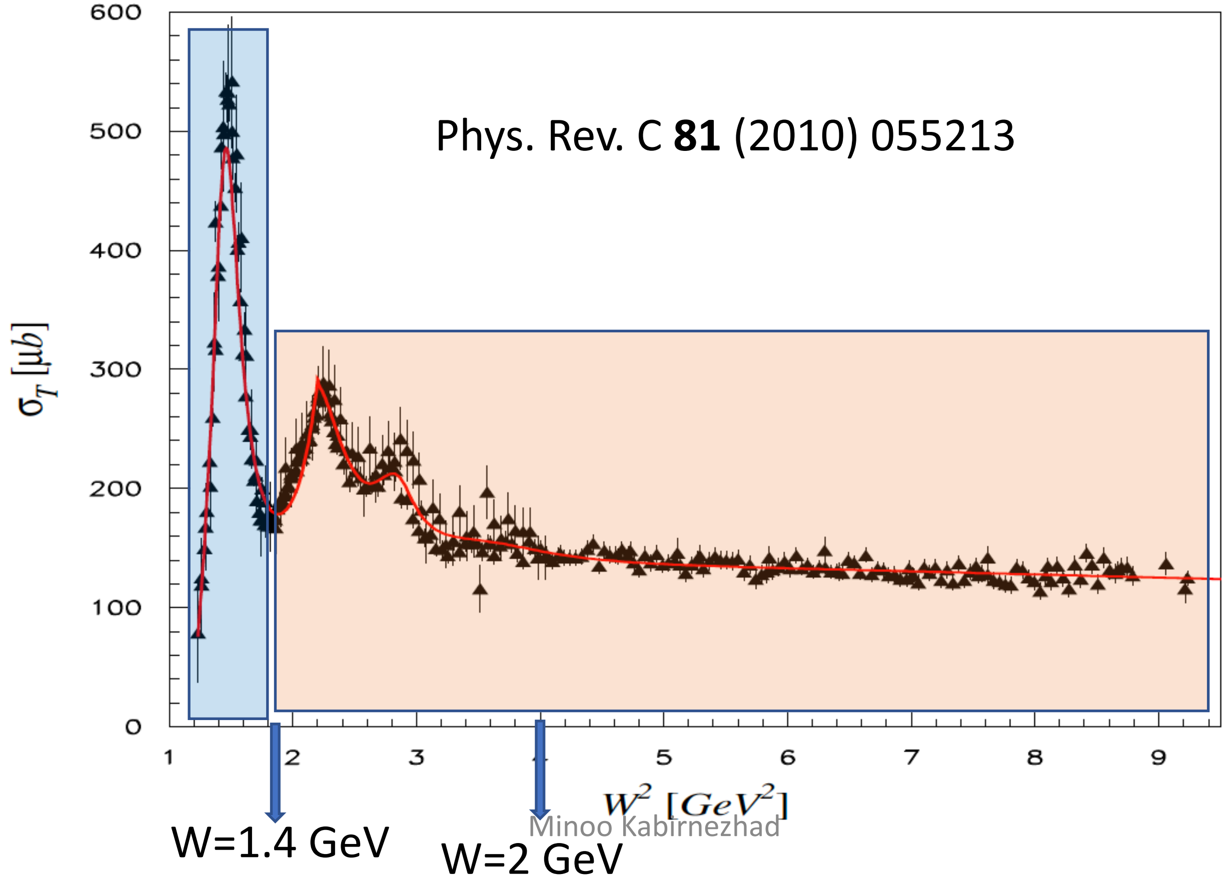
$$\frac{d^5\sigma_{ep\to e'\pi N}}{dE_{e'}d\Omega_{e'}d\Omega^*_\pi} = \Gamma_{em}\left[\frac{d\sigma_T}{d\Omega^*_\pi} + \epsilon\frac{d\sigma_L}{d\Omega^*_\pi} + \sqrt{2\epsilon(1+\epsilon)}\frac{d\sigma_{LT}}{d\Omega^*_\pi}\cos\phi^*_\pi + \epsilon\frac{d\sigma_{TT}}{d\Omega^*_\pi}\cos 2\phi^*_\pi + h_e\sqrt{2\epsilon(1+\epsilon)}\frac{d\sigma_{LT'}}{d\Omega^*_\pi}\sin\phi^*_\pi\right].$$

- In all the following plots the Y axis is
- Fits were used to determine the Q2 dependence of the transition form-factors for resonance production and nonresonant SPP.

Resonance regions

Δ(1232) region (1.08GeV<W<1.4 GeV)

- Δ resonance dominates
- no other resonances
- Only single pion can be produced



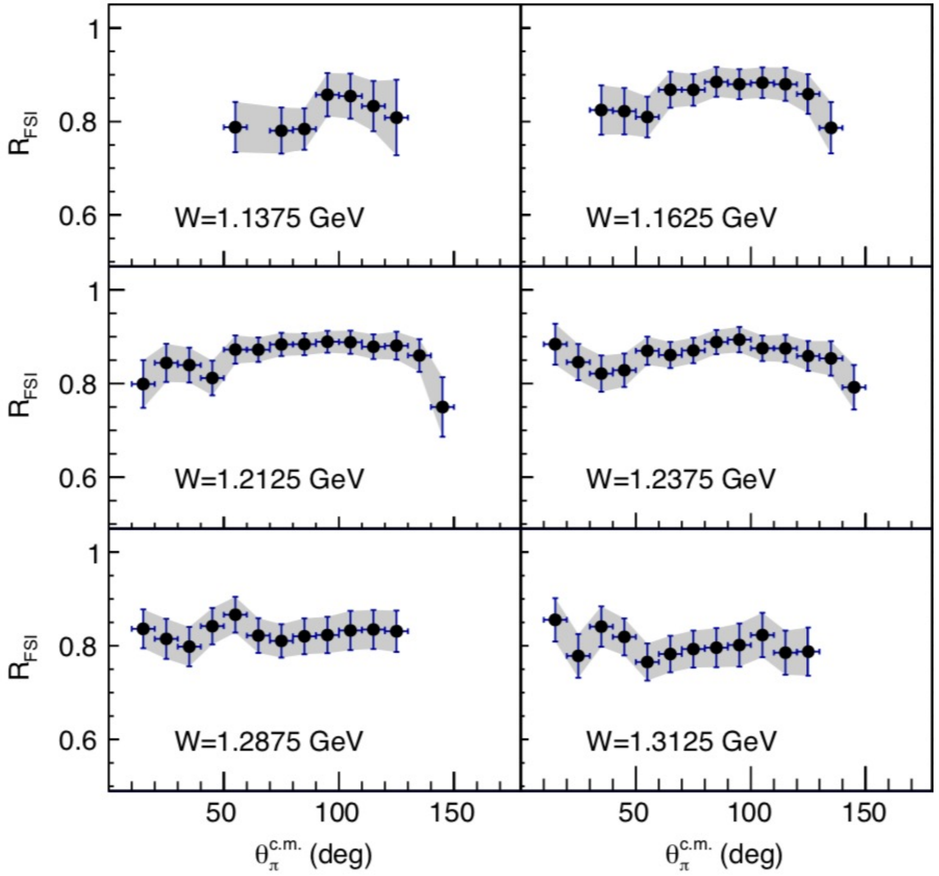
Beyond Δ(1232) region W>1.4 GeV

- No single resonance dominate
- Several comparable resonances overlap
- Multi-pion and other mesons can be produced

Deuterium effects

$$R_{FSI}(W, Q^2, \cos\theta_\pi^{\text{c.m.}}, \phi_\pi^{\text{c.m.}}) = \frac{\frac{d^2\sigma^{qf}}{d\Omega_\pi^{\text{c.m.}}}}{\frac{d^2\sigma^{ex}}{d\Omega_\pi^{\text{c.m.}}}}$$

- Recent J-lab measurements show the nuclear effects on deuterium is different from what we expected!
- Measurement of full exclusive cross section $\gamma^* \, n(p) \rightarrow p\pi^- \, (p)$ vs quasi-free cross section for $0.6 < Q^2 < 0.8$ $(GeV/C)^2$
- This could solve the ANL/BNL puzzle!

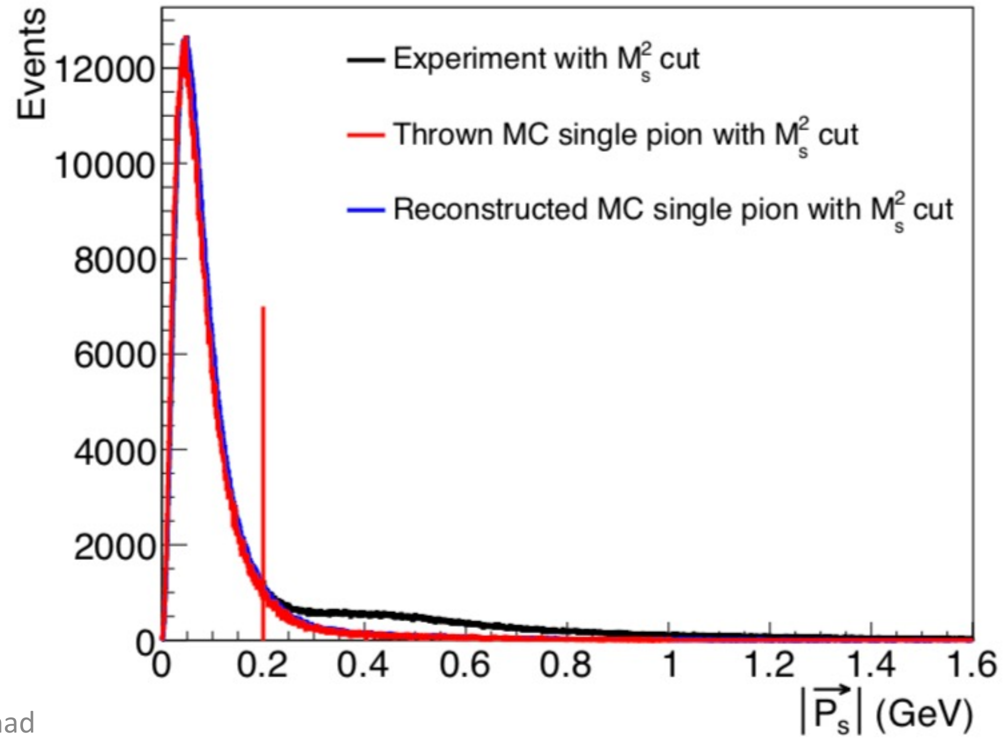


Exclusive π− Electroproduction off the Neutron in Deuterium in the Resonance Region

https://arxiv.org/abs/2203.16785

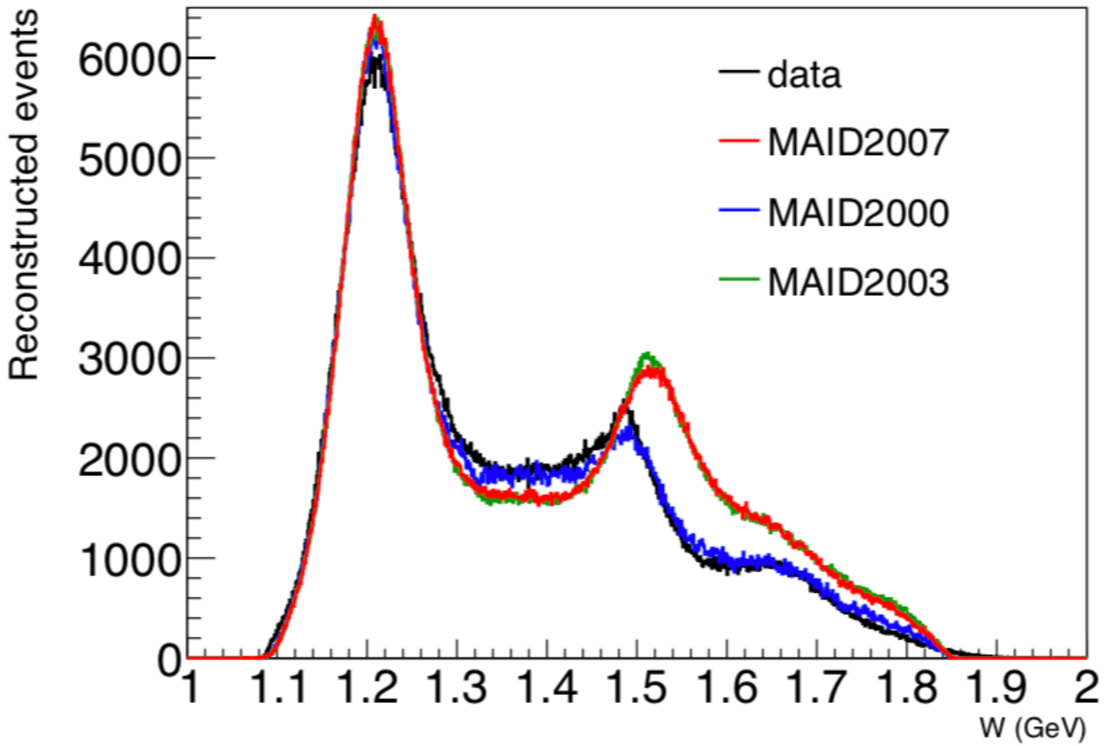
- Measurement of full exclusive cross section $\gamma^* n(p) \rightarrow p\pi^- (p)$
- Measurement of exclusive quasi-free cross section

$$R_{FSI}(W, Q^2, \cos\theta_\pi^{\mathrm{c.m.}}, \phi_\pi^{\mathrm{c.m.}}) = \frac{\frac{d^2\sigma^{qf}}{d\Omega_\pi^{\mathrm{c.m.}}}}{\frac{d^2\sigma^{ex}}{d\Omega_\pi^{\mathrm{c.m.}}}}$$

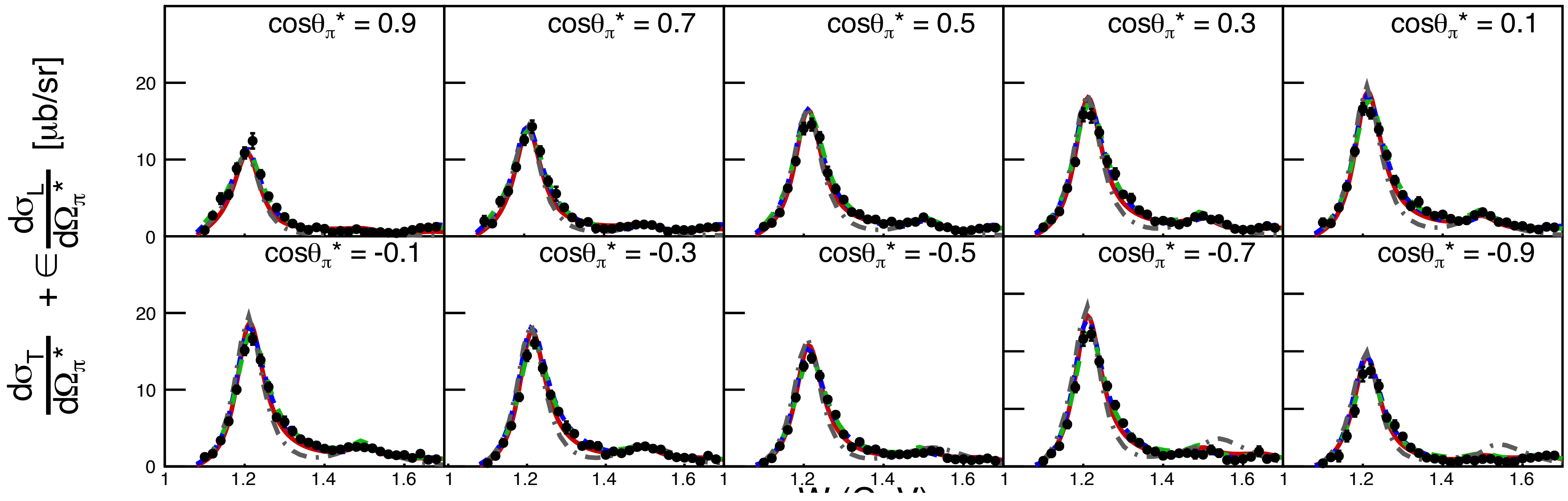


Data comparison with MAID model

- MAID is a unitary isobar model for partial wave analysis on the world data of pion photo and electroproduction in the resonance region.
- the MAID2000 model was chosen as input for the event generator.
- Even though MAID2007 is the latest version, the second resonance peak from this version is shifted relative to the experi- mental neutron data.

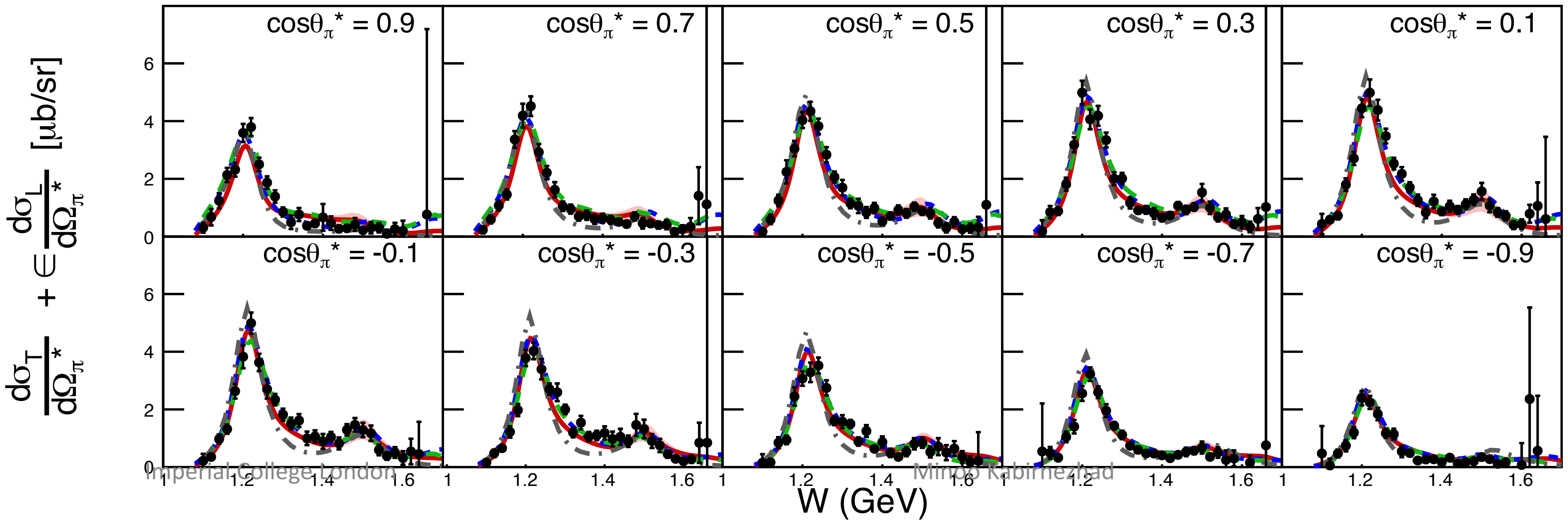


$ep \rightarrow ep + \pi^0$



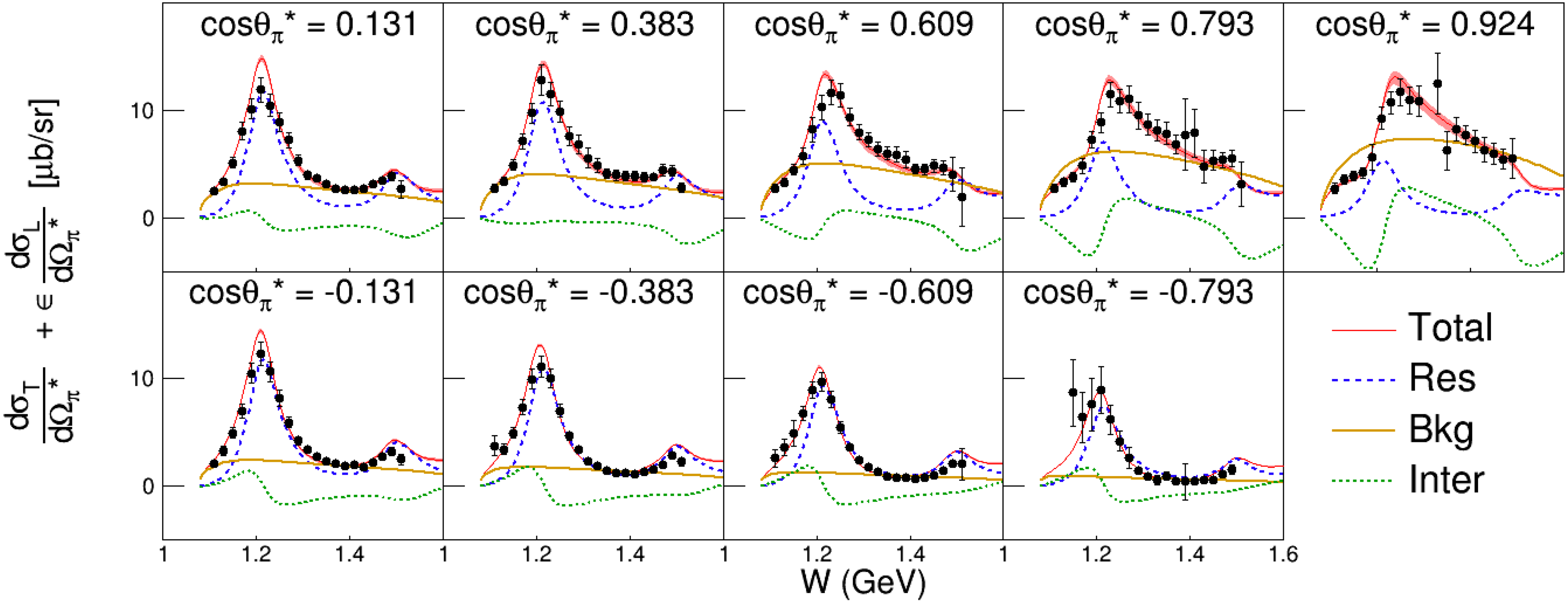
E= 1.645 GeV
Q^2 =0.65 GeV^2
1.1<W<1.68 GeV

MAID-2007
DCC model
Hybrid model



E= 2.445 GeV
Q^2 =1.45 GeV^2
1.1<W<1.68 GeV

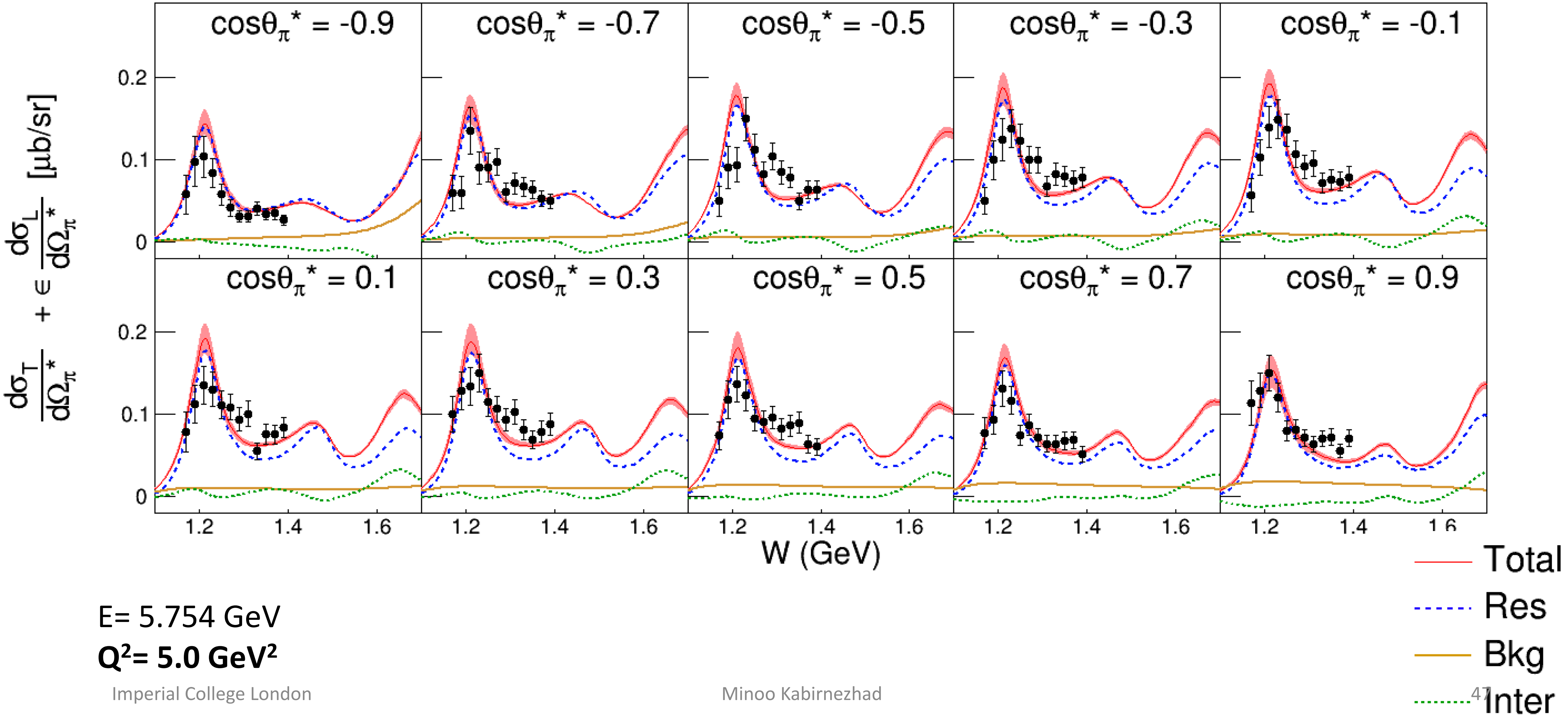
$ep \rightarrow en + \pi^{+}$



E= 1.515 GeV

Q^2= 0.5 GeV^2

$$ep \rightarrow ep + \pi^0$$



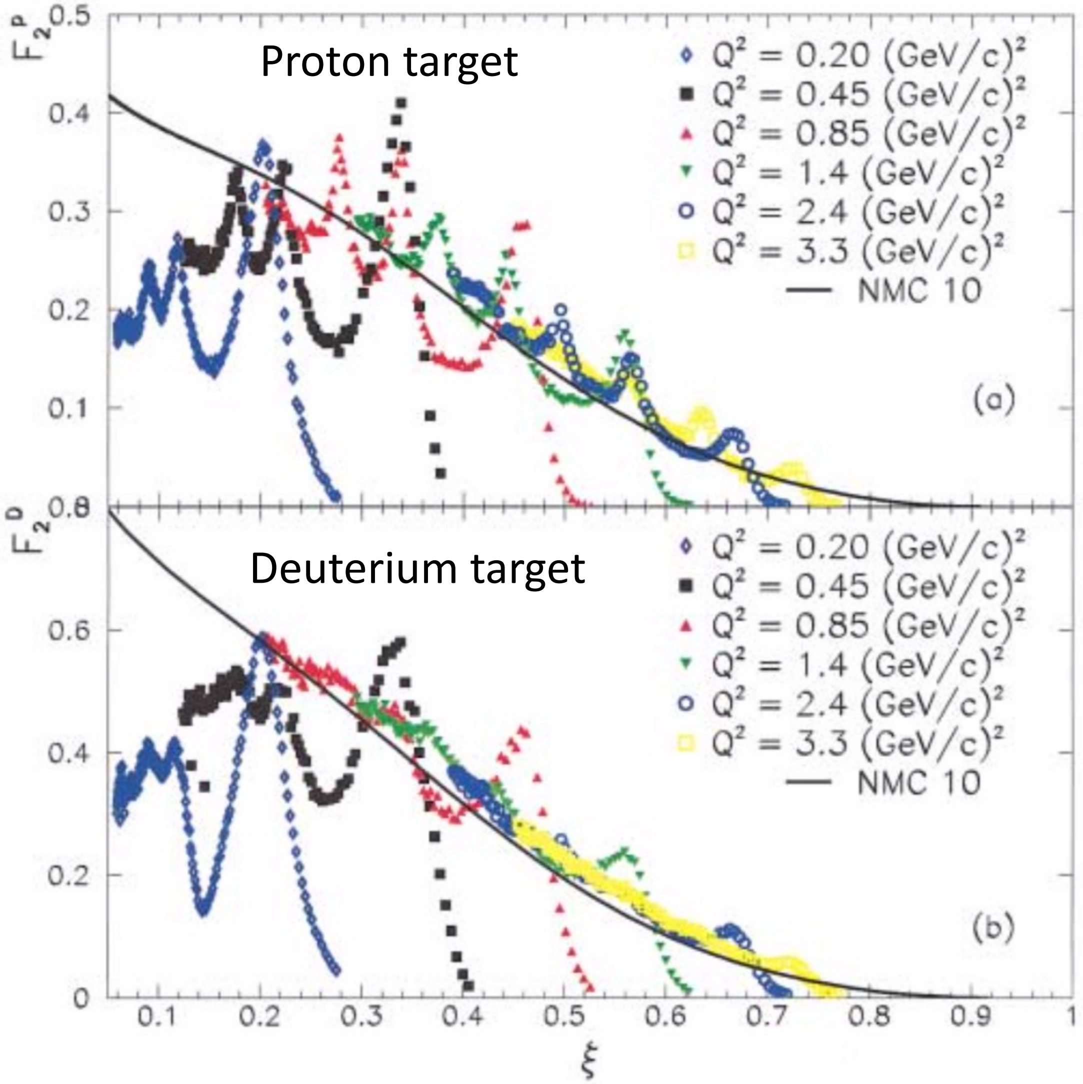
E= 5.754 GeV

Q^2= 5.0 GeV^2

From I. Niculescu *et al.*

Quark–hadron duality

- It was observed about 50 years ago.
- The resonances oscillate around an average scaling curve.
- Scaling behaviour would imply that the nucleon target appears as a collection of point-like constituents when probed at very high energies in DIS.
- Establishes a relationship between the quark–gluon description, and the hadronic description.



$$\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$$

$$x = Q^2/2M\nu$$

Quark-hadron duality and high-Q2 form-factor behaviour

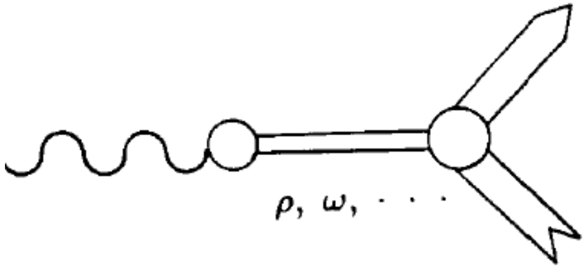
- The phenomenological model should obey the general implications of the quark-hadron duality.
- The duality makes the helicity amplitudes and the form factors to take on some specific properties in accordance with the origin of the form factors on both quark and hadron levels.
- At very high momentum transfer, perturbative QCD predicts the asymptotic behaviour of helicity amplitudes and therefore constrains form factors in the phenomenological models.

Form factors within VMD model

- Vector-meson-dominance (VMD) model is based on the strongly interacting virtual vector mesons as intermediaries in the coupling between a virtual photon and nucleon.

k		$m_{(\rho)k}$, GeV		$m_{(\omega)k}$, GeV
1	$\rho(770)$	0.7755	$\omega(782)$	0.78265
2	$\rho(1450)$	1.459	$\omega(1420)$	1.425
3	$\rho(1700)$	1.720	$\omega(1650)$	1.670
4	$\rho(1900)$	1.885	$\omega(1960)^b$	1.960
5	$\rho(2150)$	2.149	$\omega(2145)^b$	2.148

- VMD model has been a successful theory of low-Q2 form-factor. The dipole form factor would be obtained if vector mesons propagate between the virtual photon and the nucleon.



$$F_1^{\mathrm{IV,IS}}(Q^2) = \frac{m^2_{\rho,\omega}}{m^2_{\rho,\omega} + Q^2} c_{\rho,\omega} f_1(Q^2)$$

$$f_1(Q^2) \sim \Lambda^2/(\Lambda^2 + Q^2)\,, \quad \Lambda \sim 0.8\ \mathrm{GeV}/c$$

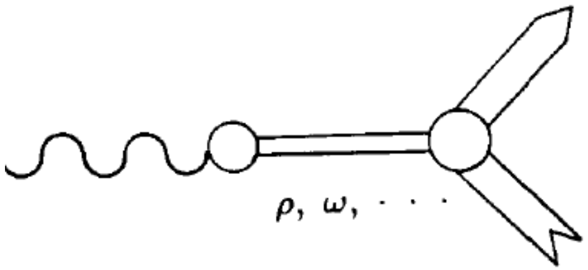
$$c_{\rho,\omega} = g_{\rho,\omega}/f_{\rho,\omega},$$

$g_{\rho,\omega}/2$ is couplings at the meson-nucleon vertices

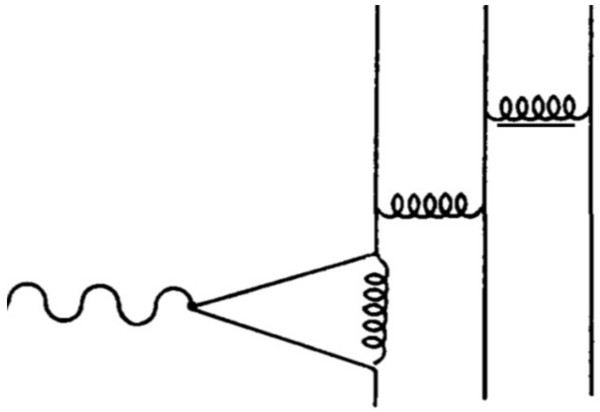
$m^2_{\rho,\omega}/f_{\rho,\omega}$ is photon-meson coupling

Form factors within VMD model

- VMD model has been a successful theory of low-Q2 form-factor. The dipole form factor would be obtained if vector mesons propagate between the virtual photon and the nucleon.



- VMD model can reproduce Q^2-evolution of form-factors to join smoothly with pQCD expectations at high Q^2.



- The VMD model was very successful to define nucleon form-factors at low and high Q^2 for elastic scattering (Gari-Krumpelmann-1992) and I used these form-factors for nonresonant interactions.

Q2-evolution of nucleon-to-resonance transition form factors in a QCD-inspired vector-meson-dominance model

G. Vereshkov and N. Volchanskiy (PRD 2007)

- They used Helicity amplitudes derived in Rarita-Schwinger formalism.
- They used VMD model for resonance form factors.
- To join predictions of such VMD models with pQCD expectations, the amplitudes should be suppressed by power and logarithmic functions.

Asymptotic behavior of spin 3/2 resonance's form factors

$$F_\alpha(Q^2) \simeq \left(\frac{4M_N^2}{Q^2}\right)^{p_\alpha} \cdot \frac{f_\alpha}{\ln^{n_\alpha} Q^2/\Lambda^2}$$

$$\Lambda \approx \Lambda_{\rm QCD}$$

$$p_1 = 3,\ p_2 = p_3 = 4,\ n_3 > n_1 > n_2,\ n_1 \simeq 3.$$

Q2-evolution of nucleon-to-resonance transition form factors in a QCD-inspired vector-meson-dominance model

G. Vereshkov and N. Volchanskiy (PRD 2007)

$$F_1(Q^2) = \frac{F_1^{(exp)}}{L_1^{(\rho)}(Q^2)} \sum_{k=1}^{K} \frac{a_{1k}^{(\rho)} m_{(\rho)k}^2}{Q^2 + m_{(\rho)k}^2},$$

$$F_2(Q^2) = \frac{F_2(0)}{L_2^{(\rho)}(Q^2)} \sum_{k=1}^{K} \frac{a_{2k}^{(\rho)} m_{(\rho)k}^2}{Q^2 + m_{(\rho)k}^2},$$

$$F_3(Q^2) = \frac{F_{23}^{(exp)} - F_2(0)}{L_3^{(\rho)}(Q^2)} \sum_{k=1}^{K} \frac{a_{3k}^{(\rho)} m_{(\rho)k}^2}{Q^2 + m_{(\rho)k}^2},$$

$$L_\alpha^{(V)}(Q^2) = 1 + C_\alpha^{(V)} \ln^{n_\alpha}\left(1 + \frac{Q^2}{\Lambda^2}\right)$$

$$L_\alpha^{(V)}(Q^2) = \left[1 + h_\alpha^{(V)} \ln\left(1 + \frac{Q^2}{\Lambda^2}\right) + k_\alpha^{(V)} \ln^2\left(1 + \frac{Q^2}{\Lambda^2}\right)\right]^{n_\alpha/2}$$

superconvergence relations to satisfy QCD asymptotic and unitarity constraints.

$$\sum_{k=1}^{K} a_{1k}^{(\rho)} = 1, \qquad \sum_{k=1}^{K} a_{1k}^{(\rho)} m_{(\rho)k}^2 = 0,$$

$$\sum_{k=1}^{K} a_{1k}^{(\rho)} m_{(\rho)k}^4 = 0;$$

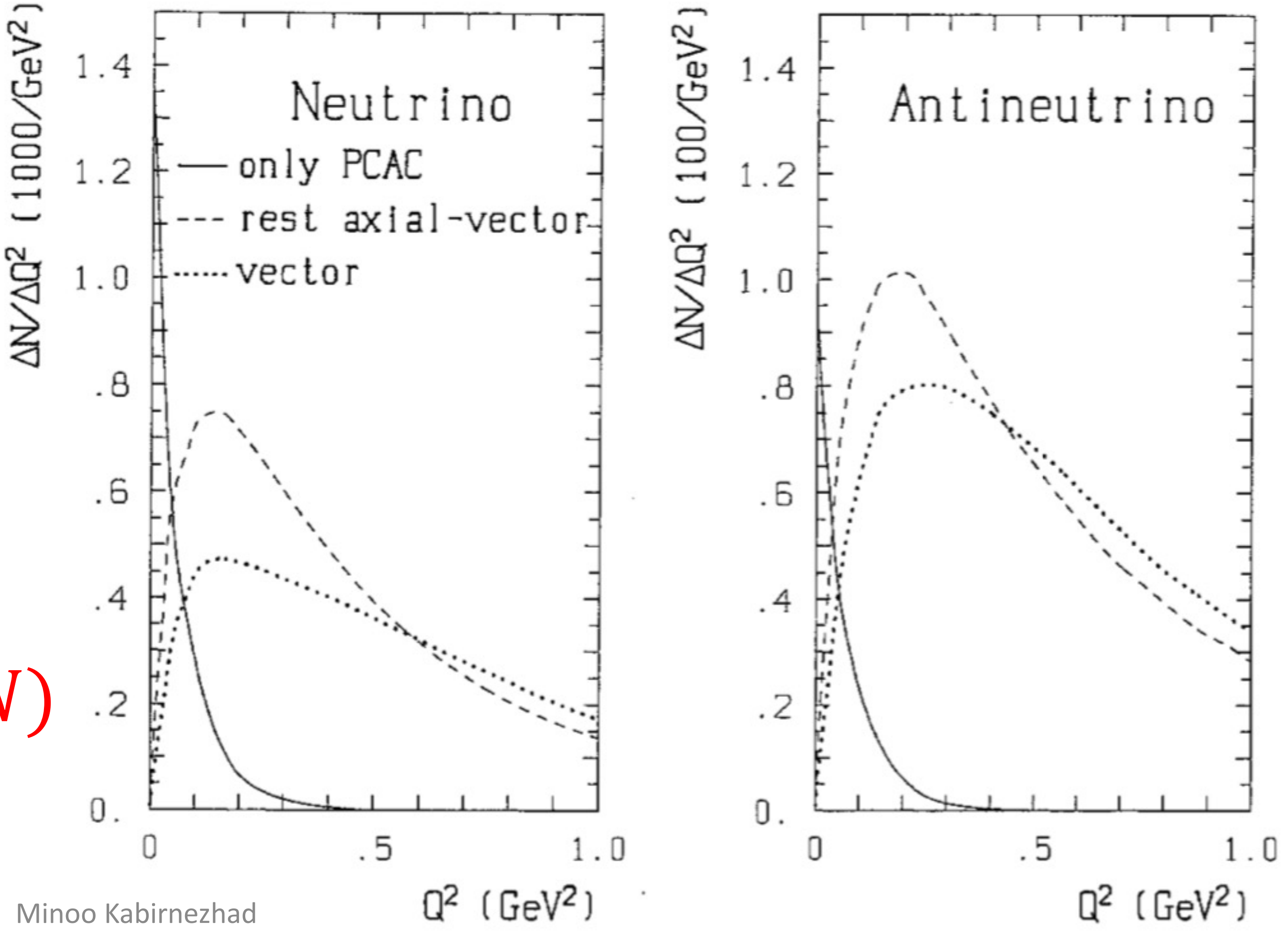
Improving the axial part

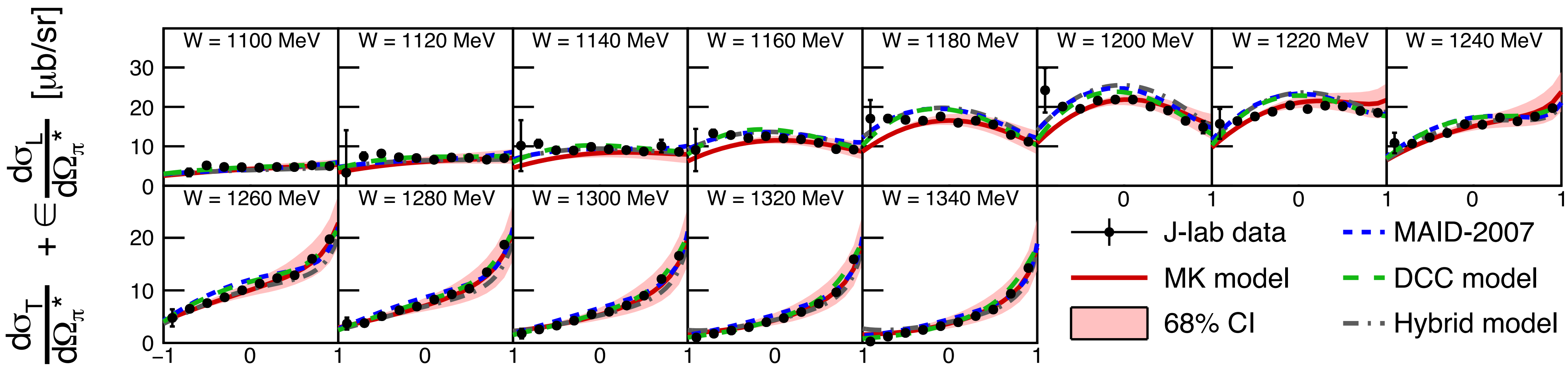
$$\frac{d\sigma}{dQ^2} = \left(\frac{d\sigma}{dQ^2}\right)^{V} + \left(\frac{d\sigma}{dQ^2}\right)^{A}$$

$$\left(\frac{d\sigma}{dQ^2}\right)^{VT} + \left(\frac{d\sigma}{dQ^2}\right)^{VL}$$

$$\left(\frac{d\sigma}{dQ^2}\right)^{AT} + \left(\frac{d\sigma}{dQ^2}\right)^{AL}$$

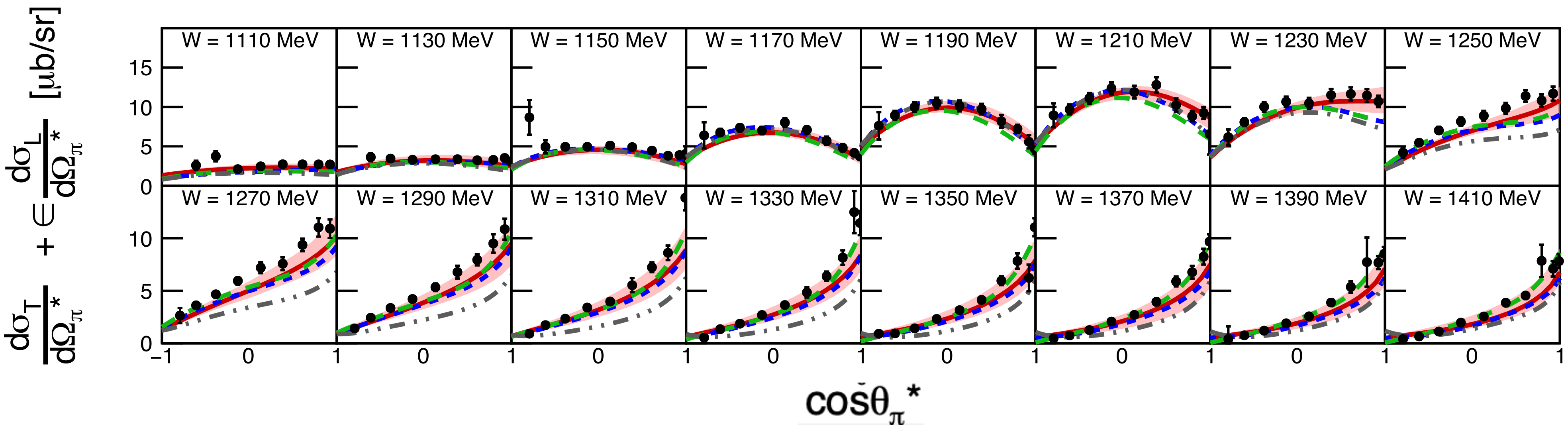
$$\left(\frac{d\sigma}{dQ^2\, dW}\right)^{AL}\Big|_{Q^2=0} \propto \sigma(\pi N \rightarrow \pi N)$$





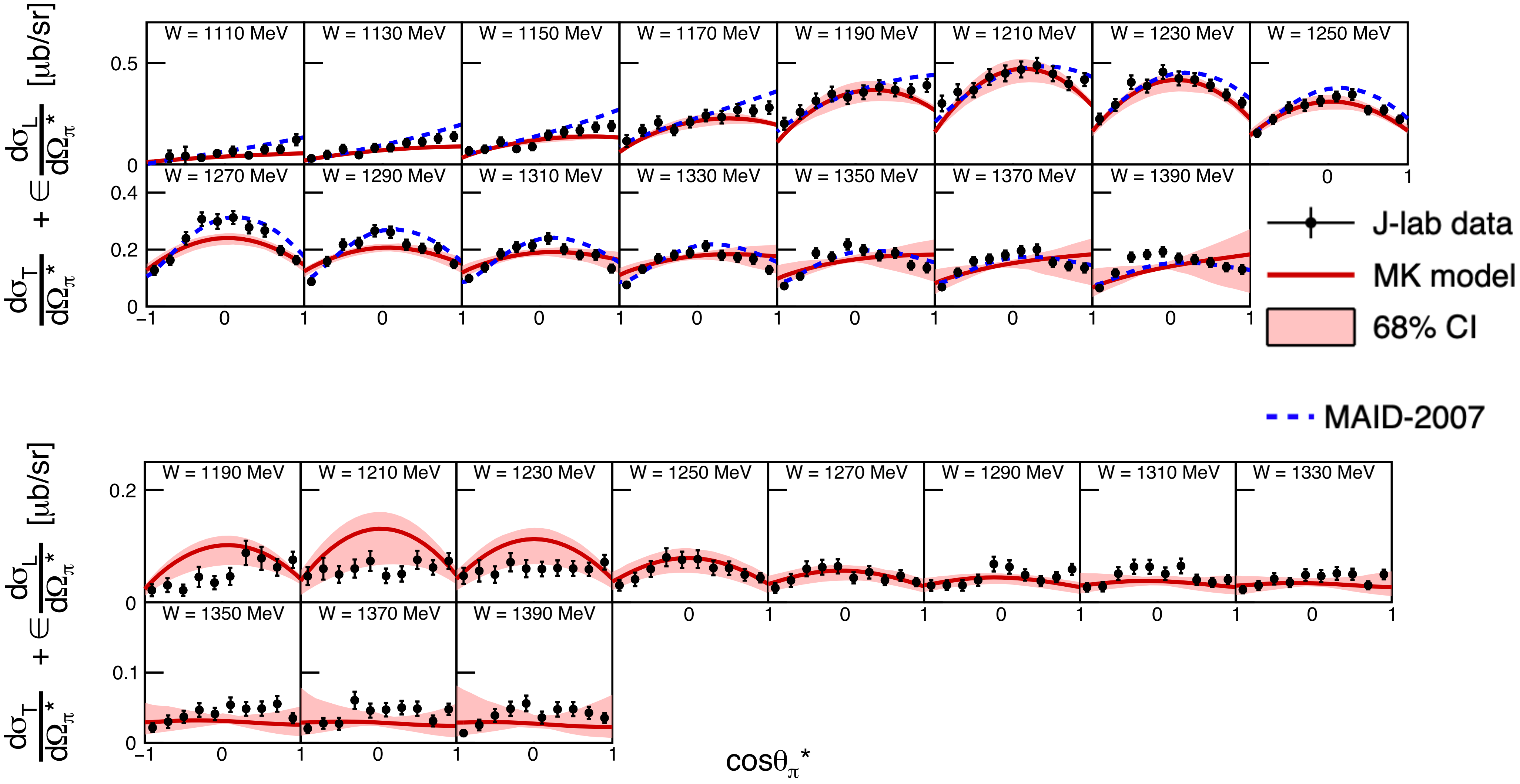
$$ep \rightarrow en + \pi^+$$

E= 1.046 GeV
Q^2=0.2 GeV^2
1.1<W<1.34 GeV



E= 1.515 GeV
Q^2=0.6 GeV^2
1.1<W<1.41 GeV

$ep \rightarrow ep + \pi^0$



E= 5.754 GeV
Q^2=3.5 GeV^2
1.1<W<1.39 GeV

E= 5.754 GeV
Q^2=6.0 GeV^2
1.1<W<1.39 GeV

Analysis of electron-induced exclusive data

- The standard cross-section formula for the single pion electro-production

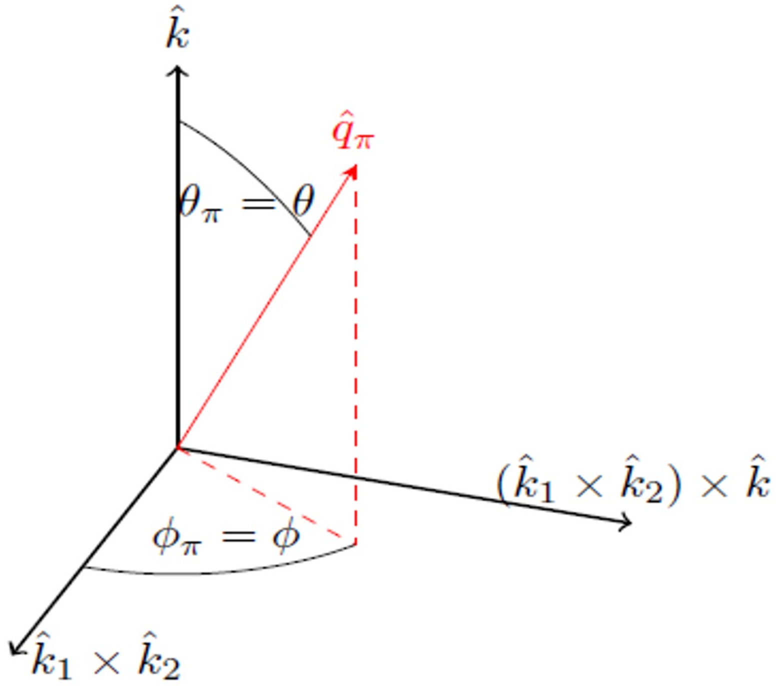
$$\frac{d^5\sigma_{ep\to e'\pi N}}{dE_{e'}d\Omega_{e'}d\Omega^*_\pi} = \Gamma_{em}\Big[\frac{d\sigma_T}{d\Omega^*_\pi} + \epsilon\frac{d\sigma_L}{d\Omega^*_\pi} + \sqrt{2\epsilon(1+\epsilon)}\frac{d\sigma_{LT}}{d\Omega^*_\pi}\cos\phi^*_\pi + \epsilon\frac{d\sigma_{TT}}{d\Omega^*_\pi}\cos 2\phi^*_\pi + h_e\sqrt{2\epsilon(1+\epsilon)}\frac{d\sigma_{LT'}}{d\Omega^*_\pi}\sin\phi^*_\pi\Big].$$

- In all the following plots the Y axis is
- Fits were used to determine the Q2 dependence of the transition form-factors for resonance production and nonresonant SPP.

MK model: Free nucleons

- Hadron current

$$J^{\mu}_{MK} = \int_V dr\ \bar{\psi}_{out}(r)\ \phi^*(r)\ \hat{O}^{\mu}_{MK}\ e^{iq.r}\ \psi_{in}(r)$$



Resonance (hadron) rest frame

MK model: Free nucleons

- Hadron current

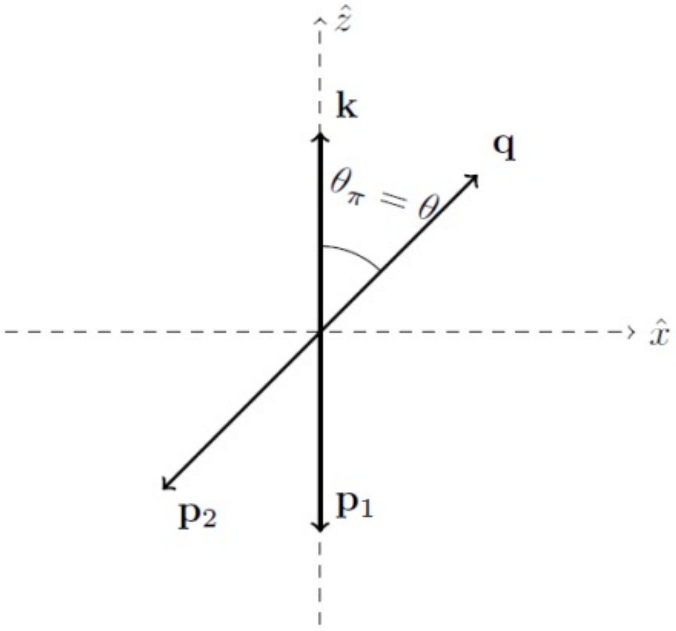
Hadron tensor

$$J^{\mu}_{MK} = \int_V dr\, \bar{\psi}_{out}(r)\, \phi^*(r)\, \hat{O}^{\mu}_{MK}\, e^{iq.r}\, \psi_{in}(r)$$

Outgoing nucleon with momentum p_2

produced pion (plane wave) with momentum q.

Incoming nucleon with momentum p_1



Resonance (hadron) rest frame

MK model: Free nucleons

- Hadron current

$$J^{\mu}_{MK} = \int_V dr\ \bar{\psi}_{out}(r)\ \phi^*(r)\ \hat{O}^{\mu}_{MK}\ e^{iq.r}\ \psi_{in}(r)$$

- The wave functions are plane-wave solution of Dirac equation:

$$\psi(r) = u(p)e^{-ip.r}$$

where $u(p)$ is the solution of Dirac equation in momentum space:

$$u_s(p) = \sqrt{E(p)+M}\begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}.\mathbf{p}}{E(p)+M}\chi_s \end{pmatrix}$$

→ Pauli spinors

$$|\uparrow\rangle_{N_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad |\uparrow\rangle_{N_2} = \begin{pmatrix} \sin\theta/2 \\ -e^{i\phi}\cos\theta/2 \end{pmatrix}$$

New project: Calculation of Nuclear effects

- Hadron current

$$J^{\mu}_{nucleus} = \int_V dr\ \bar{\psi}_S(r)\ \phi^*(r)\ \hat{O}^{\mu}_{MK-model}\ e^{iq.r}\ \psi_B(r)$$

- The goal of this work is to illustrate how the interaction between the outgoing nucleon and the residual nucleus affects the predicted initial (MK-model) cross sections.

New project: Calculation of Nuclear effects

- Hadron current

$$J^{\mu}_{nucleus} = \int_V dr\, \bar{\psi}_S(r)\, \phi^*(r)\, \hat{O}^{\mu}_{MK-model}\, e^{iq.r}\, \boxed{\psi_B(r)}$$ Bound state

- Mean Field theory provides the bound state with a real potential that bound the nucleus.
- The RMF potential is energy-independent.

Detailed description in Jake's talk

New project: Calculation of Nuclear effects

- Hadron current

$$J^{\mu}_{nucleus} = \int_V dr\, \boxed{\bar{\psi}_S(r)}\, \phi^*(r)\, \hat{O}^{\mu}_{MK-model}\, e^{iq.r}\, \psi_B(r)$$

Scattered state

- Mean field theory **doesn't** provide the scattered state.
- The wave function is the solution of Dirac equation with a phenomenological optical potential.
- All nuclear effects (SRC, RPA, FSI, etc.) can be implemented in the Dirac equation.

New project: Calculation of Nuclear effects

- Hadron current

$$J^{\mu}_{nucleus} = \int_V dr\ \bar{\psi}_S(r)\ \boxed{\phi^*(r)}\ \hat{O}^{\mu}_{MK-model}\ e^{iq.r}\ \psi_B(r)$$

Scattered pion

- Scattered pion is still a plane wave in the current RMF theory.
- It needs to be improved in order to calculate FSI pions.